Classical and nonclassical interference

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The relation between quantum interference and classical interference is discussed in terms of the Wigner function and an analogous classical expression. For two displaced coherent states and two displaced electric-field Gaussian pulses, both the quantum-mechanical and classical Wigner functions exhibit oscillatory behavior. Classical analogs of squeezing, photon-number oscillation, and the Q function are presented. [S1050-2947(98)08302-4]

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I. INTRODUCTION

A fundamental principle of quantum mechanics is the linear superposition principle [1]. Summation of quantum-mechanical amplitudes leads to a wide range of interference phenomena. Wave theory based on Maxwell equations leads to the linear superposition principle for the electric-field amplitudes that is the basis of all classical interference phenomena. Both in classical and quantum mechanics the linear superposition principle follows from the linearity of the corresponding wave equations. The fundamental difference between quantum and classical interference is that the particle-wave duality exhibited by quantum systems leads to interference between *probability* amplitudes rather than between physical realities such as the electromagnetic waves. This property is reflected in the duality principle that full which-way information and a perfect interference effect are mutually exclusive [2]. If the particle character or the wave character of a system is discussed, quantum and classical systems may exhibit striking similarities. The best example of the wave character is Young's double-slit experiment either for light or for massive particles.

It is well known that quantum interference involving many photons can be traced to the wave character of the electromagnetic field. Interference of a single photon with itself is a quantum-mechanical effect and requires measurements of higher-order correlations of the electric field. Effects such as photon antibunching, squeezing, or quantum nonlocality are examples of nonclassical behavior of quantum amplitudes [3].

Interesting features occur when interference phenomena are investigated for a quantum mesosocopic system exhibiting classical behavior. The Schrödinger cat paradox provides an example of such an interference between the two states of the cat [4]. Interference phenomena occur for a system radiating semiclassical fields. The best known example of such a semiclassical field is the coherent state $|\alpha\rangle$ of a single-mode electromagnetic field. The statistical properties of an arbitrary state of light can be described by the Glauber diagonal P representation [3]. For a coherent state, the P representation is just a sharp (Dirac's δ function) distribution in the coherent-state phase space. A superposition of two or more coherent states can exhibit nonclassical effects. The simplest case of such a superposition is given by a linear combination of two ''mirrorlike'' coherent states

$$|\Psi\rangle=\frac{1}{\sqrt{2N}}(|\alpha\rangle+|-\alpha\rangle). \tag{1}$$

Due to the nonorthogonality of the coherent states $\langle\alpha|-\alpha\rangle$, the normalization constant in this expression is $N=1+\exp(-2\alpha^2)$. The state (1), called the even coherent state (ECS), exhibits properties such as the reduction of quadrature fluctuations below the vacuum level and the oscillation of the photon-number distribution [5,6]. The appearance of these nonclassical features of the ECS has been attributed to the quantum interference of the two coherent states [7]. For this particular state, the Glauber diagonal P representation is extremely singular and nonpositive [8]. Linear superpositions of coherent states have been produced experimentally [9,10]. Coherent superpositions of neutron matter waves have been produced for particle-wave interferometry [11].

For large values of the mean photon excitation, $\overline{n}=\alpha^2\gg 1$, the coherent states represent localized Gaussian wave packets. The natural classical analog of the state given by Eq. (1) is a linear superposition of two spatial wave packets described in one dimension by complex electric-field amplitudes

$$E(x)=E_1(x)+E_2(x). \tag{2}$$

We shall ignore effects related to time dependence and polarization. This linear superposition of two or more electric fields exhibits classical interference very similar to the interference of coherent states. Although the classical field contains many plane-wave components in contrast to the single-mode of the coherent states, it has been emphasized that

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superpositions of single mode coherent states are best for seeing the main ideas of the superposition principle [7].

It is the purpose of this paper to study the relation between quantum interference effects of the ECS and classical interference of the two electric fields. We shall show that there are similarities between these two types of interference effects. The nonclassical features of the ECS such as squeezing and photon-number oscillation will have very simple classical analogies in the framework of destructive interference of classical waves. Although it makes no sense to talk about the Glauber diagonal P representation for classical waves, it is possible to formulate a Wigner representation for classical waves. Using standard paraxial optics, we shall derive the classical counterpart of the positive Q representation and show its relation to the interference effects.

This paper is organized as follows. In Sec. II the classical analog of the Wigner function (classical Wigner function) is presented, while the similarity of the quantum-mechanical Wigner function for the ECS and the classical Wigner function for two displaced Gaussian pulses is discussed in Sec. III. Section IV contains a classical analog of squeezing. Analogies of the Q distribution and photon-number oscillation are exhibited in Sec. V. The results are summarized and discussed in Sec. VI.

II. WIGNER FUNCTION

In this section we present a Wigner function approach to the description of classical and quantum interference. We shall start with the original Wigner definition [12] of the phase-space quasiprobabilty distribution for the wave function in one-dimensional configuration space:

$$W_{\psi}(x,p)=\int\frac{d\xi}{2\pi}\psi^{*}\left(x+\frac{\xi}{2}\right)e^{ip\xi}\psi\left(x-\frac{\xi}{2}\right). \tag{3}$$

There is an extensive literature devoted to the properties and various definitions of the Wigner function [13]. We discuss only the most relevant properties needed for the purpose of this paper. The marginals of the Wigner function yield the probability densities in configuration or momentum space. The Wigner function plays the role of a phase-space probability distribution

$$\int dx\int dp W_{\psi}(x,p)=1, \tag{4}$$

but cannot be guaranteed to be positive. In fact, in one-dimension the Wigner function (3) is positive everywhere if and only if the wave function is Gaussian [14]. An important result that we shall use in the following sections is the overlap relation for two Wigner functions. This theorem relates the scalar product to the overlap of two Wigner functions

$$\frac{1}{2\pi}|\langle\psi_1|\psi_2\rangle|^2=\int dx\int dp W_{\psi_1}(x,p)W_{\psi_2}(x,p). \tag{5}$$

Because the Wigner function is bilinear in the wave function, it can be used to transparently exhibit the quantum interference contribution. For a linear superposition $|\psi_1\rangle+|\psi_2\rangle$, the corresponding Wigner function is $W_{\psi_1}+W_{\psi_2}+W_{int}$, where the last term describes, in phase space, the quantum interference between the two probability amplitudes $\langle x|\psi_1\rangle$ and $\langle x|\psi_2\rangle$. The free evolution of a Wigner function during time t can be obtained applying a Galilean boost to the Wigner function at $t=0$:

$$W_{\psi}(x,p;t)=W_{\psi}(x-pt,p;t=0). \tag{6}$$

Expression (3) may be used in a similar way to construct a Wigner function for an electric-field amplitude E. One can perform such a construction in either time and frequency phase space or in position and wave-vector phase space [15,16]. For the purpose of this paper, we choose to work with the position x and wave-vector k Wigner representation. In full analogy to formula (3), in one dimension and for a complex scalar electric field $E(x)$, the classical Wigner representation has the form

$$W_E(x,k)=\int\frac{d\xi}{2\pi}E^{*}\left(x+\frac{\xi}{2}\right)e^{ik\xi}E\left(x-\frac{\xi}{2}\right). \tag{7}$$

From this definition, it follows that the marginals of the Wigner function yield the energy density $I(x)$ in space and the energy density $\tilde{I}(k)$ in the wave-vector space:

$$\int dk W_E(x,k)=E^{*}(x)E(x)=I(x),$$

$$\int dx W_E(x,k)=\frac{1}{2\pi}\tilde{E}^{*}(k)\tilde{E}(k)=\tilde{I}(k). \tag{8}$$

In this formula $\tilde{E}(k)=\int dx e^{-ixk}E(x)$ is the Fourier transform of the complex electric-field amplitude. The intensities (8) are, of course, different from the marginals of the quantum-mechanical Wigner function. In the latter case, c-number equivalents of the intensity operators and integration over both phase-space variables would be required.

Again in full analogy to the quantum-mechanical case (3), the Wigner function (7), being bilinear in the electric-field amplitudes, leads to a term that corresponds to a phase-space description of classical interference. For a linear superposition of two electric fields (2), the corresponding classical Wigner function is $W_{E_1}+W_{E_2}+W_{int}$, where the last term describes classical interference between the two electric fields E_1 and E_2.

III. QUANTUM VERSUS CLASSICAL INTERFERENCE

The Wigner function for a linear superposition of two coherent states (1) has been calculated and discussed in several of the previous references. For simplicity, we shall assume a purely real α. In configuration space, the parameter $D=2\sqrt{2}\alpha$ plays the role of a ''distance'' (in suitably selected dimensionless units) between the two coherent states $|\alpha\rangle$ and $|-\alpha\rangle$ (each having a mean number of oscillator quanta $\overline{n}=\alpha^2$). The two ''mirror'' states are said to be macroscopically distinguishable if the uncertainty regions of the two coherent states do not overlap. This is equivalent to the condition that $D\gg\sqrt{2}$. This last condition follows from the fact that the uncertainty of each of the coherent states in configuration space is of the order $1/\sqrt{2}$. The Wigner function corresponding to the ECS (1) is

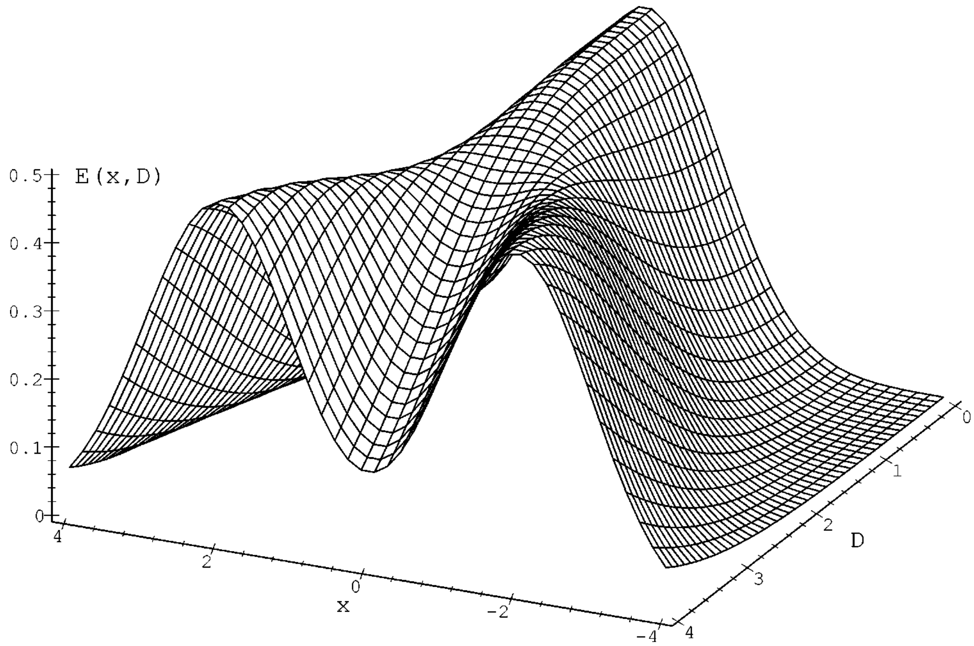

FIG. 1. Plot of the electric field $E(x)$ for the two Gaussian pulses as a function of D.

$$W_\psi(x,p) = \frac{W_0\left(x-\frac{D}{2},p\right)+W_0\left(x+\frac{D}{2},p\right)+2W_0(x,p)\cos(Dp)}{2\left[1+\exp\left(-\frac{D^2}{4}\right)\right]}, \tag{9}$$

where $W_0(x,p)=(1/\pi)e^{-x^2-p^2}$ is the Wigner function of the vacuum state $|0\rangle$ in dimensionless parameters x and p. This Wigner function of the linear superposition is not positive due to the last term that describes quantum interference. In the limit of $D=0$, the quantum state (1) is just a harmonic-oscillator vacuum state and all quantum interference effects are gone. Much has been said about the negative features of this Wigner function. In most cases the nonpositive character of this function has been associated with the nonclassical character of the ECS (1). In this paper we shall stress the relation between the nonpositive Wigner function and the interference for both the quantum-mechanical and classical cases.

In fact, below we show that a nonpositive Wigner function describes classical interference as well. We calculate the interference effects associated with a linear superposition of two electric pulses in a way similar to that of the quantum calculation. Let us imagine that in an optical resonator we superpose two Gaussian beams $E_0(x)=\exp(-x^2/2)$ spatially separated by a distance D,

$$E(x)=E_0\left(x-\frac{D}{2}\right)+E_0\left(x+\frac{D}{2}\right). \tag{10}$$

The two electric fields have equal beam waists equal to $\sqrt{2}$ in dimensionless units. These two beams are said to be distinguishable if the Gaussian packets do not overlap. In Fig. 1 we have plots of the two beams as a function of the distance separation. A clear spatial separation of the two beams is obtained if $D\gg 1$. In the limit of $D=0$, the electric field is just a Gaussian beam located at the origin. The classical Wigner function (7) for such a linear superposition of two Gaussian beams is

$$W_E(x,k) = \frac{W_0\left(x+\frac{D}{2},k\right)+W_0\left(x-\frac{D}{2},k\right)+2W_0(x,k)\cos(Dk)}{2\left[1+\exp\left(-\frac{D^2}{4}\right)\right]}, \tag{11}$$

where $W_0(x,k)=(1/\pi)\exp(-x^2-k^2)$ is the Wigner function of a Gaussian beam E_0 and the normalization has been fixed by the requirement that the total intensity is set to one in arbitrary units, i.e., $\int dxI(x)=1$.

The two formulas (9) and (11) are both obtained for single-mode fields, are identical in form, and will exhibit some striking physical similarities. The quantum and the classical Wigner functions are nonpositive, with the negative contributions arising from the interference terms. In Fig. 2 we have plotted the Wigner function (11) for the two Gaussian pulses.

The frequency of the oscillating term can be easily understood both classically and quantum mechanically if the Wigner function of a linear superposition of two plane waves e^{ik_1x} and e^{ik_2x} or two sharp pulses $\delta(x-x_1)$ and $\delta(x-x_2)$ is investigated. For the two plane waves the Wigner function is

$$W_E(x,k)\sim\delta(k-k_1)+\delta(k-k_2) + 2\,\delta\left(k-\frac{k_1}{2}-\frac{k_2}{2}\right)\cos[(k_1-k_2)x]. \tag{12}$$

For the two sharp pulses the Wigner function takes the form

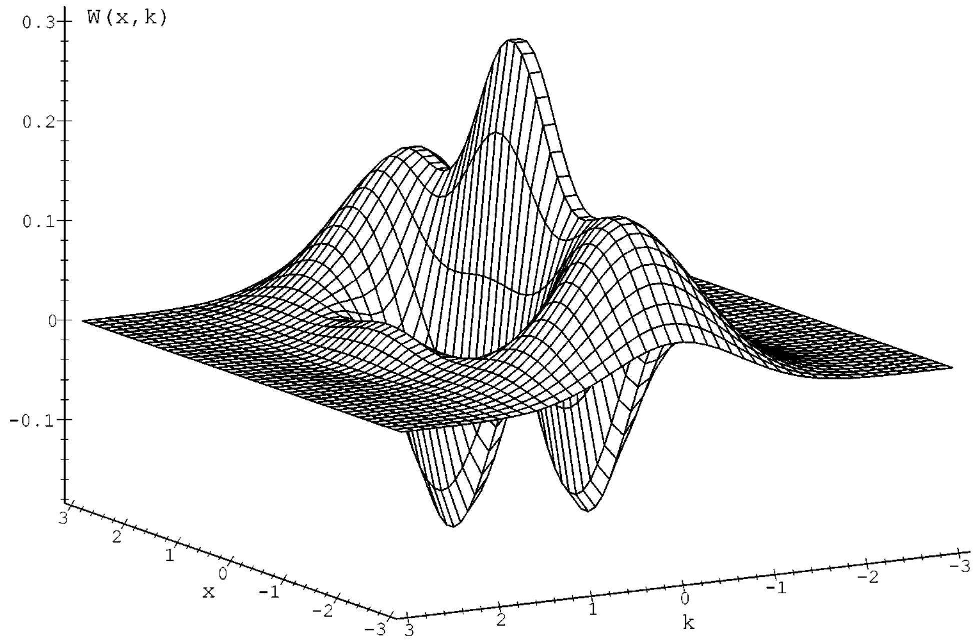

FIG. 2. Plot of the Wigner function $W(x,k)$ for the two Gaussian pulses.

$$W_E(x,k) \sim \delta(x-x_1)+\delta(x-x_2)+2\,\delta\left(x-\frac{x_1}{2}-\frac{x_2}{2}\right) \times \cos[(x_1-x_2)k]. \tag{13}$$

These two formulas exhibit a simple x and k symmetry. The first two terms describe two localized distributions in k or x space located at k_1 and k_2 or x_1 and x_2. The last terms are nonpositive and describe the interference effects of the electric fields. For two plane waves the interference term of the Wigner function is located at the mean frequency $(k_1+k_2)/2$ and oscillates in x with a frequency inverse to the wave-vector separation k_1-k_2. For two spatially sharp pulses the interference term of the Wigner function is located at the mean position $(x_1+x_2)/2$ and oscillates in k with a frequency inverse to the spatial separation $D=x_1-x_2$. Both Wigner functions (9) and (11) have an oscillating nonpositive interference term that oscillates with frequency $1/D$. This property results from the linear superposition principle and the bilinear character of the Wigner function. The structure of the interference term is the same for both the classical and the quantum superpositions.

A coherent Gaussian beam illuminating a double-slit setup produces a phase-space Wigner function given by Eq. (11). A time-resolved evolution (6) of the Wigner function given by Eq. (9) has been measured for a coherent beam of thermal helium atoms in a double-slit experiment [17].

IV. CLASSICAL SQUEEZING

In this section we shall show that squeezing of quadrature fluctuations for the quantum ECS (1) has a corresponding classical analogy in terms of the destructive interference in k space. We show that this destructive interference reduces the k-vector bandwidth below the original Gaussian beam waist. This provides a classical version of squeezing with the interpretation that the reduction of the bandwidth is entirely due to the destructive interference of the two Gaussian beams. For classical fields, the spatial waist Δx and k-vector bandwidth Δk can be defined, respectively, as statistical spreads of the energy densities $I(x)$ and $\tilde{I}(k)$. In terms of the Wigner function (7) these beam waists are given by the formulas

$$(\Delta x)^2=\int dx\int dk x^2 W(x,k),$$

$$(\Delta k)^2=\int dx\int dk k^2 W(x,k). \tag{14}$$

We have normalized the total intensity of the electric field to unity in these definitions. For an electric field (10) with $D=0$ we obtain that the beam waist is $\Delta x=1/\sqrt{2}$ and that the wave vector spread is $\Delta k=1/\sqrt{2}$ in dimensionless units. These results satisfy the Fourier uncertainty relation

$$\Delta x \Delta k=\frac{1}{2} \tag{15}$$

which is similar to the Heisenberg uncertainty relation for a single coherent state that is a minimum-uncertainty state. The Fourier uncertainty relation for the electric fields results from the wave character of the pulses. Let us apply formulas (14) to the linear superposition of the two Gaussian beams (10) for an arbitrary D. Simple integrations lead to

$$(\Delta x)^2=\frac{1}{2}+\frac{D^2}{4}\,\frac{1}{1+e^{-D^2/4}}, \quad (\Delta k)^2=\frac{1}{2}-\frac{D^2}{4}\,\frac{e^{-D^2/4}}{1+e^{-D^2/4}}. \tag{16}$$

We see that the k-space spread is reduced (''squeezed'') below the beam waist of the original Gaussian beam waist. The minimum value of the k-space spread corresponds to the maximum squeezing $(\Delta k)^2_{min}=0.221\ldots$, which is almost half of the original value. This squeezing is due to a destructive interference of the two waves (10) in a way that is identical in form to quadrature squeezing of the ECS. In k space the superposition has a narrower waist compared to the bandwidth of a single beam.

V. PHASE-SPACE OVERLAPS

Phase-space overlap techniques have been developed and used in order to establish a connection between interference effects and the Wigner function [18]. In this section we shall provide a classical description of the direct overlap property of the Wigner function (5) using a simple example from Fourier optics.

Let us assume that an arbitrary electric field $E(x,z=0)$ is propagated paraxially along the z axis through a thin lens with a focal length f. At $z=0$, where the field starts to propagate, there is a mask or filter device with transmittance $t(x_0-x)\sim\exp[-(x_0-x)^2/2]$ centered around a point x_0. The intensity of the electric field is recorded at a screen placed at the focal length $z=f$. In the paraxial approximation, the electric field at the screen is proportional to $\int dx'\exp(-iux')t(x_0-x')E(x')$, where $u=kx/f$ is the so-called spatial frequency [19]. As a result of this propagation through a Fourier device, the intensity recorded at the screen depends on the spatial frequency u and the position x_0 where the mask is centered. This intensity provides a phase-space record of the incident field probed by a filtering mask function. This is a typical measurement in Fourier optics. As a result of this, the intensity recorded on the screen is a two-parameter function

$$I(x_0,u)\sim\left|\int dx' e^{-iux'}t(x_0-x')E(x')\right|^2. \tag{17}$$

Using simple manipulations of the Fourier integrals, we can rewrite this expression in the form

$$I(x,u)=\int dx'\int du' W_t(x-x',u-u')W_E(x',u'), \tag{18}$$

where conveniently we have denoted $x=x_0$ and used a normalization $\int dx\int du I(x,u)=1$. This expression has a very simple geometrical interpretation in terms of a phase-space overlap. The recorded intensity is just a double convolution of the Wigner function of the electric field and the Wigner function corresponding to the mask shifted in phase space by u and x. This is a remarkable expression because it gives for a Gaussian filter function a classical example of the so-called Husimi distribution [20]. The Husimi distribution function is positive everywhere and results from a phase-space smoothing of the Wigner function. In quantum optics, the Husimi distribution with a smoothing function selected to be the wave function of a coherent state reduces to Glauber's positive Q function. For such a Gaussian mask function and the superposition (10), the intensity (18) has the form

$$I(x,u)=\frac{I_0\left(x+\frac{D}{2},u\right)+I_0\left(x-\frac{D}{2},u\right)+2e^{-D^2/8}I_0(x,u)\cos\left(\frac{Du}{2}\right)}{2\left[1+\exp\left(-\frac{D^2}{4}\right)\right]}, \tag{19}$$

where $I_0(x,u)=(1/2\pi)\exp(-x^2/2-u^2/2)$ is the intensity of a single Gaussian beam located at the origin with the normalization $\int dx\int du I_0(x,u)=1$. This function is an example of a classical Q distribution for a linear superposition of two Gaussian beams. An identical function is obtained if the quantum Q distribution is calculated for the ECS (1). Note that the distribution (18) is positive everywhere with the interference term oscillating with a frequency that is twice the frequency of the Wigner function. In Fig. 3 we have plotted the intensity (19) for the two Gaussian pulses. The doubling of the oscillation frequency can be easily understood, remembering that the Q function is a smoothed Wigner function with a Gaussian distribution. The interference term was making the Wigner function nonpositive. The interference term turns the Q function to zero at points $u_n=(2\pi/D)(1+2n)$ for $n=0,1,\ldots$ along the u axis. This means that the above-described device, based on simple principles of Fourier optics, allows, at least in principle, the detection of the interference pattern. This interference pattern amounts to a series of points with a vanishing intensity $I(0,u)$ at the screen locations corresponding to spatial frequencies u_n. In Fig. 3 we have plotted $I(x,u)$ for $D=4$.

If the intensity is measured just at the origin of the phase space, i.e., at $x=0$ and $u=0$, we obtain that

$$\frac{1}{2\pi}I(0,0)=\left|\int dx t^*(x)E(x)\right|^2. \tag{20}$$

The intensity $I(0,0)$ is the perfect overlap of the field Wigner function with the mask Wigner function. This integral is just a classical scalar product involving a perfect overlap of the transmittance function t with the electric-field amplitude E. This is the classical version of the quantum overlap (5).

We shall use this expression to exhibit a classical version of photon-number oscillation for the linear superposition (1). For this purpose we shall assume that a one-dimensional optical resonator holds two higher-order Gaussian beam modes. In this case the total electric field is

$$E(x)=E_n\left(x-\frac{D}{2}\right)+E_n\left(x+\frac{D}{2}\right), \tag{21}$$

where the higher-order Gaussian wave packets are $E_n(x)=H_n(x)\exp(-x^2/2)$, with $H_n(x)$ being the Hermite polyno-

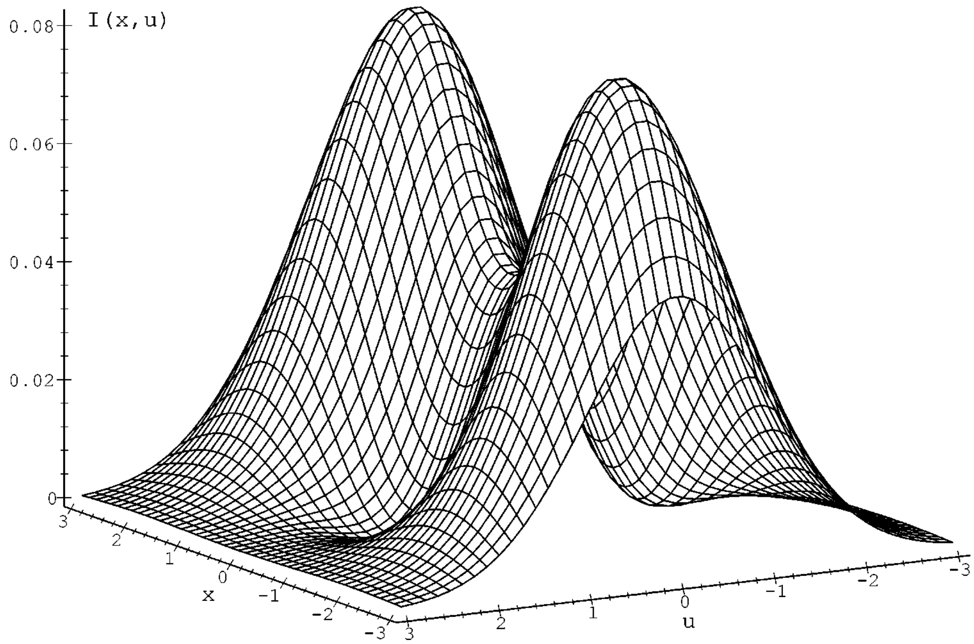

FIG. 3. Plot of the intensity $I(x,u)$ for the two Gaussian pulses.

mials with $n=0,1,\ldots$ [21]. Note that for $n=0$ we recover the expression (21) that corresponds to the ''zeroth-order'' Gaussian mode. The higher-order Gaussian beam modes will play the classical role of the n-photon states. For such an initial electric field probed by a Gaussian transmittance all the integrals in expression (18) can be calculated and as a result we obtain

$$I(0,0)\sim e^{-D^2/8}\left(\frac{D^2}{8}\right)^{2n}[1+(-1)^n]^2. \tag{22}$$

For a coherent state $\overline{n}=D^2/8$ this intensity is just

$$I(0,0)\sim e^{-\overline{n}}(\overline{n})^{2n}[1+(-1)^n]^2, \tag{23}$$

i.e., up to a normalization factor, it is equivalent to the photon-number probability for the ECS. In the case of the classical intensity, the oscillations in the above expression result from the destructive interference.

VI. CONCLUSIONS

The superposition principle is valid in both quantum mechanics and classical electrodynamics because of the linearity of the underlying equations. Interference phenomena occur when bilinear or higher-order nonlinear quantities are considered. The Wigner function is bilinear and provides a complete description of a quantum-mechanical system. An analogous quantity, the classical Wigner function, has been constructed for two classical electromagnetic pulses.

For the ECS and two displaced Gaussian pulses, both the quantum-mechanical and classical Wigner function have been shown to be the same in form, are nonpositive, and have an oscillatory term with a common frequency describing interference.

The approach has been used to show that there is a classical analogy to the squeezing of the quadrature fluctuations of the ECS. For a single Gaussian pulse, the position space and wave-vector space bandwidths are equal. As a result of the interference between two displaced pulses, the former bandwidth is increased, while the latter is decreased. Phase-space overlap techniques have been used to derive an expression for the classical analog of Glauber's Q function. Being an intensity in position and spatial-frequency phase space, it can be measured in a Fourier optics experiment. For the case of two higher-order Gaussian beam pulses, measurement of the intensity at the phase-space origin can be used to observe a classical analogy to photon-number oscillation. In conclusion, it has been shown that the classical Wigner function approach to classical interference reveals remarkable analogs of quantum-mechanical phenomena.

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[1] P. A. M. Dirac, *The Principles of Quantum Mechanics*, 4th ed. (Oxford University Press, Oxford, 1958).

[2] B.-G. Englert, Phys. Rev. Lett. **77**, 2154 (1996).

[3] See, for example, L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics* (Cambridge University Press, Cambridge, 1995).

[4] E. Schrödinger, Naturwissenschaften **23**, 807 (1935); **23**, 823 (1935); **23**, 844 (1935).

[5] W. Schleich, M. Pernigo, and F. Le Kien, Phys. Rev. A **44**, 2172 (1991).

[6] V. Bužek, A. Vidiella-Barranco, and P. L. Knight, Phys. Rev. A **45**, 6570 (1992).

[7] For a review see V. Bužek and P. L. Knight, in *Progress in Optics XXXIV*, edited by E. Wolf (North-Holland, Amsterdam, 1995).
[8] S. Ya. Kilin and V. N. Shatokhin, Phys. Rev. Lett. **76**, 1051 (1996).
[9] M. W. Noel and C. R. Stroud, Phys. Rev. Lett. **77**, 1913 (1996).
[10] M. Brune *et al.*, Phys. Rev. A **45**, 5193 (1992).
[11] P. B. Lerner, H. Rauch, and M. Suda, Phys. Rev. A **51**, 3889 (1995).
[12] E. Wigner, Phys. Rev. **40**, 749 (1932).
[13] See, for example, V. I. Tatarskij, Usp. Fiz. Nauk **139**, 587 (1983) [Sov. Phys. Usp. **26**, 311 (1983)]; M. Hillery, R. F. O'Connell, M. O. Scully, and E. P. Wigner, Phys. Rep. **106**, 121 (1984).
[14] R. L. Hudson, Rep. Math. Phys. **6**, 249 (1974).
[15] J. Ville, Cables Transm. **1**, 61 (1948); T. A. C. M. Classen and W. F. G. Mecklenbräuker, Philips J. Res. **35**, 217 (1980).
[16] K.-H. Brenner and K. Wódkiewicz, Opt. Commun. **43**, 103 (1982).
[17] Ch. Kurtsiefer, T. Pfau, and J. Mlynek, Nature (London) **386**, 150 (1997).
[18] J. A. Wheeler and W. Schleich, J. Opt. Soc. Am. B **4**, 1715 (1987); W. Schleich *et al.*, in *New Frontiers in Quantum Electrodynamics and Quantum Optics*, edited by A. O. Barut (Plenum, New York, 1990).
[19] See, for example, J. W. Goodman, *Introduction to Fourier Optics* (McGraw-Hill, New York, 1968).
[20] K. Husimi, Proc. Phys. Math. Soc. Jpn. **22**, 264 (1940).
[21] See, for example, P. W. Milonni and J. H. Eberly, *Lasers* (Wiley, New York, 1988).