Emission spectrum in driven two-level systems

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We study the dynamic behavior of an electron in a two-level system driven by dc-ac electric fields. Analytical solutions for the dipole moment and its emission spectrum are obtained under the approximation of the high-frequency driving case, from which the physical property of the system is found to be controlled by the field parameters. A number of time-periodic and quasiperiodic phenomena as well as their signature in the emission spectrum are revealed. By making the proper choice of the field parameters, it is possible to selectively eliminate some components in the spectrum. The analytical findings are confirmed numerically. [S1050-2947(98)01108-1]

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I. INTRODUCTION

In recent years the study of the dynamic effect of electrons in double quantum wells or two-level systems subject to time-dependent electric or/and magnetic fields has attracted increasing attention [1–13]. The understanding of the nature of quantum behavior of such systems is not only of fundamental importance but also provides a strong background of experimental relevance in fields as diverse as Rabi oscillations in the time domain [14] Landau-Zener transitions and Autler-Townes doublets in optical rings [15], and other contexts. Of particular interest is the case of electrons under the influence of time-periodic electric fields, e.g., a laser field or a dc-ac field. Such systems are found to be equivalent to the previous studies of an atom with spin-$\frac{1}{2}$ subject to the simultaneous action of a static magnetic field and an oscillating rf field [11–13], where many unusual, fascinating results have already been obtained. Of special interest is the tunneling effect, in which, dynamic localization and delocalization are involved. This phenomenon was reexamined in recent studies of emission properties of an electron in a double well or/and a two-level systems [9,16]. There, the signature of the localization condition in the emission spectrum for some special situation was revealed. However, a systematical study of the emission spectrum in the full field parameter space is absent.

In this paper, we report our findings on this problem for a two-level system driven by a dc-ac electric field. We find that in the case of high-frequency driving, this problem can be solved analytically. As the result, we obtain explicit expressions for the dipole moment and its emission spectrum in the full field parameter space. The time-periodic and quasiperiodic evolution behavior as well as the dynamic localization and delocalization for the system is manifested from our analytical results, which are in good agreement with those of Refs. [11–13]. To check the validity of our theory, we also provide numerical calculations. The analytical and numerical results compare very well.

The rest of this paper is set out as follows. In Sec. II, we derive a kinetic equation for the dipole moment in the case of high-frequency driving dc-ac fields. This will establish a sound foundation for further analysis, which we will address in Secs. III and IV. Section V contains the discussions and conclusions.

II. KINETIC EQUATION OF THE DIPOLE MOMENT IN THE HIGH-FREQUENCY DRIVING CASE

The Hamiltonian we consider here can be written as

$$H = -V(t)\sigma_x + \hbar\Delta\sigma_z . \tag{2.1}$$

Here Δ is the splitting parameter, $V(t)$ is the driving force:

$$V(t) = \mu E_0 + \mu E \cos\omega t, \tag{2.2}$$

where μ is the transition dipole between two levels, E_0 is a constant field, and E and ω are, respectively, the amplitude and the frequency of the driving laser field. Hereafter we set the unit $\hbar = 1$.

Equations (2.1) and (2.2) are formally equivalent to those for an atom with spin-$\frac{1}{2}$ subject to a simultaneous action of a static magnetic field and an oscillating rf field [11], which has been studied in great detail and many different results have been obtained [12,13]. With the usual Floquet theorem, the problem can be solved generally by the use of Shirley's well-known result [17]. However, in the following, we deal with this problem in terms of the kinetic equation of the time-dependent dipole moment $\mu(t)$, which is defined as

$$\mu(t) \equiv \langle \psi | \sigma_x | \psi \rangle . \tag{2.3}$$

In the case of high-frequency driving (i.e., $\epsilon \equiv \Delta/\omega \ll 1$), the time-dependent dipole moment satisfies the integrodifferential equation [9,13]

$$d\mu(\tau)/d\tau = -\epsilon^2 \,\mathrm{Re}\int_0^\tau d\tau_1 J_0(2a\sin[(\tau-\tau_1)/2]) \times \exp[ib(\tau-\tau_1)]\mu(\tau_1), \tag{2.4}$$

with the initial condition $\mu(0)=1$. Here J is the ordinary Bessel function. Note that in Eq. (2.4) we have made use of the following substitutions,

$$\tau=\omega t, \quad \mu(\tau)\rightarrow\mu(\tau)/\mu(0);$$

and the following definitions,

$$a\equiv 2\mu E/\omega, \quad b\equiv 2\mu E_0/\omega, \quad \epsilon\equiv\Delta/\omega.$$

Equation (2.4) can be solved by using the Laplace transform, the solution of the dipole moment is

$$\mu(\lambda)\equiv\int_0^\infty dt\, e^{-\lambda t}\mu(t)=\frac{1}{\lambda+\Delta^2 K(\lambda)}, \tag{2.5}$$

where

$$K(\lambda)\equiv\int_0^\infty dt\, e^{-\lambda t}J_0(2a\sin(\omega t/2))\cos[b\omega t]$$
$$=\sum_{m=0}^{\infty}\frac{\lambda J_m{}^2(a)}{\lambda^2+[(m+b)\omega]^2}+\sum_{m=1}^{\infty}\frac{\lambda J_m{}^2(a)}{\lambda^2+[(m-b)\omega]^2}. \tag{2.6}$$

It is obvious that one can obtain the time-dependent dipole moment by using Eqs. (2.5) and (2.6), and the inverse Laplace transform. However, since we are mainly concerned with the long-time evolution behavior of the system that is determined by the property of $K(\lambda)$ at small λ, we will focus on this special situation below.

The other observation on Eq. (2.6) is that the emission spectrum of the system can be classified by the number-theoretical property of the field parameter $b=2\mu E_0/\omega$. The case of b being an integer has already been presented in Ref. [16], where the signature of the localization condition for the system in the emission spectrum was manifested. A similar situation for the problem of magnetic resonance can be found in the literature [11–13], where the effect of localization was revealed in the phenomenon of level crossing. However, to provide a complete description of the problem, we will still give a brief review for this case in the following section. Then, we report our results for the general case of b being an arbitrary real number in Sec. IV.

III. THE CASE OF $2\mu E_0/\omega=N$

From Eq. (2.6) we can see that, when $b=N$, the behavior of $K(\lambda)$ is dominated by the term $m=N$ ($N=1,2,3,\dots$) in the second summation, if $J_N(a)\neq 0$. However, if $J_N(a)=0$, all terms in the summations of $K(\lambda)$ are of same order. This means that the behavior of $K(\lambda)$ in the case of $J_N(a)=0$ is different from the case of $J_N(a)\neq 0$. Therefore, in the following discussions, we present these two situations separately.

a. The case of $J_N(a)\neq 0$. In this case, considering that λ is small and using inverse Laplace transform one gets

$$\mu(\tau)=\cos(\Omega_N\tau)-\epsilon\sum_{k=1}^{\infty}(-1)^k C_{N,k}\{\cos[(k-\Omega_N)\tau]$$
$$-\cos[(k+\Omega_N)\tau]\}, \tag{3.1}$$

where

$$\Omega_N=\epsilon J_N(a), \tag{3.2}$$

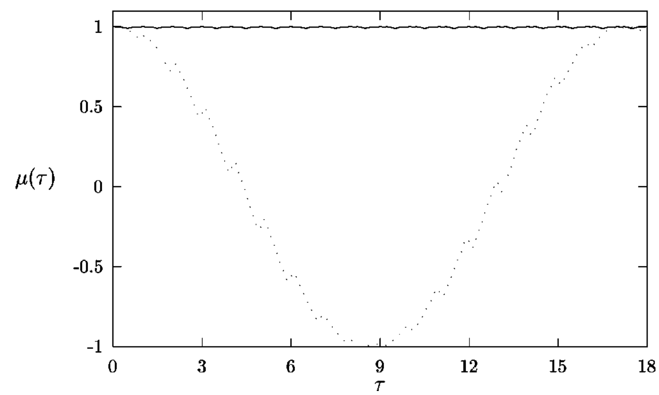

FIG. 1. Induced dipole $\mu(\tau)$ as a function of the scaled time τ (in 2π units). The solid line shows $\mu(\tau)$ for $a=2\mu E/\omega=3.83$ and $b=2\mu E_0/\omega=1$, while the dots line shows $\mu(\tau)$ for $a=2.0$ and $b=1$, respectively. Here $\epsilon=0.05$.

$$C_{N,k}=[J_{N+k}(a)+J_{N-k}(a)]/2k. \tag{3.3}$$

From Eq. (3.1) we can see that the Fourier transform $\mu(\Omega)$ has a peak at the frequency Ω_N (in this paper, Ω_N, Ω, and k are all in ω units) which was stemmed from the first term on the right-hand of Eq. (3.1). Here Ω_N is small in the high-frequency driving case (i.e., $\epsilon\equiv\Delta/\omega\ll 1$), it corresponds to the low-frequency generation (LFG). The other terms on the right-hand side of Eq. (3.1) represent the high-frequency parts of the spectrum. Note that the intensity of the LFG peak is very high, as compared to the other peaks, since the transition dipole $\mu(\tau)$ is dominated by the first term, $\cos(\Omega_N\tau)$, under the approximation of $\epsilon\ll 1$. In the extreme low-frequency limit, $\Omega_N\rightarrow 0$, the induced dipole $\mu(\tau)$ will approach unity, $\mu(\tau)\rightarrow 1$, meaning that the localization occurs. Otherwise $\mu(\tau)$ will oscillate between 1 and -1. This feature is confirmed by our numerical calculation depicted in Fig. 1, where the dipole $\mu(\tau)$ is plotted as a function of the scaled time τ (in 2π units). In the figure, we have taken $\epsilon=0.05$. The solid line in Fig. 1 shows an example where we let the parameters $a=3.83$ and $b=1$ so that $\Omega_N=\epsilon J_1(3.83)=0$ which corresponds to the localization condition [8]. It is clearly seen in this case $\mu(\tau)$ is almost equal to a constant which is close to one at all times, plus a term oscillating with small amplitude at a high frequency. This coincides with the findings of Ref. [8]. The emission properties of this case shall be discussed in detail in the following part. The dotted line in Fig. 1 illustrates the situation of $a=2.0$ and $b=1$ that results in the value of Ω_N given by Eq. (3.2) being finitely small. The time evolution of $\mu(\tau)$ shows a large-amplitude–low-frequency component and a high-frequency–low-amplitude behavior, as predicted by the theory [Eqs. (3.1) and (3.3)].

Equations (3.1)–(3.3) need more comments. First of all, the spectrum $\mu(\Omega)$ consists of a number of doublets, $\Omega=(k+\Omega_N)$ and $\Omega=(k-\Omega_N)$; here k includes both even and odd harmonics. This is different from the pure ac field driving case where the spectrum consists of doublets at even harmonics with vanishing amplitudes at odd ones [9]. Figure 2(a) shows an example for these doublets, $\Omega=(k+\Omega_N)$ and $\Omega=(k-\Omega_N)$, as well as the low-frequency component Ω_N. This plot is generated by the Fourier transform of the time-

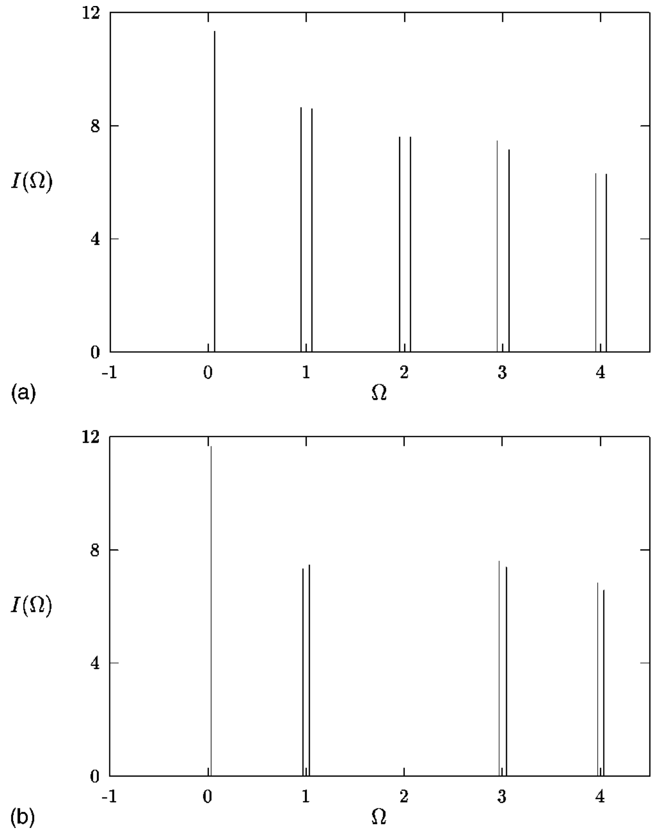

FIG. 2. Numerically calculated emission spectrum with: (a) $a=2\mu E/\omega=2.0$ and $b=2\mu E_0/\omega=1$; (b) $a=3.055$ and $b=1$. Here $\epsilon=0.05$.

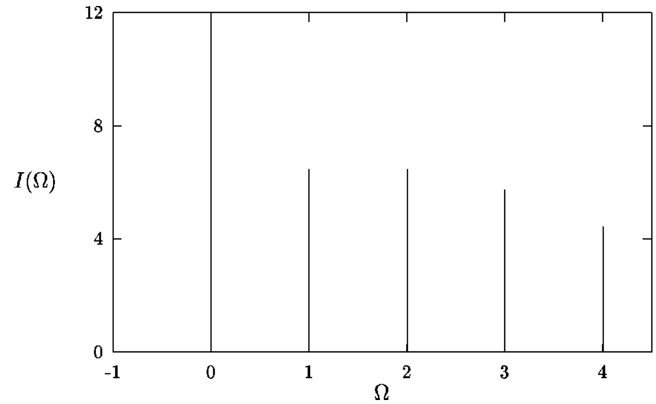

FIG. 3. Numerically calculated emission spectrum with $a=3.83$, $b=1$, and $\epsilon=0.05$.

dependent dipole moment numerically using Eq. (2.3). Note that we label the vertical axis by the relative intensity of harmonic generations in $\mu(\Omega)$.

Secondly, from Eqs. (3.1)–(3.3) we can see that some doublets will disappear when $C_{N,k}=0$. This condition can be fulfilled by proper choice of the parameters. For instance, when $a=3.055$, $b=1$, we have $J_3(3.055)+J_{-1}(3.055)=0$. Therefore, $C_{1,2}=0$. Correspondingly, the amplitude of second harmonic doublet should be eliminated. This becomes transparent in Fig. 2(b).

b. The case of $J_N(a)=0$. When $a=2\mu E/\omega$ is taken to be a zero of the Nth order Bessel function $J_N(a)$ (which is the localization condition in Ref. [8]), the Eq. (3.1) cannot give any spectrum information except of a zero-frequency term. This difficulty can be overcome by calculating the higher-order correction of $\mu(\tau)$. When $\Omega_N=0$ [or $J_N(a)=0$], one gets

$$\mu(\tau)=1-\epsilon^2\sum_{k=1}^{\infty} D_k-\epsilon^2\sum_{k=1}^{\infty} D_k \cos[k\tau] \qquad (3.4)$$

with

$$D_k=\sum_{n=-\infty}^{\infty}{}' \frac{J_n(a)}{(n+N)k}[J_{k-N}(a)+J_{n-k}(a)-J_{-k-N}(a)-J_{n+k}(a)], \qquad (3.5)$$

where the prime in the sum is used to exclude the term with $n+N=0$, and N is a constant for a given dc field, i.e., $b=2\mu E_0/\omega=N$.

Through the study of the higher-order correction of $\mu(\tau)$, we can get the following properties of emission spectrum when we choose the field parameters that make $J_N(a)=0$ (i.e., $\Omega_N=0$). In this case, because of $\Omega_N=0$, the LFG line goes to zero and the doublets coalesce to give a pure harmonic, and $\mu(\tau)$ acquires a static dipole moment [i.e., $\mu(\tau)$ equals a constant plus a time dependent part]. Hence, the emission spectrum consists of a static component and the pure harmonic k (here k includes both even and odd harmonics). This feature is confirmed by our numerical result about $\mu(\Omega)$ depicted in Fig. 3 for $\epsilon=0.05$, $a=3.83$ and $b=1$ [such choice of parameters makes $\Omega_1=\epsilon J_1(3.83)=0$].

IV. THE CASE OF $2\mu E_0/\omega\neq N$

Similar to the case of $b=N$, when $b\neq N$ one gets

$$\mu(\tau)=1+\alpha+\beta_0\cos(b\tau)+\sum_{k=1}^{\infty}\{\beta_k\cos[(k+b)\tau]-\gamma_k\cos[(k-b)\tau]-\theta_k\cos(k\tau)\}, \qquad (4.1)$$

where

$$\alpha=\epsilon^2\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty}{}' J_n(a)J_m(a)\frac{1}{n+b}\left(\frac{1}{m+b}+\frac{1}{m-n}\right), \qquad (4.2)$$

$$\beta_k=\frac{\pi\epsilon^2}{(k+b)\sin(b\pi)}J_k(a)\hat{J}_b(a) \quad (k=0,1,2,3,\ldots), \qquad (4.3)$$

$$\gamma_k=\frac{\pi\epsilon^2}{(k-b)\sin(b\pi)}J_k(a)\hat{J}_b(a) \quad (k=1,2,3,\ldots), \qquad (4.4)$$

$$\theta_k=\epsilon^2\sum_{n=-\infty}^{\infty}\frac{J_n(a)}{(n+b)k}[J_{n+k}(a)-J_{n-k}(a)] \quad (k=1,2,3,\ldots), \qquad (4.5)$$

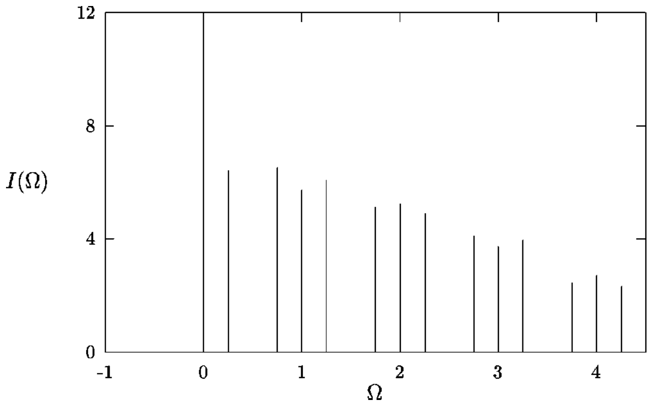

FIG. 4. Numerically calculated emission spectrum for $a=2.0$ and $b=1/4$. Here $\epsilon=0.02$.

and $\hat{J}_b(a)$ is the Anger function defined by [18]

$$\hat{J}_b(a)\equiv(1/\pi)\int_0^{\pi}d\tau\,\cos(b\tau-a\sin\tau). \tag{4.6}$$

In the above expressions, the prime in the second sum of Eq. (4.2) indicates that the term with $n-m=0$ is excluded.

In the following discussions, we divide this section into two parts: (a) the case of $b=p/q$ and (b) the case of $b=Q$ (Q is an irrational).

a. The case of $b=p/q$. When we put the dc field parameter $b=p/q$, from Eq. (4.1) we can see that, in general, the spectrum consists of a static component $\Omega=0$, the characteristic frequency of localized motion $\Omega_b=b=p/q$ (Ω_b is in ω units), and the triplets $\Omega=k$ and $\Omega=(k\pm b)$, which centered at all harmonics of ac field frequency ω. This is different from the case of $b=N$. This feature is confirmed by our numerical calculation depicted in Fig. 4. In Fig. 4, we show the emission spectrum for $\epsilon=0.02$, $b=1/4$ and $a=2.0$. It is obvious that the spectrum consists of a very high static component, the lower characteristic frequency of localized motion $\Omega_b=1/4$, and the triplets k and $(k\pm1/4)$, which are centered at all harmonics of ac field frequency ω.

Equations (4.1)–(4.6) also need further comment. First, from Eqs. (4.3), we can see that the characteristic frequency of localized motion Ω_b will disappear when $\beta_0=0$. This can be achieved by the choice of the field parameters. For instance, when $a=2.40$, we have $J_0(2.40)=0$. Therefore, $\beta_0=0$. Correspondingly, the amplitude of $\Omega_b=b$ should be eliminated. This becomes transparent in Fig. 5(a).

Second, from Eqs. (4.1), (4.3), and (4.4), it is obvious that we can eliminate any pair of the two satellites that is centered at k by the proper choice of the field parameters, since both β_k and γ_k have the same factor $J_k(a)\hat{J}_b(a)$. For instance, when we choose $a=3.83$, we have $\beta_1=0$ and $\gamma_1=0$ because of $J_1(3.83)=0$. Correspondingly, the amplitude of $\Omega=k\pm b=1\pm1/4$ should be eliminated, as we can see from Fig. 5(b).

Third, when we choose the field parameters in such a way that $\hat{J}_b(a)=0$, all of the amplitudes $\beta_k(k=0,1,2,3,\ldots)$ and $\gamma_k(k=1,2,3,\ldots)$ should disappear, and Eq. (4.1) becomes

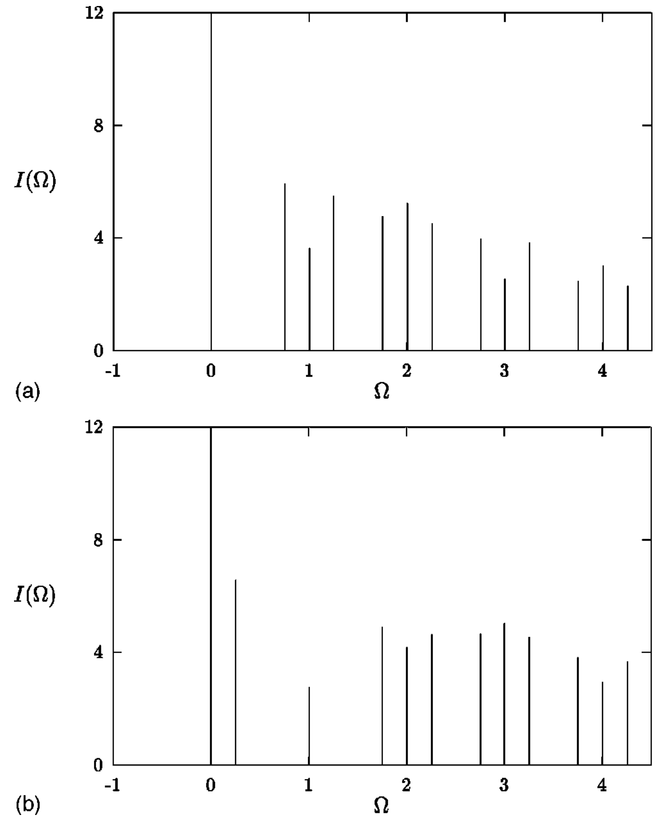

FIG. 5. (a) Numerically calculated emission spectrum for $a=2.404$ and $b=1/4$; (b) numerically calculated emission spectrum for $a=3.83$ and $b=1/4$. Here $\epsilon=0.02$.

$$\mu(\tau)=1+\alpha-\sum_{k=1}^{\infty}\theta_k\cos[k\tau]. \tag{4.7}$$

From this equation, we can see that the emission spectrum consists of a static component and the pure harmonic k (here k includes both even and odd harmonics). This feature is also confirmed by our numerical result about $\mu(\Omega)$ depicted in Fig. 6 for $\epsilon=0.02$, $b=1/2$ and $a=3.32$ [such a choice of the field parameters makes $\hat{J}_b(a)=\hat{J}_{1/2}(3.32)=0$].

b. The case of $b=Q$. Here Q is an irrational and we restrict $0<Q<1$. From Eqs. (4.1)–(4.6) we can see that, if

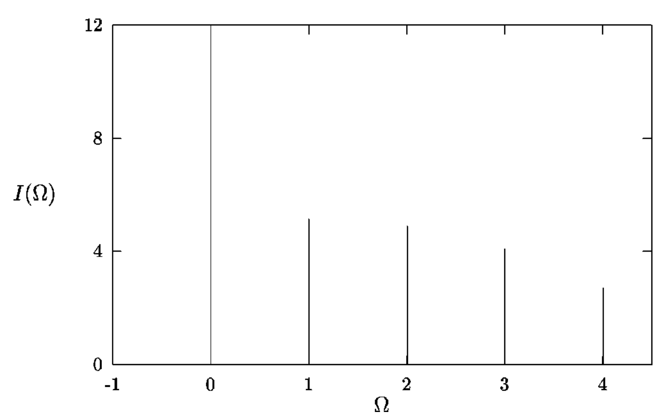

FIG. 6. Numerically calculated emission spectrum for $a=3.32$ and $b=1/2$. Here $\epsilon=0.02$.

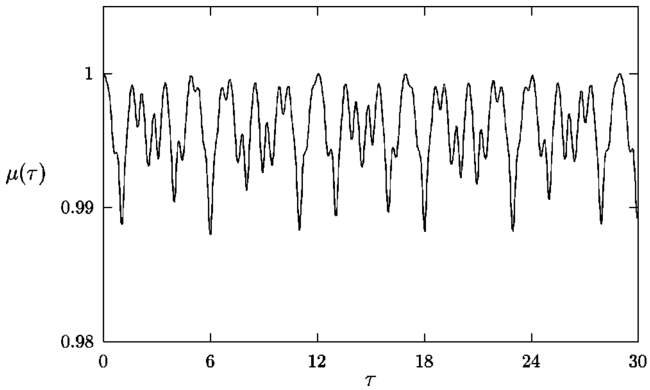

FIG. 7. Induced dipole $\mu(\tau)$ as a function of the scaled time τ (in 2π units) for $b=\sqrt{2}-[\sqrt{2}]$ and $a=2.0$. Here $\epsilon=0.02$.

$\hat{J}_b(a)=0$, one gets the same equation as Eq. (4.7). Therefore, the above discussion is still valid for this case. However, if b is an irrational and $\hat{J}_b(a)\neq 0$, the situation is different. In this case, contrary to the situation of b being rational, one cannot find any periodic characteristics in the evolution of the dipole moment $\mu(\tau)$. This feature has been shown in Fig. 7. There, we show the $\mu(\tau)$ (in 2π units) for $\epsilon=0.02$, $b=\sqrt{2}-[\sqrt{2}]$, where $[\sqrt{2}]$ means the integral part of $\sqrt{2}$ and $a=2.0$ [such a choice of the field parameters makes $\hat{J}_b(a)\neq 0$]. It can be clearly seen from Fig. 7 that the dipole moment $\mu(\tau)$ is close to one with small amplitude oscillations within any period of the driving laser. But there is not any periodic characteristics shown in this figure. This suggests the dipole moment $\mu(\tau)$ in this case is quasiperiodic. That conclusion agrees with the findings in Ref. [8(b)].

V. CONCLUSION

We have studied the evolution properties and emission spectrum of an electron in a two-level system driven by dc-ac fields under the approximation of high-frequency driving ($\epsilon\equiv\Delta/\omega\ll 1$). The problem can be mapped into that of atoms with spin-$\frac{1}{2}$ under the simultaneous action of static magnetic and oscillating rf fields [11–13]. Based on the outcome of Refs. [11–13] we carried out the analytical solutions for the dipole moment and its emission spectrum. From these results, we have shown that, when $b=2\mu E_0/\omega=N$, in general, the emission spectrum consists of a static component, low-frequency Ω_N (LFG), and doublets at frequency $(k\pm\Omega_N)$ for $k=1,2,3,\ldots$. It was found that the amplitudes of all the Fourier components of dipole moment and Ω_N depend on the field parameters a and b. We have also shown analytically and numerically that it is possible, by making the proper choice of the field parameters to selectively eliminate any one of the doublets in the spectrum. When $b=2\mu E_0/\omega=N$ and $J_N(a)=0$ (i.e., $\Omega_N=0$), the doublets coalesce to give pure harmonic and $\mu(\tau)$ acquires a static dipole moment. The emission spectrum consists of a very high static component and the lower pure harmonic, which includes both even and odd harmonics. When $b=2\mu E_0/\omega\neq N$, the emission spectrum of electron has following properties: (1) When $b=2\mu E_0/\omega=p/q$ and $\hat{J}_b(a)\neq 0$, in general, the spectrum consists of a static component $\Omega=0$, the characteristic frequency of localized motion $\Omega_b=b=p/q$, and the triplets $\Omega=k$ and $\Omega=(k\pm b)$, which is centered at all harmonics of ac field frequency ω. Meanwhile, we have also shown analytically and numerically that it is possible, by making the proper choice of the field parameters, to eliminate the characteristic frequency of localized motion and to selectively eliminate any a pair of two satellites in the spectrum. (2) When $b=2\mu E_0/\omega=p/q$ and $\hat{J}_b(a)=0$, the emission spectrum consists of a very high static component and the lower pure harmonic. (3) When $b=2\mu E_0/\omega=Q$ is an irrational and $\hat{J}_b(a)=0$, the emission spectrum also consists of a very high static component and the lower pure harmonic. (4) When $b=2\mu E_0/\omega=Q$ is an irrational and $\hat{J}_b(a)\neq 0$, the evolution of the dipole moment $\mu(\tau)$ becomes quasiperiodic.

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