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Quantum adiabatic particle transport in optical lattices

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Quantized adiabatic particle transport (QAPT) is considered and verified in a one-dimensional system using numerical simulation. More importantly, suggestions are given for ways to observe QAPT phenomena in atoms manipulated by laser beams. Although QAPT was originally considered for electrons in periodic substrate potentials, the optical systems provide a cleaner and experimentally feasible way to physically realize QAPT. The results in this paper serve as a model for the design of these experiments. [S1050-2947(98)51004-9]

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I. INTRODUCTION

It has been shown theoretically by Thouless that quantized currents can be observed in an electronic system subjected to a potential adiabatically and periodically varying in time [1]. This phenomenon of quantized currents, known as the quantum adiabatic particle transport (QAPT), requires that the Fermi energy of the system lie in a gap of the instantaneous Hamiltonian and that this gap not close during the time variation of the Hamiltonian. Although initially proven in a specialized one-dimensional case assuming independent electrons, QAPT was later shown to be quite robust in more general settings. QAPT has been shown to be preserved in systems with disorder and many-body interactions [2], and has also been proven to be in accord with relativistic quantum field theory [3].

With recent advances in atom optics, where laser beams are used to confine and manipulate trapped atoms, the possibility of actual observation of the QAPT has been greater than ever before. For sure, this is not the first use of optical setups to allow observations of paralleling solid-state effects; Bloch oscillations and Wannier-Stark ladders have also been observed in similar arrangements [4]. Although originally considered in a solid-state electronic system setting, these optical setups provide an equivalent but experimentally feasible way of demonstrating the QAPT. Atoms placed in the optical potential of a laser experience a one-dimensional periodic potential due to the dipole force [5]. In addition, the confined atoms are sufficiently dilute in concentration to be considered as independent particles, allowing detailed analyses of the system.

The aim of the present paper is then to explore through numerical simulations the possibilities of physically realizing QAPT experiments in optical systems. A simple system is studied, where the existence of QAPT is confirmed, and an interestingly counterintuitive QAPT is demonstrated. The system is then modified to make it experimentally feasible, with considerations of adiabaticity and initial-state preparation.

In Sec. II, the QAPT theory is summarized, and specific details concerning the optical system are also discussed. In Sec. III, the QAPT for a simple system is actually calculated and results from numerical tests reviewed. Section IV then discusses suggestions for actual implementation of the QAPT in the optical systems.

II. QAPT THEORY

We begin by summarizing the proof of the one-dimensional independent-particle QAPT [1]. Assume a spatially and temporally periodic potential with periods L and T, where the potential also varies adiabatically in time. Assume also that the Fermi energy lies in a gap of the energy spectrum that remains open throughout the time evolution (this gap is termed the Fermi gap). Then, we represent the particle wave function, using an expansion on the set of instantaneous eigenfunctions, known as Bloch waves. Upon integration of the current, the number of particles transported across a section during T is calculated to be

$$\int_0^T \int_0^{2\pi/L} \frac{i}{2\pi}\, dt\, dk \left[\left\langle \frac{\partial \psi_{\lambda k}}{\partial t} \middle| \frac{\partial \psi_{\lambda k}}{\partial k} \right\rangle - \left\langle \frac{\partial \psi_{\lambda k}}{\partial k} \middle| \frac{\partial \psi_{\lambda k}}{\partial t} \right\rangle \right], \tag{1}$$

where k and λ are, respectively, the wave number and band index of the Bloch wave $\psi_{\lambda k}$. The integral is proportional to the wave function's phase change as it is adiabatically varied in a closed loop traversing $(T, 2\pi/L)$ in (t,k) space. The phase is restricted to integral multiples of 2π, and thus gives rise to a quantized current.

Slight modifications of the QAPT theory are needed for the optical systems of interest. Thouless's prediction of quantized currents is not immediately applicable, since the optical Bloch bands are far from full due to the dilute concentration of laser confined atoms. Instead, similar calculations predict that the average velocity over time T and over the band will be integer multiples of the moving potential's velocity. Note that the preparation of particles in a band with uniform distribution is already experimentally possible [6].

The exact adiabatic condition required for QAPT concerns Landau-Zener (LZ) tunneling [7], a process where particles undergo interband transitions. In LZ tunneling, a particle in a certain band tunnels to a neighboring band with a probability per Bloch period of $\exp(-\pi\Delta^2/\alpha)$, where Δ is half the gap size and α is the slope of the band in (E,t) space. The condition $\pi\Delta^2/\alpha \ll 1$ will then minimize LZ tunneling and maintain the adiabaticity required of QAPT.

Finally, we mention an important and useful topological feature of QAPT theory. If the form of a potential is changed continuously such that the Fermi gap remains open, the QAPT of the bands below summed together should be in-

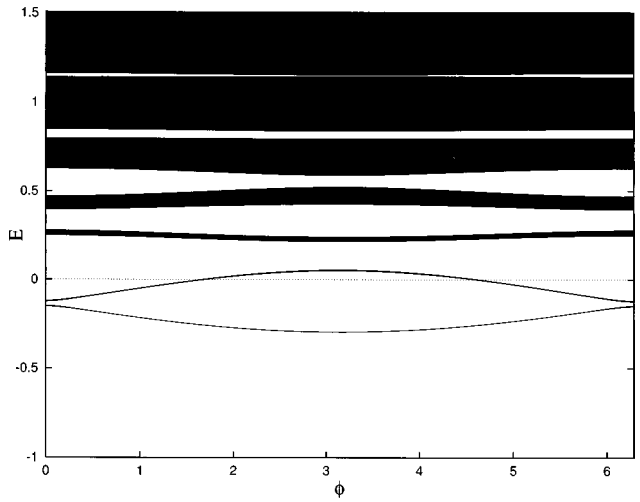

FIG. 1. Band structure given by Eq. (2) with $V_1=0.4$ and $V_2=0.2$; i.e., $V(x)=0.4\cos(x)+0.2\cos[(x-\phi)/2]$.

variant [8]. This provides a powerful method of calculating QAPT, by taking a system whose QAPT is unknown, and changing the potential to a system whose QAPT is calculable.

III. NUMERICAL DEMONSTRATION OF QAPT

We begin our numerical work by analyzing the system specified by the two-part potential:

$$V(x)=V_1\cos(x)+V_2\cos[(x-vt)/2], \tag{2}$$

where v is the velocity of the moving component. We will be using a unit system in which the spatial period is 2π, the particle mass is 1, and Planck's constant $\hbar$ is 1. The evolution of the energy bands with respect to $\phi=vt$ is plotted for the values $V_1=0.4$ and $V_2=0.2$ in Fig. 1. This is calculated by applying standard numerical eigenvalue routines to the instantaneous Hamiltonians as ϕ is varied.

From physical intuition, the average velocity of a particle in the ground band of $V(x)$ must be v, the velocity of the moving potential. The particle is trapped deep in the potential and thus is simply dragged along at the same speed. The average velocity of a particle in the first excited band can then be calculated using the above topological method.

If V_2 is continuously diminished from 0.4 to 0, so that only $V_1\cos(x)$ remains, the ground and first excited bands merge into one band, since the spatial periodicity is halved. This band of $V'(x)=V_1\cos(x)$ has zero QAPT, since $V'(x)$ has no time dependence. This then implies that the first excited band of Eq. (2) must have a QAPT velocity of $-v$, since the ground and first excited bands of Eq. (2) were split from the ground band of $V'(x)$ with zero QAPT.

To verify these QAPT predictions numerically, we integrated the time-dependent Schrödinger equation with the potential in Eq. (2) using a fourth-order Runge-Kutta routine. To satisfy the adiabatic condition, $\pi\Delta^2/\alpha\ll 1$, we used $v=\partial\phi/\partial t=0.0001$, where $\Delta\sim 0.012$ and $\partial E/\partial\phi=0.13$ was assumed. As predicted by the theory, the average velocity for the ground band particle was calculated to be 0.0001. The same procedure was then repeated for the first excited band, and again, the predicted velocity (-0.0001) was found. The importance of the adiabatic condition was also demonstrated by another trial where the velocity v was increased to 0.005. In this case, the calculated average velocity was no longer quantized.

IV. DESIGN FOR ACTUAL EXPERIMENT

The experimental counterpart for the system specified above is created using counterpropagating laser beams. The stationary component corresponds to two opposing beams at the same frequency, giving rise to a potential $V_1\cos(2k_Lx)$, where $V_1=[\hbar\Omega^2/8(\omega_L-\omega_0)]$, ω_0 being the transition frequency of the two-level atom, ω_L, k_L the laser frequency and wave numbers, and Ω the Rabi frequency, which is proportional to the square root of the laser intensity. For the moving component, two opposite beams directed at 60° off the main axis with a relative frequency difference then leads to a moving potential of $V_2\cos(k_Lx-k_Lvt)$, where $V_2=[\hbar(\Omega')^2/8(\omega_L'-\omega_0)]$. Interference between the two pairs of beams can be neglected if their frequency differences are large enough. In comparison to the dimensionless potential in Eq. (2), the variables v, V_1, and V_2 in Eq. (2) should then be scaled by $2\hbar k_L/m$, $[m/(2\hbar k_L)^2]\cdot[\hbar\Omega^2/8(\omega_L-\omega_0)]$, and $[m/(2\hbar k_L)^2][\hbar(\Omega')^2/8(\omega_L'-\omega_0)]$, respectively.

Unfortunately, the relatively low velocity required to satisfy the adiabatic condition in the simple potential specified above is experimentally difficult. However, it might be possible to observe QAPT in optical systems, by finding a potential that remains adiabatic even when moving at higher velocities. To design such a system, we utilized the form of the original potential in Eq. (2) but made the amplitude coefficients time dependent, which would be experimentally achieved by making the laser intensity time dependent:

$$V(x)=V_1(\phi)\cos(x)+V_2(\phi)\cos[(x-\phi)/2]. \tag{3}$$

The similarity of this potential to the original potential of Eq. (2) allows the theoretically predicted QAPT values to be carried over to our system. Regardless of the exact time dependence chosen for $V_1(\phi)$ and $V_2(\phi)$, as long as the gap between the ground band and first excited band remains open, the QAPT's of these two bands will be the same as if V_1, V_2 were fixed constants; i.e., v and $-v$. To decrease the amount of LZ tunneling between these two bands, the time dependence of $V_1(\phi)$ and $V_2(\phi)$ can then be chosen so that these two bands are ''flattened out,'' thus decreasing the slope $\partial E/\partial\phi$. Since the probability for LZ tunneling depends exponentially on $\alpha=\partial E/\partial t=(\partial E/\partial\phi)^*(\partial\phi/\partial t)$, this allows the velocity $v=\partial\phi/\partial t$ of the moving potential to be increased while still satisfying the adiabatic condition.

So, how exactly should the functions be chosen for $V_1(\phi)$ and $V_2(\phi)$? Bloch theory predicts that increasing V_2 will increase the gap between the ground and first excited bands, and vice versa for decreasing V_2. Similarly, adjusting V_1 will affect the size of the gap between the first and second excited bands [9]. Thus, we can control V_1 and V_2 to widen or narrow the gaps at the appropriate ϕ to achieve the desired effect of ''flattening out'' the bands.

Using this idea, we designed various potentials that remain adiabatic up to potential velocity ranges achievable today. For example, the following time dependence:

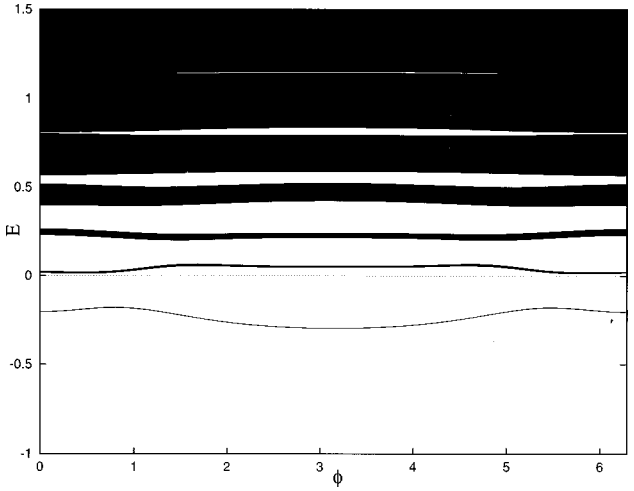

FIG. 2. Band structure of Eq. (4) where the bands are ''flatter'' and hence adiabatic enough for actual experiments. This allows a potential velocity of $v=0.03$.

$$V_1(\phi)=0.4^*[1-0.85\cos^2(\phi/2)+0.15\sin^2(\phi)],$$

$$V_2(\phi)=0.2^*[1+0.85\cos^2(\phi/2)-0.15\sin^2(\phi)] \quad (4)$$

(band structure plotted in Fig. 2) allowed the potential velocity to be increased up to 0.03 while still maintaining adiabaticity, which we defined as maintaining quantization of the average velocity within 10%, i.e., observing a particle average velocity of 0.03 ± 0.003.

Other than satisfying the adiabatic condition, one other experimental concern is the preparation of the confined atoms in the desired bands. These atoms are usually prepared in the ground band, perfect for observing the QAPT of the ground band. Yet, to observe the QAPT's of higher bands; e.g., the negative quantized velocity of the first excited band in the system $V(x)=V_1(\phi)\cos(x)+V_2(\phi)\cos[(x-\phi)/2]$, a technique is then needed to raise the particles from the ground state into the higher bands.

For this, we suggest the *use* of LZ tunneling to bring the atoms initially prepared in the ground band up into the excited band of interest. A system where significant LZ tunneling occurs can be used initially to tunnel the atoms to the desired higher band. Then, the initial potential can be switched off and joined continuously by a second potential where LZ tunneling is insignificant, thus trapping the particle in the desired band.

Consider then the following two-part potential where

$$V(x)=V_1\cos(x)+V_2\cos[(x-\phi)/2], \quad \phi<3\pi$$

$$V(x)=V_1(\phi)\cos(x)+V_2(\phi)\cos[(x-\phi)/2], \quad \phi>3\pi \quad (5)$$

$V_1(\phi)$ and $V_2(\phi)$ being the same specified in Eq. (4). Thus, if we set $v=0.03$, the initial potential will ensure that the particles tunnel into the first excited band. But then, at $\phi=3\pi$, the potential is continuously joined into the second form, which then keeps the particles in the first excited band, and QAPT for the first excited band can be observed.

V. CONCLUSION

We started by numerically verifying the phenomenon of QAPT in a simple system and demonstrated an interesting case where the forward moving potential actually causes the particle to move backwards with a negative velocity. We then designed more complex systems that make QAPT feasible experimentally with techniques currently available. Ways to maintain adiabaticity at high moving potential velocities were described, which center around reducing LZ tunneling in these systems. Yet, LZ tunneling is at the same time used to prepare atoms in the desired initial states. Hopefully, quantum adiabatic particle transport in an optical system will be realized in the near future.

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