Error-free quantum communication through noisy channels

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We suggest a method to perform a quantum logic gate between distant quantum bits (qubits) by off-resonant field-atom dispersive interactions. The scheme we present is shown to work ideally even in the presence of errors in the photon channels used for communication. The stability against errors arises from the paradoxical situation that the transmitted photons carry no information about the state of the qubits. In contrast to a previous proposal for ideal communication [Phys. Rev. Lett. **78**, 4293 (1997)] our proposal only involves single atoms in the sending and receiving devices. [S1050-2947(98)11809-7]

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I. INTRODUCTION

Quantum mechanics is known to produce a variety of phenomena in lack of classical interpretation. In recent years the fields of quantum computation and quantum communication have tried to exploit these phenomena to propose computers and communication devices that are superior to their classical counterparts. One particular example is quantum teleportation, which is based on the nonlocal features of the Einstein-Podolsky-Rosen (EPR) paradox. Quantum teleportation is the transmission of qubits without actually sending the physical system, e.g., the transfer of the state of an atom to another atom at a different location. Classically, teleportation can be performed by measuring the state of an object and sending the information to the receiver who reconstructs the state in a similar object. In the quantum world it is not that easy. Quantum mechanics forbids us to gain exact knowledge of the state of an object. However, Bennett *et al.* have suggested that it is still possible to perform teleportation [1], provided that the transmitting system does not retain any information about the state that is transmitted. Recently quantum teleportation of a photonic state has been achieved experimentally [2,3].

In practical realizations of teleportation the system may be subject to noise in the transmission channels. Recently, van Enk *et al.* have shown that the effect of the noise can be completely avoided, if we make suitable physical assumptions about the noise in the channels [4].

Eliminating noise on quantum information is considerably more complicated than eliminating noise on classical information because quantum mechanics forbids copying of information $|\psi\rangle \rightarrow |\psi\rangle|\psi\rangle$, where $|\psi\rangle$ is the state of a quantum bit (qubit) $(c_0|0\rangle + c_1|1\rangle)$ [5]. However, quantum mechanics does allow what we shall call a backup copy $|\psi\rangle \rightarrow |\psi\rangle_a + |\psi\rangle_b$, where a single quantum system is transferred to a state with projections on two different subspaces *a* and *b*, which are both equivalent to the initial state. (We use unnormalized states except where otherwise stated.) We call it backup copying because, if one-half is ''lost'' (projected out), say the *b* part, we may still have the intact quantum state in the *a* part. (The exact meaning of this statement will become clear below.)

In this paper we use the backup encoding to perform quantum communication in the presence of errors in the channel used for communication. Rather than considering teleportation as discussed by van Enk *et al.*, we perform a perfect control-not operation, which is slightly more general than teleportation. To perform the operation we use off-resonant dispersive interactions between atoms and the transmitted photons. We assume that all errors are due to imperfections in the transmission and imperfections in the dispersive interaction, whereas measurements and unitary evolutions on a single atom are assumed ideal. With this assumption we show that our scheme works ideally, even in the presence of a quite general class of errors.

We emphasize that our scheme is not a conventional quantum error correcting code [6]. We use a specific physical model of the noise to remove errors to all orders with a limited number of qubits, whereas conventional error codes introduce new qubits to correct errors up to a certain order.

II. GENERAL DESCRIPTION

The control-not gate works between two atoms or ions. The control atom is called α and the target is called β. Here α and β are two three-level atoms, where each level has a twofold Zeeman degeneracy $(J=1/2)$. The states of α are denoted by $|a_i\rangle$, $|b_i\rangle$, and $|c_i\rangle$ and the states of β are called $|d_i\rangle$, $|e_i\rangle$, and $|f_i\rangle$, where $i=0$ and 1 represents the azimuthal quantum numbers $m=-1/2$ and $m=1/2$; see Fig. 1. In practice, one may have recourse to systems with a differ-

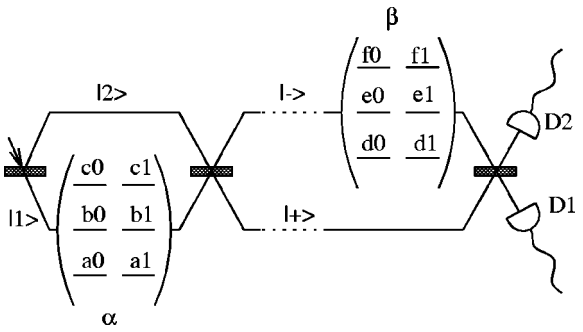

FIG. 1. Structure of the atoms α and β and the suggested setup. The two atoms have three levels (denoted by letters) with a twofold Zeeman degeneracy (denoted by 0 and 1). The sending section consists of two beam splitters and the atom α. The communication channels are the two dotted lines $|+\rangle$ and $|-\rangle$ and the receiving section is the atom β, the last beam splitter, and the two detectors.

ent arrangement of states, but our procedures is most easily explained in the suggested realization.

If we consider the quantum information to be stored in the two Zeeman degenerate ground-state levels, the action of the control-not operation can be characterized by its action on suitable basis vectors for the atomic ground state

$$\begin{bmatrix} |a_0\rangle|d_0\rangle \\ |a_0\rangle|d_1\rangle \\ |a_1\rangle|d_0\rangle \\ |a_1\rangle|d_1\rangle \end{bmatrix} \begin{matrix} \rightarrow \\ \rightarrow \\ \\ \end{matrix} \begin{bmatrix} |a_0\rangle|d_0\rangle \\ |a_0\rangle|d_1\rangle \\ |a_1\rangle|d_1\rangle \\ |a_1\rangle|d_0\rangle \end{bmatrix}. \tag{1}$$

The control-not operation interchanges the states $|d_0\rangle$ and $|d_1\rangle$ of β if and only if α is in the state $|a_1\rangle$. A comment on notation: Rather than considering the evolution of a superposition of the four basis vectors $(c_{00}|a_0\rangle|d_0\rangle+c_{01}|a_0\rangle|d_1\rangle+c_{10}|a_1\rangle|d_0\rangle+c_{11}|a_1\rangle|d_1\rangle)$, we consider the evolution of each basis vector. This emphasizes that each vector in Eq. (1) could be entangled with other qubits, as in a computational task.

Our scheme consists of local encodings and two transmissions of photons from α to β. We begin with a local backup encoding on α. We then perform the first transmission followed by a symmetrization on α and protection of relevant states of β. Another transmission is performed and finally we extract the desired quantum states.

The effect of the transmissions is to entangle the levels of the two atoms. By performing local operations we can then use this entanglement to implement the control-not operation.

The stability of our scheme arises from the horizontal symmetry among the atomic states in Fig. 1. In the transmission we only use linearly (π) polarized pulses that couple states vertically. This means that the photons contain no information about whether α is in 0 or 1. The photons only contain information about the levels of α. If, for instance, we start with a superposition $(c_0|a_0\rangle+c_1|a_1\rangle)+(c_0|b_0\rangle+c_1|b_1\rangle)$ and a photon is absorbed during a transmission, the wave function will collapse to some energy level (for instance, a), but our quantum-mechanical superposition between 0 and 1 will be intact $(c_0|a_0\rangle+c_1|a_1\rangle)$. From this ''backup'' state we can start the transmission again and continue until we are successful.

III. BACKUP CONTROL-NOT OPERATION UNDER IDEAL CONDITIONS

To perform the evolution described by Eq. (1) we suggest using an experimental setup as shown in Fig. 1. Our setup can be divided into a sending section, a receiving section, and the channels connecting them. The sending section consists of a beam splitter, the atom α, and another beam splitter. The receiving part is the atom β, a beam splitter, and two photon detectors. All beam splitters are 50-50 beam splitters. The channels are the two photon lines connecting the two sections. In a realistic implementation it might be preferable to use a delay rather than two distinct channels, but for the sake of clarity we apply two lines in our analysis.

Initially the qubits are stored in the ground states $|a_i\rangle$ and $|d_j\rangle$. We first perform a local backup encoding on α: with a linear $\pi/2$ pulse we take $|a_i\rangle$ to $|a_i\rangle+|b_i\rangle$.

We then perform the first photon transmission. A single linearly polarized photon is split into two orthogonal states $|1\rangle$ and $|2\rangle$ by a beam splitter. The field state $|1\rangle$ interacts nonresonantly with the atom α coupling the level b with a detuning Δ and a coupling constant g to a higher-lying state in the atom. The energy shift of the level b can be calculated in second-order perturbation theory to be $\hbar g^2/\Delta$. If we choose an interaction time $T=\Delta\pi/g^2$ the phase of the state vector will change by π if α is in the level b. A phase change of π may not be realistic in an experiment, and we shall relax this assumption later.

We then recombine the two photon amplitudes yielding the two states $|+\rangle=|1\rangle+|2\rangle$ and $|-\rangle=|1\rangle-|2\rangle$. Due to the dispersive interaction, the photon is entangled with atom α, and the photon enters the channel $|+\rangle$ ($|-\rangle$) if α is in level g (G). The receiving atom β is prepared with a linearly polarized $\pi/2$ pulse so that $|d_j\rangle$ is taken to $|d_j\rangle+|e_j\rangle$. The photon state $|-\rangle$ now couples $|e_j\rangle$ nonresonantly to a higher level, yielding a conditional phase shift of π, as described for atom α, and we then apply a second $\pi/2$ pulse so that if the field is in the $|+\rangle$ state β will be taken back to $|d_j\rangle$ by the last pulse, but if the field is in the $|-\rangle$ state β will be taken to $|e_j\rangle$ due to the phase change induced by the field. Since the $|+\rangle$ and $|-\rangle$ states correspond to α being in a and b, respectively, this will create the desired entanglement between α and β, but at this point the atoms are also entangled with the photon.

We get rid of the photon with a quantum eraser: The two photon states $|+\rangle$ and $|-\rangle$ interfere, yielding the two detector states $|D_1\rangle$ and $|D_2\rangle$. We assume here that the mirrors are aligned so that $|D_1\rangle$ corresponds to the incoming state $|1\rangle$ [7]. We then perform a measurement revealing whether the photon is in $|D_1\rangle$ or $|D_2\rangle$. If $|D_1\rangle$ is measured we change the sign of the level b to compensate a sign induced by the eraser.

A simple analysis shows that the transmission performs the operation

$$|a_i\rangle|d_j\rangle\rightarrow|a_i\rangle|d_j\rangle,$$

$$|b_i\rangle|d_j\rangle\rightarrow|b_i\rangle|e_j\rangle. \tag{2}$$

During transmission the Zeeman degeneracy plays no role. Subscripts i and j denoting the Zeeman state have only been written for later convenience.

Had we included the evolution of the level e, the transmission would be a control-not operation between the levels a, b, d, and e, but this control-not operation would be vulnerable to errors. Our backup scheme makes it possible to perform a perfect control-not operation between the states $|a_0\rangle$, $|a_1\rangle$, $|d_0\rangle$, and $|d_1\rangle$, also in the presence of errors. Paradoxically this may be achieved by means of the transmission described by Eq. (2) and local operations, even though the transmitted photons carry no information on the azimuthal quantum numbers.

Including the $\pi/2$ preparation of α the evolution so far is given by

$$\begin{bmatrix} |a_0\rangle|d_0\rangle \\ |a_0\rangle|d_1\rangle \\ |a_1\rangle|d_0\rangle \\ |a_1\rangle|d_1\rangle \end{bmatrix} \rightarrow \begin{bmatrix} |a_0\rangle|d_0\rangle+|b_0\rangle|e_0\rangle \\ |a_0\rangle|d_1\rangle+|b_0\rangle|e_1\rangle \\ |a_1\rangle|d_0\rangle+|b_1\rangle|e_0\rangle \\ |a_1\rangle|d_1\rangle+|b_1\rangle|e_1\rangle \end{bmatrix}. \tag{3}$$

Now, the states $|e_i\rangle$ are moved to storage states $|f_i\rangle$ and the states $|a_i\rangle$ and $|b_i\rangle$ are interchanged by linearly polarized π pulses. A second photon is transmitted, causing again the evolution in Eq. (2). Since the f states of β are not coupled to the incident photon, these states are not affected by the second transmission, and we end up with

$$\begin{bmatrix} |b_0\rangle|e_0\rangle+|a_0\rangle|f_0\rangle \\ |b_0\rangle|e_1\rangle+|a_0\rangle|f_1\rangle \\ |b_1\rangle|e_0\rangle+|a_1\rangle|f_0\rangle \\ |b_1\rangle|e_1\rangle+|a_1\rangle|f_1\rangle \end{bmatrix}. \tag{4}$$

The main result of this paper is that we are able to construct these states, even in the presence of errors. This will be shown in Sec. V. Within the quantum states in Eq. (4) we break the horizontal symmetry of azimuthal states 0 and 1 and extract the desired states on the right side of Eq. (1) by local operations.

We measure if α is in the subspace spanned by $|a_0\rangle$ and $|b_1\rangle$. This can for instance be done by interchanging $|a_1\rangle$ and $|b_1\rangle$ and making a quantum-nondemolition (QND) measurement [8] of the energy of α. If α is found in the subspace spanned by $|a_0\rangle$ and $|b_1\rangle$ we interchange the amplitudes on $|e_0\rangle$ and $|e_1\rangle$. If it is not we interchange the amplitudes on $|f_0\rangle$ and $|f_1\rangle$. We then measure if β is in the subspace spanned by $|e_0\rangle+|f_0\rangle$ and $|e_1\rangle+|f_1\rangle$. This can be done with a $\pi/2$ pulse followed by a QND measurement of the atomic energy. From the results of these measurements one can construct a sequence of pulses that takes us to the desired states.

As a specific example of the extraction procedure consider the situation where α is found in the subspace spanned by $|a_0\rangle$ and $|b_1\rangle$. The measurement collapses Eq. (4) to this subspace and we apply a pulse that interchanges $|e_0\rangle$ and $|e_1\rangle$

$$\begin{bmatrix} |a_0\rangle|f_0\rangle \\ |a_0\rangle|f_1\rangle \\ |b_1\rangle|e_0\rangle \\ |b_1\rangle|e_1\rangle \end{bmatrix} \begin{matrix} \rightarrow \\ \rightarrow \\ \\ \nearrow\!\!\!\!\searrow \end{matrix} \begin{bmatrix} |a_0\rangle|f_0\rangle \\ |a_0\rangle|f_1\rangle \\ |b_1\rangle|e_1\rangle \\ |b_1\rangle|e_0\rangle \end{bmatrix}. \tag{5}$$

Now consider the situation where we measure that β is in the subspace spanned by $|e_0\rangle-|f_0\rangle$ and $|e_1\rangle-|f_1\rangle$. Since $|e_i\rangle$ can be written $(|e_i\rangle+|f_i\rangle)+(|e_i\rangle-|f_i\rangle)$ and $|f_i\rangle$ can be written $(|e_i\rangle+|f_i\rangle)-(|e_i\rangle-|f_i\rangle)$ this is seen to introduce a minus in the first two lines in Eq. (5). By subsequently transferring $|b_1\rangle$ to $-|a_1\rangle$ and $|e_i\rangle-|f_i\rangle$ to $|d_i\rangle$ we arrive in the desired states.

This extraction procedure is illustrated in Fig. 2. We recall that the qubits are represented as superpositions of the azimuthal quantum states 0 and 1. Entanglement between the atoms is visualized by shading: Part (a) of the figure illustrates the states in Eq. (4), where β is in the level $e(f)$ if α is in $b(a)$. Our first measurement chooses states in α diagonally [part (b) in the figure and Eq. (5)]. Now, we interchange azimuthal states 0 and 1 of β if α is in 1. From the shading in the figure this is seen to correspond to interchanging $|e_0\rangle$ and $|e_1\rangle$ ($|f_0\rangle$ and $|f_1\rangle$ if the other diagonal had been measured). Finally, all states are taken to the lowest level as described in the example after Eq. (5), and we end up in the desired states.

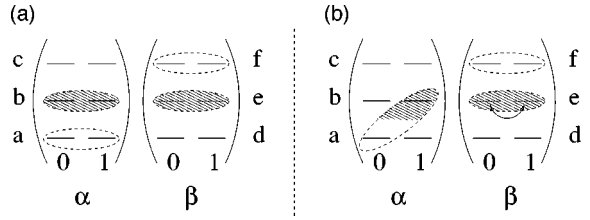

FIG. 2. Illustration of the diagonal extraction step. Part (a) corresponds to Eq. (4). The qubits are present horizontally in the Zeeman states and the levels are entangled vertically (represented by the shading). In part (b) we make a measurement that chooses states of α diagonally. With the measurement outcome in the figure, the control-not operation is achieved by interchanging $|e_0\rangle$ and $|e_1\rangle$ and transfering the atoms to the lower levels a and d as described in the text.

IV. ERROR ANALYSIS

In this section we analyze the effect of errors and in Sec. V we show how our backup scheme eliminates these errors. We will assume that measurements and unitary evolutions in single atoms are perfect. All errors will be due to imperfections in the dispersive interactions and in the channels used for communication.

A. Errors due to loss of photons

A photon is considered lost if it is not detected at the photon detectors in the end. Since the photons carry no information on the azimuthal quantum number the superposition between 0 and 1 states will not be disturbed. Using a QND measurement it can be detected whether α is in a or b and the qubit will still be present horizontally. Similarly we can measure the energy of β without disturbing the qubit and the initial states can be restored. We can then start over again and proceed with the transmission until it is successful.

B. Errors without loss of photons

Phase shift in the dispersive interaction with α. We no longer assume that our dispersive interaction causes a phase shift that is π. With a general phase shift the two levels a and b no longer give two orthogonal photon states ($|+\rangle$ and $|-\rangle$) that can be separated by a beam splitter, but we can arrange our beam splitter so that a always produces a photon in the $|+\rangle$ channel. The atom in level b, however, will yield a superposition of $|+\rangle$ and $|-\rangle$,

$$|a_i\rangle \rightarrow |a_i\rangle|+\rangle,$$

$$|b_i\rangle \rightarrow |b_i\rangle(-|-\rangle+k_+|+\rangle). \tag{6}$$

We show in Sec. V that our scheme still works because the erroneous $k_+|+\rangle$ component can be projected out with a measurement.

Errors in the channels. With the assumption that the photons cannot jump from one channel to the other and cannot be created in the channels, the most general evolution will be described by

$$|+\rangle \rightarrow \eta\widetilde{|+\rangle},$$
$$|-\rangle \rightarrow \zeta\widetilde{|-\rangle}, \tag{7}$$

where the notation means that the wave packet is changed in some way (change of shape and duration of the wave packet, etc.). The states $\widetilde{|+\rangle}$ and $\widetilde{|-\rangle}$ are assumed normalized. This evolution can be described by the non-Hermitian Hamiltonian of Monte Carlo wave functions [9] in the no jump stages of evolution.

We assume that the photon in two subsequent transmissions couples to independent and identical environments. With this assumption the evolution in Eq. (7) will be the same in the two transmissions, provided that the photon is not lost. This assumption is further justified in [4].

Errors in the interaction with β. The photon state $\widetilde{|+\rangle}$ does not interact with the atom β and we assume that local laser pulses on β are error free. This photon state will therefore not cause any transition in β. The interaction between β and $\widetilde{|-\rangle}$ is modified due to the imperfect dispersive interaction and the modified photon state. This means that β may not be transfered to the level e as desired. The effect of the interaction may be summarized as follows:

$$\widetilde{|+\rangle}|d_i\rangle \rightarrow \widetilde{|+\rangle}|d_i\rangle,$$
$$\widetilde{|-\rangle}|d_i\rangle \rightarrow \widetilde{|-\rangle}(k_d|d_i\rangle + |e_i\rangle). \tag{8}$$

Errors in the photon detection. The two orthogonal states $\widetilde{|-\rangle}$ and $\widetilde{|+\rangle}$ are measured in an orthogonal basis $\{|D_1\rangle, |D_2\rangle\}$. The beam splitter is a 50-50 beam splitter and any overall phase factors may be absorbed in the definition of $|D_1\rangle$ and $|D_2\rangle$. We can therefore write

$$|D_1\rangle = \widetilde{|+\rangle} + e^{i\delta}\widetilde{|-\rangle},$$
$$|D_2\rangle = \widetilde{|+\rangle} - e^{i\delta}\widetilde{|-\rangle}. \tag{9}$$

The phase factor δ in the two equations must be identical because $|D_1\rangle$ and $|D_2\rangle$ have to be orthogonal.

Collecting the effects of Eqs. (6)–(9) we see that (before photon detection) the transmission performs the evolution

$$|a_i\rangle|d_j\rangle \rightarrow \eta|a_i\rangle|d_j\rangle(|D_1\rangle + |D_2\rangle),$$
$$|b_i\rangle|d_j\rangle \rightarrow \zeta e^{-i\delta}|b_i\rangle|e_j\rangle(|D_2\rangle - |D_1\rangle) + |b_i\rangle|d_j\rangle \times [\zeta k_d e^{-i\delta}(|D_2\rangle - |D_1\rangle) + \eta k_+(|D_1\rangle + |D_2\rangle)]. \tag{10}$$

This expression displays unwanted disturbances of the amplitudes of our quantum-mechanical superposition. As we shall see below, these disturbances can be interchanged in the second photon transmission, thereby symmetrizing and hence eliminating their effect on the relevant amplitudes.

V. NOISY BACKUP CONTROL-NOT OPERATION

We now describe the effect of errors on the overall evolution. If a photon is lost we restore the initial situation and start the transmission again as described above. In this section we shall therefore only consider the situation where we do not lose photons.

After the first transmission we end up in states like Eq. (10). We recall that we change the sign of the level b if D_1 clicks, and after photon detection the atomic state will therefore be given by

$$\begin{bmatrix} |b_0\rangle[\zeta e^{-i\delta}(|e_0\rangle + k_d|d_0\rangle) \pm \eta k_+|d_0\rangle] \\ |b_0\rangle[\zeta e^{-i\delta}(|e_1\rangle + k_d|d_1\rangle) \pm \eta k_+|d_1\rangle] \\ |b_1\rangle[\zeta e^{-i\delta}(|e_0\rangle + k_d|d_0\rangle) \pm \eta k_+|d_0\rangle] \\ |b_1\rangle[\zeta e^{-i\delta}(|e_1\rangle + k_d|d_1\rangle) \pm \eta k_+|d_1\rangle] \end{bmatrix} + \eta \begin{bmatrix} |a_0\rangle|d_0\rangle \\ |a_0\rangle|d_1\rangle \\ |a_1\rangle|d_0\rangle \\ |a_1\rangle|d_1\rangle \end{bmatrix}, \tag{11}$$

where the sign on the $k_+|d_i\rangle$ component is + (−) if D_2 (D_1) clicks. We now interchange $|a_i\rangle$ and $|b_i\rangle$ and $|e_i\rangle$ and $|f_i\rangle$ before we perform the second transmission and subsequent photon detection. The atomic states will now read

$$\begin{bmatrix} |a_0\rangle[\zeta\eta e^{-i\delta}(|f_0\rangle + k_d|d_0\rangle) \pm \eta^2 k_+|d_0\rangle] \\ |a_0\rangle[\zeta\eta e^{-i\delta}(|f_1\rangle + k_d|d_1\rangle) \pm \eta^2 k_+|d_1\rangle] \\ |a_1\rangle[\zeta\eta e^{-i\delta}(|f_0\rangle + k_d|d_0\rangle) \pm \eta^2 k_+|d_0\rangle] \\ |a_1\rangle[\zeta\eta e^{-i\delta}(|f_1\rangle + k_d|d_1\rangle) \pm \eta^2 k_+|d_1\rangle] \end{bmatrix} + \begin{bmatrix} |b_0\rangle[\eta\zeta e^{-i\delta}(|e_0\rangle + k_d|d_0\rangle) \pm \eta^2 k_+|d_0\rangle] \\ |b_0\rangle[\eta\zeta e^{-i\delta}(|e_1\rangle + k_d|d_1\rangle) \pm \eta^2 k_+|d_1\rangle] \\ |b_1\rangle[\eta\zeta e^{-i\delta}(|e_0\rangle + k_d|d_0\rangle) \pm \eta^2 k_+|d_0\rangle] \\ |b_1\rangle[\eta\zeta e^{-i\delta}(|e_1\rangle + k_d|d_1\rangle) \pm \eta^2 k_+|d_1\rangle] \end{bmatrix}, \tag{12}$$

where the $\pm$ in the first (second) set of square brackets refers to the outcome of the photon detection in the first (second) transmission. Equation (12) shows that we have achieved the desired symmetrization of amplitude errors. Collecting terms, we get states of the form $\eta\zeta e^{-i\delta}(|a_i\rangle|f_j\rangle + |b_i\rangle|e_j\rangle) + |R_i\rangle|d_j\rangle$, where $|R_i\rangle|d_j\rangle$ are all the remaining components. The first term is the ideal states in Eq. (4), but we also have the $|R_i\rangle|d_j\rangle$ component.

We now measure if β is in the level d. If β is found in d, the qubits are restored to their initial states and the transmission is attempted again. If β is not in d, the $|R_i\rangle|d_j\rangle$ components are projected out by the measurement and we are left with the states of Eq. (4). From here the ''diagonal'' extraction proceeds as before.

VI. DISCUSSION

Above we have shown how to achieve a perfect quantum control-not operation through noisy channels. It has been shown [10] that any unitary operation on any number of qubits can be performed using single qubit operations and control-not operations. With a perfect control-not operation we are therefore able to perform any communication task.

As mentioned in the Introduction, van Enk *et al.* have used similar ideas to achieve perfect teleportation [4]. However, we believe that we suggest a simpler physical realization. The coherent control of several atoms required in the scheme of van Enk *et al.* is a very difficult experimental task, and it is a major advantage of our scheme that it only requires single atoms at each end.

In [11] van Enk *et al.* also discuss the possibility of making an error-free quantum logic gate using only single atoms. The main idea in [11] is to monitor the performance of the gate and discard unsuccessful operations. Quantum information is stored in two states of each atom, and failures are monitored by a third state of the atoms, which, however, does not enable the recovery of the quantum information as in this work and in [4].

To perform our one-atom scheme we have chosen to use nonresonant dispersive interactions. It is also possible to use other kinds of physical interactions, such as the Raman pulses in the suggestions of van Enk *et al.* Our only requirement is that the states $|d_i\rangle$ and $|e_i\rangle$ in β be coupled only when α is in level b.

We wish to emphasize another important feature of our proposal, well illustrated in Fig. 2. The use of atoms with two-plus-two relevant states, rather than pairs of atoms with two-times-two states, offers a simple geometric picture of the transfer protocol; cf. in particular the diagonal extraction in Fig. 2(b). We believe that such pictures may be useful in the development of further ideas, not only for fault-tolerant transmission.

As an example, consider computation distributed on several quantum computers [12], with signaling atoms responsible for communication. Following our proposal these atoms may be entangled vertically, prior to the calculation, and when ready for transmission, horizontal qubits may be communicated by the diagonal extraction procedure and other local operations.

Also multiparticle entanglement may be accommodated following these lines. Recently it has been shown that for quantum communication over long distances the efficiency of a channel can be enhanced if it consists of series of nodes that share EPR pairs with each of their neighboring nodes [13]. To share EPR pairs with two neighbors would normally require two atoms per node. However, with our scheme a single atom may suffice. If we start with a superposition $|a_0\rangle - |a_1\rangle$ and perform a horizontal control-not operation with one neighbor, these two nodes will share a horizontal EPR pair. By performing the steps that lead to Eq. (4) with another neighbor, a vertical EPR correlation with this neighbor is created without destroying the horizontal correlation with the first neighbor. In this way each node only requires a single atom.

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