Threshold and nonlinear behavior of lasers of Λ and V configurations

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Dynamic properties of closed three-level laser systems are investigated. Two schemes of pumping—Λ and V—are considered. It is shown that the nonlinear behavior of the photon number as a function of pump both near and far above threshold is crucially different for these two configurations. In particular, it is found that in the high pump regime laser can turn off in a phase-transition-like manner in both Λ and V schemes. [S1050-2947(98)05608-X]

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The interest in the dynamic behavior of lasers and its analogy to phase transition phenomenon have been renewed over recent years mainly due to experimental realization of microlasers [1]. The reason for this is a possibility of lowering of the threshold pump needed to start the lasing process. The value of the threshold pump is mostly determined by the fraction β of spontaneously emitted photons directed into the lasing mode [2]. It has been shown that in the limit when all spontaneous photons are emitted into the lasing mode (which corresponds to the spontaneous emission factor $\beta=1$), the laser becomes a ''thresholdless''device [3]. Such lasers are also referred to as cavity-QED lasers.

Mu and Savage [4] pointed out that when the pumping excites the lower lasing level the number of photons can decrease with pump, and the laser can even turn off at strong enough pump rate. Recently we have shown that the type of nonlinear dependence of the photon number upon the pump rate is very sensitive to the type of pumping used in a laser [5]. Figure 1 illustrates the difference between the two types of pumping on the example of a three-level atom. In the scheme shown in Fig. 1(a) an atom upon emitting a photon decays from the lower lasing state $|0\rangle$ to the ground state $|2\rangle$ with the decay rate γ_{02}, afterwards the pumping excites the atom to the upper lasing state $|1\rangle$ with the rate γ_{21}. Thus in this case the rate γ_{21} plays the role of the pump rate. In the scheme shown in Fig. 1(b) on the other hand, the pumping first excites the atom, with the rate γ_{02} from the lower lasing state $|0\rangle$, which is the ground state in this case, to the excited state $|2\rangle$, which is then depleted with the decay rate γ_{21} to the upper lasing state $|1\rangle$. Now the role of the pump rate is played by the rate γ_{02}. We refer to the schemes depicted in Figs. 1(a) and 1(b) as Λ and V configurations, respectively. One has to differentiate, however, between these Λ and V schemes and those discussed in the literature on lasing without inversion [6]. The main feature of our V-type scheme is that the pumping is used to excite the lower lasing state $|0\rangle$ directly. It is this property of the V-type schemes that makes their dynamic behavior crucially different from that of the Λ-type schemes.

In the present paper we study the dynamic behavior of the lasers with Λ and V types of pumping using modified Maxwell-Bloch equations which include a term describing spontaneous emission into the lasing mode. We follow a gradual transition from the thresholdlike to thresholdless behavior of a Λ-type scheme. We also show that if the saturation parameter (or cooperativity parameter) is larger than some critical value the lasing is not possible altogether. This implies a restriction to the spontaneous emission factor β, which has a minimal value. The V-type scheme always has a threshold, however, the pump parameter is restricted by its maximal value, which is a critical point as at this point the lasing is broken down in a phase-transition-like manner. For such a scheme the factor β is not a parameter because of its dependence on the pump rate.

We start with the set of modified Maxwell-Bloch equations, which describe both Λ- and V-type schemes shown in Fig. 1. These equations are derived by writing down an exact master equation for the atoms+field density matrix in divergent form and omitting the fluctuation terms:

$$\dot{n}=-2\kappa n+Ngx, \tag{1}$$

$$\dot{x}=-\gamma_{\perp}x+2g[(n+1)\rho_{11}-n\rho_{00}], \tag{2}$$

$$\dot{\rho}_{11}=-\gamma_{10}\rho_{11}+\gamma_{21}\rho_{22}-gx, \tag{3}$$

$$\dot{\rho}_{00}=-\gamma_{02}\rho_{00}+\gamma_{10}\rho_{11}+gx, \tag{4}$$

$$\rho_{00}+\rho_{11}+\rho_{22}=1, \tag{5}$$

with $x\equiv iz^*\rho_{10}+\text{c.c.}$, N is the total number of atoms, g is the coupling constant, κ is the cavity decay rate, $n=z^*z$ is the number of photons in the cavity. Equations (1)–(5) differ from the standard Maxwell-Bloch equations by the factor $n+1$ instead of n in Eq. (2), which takes account for spontaneous emission into the lasing mode. Such a factor occasionally appears in rate equations [7]; however, Eqs. (1)–(5) provide a more complete description as their validity is not restricted by the approximation $\gamma_{\perp}\gg\gamma_{ij}$ used to derive the rate equations. It follows from the derivation of the relaxation rates [8] that the transversal relaxation rate is related to the other rates by the following important formula:

$$\gamma_{\perp}=\tfrac{1}{2}(\gamma_{10}+\gamma_{02}+\gamma_{\rm col}) \tag{6}$$

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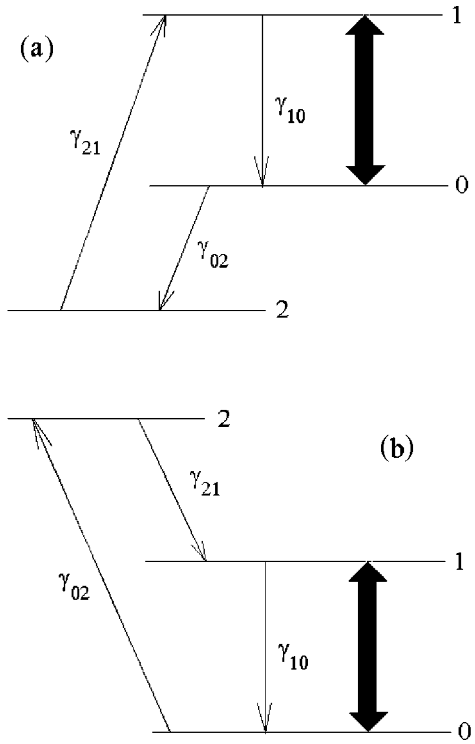

FIG. 1. Two three-level schemes of pumping: (a) Λ and (b) V configurations.

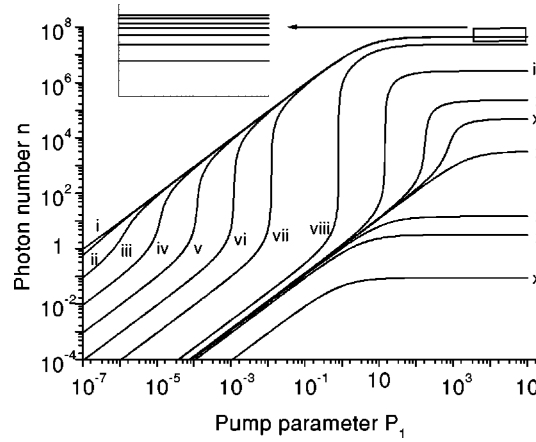

FIG. 2. Photon number as a function of the pump parameter for the Λ scheme. $\lambda=10^7$, $\alpha_2=10$, $\eta=0$, $S=0$ (i), 0.1 (ii), 1 (iii), 10 (iv), 10^2 (v), 10^3 (vi), 10^4 (vii), 3×10^5 (viii), 7×10^5 (ix), 7.4×10^5 (x), 7.43×10^5 (xi), 7.44×10^5 (xii), 8×10^5 (xiii), 10^6 (xiv), 10^7 (xv). Curve xii separates regimes with and without lasing.

with $\gamma_{\rm col}$ being the rate of collisional dephasing. This brings about additional dependence of the photon number upon the rate γ_{02}, which in the case of V configuration plays a role of a pump rate. The steady state solution for the number of photons n is

$$n=\tfrac{1}{2}(b+\sqrt{b^2+4c}) \tag{7}$$

with

$$b=\frac{\lambda(\alpha_2-1)-S(\alpha_2+1+\eta)(\alpha_2\alpha_1+\alpha_2+\alpha_1)-\alpha_1-\alpha_2}{\alpha_2+2\alpha_1},$$

$$c=\frac{\lambda\alpha_1\alpha_2}{\alpha_2+2\alpha_1}, \tag{8}$$

$$\lambda\equiv\frac{N\gamma_{10}}{2\kappa},\quad S\equiv\frac{\gamma_{10}^2}{4g^2},\quad \alpha_1\equiv\frac{\gamma_{21}}{\gamma_{10}},\quad \alpha_2\equiv\frac{\gamma_{02}}{\gamma_{10}},\quad \eta\equiv\frac{\gamma_{\rm col}}{\gamma_{10}}. \tag{9}$$

Consider first the Λ configuration [Fig. 1(a)]. In this case the role of the pump parameter is played by α_1, so $P_1=\alpha_1$. In Fig. 2 the number of photons is plotted as a function of the pump parameter P_1 for various values of the saturation parameter S. When S is not too large and the pump parameter is small enough ($P_1<1$) one can observe (i) threshold kinks, which tend to disappear as S is approaching zero and (ii) linear dependence of the photon number on the pump above threshold. In this regime our results are similar to those obtained by other authors [3,9,10]. However, the picture changes in two ways when either P_1 or S becomes large enough. The photon number saturates at large values of the pump, which is not surprising since when the pumping excites the groung state $|2\rangle$ very fast there is a bottleneck at the $|0\rangle-|2\rangle$ transition, which prevents further growth of the photon number. This kind of nonlinearity (saturation) was discussed by Hart and Kennedy [11] under similar conditions. Note that the curves for different values of the saturation parameter S have different saturation limits at $P_1\rightarrow\infty$. Another change occurs when the saturation parameter S approaches some critical value $S_{\rm cr}$ (7.44×10^5 for the set of parameters used in Fig. 2). The kinks become less defined with increasing S and at the same time the saturation photon number decreases drastically. At $S=S_{\rm cr}$ (curve xii in Fig. 2) the kinks disappear and further increasing of S results in a no lasing regime.

Consider a threshold condition for the Λ scheme that can be derived from Eqs. (1)–(5) with $n+1$ replaced by n in Eq. (2), i.e., from the standard Maxwell-Bloch equations, as a condition for the existence of a positive steady state solution for the photon number. Then the threshold condition reads

$$P_1>P_{1\rm thr}=\frac{\alpha_2 S(\alpha_2+1+\eta)}{\lambda(\alpha_2-1)-S(\alpha_2+1)(\alpha_2+1+\eta)}. \tag{10}$$

One can make two major observations from Eq. (10). The first one is the well-known fact that the threshold pump decreases with S and in the limit $S\rightarrow0$, when the kinks in the curves disappear, $P_{1\rm thr}\rightarrow0$, i.e., the laser becomes thresholdless [3,9,10]. Secondly, the saturation parameter S is restricted by its maximal value

$$S<S_{\max}=\frac{\lambda(\alpha_2-1)}{(\alpha_2+1)(\alpha_2+1+\eta)}, \tag{11}$$

which for the set of parameters used in Fig. 2 corresponds exactly to the curve xii. Therefore the curves xii–xv describe the no-lasing regime. In this sense the value $S_{\max}$ is a critical one as it separates the regimes with and without lasing. The physical meaning of inequality (11) becomes apparent if we rewrite it in the following equivalent form (provided $\gamma_{02}\gg\gamma_{10}$): $\gamma_{02}+\gamma_{\rm col}<2Ng^2/\kappa$, which, together with Eq. (6), implies a restriction on the time T_2.

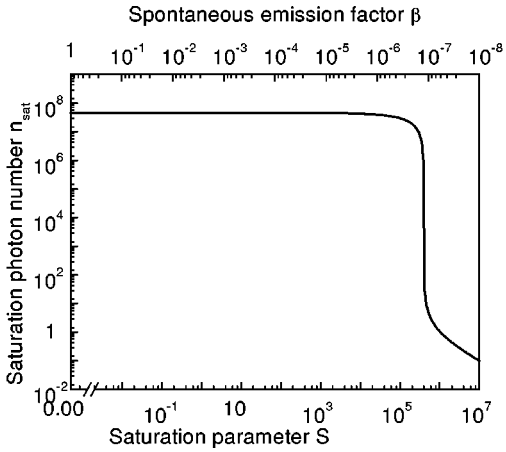

FIG. 3. Saturation photon number as a function of saturation parameter S and spontaneous emission factor β. The parameters are the same as in Fig. 2. If β factor is less than the critical value 10^{-7}, the saturation number of photons jumps down to $n_{sat}<1$ and hence the laser turns off.

Another interpretation can be obtained if we relate the parameters defined in Eq. (9) with the spontaneous emission factor β, which results in the following:

$$\beta=\frac{1}{1+S(1+\alpha_2+\eta)}, \quad \beta_{\min}=\frac{1}{1+\lambda(\alpha_2-1)/(\alpha_2+1)}. \tag{12}$$

Thus the limit $S\to 0$ corresponds to an ideal QED laser ($\beta=1$) in which case all spontaneous photons are directed into the lasing mode. Note that there exists a minimal value of β, which means that the laser cannot operate if the portion of spontaneously emitted photons directed into the lasing mode is less than $\beta_{\min}$. If α_2 and $\lambda\gg 1$, $\beta_{\min}=1/\lambda$. The physical meaning of the critical point $\beta=\beta_{\min}$ is illustrated in Fig. 3 where the saturation photon number n_{sat} (which is calculated as a limit of the photon number n at $P_1\to\infty$) is plotted as a function of β (S) parameter.

Now consider the V scheme shown in Fig. 1(b). In this case the pumping excites the lower lasing state $|0\rangle$, which is also the ground state and so the rate γ_{02} plays a role of the pump rate. Therefore it is reasonable to introduce the pump parameter $P_2=\alpha_2$. The dependence of the photon number n upon this pump parameter is essentially different from that of the Λ scheme due to relation between the transversal relaxation rate $\gamma_\perp$ and the pump parameter P_2 following from Eq. (6) and the definition of the pamp parameter P_2, namely,

$$\gamma_\perp=\frac{\gamma_{10}}{2}(1+P_2+\eta), \tag{13}$$

which brings about additional dependence on the pump parameter. This difference is clearly seen in Fig. 4 where the photon number n as well as the inversion $D=\rho_{11}-\rho_{00}$ are plotted as functions of the pump parameter P_2 for various values of the saturation parameter S. Now as the pump parameter increases the photon number grows to some critical value, at which it begins to decrease. The curves $n(P_2)$ have two kinks; the first kink corresponds to the threshold point whereas the second one corresponds to the break point at

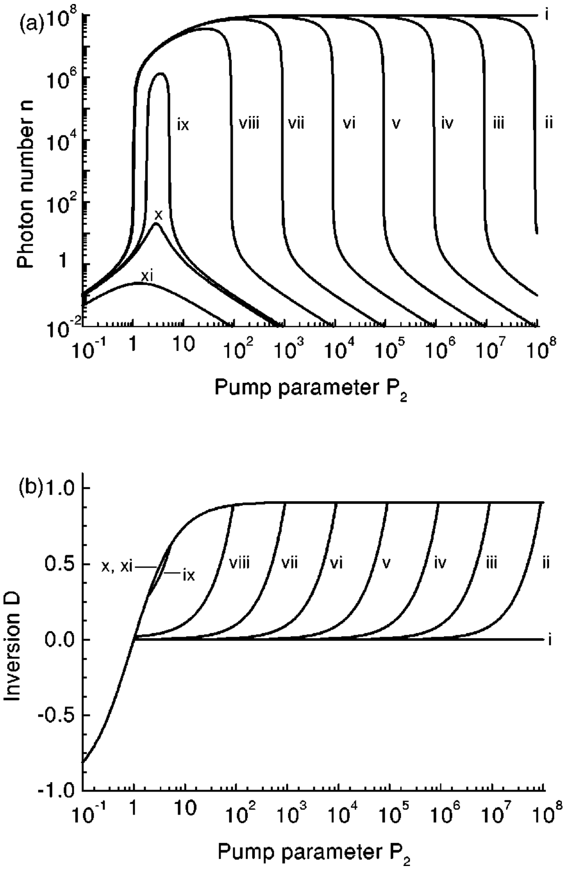

FIG. 4. Photon number (a) and inversion (b) as functions of the pump parameter P_2 for the V scheme. $\lambda=10^7$, $\alpha_2=10$, $\eta=0$, $S=0$ (i), 0.1 (ii), 1 (iii), 10 (iv), 10^2 (v), 10^3 (vi), 10^4 (vii), 10^5 (viii), 10^6 (ix), 1.25×10^6 (x), 10^7 (xi).

which lasing ceases. Note that in contrast to the Λ scheme (i) the values of the threshold pump for all curves in Fig. 4(a) are close to 1 and (ii) the kinks do not disappear at $S\to 0$ ($\beta\to 1$), i.e., the threshold exists even in an ideal QED laser.

The two critical points can be obtained from semiclassical steady-state equations by solving the inequality $n>0$, as has been done for the Λ scheme, which results in the following:

$$P_{2\mathrm{thr}}<P_2<P_{2\max} \tag{14}$$

with

$$P_{2\mathrm{thr}}=1+\frac{S}{\lambda}\frac{(1+2\alpha_1)(2+\eta)}{\alpha_1}, \tag{15}$$

$$P_{2\max}=\frac{\lambda}{S}\frac{\alpha_1}{1+\alpha_1}. \tag{16}$$

Approximate equations (15) and (16), obtained by expansion of exact expressions for $P_{2\mathrm{thr}}$ and $P_{2\max}$ at $S\ll\lambda$, work pretty well for any reasonable values of the involved parameters. The presence of the second critical point $P_{2\max}$ is caused by the dependence of the transversal relaxation rate $\gamma_\perp$ on the pump parameter P_2 [see Eq. (13)]. In fact, Eq. (16) gives rise to the same restriction for the depletion rate γ_{02} as in the case of the Λ scheme, this time, however, this is

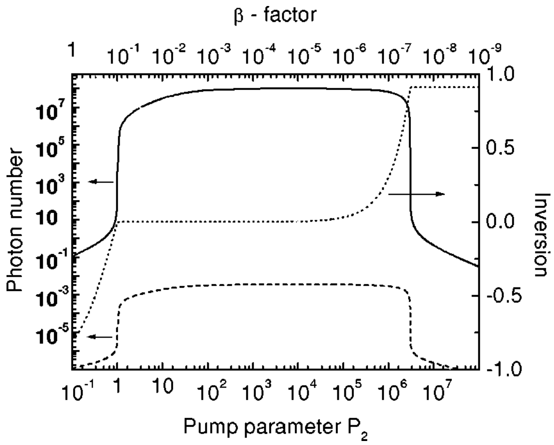

FIG. 5. Photon number (solid line), polarization (dashed line), and inversion (dotted line) as functions of the pump parameter P_2 and β factor for the V scheme.

the restriction on the pump rate. This causes a breaking down of lasing at the point $P_2 = P_{2\max}$.

Another interesting feature of the *V* scheme is that the spontaneous emission factor β is not a parameter as it depends upon the pump parameter $P_2 = \alpha_2$ [see Eq. (12)]. Therefore increasing of the pump results in decreasing of β, which is restricted by its minimal value. In this sense the breaking point $P_2 = P_{2\max}$ in the V scheme and the point $S = S_{\max}$ in the Λ scheme have the same physical origin—both stem from the restriction on T_2. Figure 5 summarizes the results for the V scheme by plotting the photon number, the polarization, and the inversion as a function of both the pump parameter P_2 and the spontaneous emission factor β in one plot. One sees what happens at $P_2 = P_{2\max}$: the inversion saturates to 1, i.e., all atoms occupy the upper lasing level, the coherence between the lasing levels is destroyed, therefore the number of photons slows down and the lasing ceases altogether. Further increase of P_2 does not affect the picture since the medium became practically transparent.

Finally, a comparison of threshold pump for the Λ and V schemes indicates different dependence on the saturation parameter S. In the Λ scheme the threshold pump $P_{1\mathrm{thr}}$ increases by several orders of magnitude as S changes. In contrast, in the V scheme the threshold pump $P_{2\mathrm{thr}}$ changes within one order of magnitude for the same range of changes of S. This difference can be seen by comparing Figs. 2 and 4(a) for the Λ and V schemes, respectively.

In summary, we have shown that the dynamic behavior of lasers of Λ and V schemes of pumping is essentially different at both low and very high pump rates. In the case of Λ configuration the photon number saturates with pumping. The threshold pump of Λ scheme is determined by the spontaneous emission factor β. If β is less than some minimal value the laser operates in a ''quiet'' regime when no lasing occurs.

In the V case the number of photons increases with pump up to some maximal value, then it begins to decrease. At the critical value of the pump rate the laser turns off because of destruction of the coherence between the lasing levels, which in turn determines the atomic lifetime. In this case the β factor is not a fixed parameter, but depends upon the pump rate.

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