Pure states in a one-atom micromaser

N. Nayak
S. N. Bose National Centre for Basic Sciences, Block JD, Sector 3, Salt Lake City, Calcutta 700 091, India
(Received 2 September 1997)

The possibilities of observing disentanglement of atomic and field states during their interaction are discussed in the framework of one-atom micromaser action. We find that, in certain cases, the atom can be in a pure state and the cavity field may be in an approximate number state. [S1050-2947(98)03001-7]

PACS number(s): 42.50.Dv, 32.80.−t

The Jaynes-Cummings model [1] involving the interaction between a single two-level atom and a single-mode radiation field has been studied extensively, as this simple system exhibits unique properties, the experimental demonstration of which would answer some fundamental questions in quantum mechanics. One such property is the quantum revival of the atomic population when the interacting cavity field is initially in a coherent state. This has been demonstrated experimentally in a micromaser setup [2] consisting of a superconducting microwave cavity into which two-level Rydberg atoms are sent at such a rate that at most one atom is present in the cavity at any time. A preassigned interaction time for each atom is maintained by a velocity selector. In addition to this quantum revival, it has been predicted by Gea-Banacloche [3] that the atom and field get disentangled in the middle of the collapse periods, that is, at these times the atom and the field are both in pure states. Further, the generation of such pure states is independent of the intial state of the atom. Gea-Banacloche's conclusions are for a cavity field in a coherent state having large average photon numbers. These results have been examined in Refs. [4,5], where it has been indicated that the atom and the field are in pure states ''only'' at the middle of the first collapse in the atomic population.

Thus the disentanglement, explained above, can be explored in the present micromaser setup [6] in the same way the atomic revival was demonstrated in Ref. [2]. In the experiment [2], atoms in the upper of the two levels concerned and having a fixed interaction time with the cavity field were sent and the state of the atom was measured after the cavity field attained a steady state. The process was repeated with varying interaction times to simulate the dynamics of the Jaynes-Cummings interaction [1] and the steady-state measurements showed a revival in the atomic population. These observations needed confirmation since the quantum revival was originally thought to be carrying a signature of the coherent-state cavity field [1]. The analysis in Ref. [7] shows that the steady-state micromaser field [8,9], unlike the field in a coherent state, produces one revival in the atomic population that was observed experimentally [2]. Likewise, the predictions of pure states in Refs. [3–5] carry signatures of a coherent state or a Fock state field. However, the steady-state cavity fields at various atom-field interaction times, necessary to reproduce the dynamics in the time scale, can be in different states. It is interesting to analyze if pure states can still be generated in such situations. This is the only way one can verify the predictions in Refs. [3–5] experimentally. We show below that the generation of pure states of the atom is possible but not at the times assigned in Refs. [3–5]. Further, we show that at times the atom is in a pure state, the radiation field can be in an approximate vacuum state.

For the present purpose, we follow the micromaser theory in Ref. [10], instead of Refs. [8,9], since it includes the effect of the cavity reservoir during the entire repetition cycle $t_c = \tau + t_{cav}$, where τ is the atom-field interaction duration and t_{cav} is the time when the cavity does not contain any atoms. It has been shown in Ref. [10] that the effect of the cavity reservoir during τ is crucial for the micromaser action even though the condition is $\tau \ll t_{cav}$. Also, the theory [10] includes the reservoir-induced atomic decay from the upper to the lower masing level. In the micromaser action, this decay factor becomes important due to the Purcell factor compared to the decay of masing levels to low-lying levels. Thus the theorey in Ref. [10] can give a firm prediction of the generation of a pure state (atom or field), as this includes the influence of decohering effects on the coherent atom-field interaction.

Reference [10] deals with a cavity into which atoms in their upper masing level are pumped at a rate $R = t_c^{-1}$ and gives the steady-state cavity field photon statistics P_n as

$$P_n = P_0 \prod_{m=1}^{n} v_m . \tag{1}$$

P_0 is obtained from the normalization $\Sigma_{n=0}^{\infty} P_n = 1$. v_n is given by the continued fraction

$$v_n = f_3^{(n)} / (f_2^{(n)} + f_1^{(n)} v_{n+1}), \tag{2}$$

with $f_1^{(n)} = (Z_n + C_n)/\kappa$, $f_2^{(n)} = -2N + (Y_n + B_n)/\kappa$, and $f_3^{(n)} = -(X_n + A_n)/\kappa$. κ is the cavity bandwidth and $N = R/2\kappa$ is the number of atoms passing through the cavity in a photon lifetime. The average blackbody photons $\overline{n}_{th}$ in the cavity at an equilibrium temperature enters the photon statistics through $A_n = 2n\kappa \overline{n}_{th}$, $B_n = -2\kappa(n + \overline{n}_{th} + 2n\overline{n}_{th})$, and $C_n = 2(n+1)(\overline{n}_{th} + 1)\kappa$. X_n, Y_n, and Z_n are given by

$$X_n = R \sin^2(g\sqrt{n}\tau) \exp\{-[\gamma + (2n-1)\kappa]\tau\},$$

$$\begin{aligned} Y_n = \tfrac{1}{2} R \big([2\cos^2(g\sqrt{n+1}\tau) - \tfrac{1}{2}(\gamma/\kappa + 2n + 1) + F_1(n-1)] \\ \times \exp\{-[\gamma + (2n+1)\kappa]\tau\} + [\tfrac{1}{2}(\gamma/\kappa + 2n + 1) \\ - F_2(n-1)] \exp\{-[\gamma + (2n-1)\kappa]\tau\} \big), \end{aligned}$$

and

$$Z_n = \tfrac{1}{2}R(\left[\tfrac{1}{2}(\gamma/\kappa+2n+3)+F_2(n)\right] \times \exp\{-[\gamma+(2n+1)\kappa]\tau\} - \left[\tfrac{1}{2}(\gamma/\kappa+2n+3) + F_1(n)\right]\exp\{-[\gamma+(2n+3)\kappa]\tau\}),$$

where g is the coupling constant of the Jaynes-Cummings interaction [1] and the decay constant γ represents reservoir-induced spontaneous emission from the upper to the lower masing level. The functions F_1 and F_2 are

$$F_i(n) = \frac{\kappa/4g}{(\sqrt{n+2}-\sqrt{n+1})^2}\Bigg[\frac{\gamma}{\kappa}(\sqrt{n+2} - \sqrt{n+1})\sin(2g\sqrt{m}\tau) - \frac{\gamma}{g}\cos(2g\sqrt{m}\tau) - [2n+3 + 2\sqrt{(n+1)(n+2)}] \times (\sqrt{n+2}-\sqrt{n+1})\sin(2g\sqrt{m}\tau)\Bigg] + \frac{\kappa/4g}{(\sqrt{n+2}+\sqrt{n+1})^2}\Bigg[\pm\frac{\gamma}{\kappa}(\sqrt{n+2}+\sqrt{n+1}) \times \sin(2g\sqrt{m}\tau) - \frac{\gamma}{g}\cos(2g\sqrt{m}\tau) \mp [2n+3-2\sqrt{(n+1)(n+2)}](\sqrt{n+2} + \sqrt{n+1})\sin(2g\sqrt{m}\tau)\Bigg],$$

where $m=n+2$ and $n+1$ for $i=1$ and 2, respectively, with the upper sign for $i=1$.

At the end of the duration τ, the probabilities of the atom being in upper and lower states are, respectively (in other words, the probabilities of the atomic states at the exit of the cavity),

$$p_a(\tau) = \sum_{n=0}^{\infty} P_n \cos^2(g\sqrt{n+1}\tau)\exp-[\gamma+(2n+1)\kappa]\tau + \frac{1}{2}\sum_{n=0}^{\infty} P_{n+1}\left\{\left[\frac{1}{2}\left(\frac{\gamma}{\kappa}+2n+3\right)+F_2(n)\right] \times \exp-[\gamma+(2n+1)\kappa]\tau - \left[\frac{1}{2}\left(\frac{\gamma}{\kappa}+2n+3\right) + F_1(n)\right]\exp-[\gamma+(2n+3)\kappa]\tau\right\} \tag{3}$$

and

$$p_b(\tau) = \sum_{n=0}^{\infty} P_{n-1}\sin^2(g\sqrt{n}\tau)\exp-[\gamma+(2n-1)\kappa]\tau + \frac{1}{2}\sum_{n=0}^{\infty} P_n\left\{\left[\frac{1}{2}\left(\frac{\gamma}{\kappa}+2n+1\right)-F_2(n-1)\right] \times \exp-[\gamma+(2n-1)\kappa]\tau - \left[\frac{1}{2}\left(\frac{\gamma}{\kappa}+2n+1\right) - F_1(n-1)\right]\exp-[\gamma+(2n+1)\kappa]\tau\right\}, \tag{4}$$

where P_n *is now the steady-state photon statistics for that particular interaction time* τ. The time evolutions of p_a and p_b are a result of the simultaneous action of the atom-field interaction and the dissipative forces. The influence of the decohering effects on the dynamics becomes very clear if we compare Eq. (3) with the expression for p_a in Ref. [7]. Thus the solutions in the present paper can give a better picture of the generation of pure states and would be close to the experimental situation.

We can examine the purity of a state by studying its entropy. The cavity field in a steady state, given by Eq. (1), is diagonal in the photon-number basis and thus we can write the entropy of the cavity field [11–13]

$$S_f = -\sum_{n=0}^{\infty} P_n \ln P_n. \tag{5}$$

The Shannon entropy is the same as the thermodynamic entropy in the present case [11,12]. The cavity field is always coupled to its reservoir. Hence we should have, according to Araki and Lieb [13], the inequality

$$|S_f - S_R| \leqslant S \leqslant S_f + S_R,$$

where S_R is the cavity reservoir entropy and S is the total entropy of the two interacting systems, that is, the cavity field and its reservoir. As the cavity is in thermal equilibrium, represented by the blackbody average photon number $\overline{n}_{th}$, we can write

$$S_R = \ln(1+\overline{n}_{th}) + \overline{n}_{th}\ln(1+\overline{n}_{th}^{-1}); \tag{6}$$

thus, for a cavity at very low temperature such that $\overline{n}_{th} \cong 0$, we can have

$$S \cong S_f, \tag{7}$$

since $S_R \cong 0$ in such a situation.

Similar arguments can be followed in the case of the atom and its reservoir. However, it has been noted [10] that with the choice of the atomic flight time τ through the cavity and the lifetime of the Rydberg levels in the experiments [2,6], we can safely ignore the influence of the atomic reservoir. As the atomic probabilities are a directly measurable quantity in the micromaser experiments, it would be appropriate to examine $\mathrm{Tr}(\rho_{atom}^2)$ to see if a pure state can be generated. We know that the Jaynes-Cummnings Hamiltonian [1] has been derived using the rotating-wave approximation. In the micromaser action, the interaction using this Hamiltonian is re-

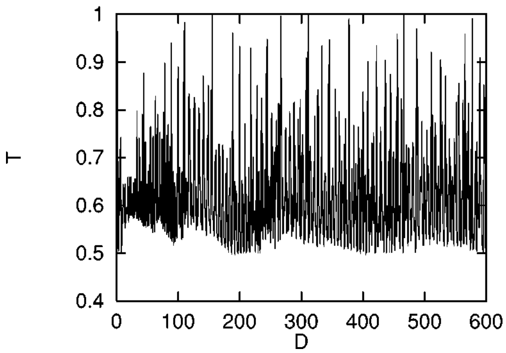

FIG. 1. Variation of T as a function of D for $\kappa/g=0.1\times10^{-5}$, $\overline{n}_{th}=0.1\times10^{-4}$, $\gamma=0$, and $N=50.0$.

peated with the very first atom interacting with a thermal field that is diagonal in photon-number state representation. The present paper considers atoms in the upper of its two masing levels. Thus the cavity radiation field always remains diagonal in the number state basis resulting in the steady state given by Eq. (1). Thus the composite atom-field states are restricted to $|a,n\rangle$ and $|b,n+1\rangle$, where a and b are upper and lower masing levels, respectively, with n being arbitrary. This leads to

$$T\equiv\mathrm{Tr}(\rho_{atom}^2)=p_a^2+p_b^2. \tag{8}$$

$T=1$ indicates that the atom is in a pure state.

We now examine S and T as functions of τ, $\overline{n}_{th}$, κ/g, and the pump parameter N. We find that the disentanglement of atomic and field states is possible for $\kappa/g\leqslant0.000\,001$ and $\overline{n}_{th}\leqslant0.000\,01$. We present these results in Figs. 1 and 2 for $N=50.0$. The atom is almost in pure states for $D=\sqrt{N}g\tau=155.5$, 267.0, and 311.0 and the corresponding values of gt are 6.997π [case (a)], 12.014π [case (b)], and 13.994π [case (c)], respectively. The condition for the field in the trapped vacuum state is to set gt equal to an integral multiple of π [10]. Indeed, we find the fields in states with the probabilities of vacuum states $P_0=0.999\,900\,25$ [case (a)] and $P_0=0.998\,925\,7$ [case (c)], whereas for case (b), the field is in a state with $P_0=0.839\,241\,2$ and $P_1=0.152\,420\,28$, as we see that gt is slightly different from the trapping condition. Also in cases (a) and (c) the values of gt are not exactly at

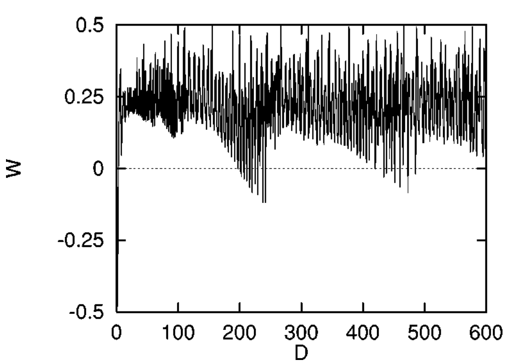

FIG. 2. Time evolution of the atomic population inversion $W=(p_a-p_b)/2$. The other parameters are the same as in Fig. 1.

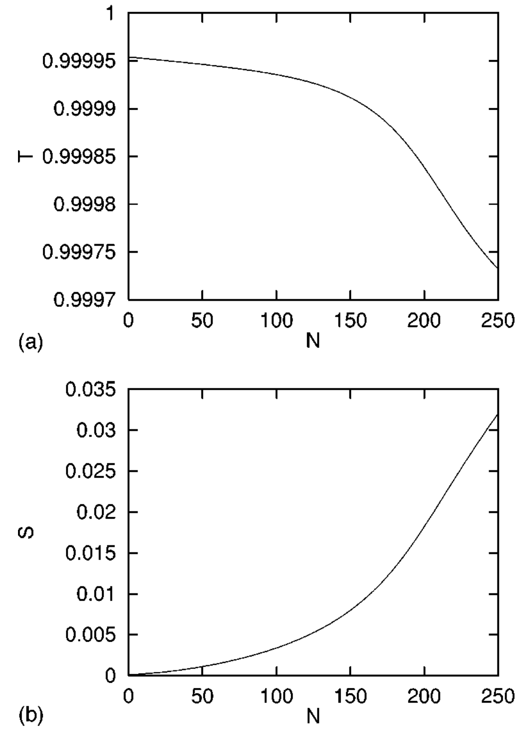

FIG. 3. (a) Evolution of atomic pure state as a function of N. The atomic flight time through the cavity is set at $gt=6.997\pi$. The cavity bandwidth and temperature are the same as in Fig. 1. (b) Variation of the radiation field entropy S as a function of N for the same situation as in (a).

the trapping conditions, but are very close to it. This is solely due to the presence of dissipation in the micromaser dynamics.

It was shown in Refs. [3–5] that disentanglements occur in the middle of collapse regions in the Jaynes-Cummings model [1]. We find in Fig. 2 that the collapse and revival overlap for the values of κ/g and $\overline{n}_{th}$ at which the disentanglement actually occur. Also, the disentgalements are away from the point where they overlap. They are resolvable if the cavity is above a certain temperature that varies with N [7]. Of course, the revival will not occur if the cavity temperature exceeds an upper limit. For $N=50$, we have seen that the collapse and revival become clear if $\overline{n}_{th}\geqslant0.1$. However, the generation of an atomic pure state is not possible at such cavity temperatures.

We need to examine the results from Fig. 1 with respect to variation in the pump parameter N. It is necessary since N determines the degree of influence of cavity dissipation due to its inclusion during the atom-field interaction [10]. It has been shown clearly in Ref. [10] that the cavity decohering effect is crucial for the micromaser action beyond a certain N. Figure 3 displays the curves of T and S with respect to N for case (a). We find that atomic pure states can be attained until $N\cong50.0$, whereas the field can be in a pure state only for $N\leqslant30.0$. This is due to the presence of dissipation during the atom-field interaction. For the cavity temperature in Fig. 3, $S_R=1.2513\times10^{-4}$ and its influence is negligible only for low N. We have neglected the effect of the atomic reservoir in this study as we have seen in Ref. [10] that this effect is negligible for $\gamma/g\leqslant0.000\,01$.

The results in Figs. 1–3 are for the atoms entering the cavity in their upper masing level. For initially polarized atoms, the equations of motion become complicated and one has to resort to an approximate method to obtain the solutions [14]. Also, the expressions for S and T are no longer simple like Eqs. (5) and (8) due to the presence of nondiagonal elements in the atomic as well as the field density matrix. However, it was shown in Refs. [3–5] that the generation of pure states is independent of the initial condition of the atom and so we do not attempt to study the case of polarized atoms here.

The results presented in this paper have been obtained by using the steady-state solution of the coarse-grained time derivative of P_n [10]. In our numerical simulation of micromaser dynamics in Ref. [15] we saw that the steady-state photon distributions are subject to small fluctuations solely due to a Poissonian pump. It is expected that such fluctuations would have negligible effects on S and T in the same way the cavity reservoir has a negligible influence for low N as indicated in Figs. 1–3.

Thus the present paper analyzes a situation in which the interesting result of disentanglement in the Jaynes-Cummings interaction can be achieved experimentally. The cavity parameters in Figs. 1–3 are already within the reach of the micromaser experiments [16]. The interaction time in these experiments is set between $gt=1.54$ and 5.13 with $g=44$ kHz, whereas the present study suggests $gt=6.997\pi$. Thus slower atoms or atoms having a stronger coupling constant are necessary to verify the disentanglement using the micromaser setup.

[1] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963); J. H. Eberly, N. B. Narozhny, and J. J. Sanchez-Mondragon, Phys. Rev. Lett. **44**, 1323 (1980); N. Nayak, R. K. Bullough, B. V. Thompson, and G. S. Agarwal, IEEE J. Quantum Electron. **QE-24**, 1331 (1988).

[2] G. Rempe, H. Walther, and N. Klein, Phys. Rev. Lett. **58**, 353 (1987).

[3] J. Gea-Banacloche, Phys. Rev. Lett. **65**, 3385 (1990).

[4] M. Orszag, J. C. Ratamal, and C. Saavedra, Phys. Rev. A **45**, 2118 (1992).

[5] V. Buzek, H. Moya-Cessa, P. L. Knight, and S. J. D. Phoenix, Phys. Rev. A **45**, 8190 (1992).

[6] G. Rempe, F. Schmidt-Kaler, and H. Walther, Phys. Rev. Lett. **64**, 2783 (1990).

[7] E. M. Wright and P. Meystre, Opt. Lett. **14**, 177 (1989).

[8] P. Filipowicz, J. Javanainen, and P. Meystre, Phys. Rev. A **34**, 3077 (1986).

[9] L. A. Lugiato, M. O. Scully, and H. Walther, Phys. Rev. A **36**, 740 (1987).

[10] N. Nayak, Opt. Commun. **118**, 114 (1995).

[11] S. J. D. Phoenix and P. L. Knight, Ann. Phys. (N.Y.) **186**, 381 (1988).

[12] C. E. Shannon and W. Weaver, *The Mathematical Theory of Communication* (Illini Books, Urbana, 1963).

[13] H. Araki and E. Lieb, Commun. Math. Phys. **18**, 160 (1970).

[14] N. Nayak, Nonlinear Opt. **18**, 31 (1997).

[15] N. Nayak, J. Opt. Soc. Am. B **13**, 2099 (1996).

[16] H. Walther (private communication).