What we can learn about single photons in a two-photon interference experiment

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We report a two-photon interference experiment in which a single-photon wave-packet concept fails to give a correct prediction, but the two-photon wave-packet, or *biphoton*, concept is helpful. Based on our experiment, we argue that single-photon wave-packet information available from two-photon measurement is limited. [S1050-2947(98)00901-9]

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Two-photon interferometry is a powerful tool to study the fundamental problems of quantum theory. For example, the Einstein-Podolsky-Rosen problem [1] is believed to be resolvable by testing Bell's inequality [2] and the Greenberger-Horne-Zeilinger theorem [3] in two-photon or multiphoton interference experiments. Two-photon interferometry also has broad applications in practical areas such as quantum cryptography [4], metrology [5], and potentially in quantum computing [6].

It is important to understand the physics of two-photon interferometry correctly. What is the difference between two-photon phenomena and phenomena involving two photons? In general, what information is available from two-photon experiments? We wish to address these questions by reporting a different two-photon interference experiment. In this experiment, it is clear that an explanation based on a single-photon wave-packet concept is misleading and only a two-photon wave-packet concept can give a satisfactory understanding.

First, let us review a typical two-photon interferometer [7,8] illustrated in Fig. 1. The entangled *signal* and *idler* photon pair generated in spontaneous parametric down-conversion (SPDC) [9,10] is mixed by a 50-50 beam splitter (BS) and detected by two detectors D_1 and D_2 for coincidences. Balancing the signal and idler path lengths by positioning the beam splitter, one can observe a ''dip'' in coincidences that indicates destructive interference. This dip has been studied in various aspects [11].

Understanding of this experiment is often based on a single-photon picture: When pathlengths are exactly equal the signal and idler wave packets overlap on the beam splitter and interference occurs. The shape of the dip is determined by temporal convolution of two single-photon wave packets and therefore provides information about them.

Although leading to numerically correct predictions for several other experiments, this mental picture is not generally true. To show this, let us consider the experiment illustrated in Fig. 2. When the BS position is $x=0$, the idler arm's length is L_0. The signal channel has two paths: One path length is L_s and the other is L_l such that $L_l-L_0=L_0-L_s\equiv\Delta L\gg l_{coh}$, where l_{coh} is the coherence length of the down-conversion beams. Because of this condition there is no interference modulation in the single detector counting rates, which remain fairly constant.

Based on the concept of interference arising from the temporal overlap of two single-photon wave packets, dips are expected to appear for two positions of the beam splitter, $x=\pm\Delta L/2$. In these two cases the idler photon has a 50% chance to overlap with the signal one. This partial distinguishability results in the contrast of these two dips should be at most 1/2. When $x=0$ there is no overlap and the temporal convolution of the signal and idler wave packets is zero. Moreover, the detectors fire at random: In 50% of the joint detections D_1 fires ahead of D_2 by $\tau=\Delta L/c$; in the other 50% the opposite happens [12]. So no interference is expected in this case according to this single-photon concept.

Figure 3 shows the experimental result, which is quite different. We observe a high contrast interference dip in the middle $(x=0)$. In addition, the dip can turn to a peak if the experimental conditions are changed (Fig. 3). The transition from the dip to the peak depends on $\phi\equiv4\pi\Delta L/\lambda$, where λ is the central signal wavelength. Fixing $x=0$ and varying ϕ we observe a nice fringe (Fig. 4) corresponding to the transition from the dip to the peak in the center of Fig. 3.

To explain the observed effects, let us have a simple quantum-mechanical calculation that will provide a basis for the two-photon wave packet concept. An average coincidence counting rate on the time interval T is given by [13]

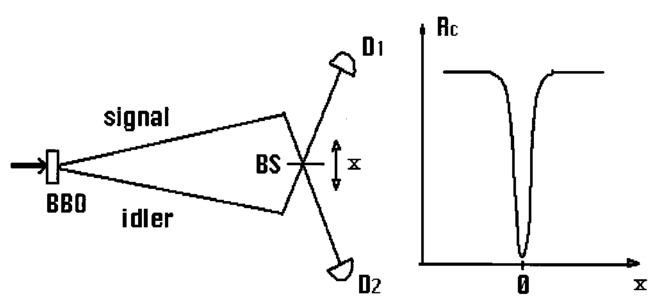

FIG. 1. Typical two-photon interferometer. The signal and idler from spontaneous parametric down-conversion are mixed by a beam splitter BS and detected by detectors D_1 and D_2 for coincidences.

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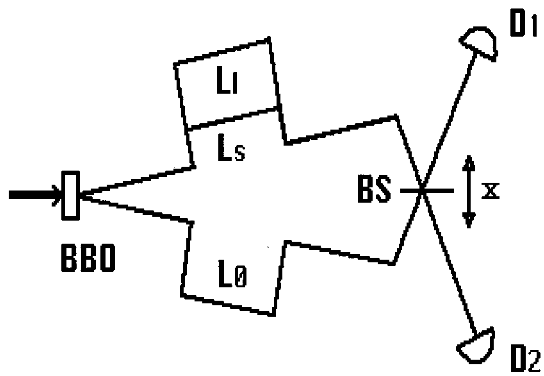

FIG. 2. Scheme of our experiment. In contrast with Fig. 1, there are two optical paths L_l and L_s in the signal beam. The idler path length is L_0.

$$R_c(x,\phi)\propto\frac{1}{T}\int_0^T\int_0^T dT_1dT_2\langle\Psi|E_1^-E_2^-E_1^+E_2^+|\Psi\rangle$$

$$=\frac{1}{T}\int_0^T\int_0^T dT_1dT_2|\langle 0|E_1^+E_2^+|\Psi\rangle|^2$$

$$\equiv\frac{1}{T}\int_0^T\int_0^T dT_1dT_2|\Psi(t_1,t_2)|^2, \tag{1}$$

where $E_{1,2}^{\pm}$ are positive- and negative-frequency components of field at detector D_1 or D_2, respectively, and $t_i\equiv T_i-l_i/c, i=1,2$, where T_i are detection times and l_i are optical path lengths. The entangled state of the SPDC pair $|\Psi\rangle$ has the form (see, e.g., [10])

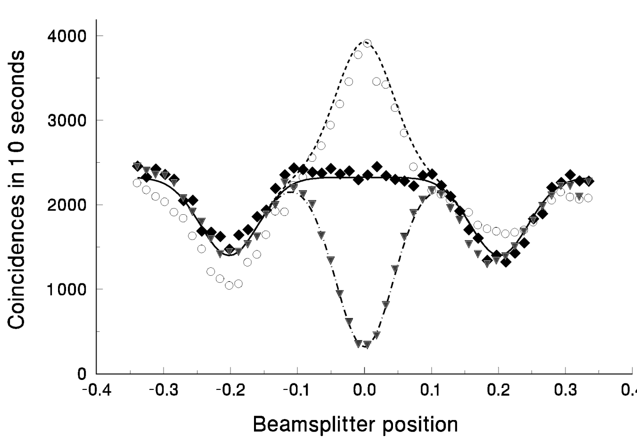

FIG. 3. Observed coincidence counting rate as a function of the beam-splitter position (in millimeters). The triple dip-peak pattern corresponds to different phases $\phi=0,\pi/2$, and π (triangles, diamonds, and circles, respectively). The side dips do not change significantly, while the central part changes from dip to peak. The raw data are fitted according to the theoretical predictions of Eq. (5).

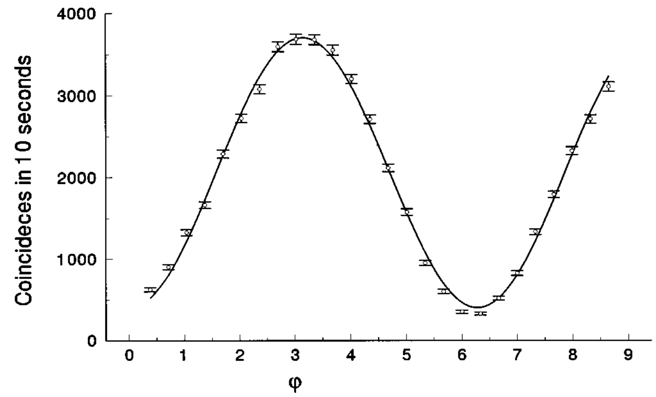

FIG. 4. ''Peak-dip'' transition of the central part $x=0$ of Fig. 3 as a function of ϕ.

$$|\Psi\rangle=\int d\omega_pF(\omega_p)d^3k\,d^3k'\,\delta(\omega_p-\omega_s(\vec{k})-\omega_i(\vec{k}'))$$
$$\times\Delta(\vec{k}_p-\vec{k}-\vec{k}')a_s^\dagger(\vec{k})a_i^\dagger(\vec{k}')|0\rangle,$$

where the subscripts s, i, and p represent the signal, idler, and pump modes, respectively; integration with respect to the pump frequency ω_p is done over the pump spectrum $F(\omega_p)$; all constants and slow functions of the integration variables are absorbed in $F(\omega_p)$. $\Delta(\vec{k_p}-\vec{k}-\vec{k'})$ takes into account the finite size of the interaction region; for an infinite interaction region it is a true δ function.

The two-dimensional function $\Psi(t_1,t_2)$ in Eq. (1) is called an *effective two-photon wave function*, or a *biphoton* [10]. This is a quantum-mechanical probability amplitude for the ''click-click'' event: Detector D_1 fires at T_1 and detector D_2 fires at T_2. For further convenience, we will introduce $t_+=t_1+t_2$ and $t_-=t_1-t_2$, so Eq. (1) becomes

$$R_c(x,\phi)\propto\frac{1}{T}\int\int dT_-dT_+|\Psi(t_-,t_+)|^2, \tag{2}$$

where the integrals are taken over the same detection time intervals as in Eq. (1).

From Fig. 2 we see that two distinct events can happen: Either detector D_1 fires ahead of detector D_2 by time $\tau\equiv\Delta L/c$ or D_2 fires ahead of D_1 by the same time τ. Although distinguishable in principle, these events are not discriminated by our coincidence circuit because its time window is much greater than 2τ. Therefore, our experiment does not involve any postselection.

The first kind of event happens either when the retarded part of the signal amplitude is transmitted to D_2 and the idler is transmitted to D_1 or when the advanced part of the signal amplitude is reflected to D_1 and the idler is reflected to D_2. Similarly, the second event happens either when the retarded part of the signal amplitude is reflected to D_1 and the idler is reflected to D_2 or when the advanced part of the signal amplitude is transmitted to D_2 and the idler is transmitted to D_1.

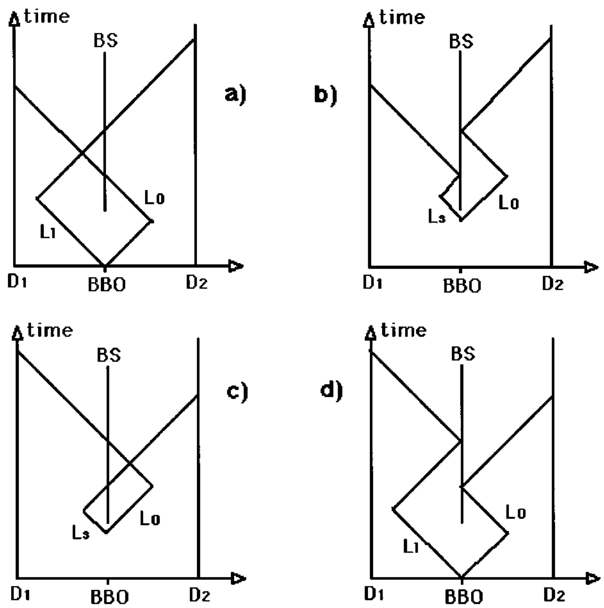

FIG. 5. (a) and (b) are two amplitudes to detect a photon pair such that D_1 fires ahead of D_2; (c) and (d) are two amplitudes to get a detection in the reversed order.

These four biphoton amplitudes are conveniently represented by Feynman-type diagrams in Fig. 5.

In all cases $t_- = T_- \pm \tau$ and $t_+ = T_+ - T_{l,s}$, $T_{l,s} \equiv (L_{l,s} + L_0)/c$; however, the "plus" or "minus" sign of τ and independently "l" or "s" is randomly realized in each trial. Therefore, the sum of the four amplitudes shown in Fig. 5 is

$$\Psi(t_-,t_+) = A(T_- - \tau, T_+ - T_l) + A(T_- - \tau, T_+ - T_s) + A(T_- + \tau, T_+ - T_s) + A(T_- + \tau, T_+ - T_l), \tag{3}$$

where each amplitude has the form [10,16]

$$A(t_-,t_+) = A_0 \exp\{-\sigma_+^2 t_+^2\} \exp\{-\sigma_-^2 t_-^2\} e^{-i\pi c t_+/\lambda_p}. \tag{4}$$

Note that in Eq. (4) there are *two* coherence times $1/\sigma_+$ and $1/\sigma_-$ that can be said to localize the biphoton in t_+ and t_- directions, respectively. This is the essence of the two-photon wave-packet concept. In our experiment $\sigma_- = c/2 l_{coh}$ [14]. It is a short coherence time: $1/\sigma_- < \Delta L/c$. On the contrary, the other coherence time is very long because it is linked to coherence length of the cw pump [10,16]: $\sigma_+ = c/2\sqrt{2} l_{pcoh} \gg \Delta L/c$. Thus the first exponent in Eq. (4) goes to unity and we consider the amplitudes in Figs. 5(a) and 5(b) [first two terms of Eq. (3)] and also those in Figs. 5(c) and 5(d) [last two terms of Eq. (3)] overlapping in both t_+ and t_- directions. When we substitute Eq. (3) into Eq. (2) and integrate over dT_-, the result breaks up into three disjoint intervals of x where interference effects are present, in complete agreement with our experiment [16]:

$$R_c \propto 1 - \cos\phi \exp\left\{-\frac{x^2}{l_{coh}^2}\right\} - \frac{1}{2}\exp\left\{-\left(\frac{x - \Delta L/2}{l_{coh}}\right)^2\right\} - \frac{1}{2}\exp\left\{-\left(\frac{x + \Delta L/2}{l_{coh}}\right)^2\right\}. \tag{5}$$

Setting ϕ to be subsequently equal to π, 0, and $\pi/2$ and varying the relative delay x we observe respectively a peak, dip or flat coincidence rate R_c distribution in the center ($x = 0$). These three cases are shown in Fig. 3. The separation between dips is equal to ΔL and the width of all dips is equal to l_{coh}. It is interesting to notice that the side dips do not depend on ϕ (within the experimental error). They correspond to the second and third terms of Eq. (5), that is, to only one of the signal paths "working."

The real experimental setup was the following. We used a 3-mm-long β-barium borate (BBO) crystal for cw-pumped type-I SPDC. The central signal and idler wavelengths $\lambda_s = \lambda_i = \lambda = 702$ nm were equal to twice of the pump wavelength λ_p. Both signal and idler were polarized in the horizontal direction and propagated at about 3.7° from the pump beam. A rod of birefringent material (crystal quartz) oriented at 45° with respect to the signal polarization is inserted into the signal channel. Its function is to provide L_l and L_s for the signal. Variation of the phase ϕ is achieved by a Pockels cell aligned with the quartz rod. A polarizer after the Pockels cell recovers the initial polarization [17]. The large-scale optical delay in the longer arm with respect to the shorter one is equal to $L_l - L_s = 2\Delta L = \Delta n L \approx 360$ μm, where Δn is birefringence and L is the length of the quartz rod. The coherence length l_{coh} of both the signal and idler is determined by the bandwidth of the interference filters placed in front of the detectors. For 3-nm full width at half maximum filters, $l_{coh} \approx 160$ μm is much shorter than the delay $2\Delta L$. The detectors are photon-counting avalanche photodiodes. The output pulses are brought to a coincidence circuit with a 10-ns acceptance window.

To conclude, we have demonstrated that the single-photon wave-packet concept is not always appropriate for two-photon interference measurements. The observed effects are described by the interference of *biphoton amplitudes* of the click-click event caused by an entangled photon pair. Therefore, its pattern carries only information concerning the biphoton. In other words, observing coincidence counts, we measure only *conditional* probability distributions for any individual photon, that is, a probability of a single-photon detection (which serves to measure a value of its observable, e.g., phase delay or polarization) conditioned on a similar measurement result for its conjugate component. If the state of the studied two-photon system is entangled, which is not a direct product of two single-photon states, then two-photon effects are not necessarily the effects of two (single) photons and may not reflect their individual properties.

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