Multilayer "dielectric" mirror for atoms

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All existing atomic mirrors are conceptually similar to metallic mirrors in standard optics in the sense that they are constructed with repulsive potentials and thus produce an imaginary "refraction index." Here a different kind of mirror for atoms that is similar to a multilayer dielectric mirror in standard optics is proposed. We show that a periodic attractive laser potential can reflect atoms of certain momenta due to the presence of gaps in the spectrum of energies, as in the photonic band-gap structures in light optics. Spontaneous emission and gravitational effects are also analyzed.
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Recently, atom optics has become an extremely fast-developing research field. Some very striking phenomena due to the wave nature of cold atoms have been reported, such as the diffraction of atoms [1], the construction of an atomic Young interferometer [2], imaging and focusing of atoms [3], and even, very recently, the atom optics equivalent of the laser, i.e., the so-called atom laser [4]. Atomic mirrors have also been constructed with evanescent laser fields [5] and, more recently, with magnetic fields [6]. Both mirrors are based on the same physical effect, i.e., the appearance of repulsive potentials. The atom is reflected if it reaches a turning point. After the turning point the probability density follows an exponential decay, just like an electric field directed onto a metallic surface. In this sense, therefore, these atomic mirrors can be considered as the atom optics equivalent of metallic mirrors in light optics. In the past few years several studies have been reported on the behavior of cold atoms in periodic light potentials (light lattices) created by stationary laser waves. The analogy between this case and solid-state physics has aroused great interest [7]. Thus Bragg scattering has been analyzed [8] and so has the use of this effect to construct atomic beam splitters and interferometers [9]. Bragg reflection is in fact a particular case of a more general situation, i.e., the so-called photonic band-gap structures (PBGSs) [10]. In dielectric periodic structures, some electromagnetic waves cannot propagate and are therefore reflected because their frequencies lie within a gap created by the periodic structure. In this paper we propose a laser arrangement that acts as a matter-wave band-gap structure in the sense that it resembles a PBGS, but for matter waves instead of electromagnetic waves. The atoms can be reflected if their incoming kinetic energy lies within a gap produced by the periodic structure. This effect allows the construction of an atomic mirror with an attractive laser potential that acts as an atom optics equivalent of a multilayer dielectric mirror in light optics. (We must note that quantum reflection, i.e., reflection on an attractive potential, is not an unknown problem; in particular, this is a textbook problem in the case of a potential step [11]. Recently, quantum reflection has been analyzed in the context of atom optics for the case of an attractive exponential potential [12]. The main different idea of this paper lies in the periodic character of the potential and the resemblance to the reflection on a PBGS.)

Let us consider two laser beams (as indicated in Fig. 1) of the same intensity, polarization, and with the same Gaussian profile, but with respective wave vectors $\vec{k}_1 = k(\cos\phi\vec{u}_x + \sin\phi\vec{u}_z)$ and $\vec{k}_2 = k(\cos\phi\vec{u}_x - \sin\phi\vec{u}_z)$. In the zone very close to $x=0$, the electric field, which one assumes to be linearly polarized in the y direction, can be written in the form

$$\vec{E} \approx E_0 \vec{e}_y e^{-(z-z_c)^2 \cos^2\phi/d^2} \cos[k(z-z_c)\sin\phi] \times \cos(kx\cos\phi - \omega t), \tag{1}$$

where $E_0/2$ is the amplitude of each laser, z_c is the center of the laser region, and d is the half-width of the Gaussians. Let us assume that the laser frequency is close to some atomic transition from the ground state to an excited state (of energies $\hbar\omega_g$ and $\hbar\omega_e$, respectively) in such a way that we can treat the atom as a two-level system. In this work we assume a large internal detuning $[\Delta = \omega - (\omega_e - \omega_g)]$ in such a way that the adiabatic approximation will be valid [13]. So to obtain a scalar Schrödinger equation to describe the atomic interaction with the laser we follow the standard formalism developed in [13], except for the inclusion of the gravita-

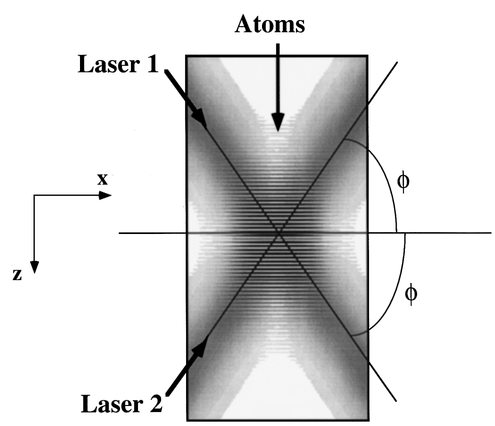

FIG. 1. Scheme of laser arrangement considered. Two Gaussian lasers propagate, forming an angle ϕ and $-\phi$ with the x axis. Atoms are dropped from a MOT onto the interference region of both lasers, where a periodic profile is formed.

tional field. As we show later, this can have non-negligible effects. The scalar equation takes the form

$$\frac{-\hbar^2}{2M}\frac{d^2}{dz^2}\psi(z)=\left[\frac{q^2}{2M}-V(z)\right]\psi(z), \tag{2}$$

where $V(z)=-Mgz-\frac{1}{2}\hbar\Delta\mp\frac{1}{2}\hbar\sqrt{\Delta^2+4\Omega(z)^2}$, with

$$\Omega(z)=\Omega_0\exp[-\cos^2\phi(z-z_c)^2/d^2]\cos[k(z-z_c)\sin\phi].$$

The sign depends on the detuning: the minus sign corresponds to $\Delta<0$ and the plus corresponds to $\Delta>0$. The coupling is given by $\Omega_0=-\mu E_0/2\hbar$, which is the Rabi frequency associated with each laser, where $\mu=\langle\vec{e}_y\vec{\mu}\rangle$, with $\vec{\mu}$ the transition dipole. q is the z momentum at $z=0$, sufficiently far away from z_c to consider that the field at $z=0$ is negligible. To make the model as realistic as possible we include the diffuse scattering resulting from spontaneous emission by adopting the lossy vector Schrödinger equation method of [14], i.e., $\Delta\rightarrow\Delta+i\gamma/2$, where γ is the spontaneous emission rate.

As may be seen, the potential $V(z)$ is periodic, its periodicity given by $\Delta z=\lambda/2\sin\phi$, where λ is the wavelength of the lasers employed. To better understand the effect of periodicity, let us remove gravitation and spontaneous emission and for the time being let us forget the smooth Gaussian dependence. Let also assume that $\Omega_0^2\ll|\Delta|^2$. With these assumptions, the laser potential is a simple cosine-squared potential, whose amplitude $(\hbar\Omega_0^2/\Delta)$ can be positive (if $\Delta>0$) or negative (if $\Delta<0$), depending on which dressed state is reached. In what follows we always consider the case $\Delta<0$ and hence the potential is always negative. It is well known that a periodic potential leads to an energy structure of allowed and forbidden bands [15]. Only the atoms whose energy lies in an allowed band can propagate inside the potential. If q does not satisfy the band condition (i.e., if $q^2/2M$ lays in a gap), then the atoms with this momentum cannot propagate inside the laser beam and are therefore reflected, reflection bands being formed. The gaps appear for repulsive and also for attractive potentials, allowing the possibility of reflecting atoms with attractive potentials. Obviously, the gravitational field, the spontaneous emission, and the Gaussian envelope of the laser profile will distort this simple picture, but the basic effect of enhancing the reflection for some momenta, due to energy gaps, remains, as we show below.

Wave functions outside the laser zone are linear combinations of Airy functions [16]. If we set $u(z)=(2M^2g/\hbar^2)^{1/3}(z+q^2/2M^2g)$, we can characterize $f_D(z)=\mathrm{Ai}[-u(z)]-i\,\mathrm{Bi}[-u(z)]$ and $f_U(z)=\mathrm{Ai}[-u(z)]+i\,\mathrm{Bi}[-u(z)]$ as the downward and the upward components of the wave function, respectively. The functions in the zone before and after the laser beam can thus be characterized by $\psi(z)=f_D(z)+rf_U(z)$ and $tf_D(z)$, respectively, where r and t are the reflection and transmission amplitudes, respectively. The aim of this work is to analyze the reflection probability $|r|^2$ as a function of the z momentum q at $z=0$.

Using convenient units of length (k^{-1}), momentum $(\hbar k)$, and frequency $(\omega_\nu=\hbar k^2/2M)$, we can rewrite the Schrödinger equation in a dimensionless form

$$\frac{d^2}{d\tilde{z}^2}\psi(\tilde{z})=-\{\tilde{q}^2+\beta\tilde{z}+\tfrac{1}{2}\tilde{\Delta}\pm\tfrac{1}{2}\sqrt{\tilde{\Delta}^2+4\tilde{\Omega}(\tilde{z})^2}\}\psi(\tilde{z}), \tag{3}$$

in which a tilde denotes dimensionless units. In Eq. (3) we have the function

$$\tilde{\Omega}(\tilde{z})=\tilde{\Omega}_0\exp[-\cos^2\phi(\tilde{z}-\tilde{z}_c)^2/d^2]\cos[(\tilde{z}-\tilde{z}_c)\sin\phi].$$

The parameter $\beta=2M^2g/\hbar^2k^3$ is a gravitational term that introduces some differences depending on the mass of the atom. Since we shall assume that $\tilde{\Omega}_0^2\ll|\tilde{\Delta}|^2$, we can define a parameter $\eta=\tilde{\Omega}_0^2/\tilde{\Delta}$ that determines the strength of the laser potential. This can be easily shown by introducing a Taylor expansion on the right-hand side of Eq. (3):

$$\frac{d^2}{d\tilde{z}^2}\psi(\tilde{z})=-\{\tilde{q}^2+\beta\tilde{z}-\tilde{V}(\tilde{z})\}\psi(\tilde{z}), \tag{4}$$

where

$$\tilde{V}(\tilde{z})=\eta\exp[-2\cos^2\phi(\tilde{z}-\tilde{z}_c)^2/\tilde{d}^2]\cos^2[(\tilde{z}-\tilde{z}_c)\sin\phi].$$

Note that the potential is formed by the product of a cosine squared and a Gaussian envelope.

In a recent paper [17] Tan and Walls developed a powerful transfer-matrix method to calculate the reflection and transmission coefficients of a plane wave interacting with a laser of arbitrary profile. However, the method was developed without considering the effects of gravitation. Here we propose another method that consists of analyzing the adiabatic scalar Schrödinger equation by finite differences. Let us take N equally spaced points $\{\tilde{z}_1,\tilde{z}_2,\ldots,\tilde{z}_{N-1},\tilde{z}_N\}$ in the laser region in such a way that for the first two points we can ensure that the laser field is zero (i.e., they are in the reflection zone) and the same for the last two points, which are in the transmission zone. For points $j=2,\ldots,N-1$ we can write the scalar Schrödinger equation using finite differences

$$\frac{\psi_{j-1}-2\psi_j+\psi_{j+1}}{\varepsilon^2}=-\{\tilde{q}^2+\beta\tilde{z}_j+\tfrac{1}{2}\tilde{\Delta}\pm\tfrac{1}{2}\sqrt{\tilde{\Delta}^2+4\tilde{\Omega}_j^2}\}\psi_J, \tag{5}$$

where $\psi_j=\psi(z_j)$, $\Omega_j=\tilde{\Omega}(\tilde{z}_j)$, and $\varepsilon=\tilde{z}_{j+1}-\tilde{z}_j$. For convenience, we use $\tilde{z}_2=0$. Using the expression for the wave function in the reflection zone, we can obtain a relationship between the first two points

$$f_U(\tilde{z}_2)\psi_1-f_U(\tilde{z}_1)\psi_2=f_D(\tilde{z}_1)f_U(\tilde{z}_2)-f_D(\tilde{z}_2)f_U(\tilde{z}_1). \tag{6}$$

For the last two points, we can use the expression for the transmission zone

$$-f_D(\tilde{z}_N)\psi_{N-1}+f_D(\tilde{z}_{N-1})\psi_N=0. \tag{7}$$

From Eqs. (4)–(6) we thus obtain N equations, which form an easily solvable tridiagonal system of equations. Once we know the solutions at $\tilde{z}_1$ and $\tilde{z}_N$, we can obtain the values of r and t.

Without considering spontaneous emission, from the expressions of the reflection and transmission zones it is possible to state that the law of conservation of the flux has the form $|r|^2+|t|^2=1$. This conservation law can be used to check our numerical analysis. In all the calculations the error is less than 10^{-6}, guaranteeing that our method is consistent. This check is evidently only a preliminary step before including spontaneous emission, which produces flux losses. We also confirmed that in the absence of gravity our method leads in all the cases to the same results of [17], i.e., is completely equivalent to a transfer-matrix method. Adiabaticity was checked in all the calculations by a comparison with a fully spinorial calculation.

In Fig. 2 we address the case of the $2s$-$2p$ ^{7}Li transition, whose parameters are $\lambda=670.8$ nm, $\omega_v=3.96\times10^5$ s^{-1}, $\gamma=3.72\times10^7$ s^{-1}, and $\beta=0.000\,293$. In this figure we consider $\eta=-5$. The position of the center of the laser region is at $z_c=300\sqrt{2}\pi k^{-1}=0.142$ mm and the half-width of the laser Gaussians is given by $d=100\pi k^{-1}=33.5$ μm. For a typical laser with a Gaussian profile, the reflection spectrum is flat and equal to zero, because for this detuning the potential is always attractive and thus there are no turning points (this is true except for very low momentum, for which quantum reflection such as that reported in [12] is produced). This is what one would observe in the case of an angle $\phi=0$. Let us analyze the effects of the periodicity that appear in the potential when ϕ is not zero. Let us consider the case of $\phi=\pi/3$. By using the previous numerical method, we show in Fig. 2(b) the reflectivity without considering gravitation and spontaneous emission. In order to obtain a physical insight into the effects behind these exact numerical results of reflectivity we cannot use the previous simple cosine-squared model, but we must take into account the Gaussian envelope, i.e., that the amplitude of the cosine-squared oscillations is not constant within the laser region. In order to take this into account, we define a band structure [Fig. 2(a)] for each value of the envelope function (V_{env}) [18]. As each value of the envelope is linked with a spatial position, then a space-dependent band structure appears. In particular we observe that for certain momenta some spatial regions are forbidden (white regions). We will call these regions spatial gaps. In this case we observe that three spatial gaps appear within the envelope (white regions). By extrapolating to zero potential, we recover the Bragg orders, as one obtains the Bragg modes for tiny refractive indices in the PBGSs. By comparing Figs. 2(a) and 2(b), we can understand why some momenta are reflected: If an atom, with some momentum, while traveling inside the laser finds a forbidden region (for its momentum) then it is reflected (if the region is very narrow there is some probability of tunneling through it, leading to partial reflection). We show now that other details of the reflectivity can be easily explained using this image of spatial gaps. In Fig. 2(c) we add the effect of spontaneous emission for $\Delta=-200\gamma=-7.44\times10^9$ s^{-1} ($\Omega_0=1.214\times10^4$ s^{-1}, which means an intensity per beam $I=54$ mW/cm^2). The spontaneous emission acts as a mechanism of flux losses and therefore affects the reflection spectrum by introducing a reduction in reflectivity. This reduction can be observed by comparing the cases without [Fig. 2(b)] and with [Fig. 2(c)] spontaneous emission. The details of this reduction can be explained using the image of spatial gaps. The longer the atom stays inside the laser region, the larger the effects of the spontaneous emission. The momentum components that reach a forbidden spatial region for small values of the envelope are reflected earlier than the components that reach a spatial gap for larger values of the envelope. Thus the first ones are affected by the laser for a shorter time and then they are less affected by the spontaneous emission than the later

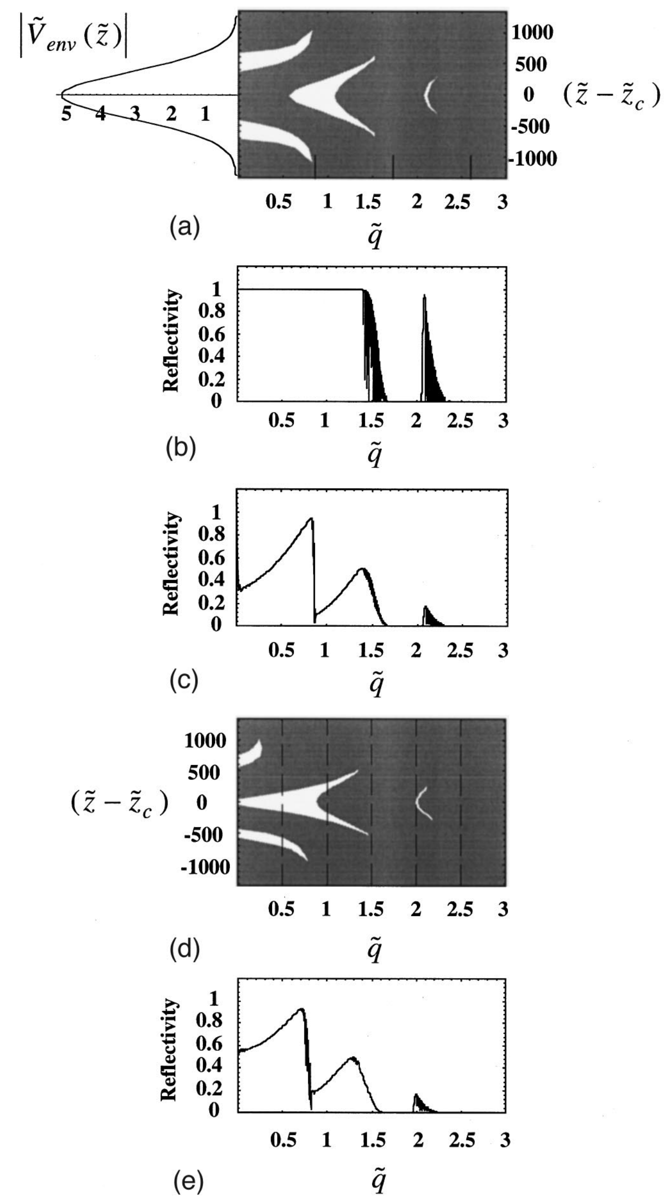

FIG. 2. Results, as a function of the z momentum q at $z=0$, for the case of the $2s$-$2p$ ^{7}Li transition, considering $\eta=-5$, $\tilde{z}_c=300\sqrt{2}\pi$, $\tilde{d}=100\pi$, and $\phi=\pi/3$. (a) Band structure as a function of z, i.e., for different values of the envelope function $|V_{\mathrm{env}}|$. Note that in this case three gaps (white regions) are opened. Small vertical bars indicate the position of the Bragg modes. The atoms are assumed to travel in the graph initially from bottom to top, beginning with momentum q at the bottom ($z=0$) of the figure. (b) Reflectivity calculated without gravitation and without spontaneous emission. (c) Reflectivity calculated without gravitation but with spontaneous emission for $\Delta=-200\gamma$. (d) Band structure as a function of z, including gravitational effects. Forbidden regions are in white. Tilting is clear compared to (a). The atoms are assumed to travel in the graph initially from bottom to top, beginning with momentum q at the bottom ($z=0$) of the figure. (e) Reflectivity for the same case as in (c) but with gravitational acceleration.

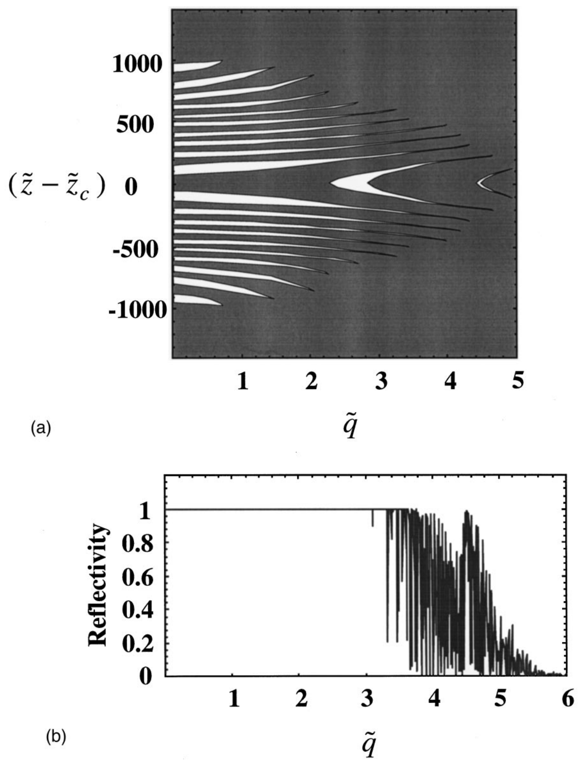

FIG. 3. Results, as a function of the z momentum q at $z=0$, for the case of the $2s$-$2p$ ^{7}Li transition, considering $\tilde{z}_c=300\sqrt{2}\pi$, $\tilde{d}=100\pi$, and $\eta=-200$. (a) Band structure as a function of z, without including gravitational effects. Note that several gaps (white regions) are opened. The atoms are assumed to travel in the graph initially from bottom to top, beginning with momentum q at the bottom $(z=0)$ of the figure. (b) Reflectivity without considering spontaneous emission (valid for large detuning compared to the spontaneous emission or for metastable transitions) and without considering gravitation (whose effect for the case of ^{7}Li atoms is basically a very small displacement to lower momenta).

ones. This effect is clearly observed on comparing Figs. 2(a) and 2(c). For larger detunings, the effects of spontaneous emission are smaller and can in fact be almost completely negligible if we use $|\Delta|\gg\gamma$. The use of metastable transitions, such as Raman transitions [19], can also allow spontaneous emission to be neglected. In Fig. 2(d) we add the effects of gravitational acceleration. Note now that the symmetry in z is broken due to the tilting of the bands, whose main effect is a displacement and a broadening of the bands [Fig. 2(e)] [20]. The effects of gravity are very small for two reasons. First, for a fixed distance z, it is easy to show from simple Newton laws that the larger the initial momentum at $z=0$, the smaller the effect of gravity over the distance z (for example, if $\tilde{q}=2$ at $z=0$, the final momentum at z_c is $\tilde{q}=2.09$). For small momentum, the effects of gravity would be important; however, the reflection is not produced at z_c but earlier, when the atoms find a forbidden region, and so the effects of the gravitation are again small. The effects of gravity are appreciable only for very low momenta $(\tilde{q}<0.5)$, as seen in Fig. 2(e).

If we increase $|\eta|$ more gaps are opened inside the laser region, as seen in Fig. 3(a) for the same case as Fig. 2, but now $\eta=-200$. Increasing the potential means increasing the laser intensity, but also increasing the laser detuning to satisfy the adiabatic condition. Figure 3(b) shows the reflectivity for the case of Fig. 3(a). Spontaneous emission is not considered, which is a valid approximation for very large detunings or for metastable transitions. Gravity is not considered because, as remarked above, its effect is only important for extremely low momenta $(\tilde{q}<0.5)$. As in the case of Fig. 2, the atoms are reflected if, when traveling inside the Gaussian envelope, they find a forbidden region. Thus, in this case we observe that a momentum up to 5 times the recoil momentum can be reflected. The figure also shows the appearance of oscillations in reflectivity for $\tilde{q}>3$. The forbidden regions are very narrow for $\tilde{q}>3$ and hence the atoms can partially tunnel through them. After tunneling through a narrow forbidden region the atoms find the next gap, from which they can be partially reflected back to the first forbidden region, where they can be partially reflected, and so on, leading to multiple reflections between the gaps. The oscillations in the reflection spectrum are due to these multiple reflections.

To conclude, let us sum up a few remarks concerning the validity of Eq. (1) and the detection of the reflected atoms. Atoms that travel out of the region of $x=0$ but close to it also undergo a periodic potential that leads to similar qualitative effects. However, the results obtained in this work are strictly only for $x=0$ or for a very close neighborhood and can be quantitatively distorted by the atoms falling out of this zone. Let us analyze a detection arrangement that could lead to a measurement of the reflection that could quantitatively coincide with our theoretical calculations. For $\phi\leqslant\pi/3$, in a region of $|x|<d/5$ the profile of the amplitude of the field is approximately constant and equal to the profile at $x=0$; Eq. (3) is therefore valid in this neighborhood. Due to the x dimensions of the trapped cloud and because the initial atomic beam (although very cold) has an x-momentum distribution and therefore spreads in x during flight, part of the beam falls out of the previous very narrow region. We can detect the reflection of the atoms with a weak probe laser by studying the fluorescence of the atoms. This probe beam can be considered as directed in the y direction, placed above the arrangement and sufficiently focused to obtain a very narrow spot of width $0.4d$ in the x direction in the zone of dropping and therefore the detected fluorescence is mainly produced by atoms traveling inside the laser within the narrow region $|x|<d/5$. Additionally, if the laser frequency is chosen with a large detuning from the transition the effects of the pressure of radiation (i.e., the effects of the absorption of momentum $k\cos\phi$ in the x direction), which tends to move the atom out of $x=0$, are negligible. The Gaussian dependence also affects the y direction, but this dependence is so smooth that it can be neglected in the region we are studying. This is the reason for simply taking into account a two-dimensional model in the x-z plane. Finally, we remark that other experimental arrangements that lead to a spatially periodic potential confined in a finite region can afford similar qualitative results.

To summarize, we have shown that an attractive periodic laser profile can act as an atomic mirror for some specific momenta. The reflection on this laser arrangement has been analyzed using an exact numerical calculation, based on finite differences, and completely equivalent to a transfer-matrix method. We have also developed a simple model based on the spatial dependence of the induced band structure, which allows a physical understanding of the relevant processes behind the numerical results of the atomic reflection. Until now, all reported atomic mirrors have been constructed with repulsive laser potentials and are therefore the equivalent of the usual metallic mirrors in optics. For these ''metallic'' mirrors for atoms, reflection can be understood as the effect of a classical repulsive force. Here we have shown a conceptually different kind of atomic mirror, constructed with an attractive periodic potential. These mirrors underscore the similarities between light and atom optics because they are the equivalent for atoms of a photonic bandgap structure in light optics. Reflection is achieved due to purely quantum effects, i.e., the periodicity of the laser potential hinders some momentum components from propagating inside the laser beam and they are therefore reflected. Like their counterparts in usual optics, these mirrors produce reflection only for certain momenta, i.e., they have a reflectivity that can be strongly dependent on the de Broglie wavelength of the incoming atom. This important feature may be of great interest for certain atom optics devices.

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[18] This approximation leads to a good explanation of the details of the exact numerical results. Its validity is based on the following considerations. In the cases analyzed in the present paper the width of the Gaussian envelope of the potential at $1/e$ is 245 cosine-squared periods. Since the width of the Gaussian is much larger than the periodicity of the cosine squared, we can consider that within small intervals of the envelope we have a large number of cosine-squared oscillations of approximately constant amplitude, for which we can define a band structure. We can extend this reasoning and define a band structure for each value of the envelope function (V_{env}). This is the band structure calculated for a cosine-squared potential of infinite periods of constant amplitude V_{env}.

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Bragg condition. The atomic wave is then reflected and its momentum is reversed. The atom travels again under Newton's law until it reaches another Bragg condition and is then reflected again. BOs can therefore be understood as oscillations between two Bragg reflections. In our problem, BOs would be produced if the atoms were spatially confined between two gravitationally tilted forbidden regions. Multiple reflections would then occur at the edges of the forbidden regions, leading to an oscillatory motion. However, in our case, the atoms come from outside the laser. If the atoms reach a forbidden region, they cannot be transmitted and are reflected back to free space. The atoms are therefore not between two gaps and hence cannot develop such multiple oscillations. Accordingly, the gravitational field does not lead to Bloch oscillations and so no effect of the WS ladders is observed in the reflection spectrum. Only if the first forbidden region were very narrow could the atom tunnel through the forbidden region (in a similar way to Landau-Zener tunneling) and enter an allowed region, finally reaching another forbidden region and producing multiple reflections. [For a review of the BOs, and WS ladders in the context of solid-state physics see E. E. Mendez and G. Bastard, Phys. Today **46** (6), 34 (1993)].