Dynamics of Bose-Einstein condensed gases in highly deformed traps

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We provide a unified investigation of normal modes and sound propagation at zero temperature in Bose-Einstein condensed gases confined in highly asymmetric harmonic traps and interacting with repulsive forces. By using hydrodynamic theory for superfluids we obtain explicit analytic results for the dispersion law of the low-energy discretized modes for both cigar- and disk-shaped geometries, including the regime of large quantum numbers where discrete modes can be identified with phonons. The correspondence with sound propagation in cylindrical traps and the one-dimensional nature of cigar-type configurations are explicitly discussed. [S1050-2947(98)02509-8]

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The recent experimental realization [1] of Bose-Einstein condensation in dilute gases of alkali atoms confined in magnetic traps has opened unique perspectives in the study of the dynamical and statistical behavior of mesoscopic quantum systems. These highly degenerate quantum gases have been shown to exhibit both the dynamic features of microscopic many-body systems, associated with the occurrence of quantized collective excitations (normal modes), and the ones of macroscopic quantum fluids, characterized by the propagation of sound in the collisionless regime (zero sound).

The frequencies of the collective excitations have been the object of various experimental [2–4] and theoretical studies, based on both analytic [5–8] and numerical [9] investigations. The magnitude of these frequencies is fixed by the oscillator frequency of the trapping potential (typically a few tens of Hz), the exact value depending on the nature of the excitation (angular momentum, etc.) and on the effects of two-body interactions. In the experiment of [4] the large number of Bose-Einstein condensed atoms and the elongated geometry of the trap have allowed for very precise *in situ* images of the oscillations of the axial radius and consequently for high precision in the measurement of the collective frequencies, whose values turn out to be in excellent agreement with the predictions of theory [5]. These experiments, however, do not probe directly the phononic nature of the excitation because of the discretization of frequencies imposed by the harmonic trapping.

The fact that phonons can propagate in the medium in a continuous way, according to Bogoliubov theory, has been recently shown in the experiement of [10] where wave packets have been produced and directly observed in a Bose-Einstein condensed gas of sodium atoms confined in a highly asymmetric, cigar type trap.

The purpose of this paper is to discuss the correspondence between these two dynamic features (occurrence of discretized collective modes and phononlike propagation) and to give analytic results for the dispersion law of the collective modes in the case of highly deformed traps, including the regime of large quantum numbers where the discrete modes can be identified with phonons.

The proper theory to describe the dynamic behavior of interacting Bose gases is the time-dependent Gross-Pitaevskii [11] equation for the order parameter. This equation can be written in a convenient form by expressing the order parameter in terms of its modulus and phase, $\Phi = \sqrt{\rho} e^{i\phi}$, and looking for equations for the density ρ and for the velocity field $\mathbf{v} = \nabla\phi$. The equations of motion then take the following form [5]:

$$\frac{\partial}{\partial t}\rho + \nabla(\mathbf{v}\rho) = 0 \tag{1}$$

and

$$m\frac{\partial}{\partial t}\mathbf{v} + \nabla\left(V_{\rm ext} + g\rho - \frac{\hbar^2}{2m\sqrt{\rho}}\nabla^2\sqrt{\rho} + \frac{1}{2}m\mathbf{v}^2\right) = 0, \tag{2}$$

where $g = 4\pi\hbar^2 a/m$ is the interaction coupling constant, fixed by the s-wave scattering length a and

$$V_{\rm ext} = \tfrac{1}{2}m\omega_\perp^2 r_\perp^2 + \tfrac{1}{2}m\omega_z^2 z^2 \tag{3}$$

is the trapping potential, for which we have made the choice of an axially symmetric oscillator ($r_\perp = \sqrt{x^2+y^2}$ is the radial coordinate). Equation (1) is the equation of continuity, while Eq. (2) establishes the irrotational nature of the motion. These equations have the typical structure of the dynamic equations of superfluids at zero temperature (see, for example, [12]).

If the repulsive interaction among atoms is enough strong, than the density profiles become smooth and one can safely neglect the kinetic pressure term proportional to $\hbar^2$ in the equation for the velocity field. This yields, for the static solution of Eq. (2), the so-called Thomas-Fermi expression

$$\rho_0(\mathbf{r}) = \frac{1}{g}[\mu - V_{\rm ext}(\mathbf{r})] \tag{4}$$

if $\mu \geqslant V_{\rm ext}(\mathbf{r})$, and zero elsewhere. Here μ is the chemical potential, fixed by the normalization of $\rho(\mathbf{r})$. In the case of harmonic trapping one has

$$\mu = \frac{1}{2}\hbar\omega_0\left(15N\frac{a}{a_{\rm HO}}\right)^{2/5}, \tag{5}$$

where $\omega_0=(\omega_z\omega_\perp^2)^{1/3}$ is the geometrical average of the oscillator frequencies and $a_{\rm HO}=\sqrt{\hbar/m\omega_0}$ is the corresponding oscillator length. The Thomas-Fermi approximation (4) for the ground state is accurate to the extent that the conditions $\mu\gg\hbar\omega_z,\hbar\omega_\perp$ are satisfied. The density (4) has the shape of an ellipsoid with radial ($R_\perp$) and axial (Z) radii defined by $m\omega_\perp^2R_\perp^2=m\omega_z^2Z^2=2\mu$.

In the following we will neglect the kinetic energy pressure term also in the solution of the time dependent equations (1) and (2). After linearization these equations then take the simplified, hydrodynamic form

$$\frac{\partial^2\delta\rho}{\partial t^2}=\nabla(c^2(\mathbf{r})\nabla\delta\rho), \tag{6}$$

where $\delta\rho=\rho(\mathbf{r},t)-\rho_0(\mathbf{r})$ and $c(\mathbf{r})$, defined by

$$mc^2(\mathbf{r})=\mu-V_{\rm ext}(\mathbf{r}), \tag{7}$$

can be interpreted as a local sound velocity. The validity of the HD equations (6) and (7) is based on the assumption that the spatial variations of the density are smooth not only in the ground state, but also during the oscillation. In a uniform system ($V_{\rm ext}=0$) this is equivalent to imposing that the collective frequencies should be much smaller than the chemical potential. In this case the solutions of the HD equation (6) are sound waves propagating with the Bogoliubov velocity $c=\sqrt{\mu/m}$, where $\mu=g\rho_0$ and ρ_0 is the equilibrium density. Due to the nonuniform nature of the trapping potential the solutions of Eqs. (6) and (7) exhibit new interesting features. The corresponding spectrum is discretized and its explicit form, in the case of spherical trapping, was first derived in [5]. The fact that these equations have analytic solutions reflects the occurrence of underlying, nontrivial symmetries as recently discussed in [8]. For the lowest multipolarities it is possible to obtain analytic results also for arbitrary values of ω_z and $\omega_\perp$. In particular, for the lowest $m=0$ value of the zth component of angular momentum and even parity, one finds [5]

$$\omega^2(m=0)=2\omega_\perp^2+\frac{3}{2}\omega_z^2\mp\frac{1}{2}\sqrt{9\omega_z^4-16\omega_z^2\omega_\perp^2+16\omega_\perp^4}. \tag{8}$$

The dispersion (8) has been also derived in [7] using different approaches. Result (8) can be generalized to a triaxially deformed trap of the form $V_{\rm ext}=(\omega_x^2x^2+\omega_y^2y^2+\omega_z^2z^2)/2m$. In this case the collective frequencies are given by the solution of the equation

$$\omega^6-3\omega^4(\omega_x^2+\omega_y^2+\omega_z^2)+8\omega^2(\omega_x^2\omega_y^2+\omega_y^2\omega_z^2+\omega_z^2\omega_x^2)-20\omega_x^2\omega_y^2\omega_z^2=0. \tag{9}$$

For example, using the values $\omega_x^2=2\omega_b^2=4\omega_z^2$, characterizing the trap recently used in [18] to achieve Bose-Einstein condensation, we find solutions at $\omega^2=5\omega_z^2$ and $\omega^2=(8\pm4\sqrt{2})\omega_z^2$.

In the limit of highly deformed, cigar-shaped geometry ($\omega_z\ll\omega_\perp$), Eq. (8) gives the result $\omega=\sqrt{5/2}\omega_z$ and $2\omega_\perp$ for the low- and high-energy solutions, respectively. As anticipated in the Introduction, in the experiment of [4], carried out on a very asymmetric trap ($\omega_z/\omega_\perp=17/230$), it has been possible to measure at low temperature the frequency of the low-energy $m=0$ mode of even parity with very high precision [$\omega_{\rm expt}=1.569(4)\omega_z$] in excellent agreement with theory ($\sqrt{5/2}=1.58$). This agreement is not a surprise because the conditions of applicability of hydrodynamic theory are very well satisfied in this experiment. In fact the parameters of the trap ($a/a_{\rm HO}\sim10^{-3}$) and the number of atoms ($N\sim10^7$) are such that $\mu\sim30\hbar\omega_\perp\sim400\hbar\omega_z$ and hence the validity of the Thomas-Fermi approximation is well ensured. Furthermore, since the lowest frequencies are of order ω_z the condition $\hbar\omega\ll\mu$, relevant for the applicability of hydrodynamic theory, is also very well satisfied.

It is useful to derive the dispersion law also for the excitations with higher quantum numbers in axially symmetric cigar-shaped traps. The excitations we are interested in have frequencies of order ω_z, much smaller than $\omega_\perp$. Let us rewrite the linear equations (6) and (7) in the form

$$\omega^2\delta\rho=-\frac{1}{m}\nabla_\perp[(\mu-V_{\rm ext})\nabla_\perp\delta\rho]-\frac{1}{m}\nabla_z[(\mu-V_{\rm ext})\nabla_z\delta\rho]. \tag{10}$$

This equation shows that in the limit $\omega_z\ll\omega_\perp$ the low-energy solutions cannot have any dependence on the radial coordinates. This would in fact yield high frequency components of order $\omega_\perp$ in the solution, due to the first term in the right-hand side of Eq. (10). It is then natural to expand the $m=0$ solutions of Eq. (10) in the form

$$\delta\rho(\mathbf{r})=\delta\rho_0(z)+\lambda^2r_\perp^2\delta\rho_1(z)+\cdots, \tag{11}$$

where $\lambda=\omega_z/\omega_\perp$ is the deformation parameter of the trap. After inserting Eq. (11) into the hydrodynamic equation (10) and integrating over the radial coordinates we obtain, for $\lambda\to0$, the following differential equation for $\delta\rho_0$:

$$\omega^2\delta\rho_0(z)=-\tfrac{1}{4}\omega_z^2(Z^2-z^2)\nabla_z^2\delta\rho_0(z)+\omega_z^2z\nabla_z\delta\rho_0(z). \tag{12}$$

where $-Z\le z\le Z$. The discretized solutions of Eq. (12) are polynomials of the form $\delta\rho_0^{(k)}(z)=(z^k+\alpha z^{k-2}+\cdots)$, satisfying the orthogonality condition $\int_{-Z}^{+Z}dz(Z^2-z^2)\delta\rho_0^{(k)}(z)\delta\rho_0^{(k')}(z)=0$ for $k\neq k'$. They obey the dispersion relation

$$\omega^2=\tfrac{1}{4}k(k+3)\omega_z^2 \tag{13}$$

already derived in [8] using a different approach. The number of nodes of these solutions is equal to $k/2$ for even k, and to $(k+1)/2$ for odd k.

It is also interesting to look for solutions of Eq. (12) localized in the center of the trap ($z\sim0$). These are sound waves propagating with velocity

$$c_{\rm 1D}=\sqrt{\mu/2m}, \tag{14}$$

where we have used the identity $\mu=\frac{1}{2}m\omega_z^2Z^2$ for the chemical potential. Notice that in the Thomas-Fermi approximation the chemical potential is always related to the central density by the relation $\mu=g\rho_0(0)$ [see Eq. (4)], so that the sound velocity $c_{\rm 1D}$ is smaller by a factor $\sqrt{2}$ with respect to the Bogoliubov velocity calculated in the center of the trap. The occurrence of this factor was first pointed out in [13] and has a simple physical meaning. In fact, in deriving the relevant hydrodynamic equation (12), we have integrated Eq. (10) over the radial variables, so that the new sound velocity corresponds to an average whose value is smaller than the one in the center of the trap.

To better understand the propagation of sound waves in the case of highly elongated traps let us consider a trap with cylindrical geometry and harmonic confinement in the radial direction. The hydrodynamic equations in this case are simply obtained by setting $\omega_z=0$ in Eq. (3) and take the form

$$\omega^2\delta\rho=-\tfrac{1}{2}\omega_\perp^2\nabla_\perp[(R_\perp^2-r_\perp^2)\nabla_\perp\delta\rho]-\tfrac{1}{2}\omega_\perp^2(R_\perp^2-r_\perp^2)\nabla_z^2\delta\rho \tag{15}$$

defined in the interval $-L<z<L$, where $2L$ is the length of the cylinder, and $0<r_\perp<R_\perp$. It is worth noting that in the cylindrical geometry the validity of the Thomas-Fermi approximation for the ground state is guaranteed by the condition $\mu\gg\hbar\omega_\perp$ or, equivalently, $Na/L\gg1$ (in this case we always assume $L\gg R$). If we impose periodic boundary conditions at $z=\pm L$ the solutions of Eq. (15) can be written in the form $\delta\rho=[\delta\rho_0(z)+r_\perp^2\delta\rho_1(z)+\cdots]$ with $\delta\rho_0$ and $\delta\rho_1$ proportional to e^{iqz}. After integration in the radial variables Eq. (15) takes, to the lowest order in $q^2R_\perp^2$, the simplified form $\omega^2\delta\rho_0=-(\mu/2m)\nabla_z^2\delta\rho_0$, yielding the dispersion $\omega=c_{1\mathrm{D}}q$ with the sound velocity given by Eq. (14). The numerical solution of Eq. (15) with larger q has been carried out in [13]). It is not difficult to calculate the first correction to the linear behavior. One finds $\delta\rho_1(z)=q^2\delta\rho_0(z)/8$ and

$$\omega^2=c_{1D}^2q^2\left(1-\frac{1}{48}q^2R_\perp^2\right). \tag{16}$$

Result (16) explicitly reveals that the linear dispersion holds if the wavelength is much larger than the radial size of the condensate and that the sound has a negative dispersion [13].

Coming to the dynamic behavior in the presence of harmonic trapping one expects to observe wave packets propagating with the 1D sound velocity $c_{1\mathrm{D}}$ if the conditions $qZ\gg1$ and $qR_\perp\ll1$ are simulataneously satisified. Of course the condition $\hbar q\ll mc$ must be also satisfied because it ensures the applicability of hydrodynamic theory. The condition $qZ\gg1$ guarantees that the medium can be treated as locally uniform in the z direction and that one can consequently observe wave packets propagating in the central region of the trap. The condition $qR_\perp\ll1$ instead ensures that we are not exciting the motion in the radial direction and that the dispersion will look ''one dimensional'' and given by the first term of (16). In the experiment of [10] Z is a few hundred micrometers, $R_\perp=\lambda Z$ is a few tens of micrometers and $mc/\hbar\sim2-4$ $(\mu\mathrm{m})^{-1}$ depending on the value of the peak density. It is then possible that the wave packets observed in [10] are, at least partially, built up with wave vectors satisfying the above conditions. A detailed discussion of the propagation of wave packets, with the inclusion of nonlinearity effects, has been recently reported in [14].

Let us complete our discussion by calculating the first corrections to the dispersion relation (13). By solving the HD equations (15) to the next order in λ^2 we obtain, after some straightforward algebra, the result $\delta\rho_1(z)=-\frac{1}{8}\nabla_z^2\delta\rho_0(z)$ (notice the analogy with the result $\delta\rho_1=q^2\delta\rho_0/8$ holding in cylindrical geometry) and

$$\omega^2=\frac{1}{4}k(k+3)\omega_z^2\left(1-\frac{\lambda^2}{48}(k-1)(k+4)\right). \tag{17}$$

Some remarks are in order here. First one recovers the limit of the sound wave dispersion (16) holding in cylindrical geometry in the limit of large quantum numbers $k\gg1$, by the proper identification

$$c_{1\mathrm{D}}^2q^2=k^2\omega_z^2/4 \tag{18}$$

yielding $k^2=q^2Z^2$. This is consistent with the already discussed condition $qZ\gg1$ needed to observe phonons propagating in the zth direction. In the same limit the first corrections in the dispersion (16) and (17) coincide since one has $q^2R_\perp^2=k^2\lambda^2$. This completes the correspondence between the propagation of discretized modes and sound and shows the analogy between the dynamic behavior in the cylindrical and elongated harmonic oscillator geometries.

Concerning the frequencies of the discretized modes predicted by Eq. (17) it is worth pointing out that the lowest mode $(k=1)$ corresponds to the center-of-mass motion and its frequency coincides with the oscillator frequency ω_z. This frequency is unaffected by the presence of two-body interactions. The second mode $(k=2)$ is the "quadrupole" collective excitation observed in [4]. It corresponds to the low-energy solution (8) in the $\lambda\to0$ limit. The direct experimental observation of the higher modes, as well as of the first correction in λ^2 predicted by Eq. (17) would complete the scenario of the low-energy excitations in the elongated geometry.

It is finally interesting to discuss the one-dimensional nature of these systems. One should first point out that all the results discussed in this paper have been derived starting from 3D configurations. In particular, in order to derive the dispersion law (17), we have assumed the validity of the Thomas-Fermi approximation in both axial and radial directions, so that Eq. (17) holds if $\mu\gg\hbar\omega_\perp\gg\hbar\omega_z$. This means that the ground-state wave function of the system is built up including many excited single-particle states in both axial and radial directions. A full 1D problem should involve only the lowest oscillator wave function in the radial direction and in this case the corresponding excitation spectrum in the Thomas-Fermi regime would be [15] $\omega^2=\frac{1}{2}k(k+1)\omega_z^2$ instead of Eq. (17). This dispersion is easily derived from Eq. (10) ignoring the radial coordinates in the equation and holds if $\hbar\omega_\perp\gg(\mu-\hbar\omega_\perp)\gg\hbar\omega_z$. At present this regime is far from experimental possibilities. Nevertheless, even remaining in the 3D Thomas-Fermi regime, it is clear that for highly elongated configurations the low-energy dynamic behavior $(qR_\perp\ll1)$ looks one dimensional, the radial directions providing only a renormalization of the sound velocity. So all the statistical and thermodynamic properties of 1D systems should apply to these configurations provided the temperature is smaller than the radial oscillator energy. This includes in particular the Luttinger liquidlike behavior recently suggested for these systems [16] and the two-step Bose-Einstein condensation recently discussed in the context of the ideal Bose gas [17]. Furthermore, due to the coupling with the radial modes, the transition from the phonon to the single-particle regime exhibits new interesting features. In particular the first correction to the phonon dispersion is negative [see Eqs. (16) and (17)], differently from the traditional Bogoliubov behavior $\omega^2=c^2q^2+(q^2/2m)^2$.

In analogous way we can carry out the analysis in the disk geometry ($\omega_z \gg \omega_\perp$). In this case, to the lowest order in $1/\lambda^2$, the density fluctuations will depend only on the radial coordinates and, after integration of Eq. (10) in the variable z, the relevant equation for $\delta\rho(\mathbf{r}_\perp)$ takes the form

$$\omega^2\delta\rho(\mathbf{r}_\perp)=-\tfrac{1}{3}\omega_\perp^2(R_\perp^2-r_\perp^2)\nabla_\perp^2\delta\rho(\mathbf{r}_\perp)+\omega_\perp^2\mathbf{r}_\perp\nabla_\perp\delta\rho(\mathbf{r}_\perp). \quad (19)$$

Notice that in this case wave packets in the center of the trap will propagate with the 2D sound velocity

$$c_{2\mathrm{D}}=\sqrt{\frac{2}{3}\frac{\mu}{m}}. \quad (20)$$

The discretized solutions of Eq. (19) have the form

$$\delta\rho^{(n,m)}=(r_\perp^{2n}+\alpha r_\perp^{2n-2}+\cdots)r_\perp^m e^{im\phi} \quad (21)$$

where m is the zth component of angular momentum and n fixes the number of radial nodes. They satisfy the orthogonality condition $\int_{r_\perp\leq R_\perp}d\mathbf{r}_\perp(R_\perp^2-r_\perp^2)^{1/2}\delta\rho^{(n,m)}\delta\rho^{(n',m')}=0$ for $n,m\neq n',m'$. The resulting dispersion takes the form

$$\omega^2=(\tfrac{4}{3}n^2+\tfrac{4}{3}nm+2n+m)\omega_\perp^2. \quad (22)$$

The cases $n=0,m=2$ ($\omega=2\omega_\perp$), and $n=1,m=0$ ($\omega=\sqrt{10/3}\omega_\perp$) correspond to the modes of even parity observed in the experiments of [2] (the $m=0$ state corresponds to the low-energy solution (8) in the $\omega_\perp\ll\omega_z$ limit). One should, however, note that in this experiment the deformation of the trap and the number of atoms in the condensate were not very large ($\lambda=\sqrt{8}$ and $N\sim10^4$). As a consequence the conditions required to apply the dispersion law (22) ($\lambda\gg1$ and validity of the Thomas-Fermi approximation) are not very well satisfied in this case.

We finally note that also in deriving the dispersion law (22) we have assumed the validity of the Thomas-Fermi approximation in both axial and radial directions ($\mu\gg\hbar\omega_z\gg\hbar\omega_\perp$). The hydrodynamic dispersion in a true 2D trap would in fact follow a different dispersion law given by [15] $\omega^2=(2n^2+2nm+2n+m)\omega_\perp^2$. This 2D hydrodynamic dispersion law holds if the conditions $\hbar\omega_z\gg(\mu-\frac{1}{2}\hbar\omega_z)\gg\hbar\omega_\perp$ are satisfied.

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