Nonadiabatic cooling and optimal control in off-resonance dipole optical potentials

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We investigate coherent control of atomic translational motion in an applied off-resonance dipole optical potential. Since spontaneous emission can be neglected in this regime, the problem is treated as one of coherent momentum transfer. We consider both adiabatic and nonadiabatic regimes of cooling in the framework of the Wigner function formalism. For the adiabatic case, an approximate solution is obtained corresponding to the cooling envelope for an arbitrary time dependence of the external field. The nonadiabatic cooling process is formulated in terms of optimal control theory in order to define the most favorable regime of cooling under imposed constraints on the intensity of the control field. We find that the applied control field yields a significant reduction of the effective temperature of the atoms. [S1050-2947(98)01808-3]

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I. INTRODUCTION

The manipulation of the atomic center-of-mass momentum distribution is important for the adiabatic cooling of atomic beams [1] and atomic clouds in optical lattices [2]. For atomic beams, the experimental realizations of adiabatic cooling are based on the effects of trapping and channeling of atoms in the field of an intense standing wave [3]. The effects of localization of atoms on the scale less than the optical wavelength [4] and of quantization of the atomic motion in one-dimensional (1D), 2D, and 3D optical molasses has also been observed and extensively studied [5].

A detailed theoretical account of the adiabatic cooling process is available for atoms undergoing no spontaneous emission in microcavities and optical traps [6,7]. By applying band theory to a two-level atom in a periodic optical dipole potential [2,8,9], a lower limit of the order of the recoil energy was predicted for the minimum temperature of the atomic cloud. The approach in [6,7] is based on the numerical simulation of two-level atoms in the dipole potential and it is difficult to analyze the dependence of the cooling process on the parameters of the optical potential. The adiabatic regime of cooling corresponds to the case when the effective optical potential is changing slowly in comparison to the oscillation frequency of the atom trapped in the standing-wave dipole potential and the constraint to slow dynamics can be a serious restriction since the spontaneous emission should be negligible during the cooling process. On the other hand, the use of the nonadiabatic time-dependent off-resonance dipole potentials was shown to be extremely successful for interacting with the atomic center-of-mass motion [10,11]. The manipulation of the atomic distribution in optical lattices with nonadiabatic potentials was studied under various circumstances [12,13].

The goal of the present theoretical analysis is to define optical characteristics of a radiation field in order to achieve a cooling of an atomic system. We propose a general approach, valid for both the adiabatic and nonadiabatic limits of cooling in the framework of density-matrix formalism. This approach enables the formulation of the problem in terms of optimal control theory in order to identify the most favorable regime of cooling under imposed laboratory constraints.

We assume that the atoms are subjected to off-resonance laser field and experience the energy shift of the ground state $V(x,t)$ proportional to the intensity of the field

$$V(x,t)=V_0(t)[1-\cos(2qx)], \tag{1}$$

where x denotes the atomic center-of-mass coordinate, $V_0(t)$ is the amplitude of the dipole potential (proportional to the intensity of the laser field), and q is the wave vector of the laser field.

One should note that the arguments of [14] are applicable here and the phase density of the atomic system is conserved. If the goal is a redistribution in momentum space, then this limitation is not significant. Since the phase-space density of the system is conserved, this implies that the compression in momentum space will cause an increase in the coordinate spread for the atomic cloud.

II. BASIC EQUATIONS

We will employ the Wigner function formalism, which enables one to study both coordinate and momentum probability distributions simultaneously. The evolution of the Wigner function has to be followed for the noninteracting particles in the external potential given by Eq. (1). The Wigner function is defined as a Fourier transform of the density matrix with respect to the relative momentum variable

$$\rho(p,x,t)=\int d(p-p')\exp[ix(p-p')]\rho\left(\frac{p+p'}{2},p-p',t\right). \tag{2}$$

For the atomic system moving in the potential $V(x,t)$, the time evolution of the Wigner function $\rho(p,x,t)$ is given by

$$\left(\frac{\partial}{\partial t}+\frac{p}{m}\frac{\partial}{\partial x}\right)\rho(p,x,t)$$

$$=-\frac{1}{2\pi i h}\int dp'\int dz\,\exp[iz(p-p')]$$

$$\times\left\{V\left(x+h\frac{z}{2},t\right)-V\left(x-h\frac{z}{2},t\right)\right\}\rho(p',x,t). \quad (3)$$

In the case when the atom is in the vicinity of one of the minima of the dipole potential, it is reasonable to expand the effective potential to second order around the minimum. This is a good approximation in many situations, such as in adiabatic cooling [6]. It is required that each atom be located around one of the minima of the potential given by Eq. (4) and then the effective potential becomes quadratic $V(x,t)=k(t)x^2/2$, where x is a distance from the minimum and $k(t)=4/mV_0(t)q^2$. In this case, Eq. (6) for the Wigner function exactly reduces to the Liouville equation for the distribution function of the classical harmonic oscillator. Therefore, we obtain

$$\left(\frac{\partial}{\partial t}+\frac{p}{m}\frac{\partial}{\partial x}-k(t)x\frac{\partial}{\partial p}\right)\rho(p,x,t)=0. \quad (4)$$

The initial condition corresponding to the squeezing oscillations of the atomic phase-space distribution can be obtained from the thermal distribution by means of the abrupt nonadiabatic change of the dipole potential. The nonadiabatic change of the dipole potential produces the classical squeezed states [16] and initiates the squeezing oscillations of the atomic phase-space distribution. In this case, the initial condition for Eq. (4) is given by [6]

$$\rho_0(p,x)=\frac{1}{\sqrt{2\pi\sigma_p^2(0)}}\frac{1}{\sqrt{2\pi\sigma_x^2(0)}}$$

$$\times\exp\left[-\frac{p^2}{2\sigma_p^2(0)}\right]\exp\left[-\frac{x^2}{2\sigma_x^2(0)}\right]. \quad (5)$$

The solution of Eq. (5) with this initial condition is obtained in the form

$$\rho(p,x,t)=\frac{1}{\sqrt{2\pi\sigma_p^2(0)}}\frac{1}{\sqrt{2\pi\sigma_x^2(0)}}$$

$$\times\exp[-\alpha(t)p^2-\beta(t)px-\gamma(t)x^2]. \quad (6)$$

Indeed, the substitution of Eq. (6) into Eq. (4) gives us the closed set of equations

$$\frac{d\alpha(t)}{dt}=-\frac{1}{m}\beta(t), \quad (7a)$$

$$\frac{d\beta(t)}{dt}=2k(t)\alpha(t)-\frac{2}{m}\gamma(t), \quad (7b)$$

$$\frac{d\gamma(t)}{dt}=k(t)\beta(t), \quad (7c)$$

Taking into account Eq. (5), we obtain the initial conditions for Eq. (7),

$$\frac{1}{\alpha(0)}=2\sigma_p^2(0),\quad \frac{1}{\gamma(0)}=2\sigma_x^2(0),\quad \beta(0)=0. \quad (8)$$

One should note that the system given by Eq. (7) has an integral of motion

$$J\equiv\alpha(t)\gamma(t)-\frac{\beta^2(t)}{4}, \quad (9)$$

where J is a constant determined by the initial conditions. Combining Eqs. (6), (8), and (9), one can verify that $\int dx\int dp\ \rho(p,x,t)\equiv 1$, i.e., the distribution (6) is normalized. Making use of the distribution (9), we obtain the information entropy S in the form

$$S=-\int dx\int dp\ \rho(p,x,t)\ln[\rho(p,x,t)]$$

$$=-\tfrac{1}{2}\ln(J)+(1-\ln\pi). \quad (10)$$

From Eq. (10) it follows that J is related to the information entropy of the atomic system, which is conserved in accordance with the Liouville theorem [14].

According to Eq. (6), the dispersion of the momentum and coordinate probability distributions at time t are given by

$$\sigma_p^2(t)\equiv\langle p^2\rangle=\frac{1}{2}\frac{\gamma(t)}{J},\quad \sigma_x^2(t)\equiv\langle x^2\rangle=\frac{1}{2}\frac{\alpha(t)}{J}, \quad (11)$$

where angular brackets denote the averaging with the distribution given by Eq. (9). Differentiating Eq. (7a), we reduce Eqs. (7) to

$$\frac{d^2\alpha(t)}{dt^2}+\frac{2k(t)}{m}\alpha(t)=\frac{2}{m^2}\gamma,$$

$$\frac{d\gamma(t)}{dt}=-mk(t)\frac{d\alpha(t)}{dt}, \quad (12)$$

with the initial conditions defined by Eq. (8).

III. NONADIABATIC REGIME OF COOLING

The traditional mechanism of laser cooling based on the squeezing of the atomic phase-space distribution is an adiabatic mechanism of cooling that has been discussed in details in the literature [6]. In this case, the frequency of the atomic oscillations is decreasing adiabatically slowly by means of the reduction of the effective dipole potential. The resulting evolution of the momentum and coordinate distributions can be found from Eq. (12), assuming the equilibrium initial conditions. Looking for the envelope solution of Eq. (12), we neglect the term with the second derivative $d^2\alpha/dt^2$. Taking into account Eq. (11), we obtain

$$\frac{\sigma_x^2(t)}{\sigma_x^2(0)}=\left(\frac{k(t)}{k(0)}\right)^{-1/2}, \quad \frac{\sigma_p^2(t)}{\sigma_p^2(0)}=\left(\frac{k(t)}{k(0)}\right)^{1/2}. \tag{13}$$

It is easy to verify that Eq. (16) is equivalent to the following change of temperature: $T(t)/T(0)=\sqrt{k(t)/k(0)}$. In Ref. [6] the adiabatic regime of cooling with exponentially decaying effective potential $k(t)=k_0\exp(-st)$ has been studied by means of the numerical simulations. The envelope analytical solution given by Eq. (13) reproduces the results of simulation [Ref. [6], Fig. 1(d)] with remarkably good accuracy.

As opposed to the adiabatic one, the nonadiabatic regime opens the possibility of achieving a significant cooling effect within a short-time interval. The simplest nonadiabatic cooling scheme corresponds to the abrupt change of the effective potential

$$k(t)=\begin{cases} k_0, & t\leqslant 0 \\ k_1, & t>0. \end{cases} \tag{14}$$

Assuming that for $t<0$ the system is in equilibrium, we obtain from Eqs. (12) and (8)

$$\sigma_p^2(t)=\frac{1}{2}\,\sigma_p^2(0)\left[\left(1+\frac{k_1}{k_0}\right)+\left(1-\frac{k_1}{k_0}\right)\cos(2\omega_1 t)\right],$$
$$\sigma_x^2(t)=\frac{1}{2}\,\sigma_x^2(0)\left[\left(1+\frac{k_0}{k_1}\right)+\left(1-\frac{k_0}{k_1}\right)\cos(2\omega_1 t)\right], \tag{15}$$

where $\omega_0^2=k_0/m$ and $\omega_1^2=k_1/m$. One can see that for $k_1 \ll k_0$ a substantial reduction of the momentum dispersion can be achieved at $t=\pi/2\omega_1$. The oscillations of the momentum and coordinate dispersions of the atomic distribution described by Eq. (15) are known [15,18,19].

In order to demonstrate the crossover from the adiabatic to the nonadiabatic regime of cooling, we will employ an exact solution for the power-law profile of the effective potential $k_0/(1+\mu_0\omega_0 t)^2$ (μ_0 is a cooling rate parameter), valid in both adiabatic and nonadiabatic regimes. In the Appendix we present an exact solution of Eq. (12) for this potential for arbitrary initial conditions. Making use of Eq. (A2) and assuming the equilibrium initial conditions, we obtain

$$\frac{\sigma_p^2(t)}{\sigma_p^2(0)}=\frac{1}{1+\mu_0\omega_0(0)t}\left[\frac{1-(\mu_0^2/4)\cos(2\tilde{\tau})}{1-\mu_0^2/4}+\frac{(\mu_0/2)\sin(2\tilde{\tau})}{\sqrt{1-\mu_0^2/4}}\right],$$
$$\frac{\sigma_x^2(t)}{\sigma_x^2(0)}=[1+\mu_0\omega_0(0)t]\left[\frac{1-(\mu_0^2/4)\cos(2\tilde{\tau})}{1-\mu_0^2/4}-\frac{(\mu_0/2)\sin(2\tilde{\tau})}{\sqrt{1-\mu_0^2/4}}\right], \tag{16}$$

where $\tilde{\tau}(t)=(1-\mu_0^2/4)^{1/2}(1/\mu_0)\ln[1+\mu_0\omega_0(0)t]$. The time evolution of the momentum dispersion is presented in Fig. 1

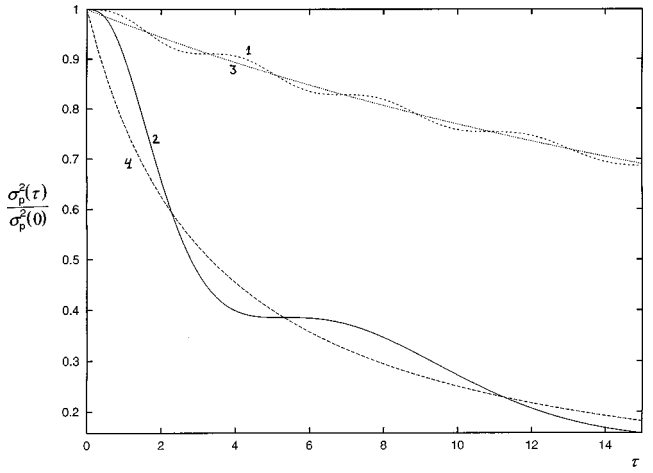

FIG. 1. Comparison between an exact solution for the momentum dispersion $\sigma_p^2(t)$ given by Eq. (19) with $\mu_0=0.03$ and 0.3 (lines 1 and 2, respectively) and the envelope solution given by Eq. (16) (lines 3 and 4, respectively), as a function of $\tau=\omega_0 t$.

for two values of the cooling rate μ_0. One can see that for $\mu_0=0.03$ the amplitude of the oscillations of momentum dispersion is small enough and the exact solution deviates only slightly from the envelope solution represented by a dashed line. This corresponds to the adiabatic regime of cooling. At the same time, for the larger values of the cooling rate, the amplitude of the oscillations increases and the exact solution deviates significantly from the envelope solution. In the nonadiabatic regime, the decay rate of the oscillation amplitude is comparable to the frequency of the oscillations. With the increase of μ_0, a crossover from underdamped to the overdamped oscillations occurs and the analytical solution (A3) becomes invald.

IV. OPTIMAL CONTROL ANALYSIS

As we discussed above, in the nonadiabatic limit the evolution of the atomic distribution is quite sensitive to the temporal profile of the effective potential. Namely, the effect of the fast changing of the effective potential is twofold: The energy and therefore the average value of the dispersion of the momentum distribution may increase, but at the same time the amplitude of the oscillations is also increasing. The evolution of the distribution becomes quite complex and may lead to decreasing or increasing of the dispersion of momentum distribution for different time profiles of the effective potential.

For this reason it is useful to apply the formalism of optimal control theory (OCT) (see, e.g., Ref. [17]) to the problem of nonadiabatic cooling. The purpose of the OCT analysis is to define the temporal profile of the control field $k(t)$ that drives the system from a given initial state into a target state characterized by target values γ_g and α_g, which define the target values of the momentum and coordinate dispersions at a given moment of time t_g. Taking into account Eq. (10) for the parameters of the atomic momentum distribution, we define an objective functional in the form

$$J=\frac{1}{2}\int_0^{t_g}dt'[k(t')]^2+\frac{K}{2}[\gamma(t_g)-\gamma_g]^2$$
$$+\frac{K}{2}\beta^2(t_g)+\frac{K}{2}[\alpha(t_g)-\alpha_g]^2$$
$$+\int_0^{t_g}dt'\lambda(t')\left[\frac{d}{dt'}\alpha(t')+\frac{1}{m}\beta(t')\right]$$
$$+\int_0^{t_g}dt'\mu(t')\left[\beta(t')-2k(t')\alpha(t')+\frac{2}{m}\gamma(t')\right]$$
$$+\int_0^{t_g}dt'\nu(t')\left[\frac{d}{dt'}\gamma(t')-k(t')\beta(t')\right], \qquad (17)$$

where the functions $\lambda(t)$, $\mu(t)$, and $\nu(t)$ are Lagrange multipliers and have to be defined from the extremum conditions and the constant K controls the accuracy of the target conditions at $t=t_g$. Taking into account the integral of motion given by Eq. (12), we conclude that the product $\alpha(t_g)\gamma(t_g)$ is minimal when $\beta(t_g)=0$. Since we want to minimize the increase of the coordinate dispersion, the target functional given by Eq. (18) requires that $\beta_g=0$. An additional cost assumed in Eq. (18) is represented by the first term on the right-hand side associated with the requirement to minimize the intensity of the control laser field. The desired solution corresponds to the minimum of the functional (17). The first variation of the objective functional yields the set of equations

$$\frac{d\alpha(t)}{dt}=-\frac{1}{m}\beta(t),$$
$$\frac{d\beta(t)}{dt}=2k(t)\alpha(t)-\frac{2}{m}\gamma(t),$$
$$\frac{d\gamma(t)}{dt}=k(t)\beta(t),$$
$$\frac{d\nu(t)}{dt}=\frac{2}{m}\mu(t), \qquad (18)$$
$$\frac{d\mu(t)}{dt}=-k(t)\nu(t)+\frac{1}{m}\lambda(t),$$
$$\frac{d\lambda(t)}{dt}=-2k(t)\mu(t),$$
$$k(t)=2\mu(t)\alpha(t)+\nu(t)\beta(t),$$

with the initial and final conditions

$$\alpha(0)=\alpha_0,\quad \beta(0)=0,\quad \gamma(0)=\gamma_0,$$
$$\lambda(t_g)=-K\left(\alpha(t_g)-\frac{\alpha_0\gamma_0}{\gamma(t_g)}\right), \qquad (19)$$
$$\mu(t_g)=-K\beta(t_g),\quad \lambda(t_g)=-K[\gamma(t_g)-\gamma_g].$$

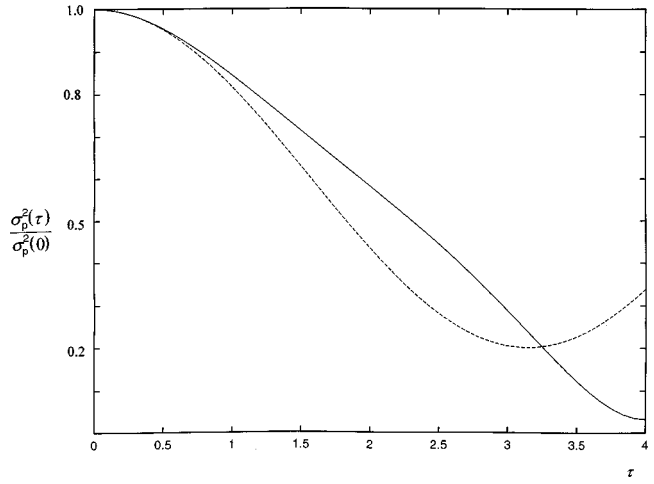

FIG. 2. Optimal control solution for the momentum dispersion $\sigma_p^2(t)$ (solid line) compared to a reference solution (dashed line). The initial conditions correspond to $\sigma_p^2(0)/(m\omega_1)^2\sigma_k^2(0)=3$. The target time $\omega_1 t_g=2$.

In order to improve the convergence of the numerical solution evolving from the initial equilibrium state to the target state, we take advantage of the fact that a substantial cooling effect can be already achieved by an abrupt change of the potential, as we discussed in Sec. III. This steplike potential creates the nonequilibrium initial conditions for the atomic distribution at the moment $t=0$, which cause the oscillations of the dispersions of the atomic distribution. With the properly chosen steplike potential, such oscillations brings the system closer to the desired target state compared to the initial equilibrium state. Driving the system into the target state is achieved by means of the continuous control field applied for $t>0$. This means that we choose the nonequilibrium initial conditions (19) for Eq. (18).

The system given by Eqs. (18) and (19) was solved by means of a two-point iteration method [20]. This method avoided any divergence of the iteration by means of the decreasing of the norm of the operator in the Picard iteration procedure.

A solution of the optimal system (18) for the target time $\omega_1 t_g=2$ is given in Figs. 2 and 3. In Fig. 2 we compare the evolution of the momentum dispersion with the optimal control field to the reference solution with a constant field. The solution presented in Fig. 2 with a dashed line corresponds to the case when the squeezing oscillations have been initiated at $t=0$ by means of the nonadiabatic change of the effective potential, which remains constant at $t>0$. The solid line corresponds to the optimal control solution of the squeezing oscillations. We conclude that the use of optimal control provides a significant reduction of the dispersion of the momentum distribution at a given moment of target time. Figure 3 presents the optimal control field. One can see that the temporal structure of the optimal potential is nonmonotonic. The reason for this is a competition between the reduction of the energy of oscillations favorable for cooling and the unfavorable reduction of the amplitude of the oscillations due to the decrease of the potential, combined with the requirement of minimization of the control field. As we discussed before, it is mainly this nonadiabatic regime of cooling that makes the evolution of the distribution complex and the application of OCT becomes essential.

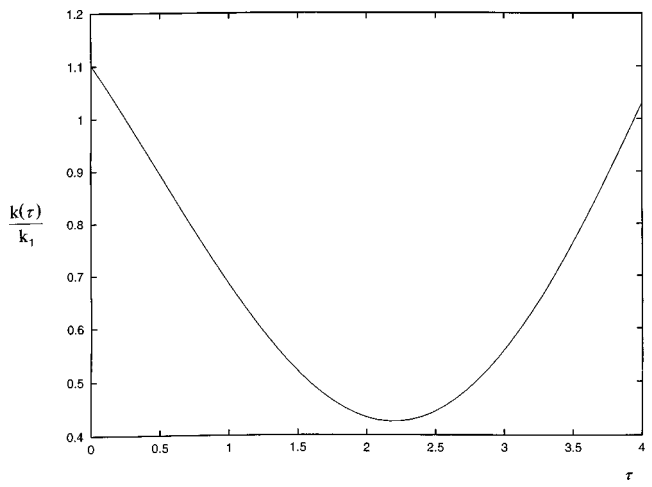

FIG. 3. Time evolution of the dimensionless optimal control field $k(t)/k_1$, where k_1 is a constant reference field corresponding to Fig. 2.

According to the above discussion, the effective potential should change nonadiabatically in order to initiate the squeezing oscillations. The typical nonadiabatic cooling time is of the order of the period of the squeezing oscillations of the atomic distribution $t_{\rm na}\cong 1/\omega_1$ as opposed to the adiabatic cooling time $t_a\cong 1/s$, where s is the typical rate of adiabatic decay of the effective potential. In the adiabatic regime, $s \ll \omega_1$ and therefore $t_a \gg t_{\rm na}$. In the optical lattices, $\omega_1 \sim 10^6$ and therefore $t_{\rm na}\sim 10^{-6}$ s, whereas $t_a \sim 10^{-4}$ s. For example, in the situation considered in Figs. 2 and 3, $t_{\rm na}=t_g=2/\omega_1$. Therefore, operating in the nonadiabatic regime provides a possibility of a much faster cooling than in the usual adiabatic one.

V. CONCLUSION

In conclusion, we proposed a general approach for the coherent manipulation of the dispersion of the center-of-mass atomic momentum distribution, in the framework of the Wigner function formalism, valid in both the adiabatic and nonadiabatic regimes of cooling. One should note that the extension of the above formulated results to the 3D periodic potential is straightforward since in the coherent regime different degrees of freedom are completely independent. In the adiabatic regime, we obtained an analytic solution for the envelope of the momentum and coordinate distributions. We also obtained analytical results for the atomic distribution valid beyond the adiabatic limit for some particular profiles of the time-dependent effective dipole potential. In the nonadiabatic regime, the evolution of the atomic distribution becomes complex and it is natural to employ optimal control theory to obtain cooling at a given target time. The general approach was formulated in order to define the most favorable regime of cooling under imposed costs. We found that the use of optimal control enables a significant reduction of the width of the momentum distribution at a given moment of target time.

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APPENDIX: EXACT SOLUTION FOR A MODEL TIME-DEPENDENT EFFECTIVE POTENTIAL

The cooling equations can be solved exactly for the power-law decaying effective potential, given by

$$k(t)=k(0)\,\frac{1}{[1+\mu_0\omega_0(0)t]^2} \tag{A1}$$

and the solution we obtained is valid in the region of parameter $0\leqslant\mu_0\leqslant 2$. With the substitutions $\alpha(t)=[1+\mu_0\omega_0(0)t]\tilde{\alpha}(\tilde{\tau})$ and $\gamma(t)=\{1/[1+\mu_0\omega_0(0)t]\}\tilde{\gamma}(\tilde{\tau})$ and the dimensionless time defined by $\tilde{\tau}(t)=(1-\mu_0^2/4)^{1/2}(1/\mu_0)\ln[1+\mu_0\omega_0(0)t]$, Eq. (12) is reduced to

$$\begin{aligned}&\frac{d^3\tilde{\alpha}(\tilde{\tau})}{d\tilde{\tau}^3}+4\,\frac{d\tilde{\alpha}(\tilde{\tau})}{d\tilde{\tau}}=0,\\ &\frac{d\tilde{\gamma}(\tilde{\tau})}{d\tilde{\tau}}-\lambda\,\tilde{\gamma}(\tilde{\tau})=-m^2\omega_0^2\left(\frac{d\tilde{\alpha}(\tilde{\tau})}{d\tilde{\tau}}-\lambda\,\tilde{\alpha}(\tilde{\tau})\right),\end{aligned} \tag{A2}$$

where $\lambda\equiv\mu_0/\sqrt{1-\mu_0^2/4}$. Equation (A2) should be solved with the initial conditions given by Eq. (8). Returning to the variables $\alpha(t)$ and $\gamma(t)$, a solution of Eq. (12) is obtained in the form

$$\begin{aligned}&\alpha(t)=[1+\mu_0\omega_0(0)t][c_1\sin(2\tilde{\tau})+c_2\cos(2\tilde{\tau})+c_3],\\ &\gamma(t)=\frac{m^2\omega^2(0)}{[1+\mu_0\omega_0(0)t]}\,[c_4\sin(2\tilde{\tau})+c_5\cos(2\tilde{\tau})+c_6]\end{aligned} \tag{A3}$$

and the coefficients are

$$\begin{aligned}c_1&=-\frac{\alpha_0}{2}\,\frac{\mu_0}{\sqrt{1-\mu_0^2/4}},\\ c_2&=\frac{1}{1-\mu_0^2/4}\left[\frac{\alpha_0}{2}\,(1-\mu_0^2/2)-\frac{\gamma_0}{2m^2\omega_0^2(0)}\right],\\ c_3&=\frac{1}{1-\mu_0^2/4}\left[\frac{\alpha_0}{2}+\frac{\gamma_0}{2m^2\omega_0^2(0)}\right],\\ c_4&=\frac{\gamma_0/2}{m^2\omega_0^2(0)}\,\frac{\mu_0}{\sqrt{1-\mu_0^2/4}},\\ c_5&=\frac{1}{1-\mu_0^2/4}\left[\frac{\gamma_0}{2m^2\omega_0^2(0)}\,(1-\mu_0^2/2)-\frac{\alpha_0}{2}\right],\\ c_6&=\frac{1}{1-\mu_0^2/4}\left[\frac{\alpha_0}{2}+\frac{\gamma_0}{2m^2\omega_0^2(0)}\right].\end{aligned} \tag{A4}$$

In the limit $\mu_0\to 0$ the effective potential (A1) is a constant and the solution given by Eq. (A3) reduces to the one for the constant effective potential, given by Eq. (15). Note that the dispersions of the momentum and coordinate distributions given by Eq. (A3) contain both oscillating and power-law

factors. The oscillating factors in Eq. (A3) are analogous to the ones in Eq. (15), but the frequency of the oscillations is a function of time. The power-law factors in Eq. (A3) describe the adiabatic cooling of the atomic cloud when the effective potential is decreasing slowly $\mu_0 \ll 1$. When the parameter μ_0 is increasing, the cooling process goes faster and the oscillations of the momentum dispersion become more pronounced.

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