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Dependence of transient dynamics in a class-C laser upon variation of inversion with time

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The transient statistics of a gain-switched coherently pumped class-C laser displays a linear correlation between the first passage time and subsequent peak intensity. Measurements are reported showing a positive or negative sign of this linear correlation, controlled through the switching time and the laser detuning. Further measurements of the small-signal laser gain combined with calculations involving a three-level laser model indicate that this sign fundamentally depends upon the way the laser inversion varies during the gain switching, despite the added dynamics of the laser polarization in the class-C laser. [S1050-2947(97)07112-6]

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INTRODUCTION

There has been extensive study of the statistical nature of transient dynamics of a laser that result from controlled changes in laser parameters [1–4]. In most studies, the parametric changes involve increasing the laser pumping (gain switching) or losses (Q switching) over time scales that are either fast or slow compared to the response time of the particular system [5,6]. The time taken for the laser intensity to build up to a macroscopically observable level after a parameter switch is referred to as the first passage time, or FPT. The statistical fluctuations observed in the transient dynamics of lasers for repeated sweeps of the control parameter have been identified as macroscopic fluctuations resulting from quantum noise [7–10]. Specifically, statistical fluctuations are observed in the FPT and these reflect the quantum statistics in laser variables such as the field and population inversion, at the time of parametric switch.

The FPT statistics, such as mean FPT and standard deviation, are primarily sensitive to statistical events taking place during the initial linear amplification of the intensity. When the intensity grows to an observable level, saturation effects become significant. This is also referred to as the nonlinear amplification regime. During this time, the character of the resulting intensity transient response depends on the dynamics of the laser variables as determined by their characteristic time scales [11,12]. In studies to date, lasers of classes A and B have been studied [5,13] wherein the statistics of the FPT are observed to be correlated to characteristics of the laser transient.

One such correlation is that between the FPT and the subsequent peak intensity obtained. Recent work [6] has clearly demonstrated for the class-B laser that the sign of the slope of the linear correlation is determined by the switching mechanism. If the laser is gain switched, the gain may still be increasing at the time of first passage, depending upon the system relaxation speed and the speed of switching. In such a case the peak intensity obtained is therefore greater for longer FPT leading to a positive correlation. In the case of loss switching, the gain is already present when the losses are swept. Fluctuations upon the gain from one switch to the next (e.g., due to modulation on the pumping mechanism) results in either an earlier FPT and larger peak intensity when the gain is larger than average or conversely a longer FPT and smaller peak intensity when the gain is smaller. This results in a negative correlation.

In this paper we report experimental observations of fluctuations in the FPT of a class-C laser when the laser is gain switched. This is novel on two counts. First, the laser gain in the class-C laser is determined by the polarization and not simply by the inversion, as in classes A and B, for which the polarization relaxation is fast compared to the inversion. It is conceivable, therefore, that the linear correlation mechanisms discussed above are complicated by the polarization. Second, although the laser is solely gain switched we observe both positive and negative linear correlation between the FPT and the subsequent peak intensity. This is in contrast to the generalized behavior reported in [6], wherein gain and loss switching mechanisms are contrasted. We can control the slope of the linear correlation via the laser detuning and also by whether the pump is switched on quickly or swept slowly relative to the time scale of the laser transient dynamics.

The two-level Lorenz-Haken model [14] (previously used to describe the observed dynamics of the laser we use [15,16]) adequately accounts for the observed positive slope of the correlation between FPT and subsequent peak intensity when the pump parameter is slowly swept. In this case, the form of both the small-signal laser polarization and inversion with time is essentially the same as that of the pump intensity. This is consistent with similar results discussed by [6] for the class-B laser. With a slow sweep of the pump, the peak intensity is greater for FPT events that occur later, giving rise to a correlation with positive slope.

For the case of quick switching, both positive and negative slopes are observed for near-resonant and off-resonant tuning of the laser, respectively. Small-signal gain measurements for the quickly switched pump demonstrate that the form of the small-signal gain differs significantly from that of the pump. There is an overshoot in the gain that is consistent with undamped coherence between the pumping levels. A simplified three-level laser model exhibits this effect and also shows both positive and negative slopes with near-resonant and off-resonant tuning of the laser, respectively, as

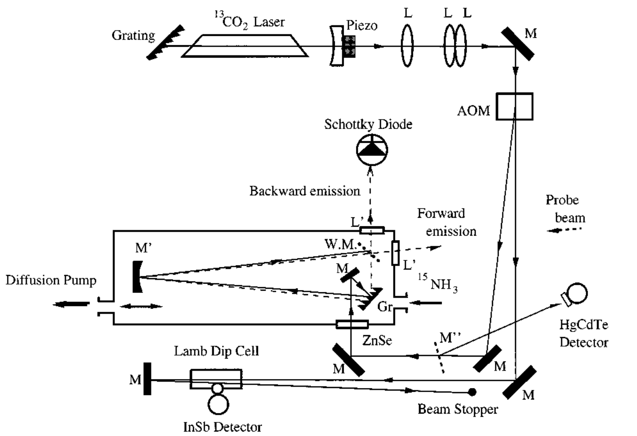

FIG. 1. Experimental setup. L: ZnSe lenses focus the pump beam through the acousto-optic modulator (AOM); M: flat mirror; M': 2.0 m ROC movable mirror controls the ring laser tuning; M'': beam splitter reflects portion of the pump beam to HgCdTe detector; G_r: grating couples the CO_2 radiation into the ring resonator (80 lines/mm); WM: wire mesh output coupler (5 lines per mm). See text.

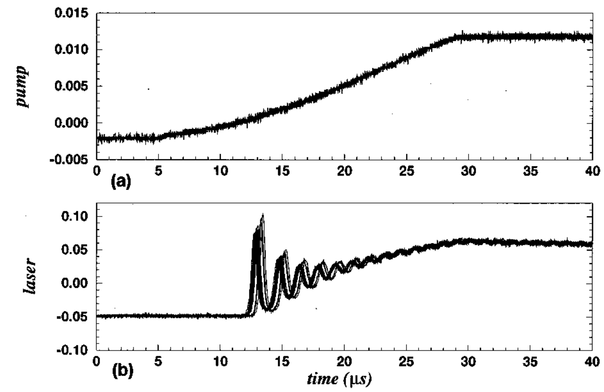

FIG. 2. FPT measurements with slowly swept pump. The pump power (a) is ramped up slowly with respect to the period of oscillations of the laser dynamics at turn-on (b). Pump and laser intensity units are arbitrary. 15 overlayed traces of both the pump and intensity are depicted. Maximum pump power is 1.0 W. Laser gas pressure is 30 μbar.

observed experimentally. Ultimately however, the sign of the slope is shown to depend primarily upon the variation of the laser inversion, which in the class-C system can be significantly different from the gain.

I. EXPERIMENTAL SETUP

The laser used in these experiments is the 153-μm $^{15}NH_3$ far-infrared (FIR) ring laser, which has been described before [15,16]. Figure 1 shows the experimental setup for the ring laser transient measurements. An acousto-optic modulator (AOM) is used to quickly and repeatedly switch the pump laser beam. The diffracted beam from the AOM is used to pump the FIR ring laser as this allows the pump power to be reduced to zero. This ensures that there is no initial pumping of the laser levels. A portion of the pump beam is reflected from partial reflector M'', to a fast HgCdTe detector to monitor the form of the pump power with time. Pump radiation is coupled into the ring laser via a ruled grating G_r, with rule spacing designed to specularly reflect the FIR radiation. The frequency of the pump laser is tuned via a piezomounted output coupling mirror and is monitored using a Lamb Dip cell. The ring laser is tuned mechanically via a translatable curved mirror. The intensity of the ring laser output is measured using a Schottky-barrier diode. The ZnSe lenses are used to control the divergence and size of the pump beam as it couples into the ring laser. These lenses also position the beam waist at the AOM to minimize loss of power via diffraction by the AOM aperture.

In addition to the ring laser and its respective $^{13}CO_2$ pump there is a second $^{15}NH_3$ FIR laser with linear resonator that is used to make measurements of the single-pass gain (not shown). For the single-pass gain measurements, the second FIR laser signal is injected through the forward emission port of the ring laser (indicated in Fig. 1) and the wire mesh is removed. In this configuration, the ring laser effectively becomes an amplifying or absorbing medium depending upon whether the pump is on or off, respectively. By switching the pump on while the FIR injected signal is present, the transient small-signal gain can be measured.

II. FPT MEASUREMENTS: SLOWLY SWEPT PUMP

Figure 2 shows experimental results of multiple FPT events (b) for a repeated slow sweeping of the pump (a). The sweep of the pump is referred to as slow compared to the relaxation constants of the system, as evidenced by the period of the ringing oscillations of the intensity. The mean FPT is approximately 7.5 μs and the standard deviation of arrival times is roughly 0.3 μs. It can be seen that there is a positive correlation slope between the FPT and the subsequent peak intensity for the slowly swept pump. This is as expected since the pump power is increasing even as the peak intensity is achieved. Similar results were obtained by Balestri *et al.* [12] using a class-B, CO_2 laser. In the class-B laser, the effect can be simply understood with reference to the laser inversion, since only the field and inversion are required to describe the dynamics and the laser gain is equivalent to the inversion. However, in the class-C laser, the laser gain is represented by the polarization and since the polarization is not adiabatically eliminated, the dynamics of the laser gain are consequently distinct from those of the inversion.

Previously, the laser used in these experiments has been qualitatively described (and to a lesser degree, quantitatively) by the Lorenz-Haken two-level laser model [16,15]. We use this model here to gain a qualitative description of the correlation between the FPT and the subsequent peak intensity for slow sweeping of the pumping parameter in the class-C laser.

III. TWO-LEVEL LASER MODEL

The Lorenz-Haken laser equations [14] with an additional stochastic term in order to simulate spontaneous emission are given by

$$\dot{E}=-\kappa(E-P)+\xi(t),$$
$$\dot{P}=-\gamma_{\perp}(P-ED), \tag{3.1}$$
$$\dot{D}=\gamma_{\parallel}(\Lambda+1-D-\Lambda EP),$$

where E,P,D correspond to the laser electric field and medium polarization and inversion, and $\kappa,\gamma_{\perp},\gamma_{\parallel}$ are the respective relaxation constants. Λ is the pump parameter where $\Lambda=0$ is the threshold for lasing. The stochastic term $\xi(t)$ is a δ-correlated Gaussian noise term with zero mean and variance ϵ. These equations are isomorphic to the equations of Lorenz [17]:

$$\dot{x}=-\sigma(x-y),$$
$$\dot{y}=-y+xz, \tag{3.2}$$
$$\dot{z}=b(r-z)-xy,$$

with the following correspondences between symbols:

$$t\rightarrow t'\sigma/\kappa,\quad E\rightarrow\alpha x,\quad P\rightarrow\alpha y,\quad D\rightarrow z,$$
$$\gamma_{\parallel}=\kappa b/\sigma,\quad \gamma_{\perp}=\kappa/\sigma,\quad \Lambda=r-1,\quad \alpha=1/\sqrt{b(r-1)}. \tag{3.3}$$

It is important to note that the Lorenz-Haken equations are normalized to the steady-state values of the field, polarization, and inversion for a given pump parameter. Consequently the normalizations change as the pump is varied and for this reason the isomorphic Lorenz equations are chosen to model the laser dynamics under a slow sweep of the pump parameter r. In such a case, the laser intensity is represented by x^2 and the Lorenz dimensionless time units correspond to real time, as above, via $t'\rightarrow t/(\sigma/\kappa)=t\gamma_{\perp}$. In this paper, the value chosen for $\gamma_{\perp}$ in the Lorenz-Haken model is 1 MHz and thus the time scale corresponds to time in units of microseconds. Other parameters used are $\kappa=12$ MHz, $\gamma_{\perp}=1$ MHz, $\gamma_{\parallel}=2$ MHz, corresponding to $\sigma=12$, $b=2$.

Figure 3(a) shows the variation of pump parameter with time chosen to compare with Fig. 2. Figure 3(b) shows the intensity (corresponding to x^2) calculated for multiple FPT events clearly showing the direct correlation between the FPT and the subsequent peak intensity. This qualitatively compares well with the experimentally observed FPT traces shown in Fig. 2(b). The behavior can be simply understood in terms of the small-signal gain and the medium polarization and inversion.

A small-signal gain measurement consists of passing a probe field of the same frequency as the laser transition through the medium as it is pumped. The relative increase in the probe field is the small-signal gain of the active medium. In the above Lorenz-Haken equations, the source term in the electric field equation is the polarization variable P. This is intuitively as expected, since the medium dipole is ultimately responsible for radiative additions to the electric field already present. It is therefore the polarization that is represented in a small-signal gain measurement. This single-pass probe field measurement can be modeled by removing the laser field equation, corresponding to the removal of the wire mesh reflector in the real laser cavity. The value of the field is then set to some small value representing the probe signal. The Lorenz variables y,z then correspond to the small-signal gain and inversion of the system. Figures 3(c) and 3(d) show such calculations of the small-signal gain and inversion, for the same form of pump with time as in Fig. 3(a).

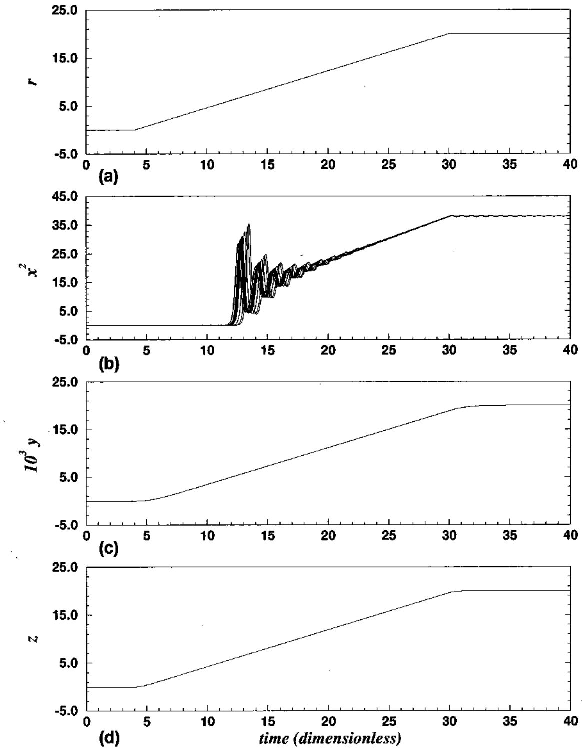

FIG. 3. Intensity (b) calculated from Lorenz equations for slow increase of pump (a) with time (dimensionless—1 unit in the Lorenz system corresponds to 1 μs in the Lorenz-Haken system for the choice $\gamma_{\perp}=1$ MHz). 10 traces. $r_{\max}=20$, $\epsilon=1.0\times10^{-3}$. (c),(d): Lorenz-Haken variables y,z corresponding to small signal polarization and inversion, calculated with $\dot{x}=0$ and $x=0.001$. All variables are dimensionless.

The small-signal variables of polarization and inversion are more instructive for description of the laser intensity peaks since the respective nonlinear variables in the full Lorenz-Haken equations saturate in response to the increasing intensity of the laser. The state of the small-signal variables at the time of nonlinear amplification is responsible for the height of the intensity peak, and hence the variation of the small-signal parameters will more clearly show the dependence of these parameters upon the correlation between the intensity peak and FPT.

In the figure, since the pump is increased slowly relative to the laser parameters, the polarization and inversion closely follow the form of the pump with time. When the intensity in the laser peaks, the small signal gain and inversion are still increasing with the pump. Indeed, the laser intensity exhibits a ringing oscillation, which is dampened to a quasi-steady-state level that steadily rises with the pump until the maximum value. Clearly then, if spontaneous emission events lead to a relatively short FPT, the small-signal gain and inversion at the peak of laser intensity is less than that for a FPT, which is relatively long. Consequently, the maximum

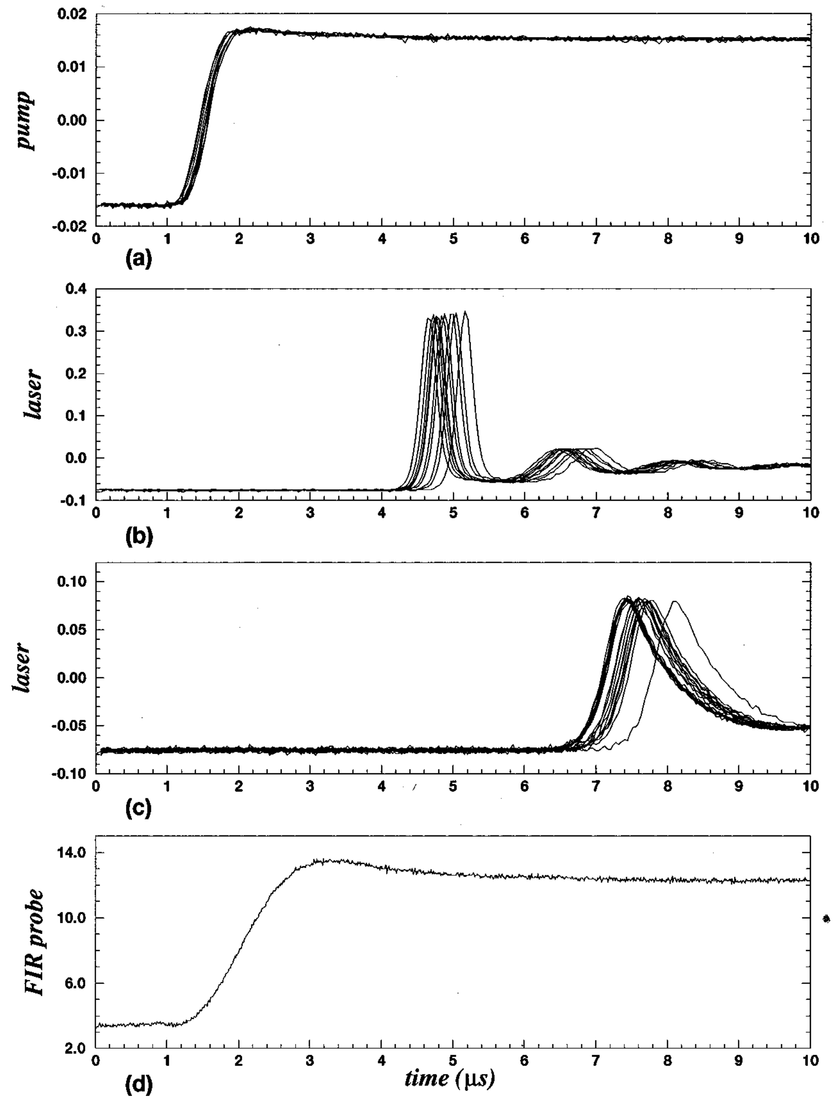

FIG. 4. FPT measurements with the pump power (a) quickly turned on (in less than 1 μs). (b) 15 overlayed traces of the intensity response for near-resonant tuning of the laser. (c) Intensity response with off-resonant tuning. Maximum pump power is 1.0 W. Laser gas pressure is 22 μbar. (d) Small-signal gain measurement of laser medium using a FIR probe laser field. All measurements made with maximum pump power $=1.0$ W, gas pressure $=22$ μbar. All y-axis scale units are arbitrary.

peak intensity monotonically increases with the first passage time.

Since the gain and inversion both have the same form, however, these results do not make clear which variable is responsible for the observed correlation. As investigated in the following section, when the laser is switched quickly, the different relaxation constants for the gain and inversion lead to different variations of these variables with time and consequently elucidate which laser variable is responsible for the peak-intensity–FPT correlation.

IV. FPT MEASUREMENTS: QUICKLY SWITCHED PUMP

Figure 4 shows experimental results of multiple FPT events (b),(c) for a fast switch on of the pump (a). The turn-on time is less than 1 μs. The intensity traces for (b) and (c) are for near-resonant and off-resonant tuning of the laser cavity, respectively. All other parameters are held constant. In this case where the pump is quickly switched, both positive and negative slopes are found of the correlation between FPT and maximum peak intensity. Figure 4(d) shows an experimental determination of the small-signal gain, as outlined in the previous section and experimental description. The wire mesh in Fig. 1 was removed and the output from a second $^{15}NH_3$ FIR laser was introduced into the laser, as indicated. The signal was measured as usual with the Schottky detector and the pump was switched as before. Figure 4(d) shows that the small-signal gain does not closely follow the form of the pump with time. The gain takes over 1 μs to rise and exhibits an overshoot before a gradual decrease to steady state.

The slower response of the gain with respect to the form of the pump in the quickly switched case is expected since the rise of the gain is ultimately limited by the medium re-

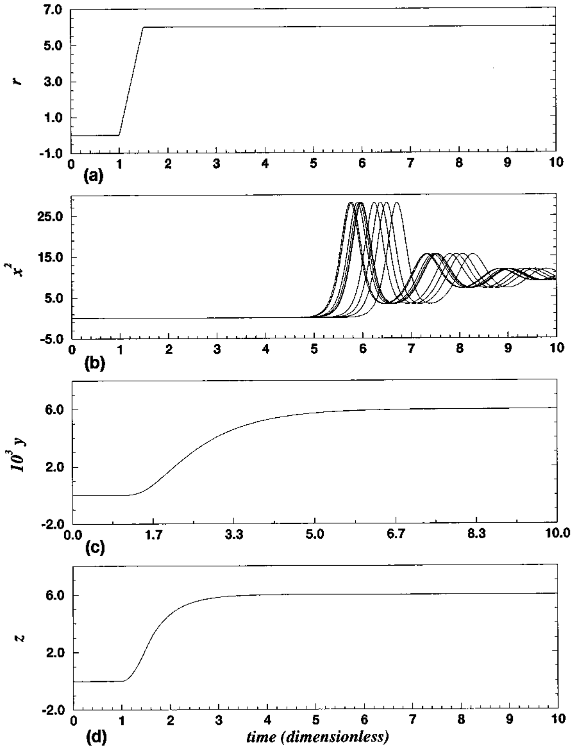

FIG. 5. Intensity (b) calculated from Lorenz equations for rapid increase of pump (a) with time (dimensionless). 10 traces. $r_{\max}=6.0$, $\epsilon=1.0\times10^{-3}$. (c),(d): Lorenz variables y,z corresponding to small signal polarization and inversion, calculated with $\dot{x}=0$ and $x=0.001$. All variables are dimensionless.

laxation parameters. Figure 5 shows calculations, again using the Lorenz equations, for such a fast switch of the pump. As in Fig. 3, Figs. 5(c),(d) represent the small signal values of the polarization and inversion, wherein (c) corresponds to the laser gain measurement in Fig. 4(d). The slower response of the gain with respect to the pump is reproduced. However, there is no overshoot and furthermore, the intensity curve peaks show no variation with FPT. The experimentally observed overshoot of the small-signal gain suggests that undamped coherence is also present between the pumping levels of the medium and hence the two-level approximation implicit in the Lorenz-Haken equations is inadequate to describe the behavior for the case of quickly switched pump power. In order to qualitatively account for the observed overshoot of measured gain together with the positive and negative correlations between laser peak intensity and FPT, a three-level laser model is at least required.

It may be noted at this point that although the gain does exhibit an initial overshoot, the positive and negative correlations in the peak intensities occur at times much later than the overshoot in the gain. Although the initial overshoot of the gain suggests that pump level coherence is responsible for the appearance of a correlation in the peak heights, it does not explain the respective positive and negative slopes of the peak intensity-FPT correlation. This point will be pursued upon introduction of the three-level model.

V. THREE-LEVEL LASER MODEL

The two-level system provides a sufficient description of the laser dynamics for the slowly swept pump. While a three-

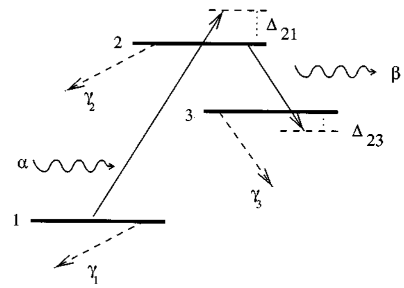

FIG. 6. Schematic representation of three-level system. The transition between levels 1 and 2 is the pump transition and the laser transition is between levels 2 and 3.

level model would of course also reproduce the observed behavior for slow sweeping of the pump, the effects of coherent pumping are not manifested under these conditions and the two-level model adequately accounts for the observed behavior. This is not the case for a quickly switched pump, as experimentally observed in the previous section. A three-level system is necessary to take into account pump coherence.

The three-level system used is schematically represented in Fig. 6. We consider the general semiclassical equations describing the evolution of the atomic variables and field in an open three-level Λ-system [18]

$$\dot{\rho}_{11}=\gamma_1(\mathring{\rho}_{11}-\rho_{11})+2\,\mathrm{Im}\{\alpha^*\rho_{21}\},$$

$$\dot{\rho}_{22}=\gamma_2(\mathring{\rho}_{22}-\rho_{22})-2\,\mathrm{Im}\{\alpha^*\rho_{21}+\beta^*\rho_{23}\},$$

$$\dot{\rho}_{33}=\gamma_3(\mathring{\rho}_{33}-\rho_{33})+2\,\mathrm{Im}\{\beta^*\rho_{23}\},\tag{5.1}$$

$$\dot{\rho}_{21}=-(\gamma_{21}+i\Delta_{21})\rho_{21}+i\alpha(\rho_{22}-\rho_{11})-i\beta\rho_{31},$$

$$\dot{\rho}_{23}=-(\gamma_{23}+i\Delta_{23})\rho_{23}+i\beta(\rho_{22}-\rho_{33})-i\alpha\rho_{31}^*,$$

$$\dot{\rho}_{31}=-(\gamma_{31}+i\Delta_{31})\rho_{31}-i\beta^*\rho_{21}+i\alpha\rho_{23}^*,$$

$$\dot{\beta}=-iG\rho_{23}-\kappa\beta+\xi(t),$$

where ρ_{ij} $(i,j=1,2,3)$ are slowly varying envelopes of density-matrix variables normalized to the density of molecules in the three-level system and $\mathring{\rho}_{ij}$ are the zero field values. The parameters, γ_i, γ_{ij} $(i,j=1,2,3)$, and κ are the population, coherence, and cavity decay rates, respectively. α and β are Rabi frequencies that characterize the interaction of the molecules with the pump and laser fields and G accounts for the molecule generated field coupling. As before, $\xi(t)$ is a Gaussian noise term to simulate spontaneous emission, with zero mean and variance ϵ.

Parameters used are as follows: $\gamma_1=0.5$ MHz; $\gamma_2=\gamma_3=2.0$ MHz; $\gamma_{21}=1.0$ MHz; $\gamma_{23}=1.0$ MHz; $\gamma_{31}=5.0$ MHz. In the limit where the pump level coherences decay quickly, these values correspond to those used in the two-level model, for which $\gamma_\perp=\gamma_{23}$, $\gamma_\parallel=\gamma_2=\gamma_3$. The pump is chosen to be resonantly tuned $(\Delta_{21}=0)$ and $\Delta_{31}=\Delta_{21}-\Delta_{23}$.

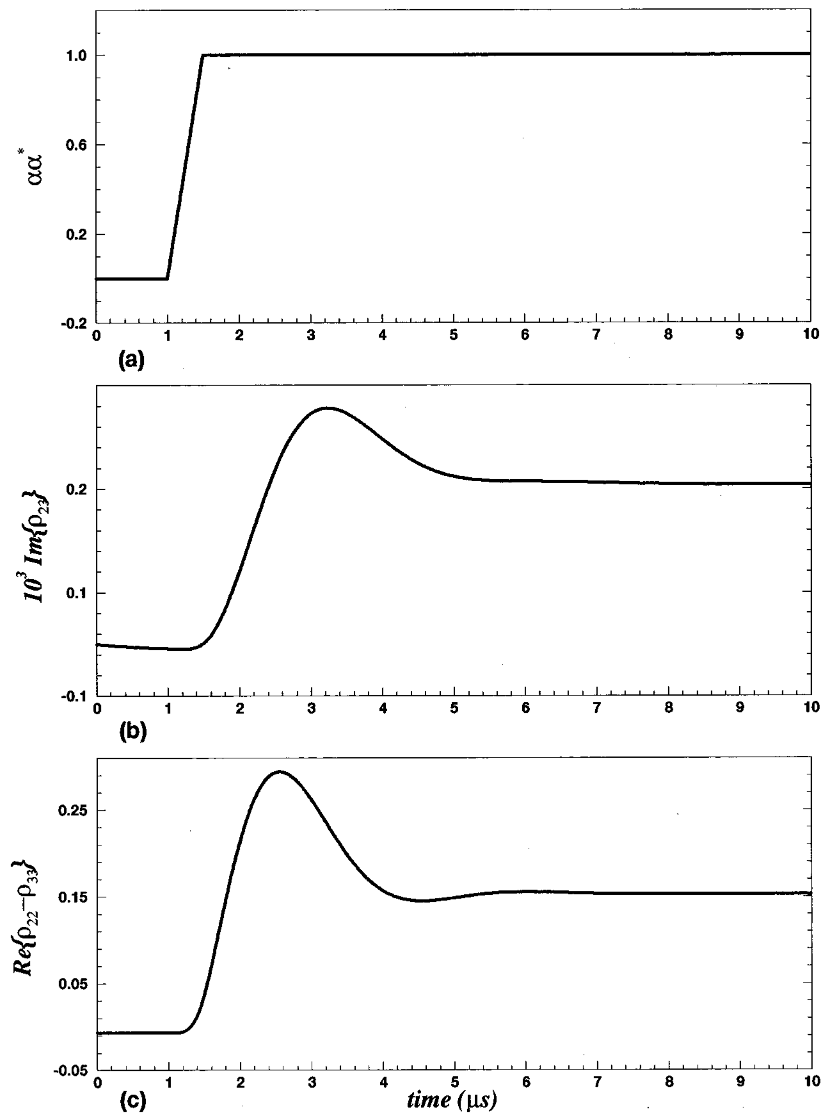

FIG. 7. Small signal gain measurement of laser polarization (b) and inversion (c) represented by Im$\{\rho_{23}\}$ and Re$\{\rho22-\rho33\}$ calculated from 3-level equations for the given form of pump (a) with time (μs). See text for medium parameters. $\beta = 0.001$.

Figure 7 shows calculations using Eqs. (5.1) of the three-level small-signal gain and inversion, determined similarly to those of Figs. 3(c),(d) and 5(c),(d). For the current equations, the coherence between the laser levels, ρ_{23} corresponds to the laser polarization as this is also the source term in the laser field equation. Once again, setting β small and $\dot{\beta}=0$ corresponds to a small signal gain measurement, with ρ_{23} being the small-signal gain in figure (b) and $\rho_{22}-\rho_{33}$ being the laser inversion in (c).

The pump strength and coherence decay rates cited above were chosen for qualitative comparison. In this choice, correspondence between the form of the small-signal gain in Fig. 7(b) and the measured gain in Fig. 4(d) was primarily in view. The pump strength controls the time scale of Rabi oscillations and was chosen so that the time scale of the overshoot corresponded to the measured one. The pump coherence decay rates were chosen to be smaller than the level decay rates in order to ensure that any oscillations of the coherence were damped more quickly than the level populations. In this way, the observed small-signal gain 7(b) exhibits an overshoot and damped decay in accord with Fig. 4(d) whereas careful inspection of the inversion 7(c), by contrast shows a secondary oscillation (from 5 to 7 μs). These dynamics are sufficient to qualitatively explain the experimentally observed peak-height–FPT correlations.

Figures 8(b) and 8(c) show the laser intensities calculated from the complete set of Eq. (5.1), restoring the field equation. As in the experiment, two detunings are used: resonant tuning in (b) ($\Delta_{23}=0.0$ MHz) and off-resonant in (c) ($\Delta_{23}=10.0$ MHz), respectively. Comparing these with Fig. 7(c), it appears that the inversion is ultimately responsible for the peak intensity–FPT correlation. In Fig. 8(b) the FPT peaks occur between $5.0-6.2$ μs and the laser inversion in Fig. 7(c) is increasing under a relaxation oscillation during this time. For detuning in 8(c), the FPT peaks occur later ($6.2-7.0$ μs) and during this time, the small-signal inver-

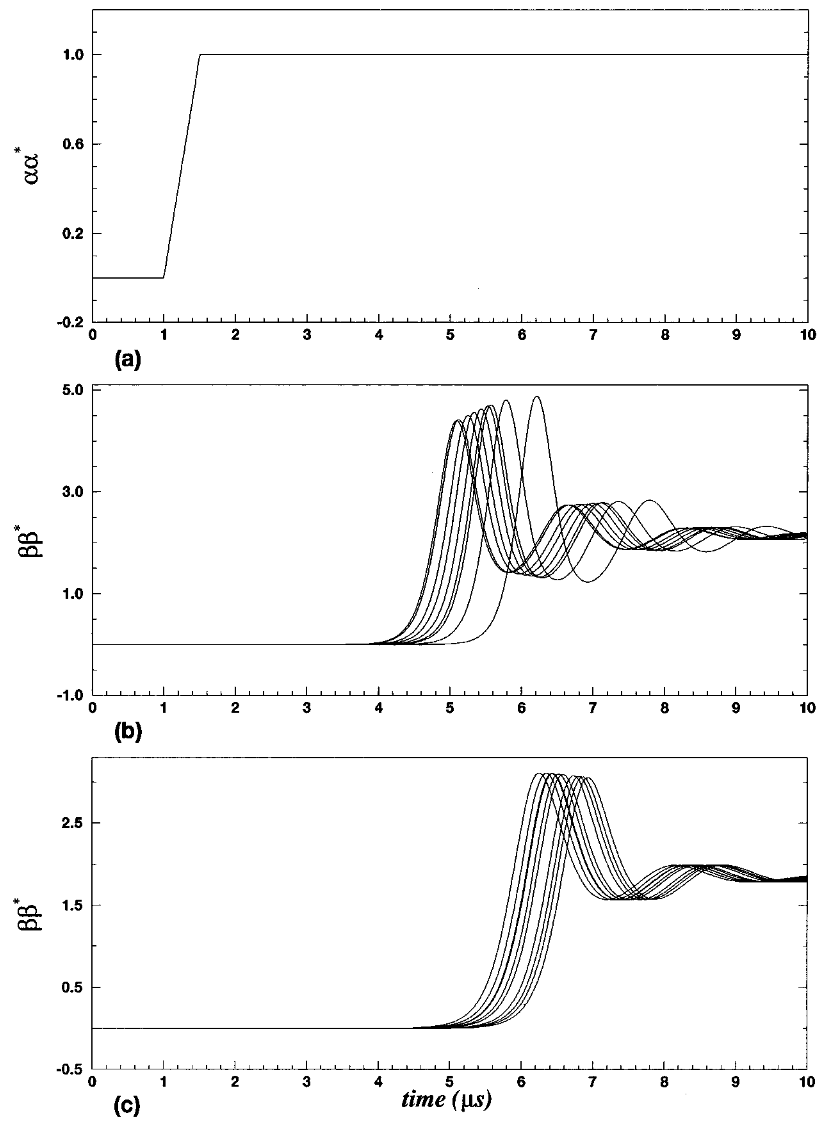

FIG. 8. Laser variables calculated from three-level laser equations for the given form of pump (a) with time (μs). See text for medium parameters. (b) Laser intensity, $\beta\beta^*$, for resonant ($\Delta_{23}=0.0$ MHz) and (c) off-resonant ($\Delta_{23}=10.0$ MHz) tuning of the laser cavity, respectively. Cavity decay, $\kappa=12$ MHz; coupling constant, $G=700$; $\epsilon=1.0\times10^{-3}$.

sion is decreasing. For all of this time the small-signal gain 7(b) is decreasing and is clearly not directly responsible for the correlation. In all calculated instances where either a positive or negative correlation was observed, the sign of the correlation was found to be directly related to the variation of the small-signal inversion, even in situations for which the polarization exhibited the opposite variation. If the inversion displayed no variation at the FPT, then no correlation was observed in the calculated intensity peaks with FPT, once again, even if the polarization exhibited a variation.

Although these calculations are quite simplistic and empirical in their approach to the dynamics of the laser, the qualitative behavior strongly suggests that the correlation of peak intensity and FPT continues to be a reflection of the variation of the laser inversion, even in the class-C laser for which the laser gain is determined by the polarization of the medium. The strong role of the inversion might perhaps be expected since the inversion in the laser is where the energy is stored and must be ultimately responsible for the peak power of any laser pulses that occur.

CONCLUSION

We have performed measurements in the class-C, $^{15}NH_3$ FIR ring laser demonstrating a correlation between the peak laser intensity and the first passage time, which depends upon the relative speed of the pump switch and the laser detuning. We have shown that the variation of the laser inversion at the first passage time is ultimately responsible for any observed correlation, rather than the polarization, which in a class-C laser is the variable corresponding to gain. The two- and three-level models used, while relatively simple approximations to the true laser dynamics nevertheless provide a good qualitative understanding of the experimental observations.

[1] F. T. Arecchi, V. Degiorgio, and B. Querzola, Phys. Rev. Lett. **19**, 1168 (1967).
[2] F. T. Arecchi and V. Degiorgio, Phys. Rev. A **3**, 1108 (1971).
[3] S. Zhu, A. W. Yu, and R. Roy, Phys. Rev. A **34**, 4333 (1986).
[4] A. Valle, L. Pesquera, and M. A. Rodriguez, Phys. Rev. A **45**, 5243 (1992).
[5] M. Ciofini, R. Meucci, and F. T. Arecchi, Phys. Rev. A **42**, 482 (1990).
[6] S. Balle, M. San Miguel, N. B. Abraham, J. R. Tredicce, R. Alvarez, E. J. D'Angelo, Alok Gambhir, K. Scott Thornburg, and R. Roy, Phys. Rev. Lett. **72**, 3510 (1994).
[7] F. Haake, Phys. Rev. Lett. **41**, 1685 (1978).
[8] F. T. Arecchi and A. Politi, Phys. Rev. Lett. **45**, 1219 (1980).
[9] R. Roy, A. W. Yu, and S. Zhu, Phys. Rev. Lett. **55**, 2794 (1985).
[10] F. T. Arecchi, W. Gadomski, R. Meucci, and J. A. Roversi, Phys. Rev. A **39**, 4004 (1989).
[11] R. Meucci, M. Ciofini, and Peng ye Wang, Opt. Commun. **91**, 444 (1992).
[12] S. Balestri, M. Ciofini, R. Meucci, and F. T. Arecchi, Phys. Rev. A **44**, 5894 (1991).
[13] A. Mecozzi, S. Piazzaolla, A. D'Ottavi, and P. Spano, Phys. Rev. A **38**, 3136 (1988).
[14] H. Haken, Phys. Lett. **53A**, 77 (1975).
[15] M. Y. Li, Tin Win, C. O. Weiss, and N. R. Heckenberg, Opt. Commun. **80**, 119 (1990).
[16] Tin Win, M. Y. Li, J. T. Malos, N. R. Heckenberg, and C. O. Weiss, Opt. Commun. **103**, 479 (1993).
[17] E. N. Lorenz, J. Atmos. Sci. **20**, 130 (1963).
[18] M. A. Dupertuis, R. R. E. Salomaa, and M. R. Siegrist, Opt. Commun. **57**, 410 (1986).