Electron-atom scattering at small angles in a CO_2 laser field

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Electron-atom scattering at small scattering angles ($0°-25°$) in the presence of an intense radiation field is investigated employing second Born approximation. We perform exact numerical calculation of differential cross sections of electron-He scattering with one-, two-, and three-photon exchange and compare our results with the Kroll-Watson approximation (KWA) results and the recent experiments; our results are much better than KWA's for small angle scattering compared with experiments. [S1050-2947(98)09808-4]

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I. INTRODUCTION

The free-free absorption and emission of radiation in electron-atom collision have been studied for 60 years as an important process in understanding stellar atmospheres, laboratory discharges, and plasmas. With the availability of lasers it has become possible to make detailed studies of these differential cross sections (DCS), not only for single-photon exchanges, but also for multiphoton process. The major experimental effort in this work has been carried out by Weingartshofer, Wallbank, and co-workers over the past 20 years using a CO_2 laser for the scattering of a low energy (~ 10 eV) electron by inert gas atoms (mainly He and Ar). The theoretical treatments most widely applied to free-free transitions are based on the Kroll-Watson approximation (KWA) [1]. The well known KW formula is

$$\left(\frac{d\sigma}{d\Omega}\right)^{\nu}=\frac{K_f(\nu)}{K_i}J_{\nu}^2(\lambda_{fi})\left(\frac{d\sigma}{d\Omega}\right)^{\mathrm{el}}, \tag{1}$$

where $(d\sigma/d\Omega)^{\nu}$ denotes the elastic differential cross section for transferring ν photons ($\nu>0$ for absorption and $\nu<0$ for emission); $K_f(\nu)$ and K_i express the final and the initial wave number respectively, the relation of which is $\hbar^2K_f^2/2m=\hbar^2K_i^2/2m+\nu\hbar\omega$; $J_{\nu}(\lambda_{fi})$ is the ν order normal Bessel function, the argument $\lambda_{fi}=\vec{\alpha}_0\cdot(\vec{K}_f-\vec{K}_i)$, here $\vec{\alpha}_0=e\vec{E}_0/m\omega^2$ for a spatially homogeneous linear polarization laser field with electric field amplitude $\vec{E}_0$ and frequency ω; e and m are the charge and mass of the electron, respectively; $(d\sigma/d\Omega)^{\mathrm{el}}$ is the elastic differential cross section in the absence of a laser field.

The early experiments [2,3] were performed at large scattering angles and the results qualitatively agree with KWA. For small angle the argument of the Bessel function appearing in the KW formula is small, so according to the KW formula, the DCS should be small for small scattering angle. However, the more recent measurements performed by Wallbank and Holmes [4–6] for small angle scattering have obtained much greater DCS values than the expectations of KWA. This obvious disagreement has led to considerable theoretical research. Wallbank and Holmes [5] suggested that the effect of the polarization of an atom by the laser field might account for these large discrepancies, but the theoretical estimates performed by some other authors [7–9] have proved that the atomic polarization effect induced by a laser field is small enough to ignore; Varró and Ehlotzy [10] have presented a collective potential, which was a coherent superposition of the contributions of the individual laser-induced polarization potential of atoms in the laser beam, to explain the experimental results. However, Robicheaux [11] found that there was an error in the derivation of Varró and Ehlotzy and the collective potential could not explain these experimental results; Geltman [12] has carried out a detailed study of laser-assisted collisions using perturbation theory and found their results are in better agreement with experiments at small angle; Milošević and Ehlotzy [13] found that the double scattering effect could considerably improve the multiphoton transfer differential cross sections.

In this work we employ second Born approximation to calculate the differential cross sections of the laser-assisted electron-He scattering for one-, two-, and three-photon exchanges, and make comparisons with KWA and experiments. By the results of this research we intend to probe the relations of KWA and Born approximations and the reason for such large discrepancies between KWA and recent experiments.

In Sec. II of this paper we give a detailed derivation of the second Born approximation (SBA) for the electron scattering in a laser field and compare it with the Kroll-Watson theory and the first Born approximation (FBA); in Sec. III we present our results and discussion.

II. SECOND BORN APPROXIMATION

Consider an electron moving in the radiation field with a vector $\vec{A}(t)$ and scattered by the atom potential V; the time

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dependent Schrödinger equation is

$$i\hbar \frac{d}{dt}|\Phi(t)\rangle=\left(\frac{P^2}{2m}+V+H_{e-PT}(t)\right)|\Phi(t)\rangle \tag{2}$$

where $H_{e-PT}=-(e/mc)\vec{A}\cdot\vec{P}$ is the interaction of the incident electron with a radiation field; here the $A^2(t)$ term is removed by a travail contact transformation [4]. Initially (at time $t\rightarrow-\infty$) the electron is far from the atom, i.e., $V=0$, the solution to Eq. (2) in a monochromatic, linearly polarized field $\vec{A}=\vec{A}_0 \cos \omega t$ ($\vec{A}_0$ is the electric field vector) is just the Volkov solution [14]

$$|\chi_K(t)\rangle=\exp[-i(\vec{K}\cdot\vec{r}-\vec{K}\cdot\vec{\alpha}_0\sin\,\omega t-E_K t/\hbar)], \tag{3}$$

where $\vec{K}=\vec{P}/\hbar$ is the electron wave vector, and $E_K=\hbar^2K^2/2m$ is its energy. Using Green function representation

$$G(r,t;r',t')=-\frac{i}{(2\pi)^3\hbar}\int d\vec{K}\chi_K(r,t)\chi_K^*(r',t')u(t-t'). \tag{4}$$

Here $u(x)$ is the step function, and one can obtain the solution of Eq. (2) with electron momentum $\hbar\vec{K}_0$:

$$|\Phi_{K_0}(r,t)\rangle=|\chi_{K_0}(r,t)\rangle+\int dr'\int dt' G(r,t;r',t')V(r')\Phi_{K_0}(r',t'). \tag{5}$$

It is well known that the scattering matrix of the electron scattering by atom potential V with the transition from initial momentum $\hbar\vec{K}_i$ to the final one $\hbar\vec{K}_f$ is

$$T=\langle\chi_{K_f}|V|\Phi_{K_i}\rangle. \tag{6}$$

If the $\Phi_{K_0}(r',t')$ in the second term of Eq. (5) is replaced by $\chi_{K_0}(r',t')$, using Eqs. (3)–(6), we can obtain the scattering matrix $T_{fi}^{\langle 2\rangle}(\nu)$ with exchange of ν photons in second Born approximation:

$$T_{fi}^{\langle 2\rangle}(\nu)=T_{fi}^{(1)}(\nu)+T_{fi}^{(2)}(\nu), \tag{7}$$

where

$$T_{fi}^{(1)}(\nu)=J_\nu(\lambda_{fi})\tilde{V}(\vec{K}_{fi}), \tag{8}$$

$$\begin{aligned}T_{fi}^{(2)}(\nu)&=\int dt\int d\vec{r}\int dt'\int d\vec{r}'\chi_{K_f}^*(r,t)V(r)G_{\vec{K}_q}(r,t;r',t')V(r')\chi_{K_i}(r',t')\\ &=\frac{1}{2\pi}\int_{-\infty}^{\infty}dt\int_{-\infty}^{t}dt'\int d\vec{K}_q\exp\left[i\left(\frac{(E_f-E_q)t}{\hbar}-(\vec{K}_f-\vec{K}_q)\cdot\vec{\alpha}_0\sin(\omega t)\right)\right]\tilde{V}(\vec{K}_{fq})\\ &\quad\times\sum_n T_{qi}^{(1)}(n)\tilde{V}(\vec{K}_{qi})\exp\left[i\left(\frac{(E_q-E_i)t}{\hbar}-n\omega t'\right)\right].\end{aligned} \tag{9}$$

By integrating Eq. (9) for t and t' we finally obtain

$$\begin{aligned}T_{fi}^{(2)}(\nu)=&\frac{1}{(2\pi)^3}\sum_n\int_0^\infty dK_q\int_0^\pi d\theta\int_0^{2\pi}d\phi K_q^2\\ &\times\sin\theta[K_q^2-K_i^2-n\hbar\omega+i0^+]^{-1}\\ &\times T_{fq}^{(1)}(\nu-n)T_{qi}^{(1)}(n),\end{aligned} \tag{10}$$

where

$$T_{\alpha\beta}^{(1)}(n)=J_n(\lambda_{\alpha\beta})\tilde{V}(\vec{K}_{\alpha\beta}). \tag{11}$$

Here,

$$E_\alpha=\frac{\hbar^2K_\alpha^2}{2m},\quad \vec{K}_{\alpha\beta}=\vec{K}_\alpha-\vec{K}_\beta,\quad \lambda_{\alpha\beta}=(\vec{K}_\alpha-\vec{K}_\beta)\cdot\vec{\alpha}_0,\quad \alpha,\beta=i,q,f. \tag{12}$$

$\tilde{V}(\vec{K})$ denotes the Fourier transform of the atom potential. For simplification, we do not consider the polarization potential which describes the atom's polarization effect induced by the incident electron, but only consider the analytic screening electric static potential with the following form [15]:

$$V(r)=-\frac{Z}{r}\sum_{i=1}^{2}A_i\exp(-\alpha_i r), \tag{13}$$

where the parameters A_i and α_i are given by Salvat *et al.* [15], Z is the nuclear charge number, then

$$\tilde{V}(\vec{K})=-4\pi Z\sum_{i=1}^{2}\frac{A_i}{\alpha_i^2+|\vec{K}|^2}. \tag{14}$$

From Eqs. (9)–(11), one can see that $T_{qi}^{(1)}(n)$ represents the scattering amplitude of the electron transformation from initial momentum $\hbar\vec{K}_i$ to $\hbar\vec{K}_q$ at time t' absorbing n photons, $J_n(\lambda_{qi})$ is the amplitude for this absorbing process, then it reabsorbs $(\nu-n)$ photons with amplitude $J_{\nu-n}(\lambda_{fq})$ at time t, hence, the net result is that the electron absorbs ν photons. In the whole process, $\vec{K}_i\neq\vec{K}_q\neq\vec{K}_f$ because the electron is affected by photons and the potential V.

The ν-photon differential cross section can be easily obtained by the following formula:

$$\left(\frac{d\sigma}{d\Omega}\right)^{\nu}=\frac{K_f(\nu)}{K_i}\left(\frac{m}{2\pi\hbar^2}\right)^2|T_{fi}^{(2)}|^2. \qquad (15)$$

We can see that the first term of Eq. (7) is just the FBA amplitude, then the DCS obtained by FBA can expressed as

$$\left(\frac{d\sigma}{d\Omega}\right)^{\nu}=\frac{K_f(\nu)}{K_i}J_{\nu}^2(\lambda_{fi})\left(\frac{d\sigma}{d\Omega}\right)^{B}. \qquad (16)$$

Here $(d\sigma/d\Omega)^B$ is the differential cross section of field-free scattering by the first Born approximation. This formula was obtained by Bunkin and Fedorov [16].

One can see that the formula for the FBA differential cross section is the same as the KW formula except that the field-free DCS is the exact elastic DCS in the KW formula but the field-free-first-Born DCS in FBA. The dynamic part is expressed solely in terms of a differential cross section for elastic scattering readily attainable in a field-free situation, and the laser-assisted relative signal is plainly represented as a simple analytic function $J_{\nu}^2(\lambda_{fi})$ which only depends on the polarization of laser field and the momentum transfer of the incident electron, but is independent of the atom potential. However, we can clearly see from Eqs. (7)–(15) that the dynamic part includes the double scattering effect in SBA, and hence the laser-assisted relative signal depends upon not only the laser field, but also the atom potential. This feature is consistent with the perturbation theory [12].

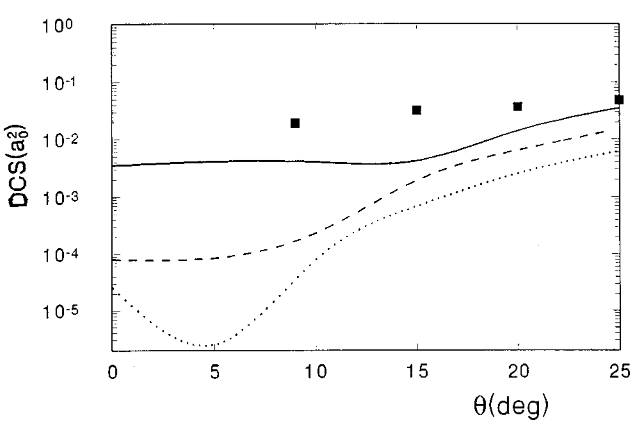

FIG. 1. Absolute differential cross section (DCS) for scattering from helium as a function of the scattering angle θ for single-photon absorption. The laser field frequency is $\omega=0.117$ eV and its intensity is $I=1.5\times10^8$ W cm^{-2}; the incident electron energy is $E_i=10.5$ eV and the laser electric field vector is taken to be parallel to the incident direction. Solid line, SBA result; dashed line, KWA result; dotted line, FBA result. Solid box, experiment, Ref. [6].

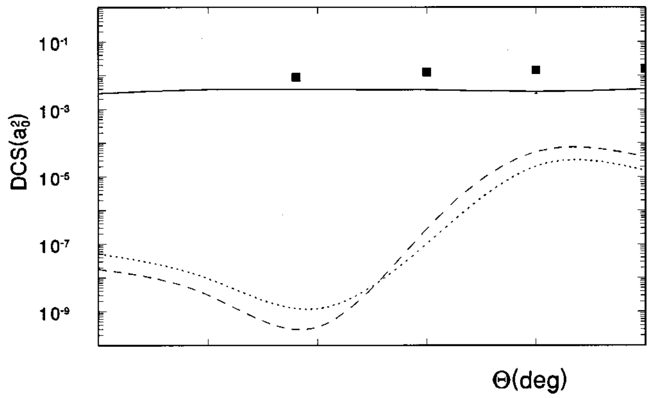

FIG. 2. Same as Fig. 1 but for two-photon absorption.

III. RESULTS AND DISCUSSION

The main difficulty arising in evaluating the second order term $T_{fi}^{(2)}(\nu)$ is the presence of the intermediate wave vector $\vec{K}_q$ in the argument of the Bessel function. In order to reduce the difficulty of calculating $T_{fi}^{(2)}(\nu)$, we choose the geometry in which the polarization vector is parallel to the direction of the incident electron $(\vec{E}_0\|\vec{K}_i)$, thus $T_{fi}^{(2)}(\nu)$ can be easily evaluated by numerical integration.

We have calculated the differential cross sections of the electron-He scattering for the incident energy 10.5 eV at small angles $(0°-25°)$ with one-, two-, and three-photon exchange in the laser field with intensity 1.5×10^8 W/cm^2 and frequency $\omega=0.117$ eV by SBA. For comparisons, we have also calculated the differential cross sections using KWA and FBA at the same condition. The field-free elastic differential cross sections in the KW formula are obtained using our previous optical method [17]. These results along with the experimental results obtained by Wallbank and Holmes [6] are shown in Figs. 1–3. One can see that the SBA results are apparently better than KWA and FBA compared with experiments. The KWA and FBA results are several orders of magnitude lower than experiments; the reason is that the argument of the Bessel function is very small in the small angle scattering situation. While the second amplitude in SBA is integrated for polar angles after the first collision [Eq. (10)], the arguments of the Bessel function which appeared under the integral can be large even for small scattering angle, so the second order amplitude is larger and

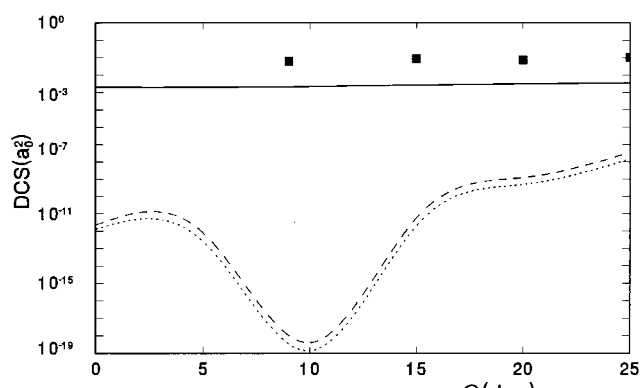

FIG. 3. Same as Fig. 1 but for three-photon absorption.

becomes the main term in this situation. That can be seen clearly from the comparison between SBA and FBA in Figs. 1–3. We can also see from the figures that with increase of scattering angle beyond 10°, the DCS's of KWA increase gradually to approach the experimental data, and the discrepancies between SBA and KWA are decreasing. This indicates that KWA is a convenient and fairly proper method to predict the DCS at larger scattering angle, but for small angle, it is too poor.

Moreover, from Figs. 1–3, one can see that the results of FBA almost agree with that of KWA. The reason is as follows. The physical basis of the derivation of the KW formula arises from an S-matrix expansion for the scattering using Volkov basis states (free electron in the laser field). If only the first term was retained, the result is just the FBA. Kroll and Watson have shown that the entire Born series can be formally summed under the low-frequency-approximation condition, which neglected the intermediate process of photon exchange and only the direct exchange of photons was considered. From this point of view, KWA coincides with FBA. On the other hand, the exact elastic scattering cross section appears on the right hand side of Eq. (1) due to the sum of the entire Born series under the low-frequency-approximation condition, and so there exist differences between KWA and FBA.

From Figs. 1–3, we also find that discrepancies between SBA results and experiments still exist; our results are a little less than experiments. Along with the increase of the number exchange photon, the discrepancies become larger. We think that the reasons accounting for these discrepancies are the following. (I) The experiments [6] still contain some defaults. Madsen and Taulbjerg [18] claim that the collimation of the incident electron beam is rather poor in experiments and the acceptance angle of scattered electrons is very wide. However, Wallbank and Holmes have not specified the corresponding uncertainties in the angular definitions except that the angular resolution of the electron detector is 2°. There are, however, other important contributions to the angular uncertainties. Especially, it appears that the extent of the collision region might proved other few degrees of uncertainty in the definition of the exit angle. The divergence of the incident beam cannot be ignored either. In early experiments with a similar piece of apparatus, Weingartshofer *et al.* [3] profess that the total uncertainty of the experimental scattering angle is 8°. All these factors lead to the fact that the experimental results are large. (II) In this work, some factors are not considered. (1) It is well known that the atom polarization effect induced by the incident electron is very important in field-free scattering. Naturally, this effect cannot be ignored in laser-assisted collision either, however, the screening electric static potential used in the present work [Eq. (13)] does not include this effect. (2) We only considered the homogeneous, linearly polarized laser field, however, this ideal laser field does not exist in fact. So, we have not mimicked the experimental conditions completely. Though so many factors have not been considered, the present SBA has obtained encouraging results and greatly improved the DCS of small angle scattering.

In summary, we obtain results which reasonably agree with experiments [6]; this indicates that the experiments can be explained by second order Born approximation, which is consistent with the double scattering results [13].

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