# Mimicking a squeezed-bath interaction: Quantum-reservoir engineering with atoms

N. Lütkenhaus, J. I. Cirac, and P. Zoller

Institut für Theoretische Physik, Universität Innsbruck, Technikerstraße 25, A-6020 Innsbruck, Austria

(Received 16 June 1997)

The interaction of an atomic two-level system and a squeezed vacuum leads to interesting effects in atomic dynamics, including line narrowing in resonance fluorescence and absorption spectra, and a suppressed (enhanced) decay of the in-phase and out-of-phase components of the atomic polarization. On the experimental side these predictions have so far eluded observation, essentially due to the difficulty of embedding atoms in a  $4\pi$  squeezed vacuum. In this paper we show how to "engineer" a squeezed-bath-type interaction for an effective two-level system. In the simplest example, our two-level atom is represented by the two ground levels of an atom with an angular momentum  $J=1/2 \rightarrow J=1/2$  transition (a four-level system), which is driven by (weak) laser fields and coupled to the vacuum reservoir of radiation modes. Interference between the spontaneous emission channels in optical pumping leads to a squeezed-bath-type coupling and thus to symmetry breaking of decay on the Bloch sphere. With this system it should be possible to observe the effects predicted in the context of squeezed-bath-atom interactions. The laser parameters allow one to choose properties of the squeezed-bath interaction, such as the (effective) photon-number expectation number N and the squeezing phase  $\phi$ . We present results of a detailed analytical and numerical study. [S1050-2947(97)04012-2]

PACS number(s): 42.50.Ct, 32.80.-t, 42.50.Lc

## I. INTRODUCTION

The interaction of atomic systems with squeezed light leads to interesting effects in atomic dynamics [1]. In particular, Gardiner [2] has shown that the atomic Bloch vector of a two-level system coupled to a squeezed bath, which is characterized by a mean photon number N and squeezing parameter M, obeys the equations

$$\frac{d}{dt} \langle S_x \rangle = -\gamma_x \langle S_x \rangle,$$

$$\frac{d}{dt} \langle S_y \rangle = -\gamma_y \langle S_y \rangle,$$
(1)
$$\frac{d}{dt} \langle S_z \rangle = -\gamma_z \langle S_z \rangle - \gamma.$$

The decay constants in this equation are given by

$$\gamma_{x} = \gamma \left( N + \frac{1}{2} - M \right),$$
  
$$\gamma_{x} = \gamma \left( N + \frac{1}{2} + M \right),$$
 (2)

$$\gamma_z = \gamma(2N+1),$$

with  $\gamma$  is the spontaneous emission rate in free vacuum. We will refer to Eq. (1) as the Gardiner-Bloch equation. According to Eqs. (1) and (2), the two quadrature components of the atomic polarization  $\langle S_x \rangle$  and  $\langle S_y \rangle$  will decay with different rates  $\gamma_x$  and  $\gamma_y$ , respectively. In the limit of large photon number  $N \gg 1$  and maximal squeezing  $M^2 = N(N+1)$ , the decay of  $\langle S_x \rangle$  will be suppressed according to  $\gamma_x \rightarrow \gamma/8N$  ( $\ll \gamma$ ) in comparison to spontaneous emission in free

vacuum. At the same time the decay of  $\langle S_y \rangle$  will be enhanced:  $\gamma_y \rightarrow 2 \gamma N$  ( $\geq \gamma$ ). As studied in numerous theoretical papers, these phase-sensitive decay rates will also be visible in the spectrum of resonance fluorescence [3,4] and the atomic absorption spectrum [5]. A study in the context of the Jaynes-Cummings model has been done in [6]. From a physical point of view, the suppression of the decay below the free vacuum level is due to the reduced quantum fluctuations of one of the quadrature components of squeezed light. Similar effects have been investigated for the case of three-level systems [7,8]. For experimental realization one has to include finite bandwidth effects. This has been done, for example, in [9–13].

On the experimental side, there have been only a few experiments where the dynamics of atoms in squeezed light has been studied in the laboratory. Most notable are the experiments by Kimble and co-workers who, for example, reported experimental observation of the linear intensity dependence [14,15] of two-photon absorption rate in squeezed light [16]. The predictions of a suppressed and phase-sensitive decay of the atomic polarization according to Eqs. (1) and (2) have so far eluded observation, essentially due to the difficulty of embedding atoms in a squeezed vacuum in a complete  $4\pi$  solid angle. If we denote by  $\epsilon$  the fraction of the solid angle filled by the squeezed vacuum modes, then the polarization decay rates reduce to [2]

$$\begin{split} \gamma_x &= \gamma \bigg[ \, \epsilon \bigg( \, N + \frac{1}{2} - M \, \bigg) + (1 - \epsilon) \, \frac{1}{2} \bigg], \\ \gamma_y &= \gamma \bigg[ \, \epsilon \bigg( \, N + \frac{1}{2} + M \, \bigg) + (1 - \epsilon) \, \frac{1}{2} \bigg]. \end{split}$$

Clearly, if  $\epsilon$  differs significantly from 1 the influence of the squeezed vacuum will be reduced accordingly. One possible solution to achieve a large effective  $\epsilon$  close to 1 is to consider systems that are effectively one dimensional due, for

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FIG. 1. Realization of a level scheme involving two ground states and two upper states that are Zeeman sublevels. The atomic levels are coupled by two right- and left-circularly polarized laser fields with Rabi frequencies  $\epsilon_{+}\Omega$  and  $\epsilon_{-}\Omega$ . These laser fields are weak in the sense that  $\Gamma \gg \epsilon_{\pm}\Omega$ . We will consider this scheme in two situations. (a) In the *ideal situation* there is only spontaneous decay along channels with  $\Delta m_{j}=0$ . These decays interfere and give rise to squeezed-bath-like effects. (b) In a *realistic model* each upper level decays with a total decay rate  $\Gamma$  with a branching probability for the decay channels determined by the Clebsch-Gordan coefficients  $g_{c}$  and  $g_{l}$ . The processes along the cross lines of decay do not interfere since they give rise to right- and left-circularly polarized photons, respectively. The effect of these cross-decay channels will give rise to collisionlike effects of the reservoir. In Sec. V B we show how to suppress the cross decay.

example, to a strong cavity-atom interaction as proposed (see, e.g., Refs. [17] and [18]).

In this paper we will show how to "engineer" a squeezed-bath-type interaction leading to a Gardiner-Bloch equation (1) by studying the dynamics of a *driven multilevel* atom coupled to "normal" vacuum. In the simplest version we will consider a four-level system: for example, an angular momentum  $J_g = 1/2 \rightarrow J_e = 1/2$  transition with two (degenerate) ground  $(|g_{m=\pm 1/2}\rangle)$  and excited states  $(|e_{m=\pm 1/2}\rangle)$ , as illustrated in Fig. 1. If this atomic transition is driven by  $\sigma_+$ and  $\sigma_{-}$ -polarized laser light, the spontaneously emitted linearly polarized  $\pi$  photons emitted in the transition  $|e_{+1/2}\rangle \xrightarrow{\pi} |g_{+1/2}\rangle$  and  $|e_{-1/2}\rangle \xrightarrow{\pi} |g_{-1/2}\rangle$  will *interfere* since they are indistinguishable. For weak driving fields far below saturation we can adiabatically eliminate the excited states. The dynamics of the two ground states  $|g_{\pm 1/2}\rangle, |g_{\pm 1/2}\rangle$  then obeys a Master equation with damping terms due to optical pumping processes between the two ground states. This Master equation has a structure analogous to the coupling of two-level atoms to a squeezed bath. Interfering processes to mimic squeezed state detection statistics have been used previously by Lewenstein and co-workers [19,20]. It is used in the context of atomic spin measurements in [21]. Reservoir engineering to influence resonance fluorescence by changing the density of states in a cavity has been investigated in [22]. In the context of ion motion, reservoir engineering has been shown in [23].

The paper is organized as follows. In Sec. II we will summarize the properties of the master equation for a two-level atom coupled to a squeezed bath for reference and comparison in later sections. In Sec. III we discuss the reduction of the multilevel Master equation to an effective two-level master equation with squeezed-bath-type couplings of the form (1). Section IV presents numerical results for resonance fluorescence and absorption spectra in four-level systems and compares with the corresponding squeezed bath results. Nonideal effects, leading to thermal (as opposed to squeezed) bath coupling, as given, for example, by the crossdecay terms in spontaneous emission in Fig. 1, will be investigated in Sec. V; in addition, we will suggest mechanisms on how to suppress these unwanted effects. We conclude in Sec. VI.

# II. TWO-LEVEL ATOM IN A SQUEEZED VACUUM: A SUMMARY

In this section we will briefly review the basic effects and properties of a two-level system coupled to a squeezed bath. In particular, we discuss solutions of the Gardiner-Bloch equations [2] and the main features of the spectrum of resonance fluorescence and the atomic absorption spectrum. We summarize these results for reference in the following sections.

# A. The Gardiner-Bloch equation

We consider a two-level atom with ground state  $|g\rangle$  and excited state  $|e\rangle$ . The coupling of the atom to a squeezed vacuum is described by the interaction Hamiltonian

$$H_{\rm int} = \sigma \Gamma^{\dagger} + \sigma^{\dagger} \Gamma, \qquad (3)$$

with an atomic lowering operator  $\sigma = |g\rangle \langle e|$  and a bath operator  $\Gamma$ , which is given in terms of the coupling constants  $\kappa_{k,\lambda}$  and the photon annihilation operator  $a_{k,\lambda}$  as

$$\Gamma(t) = \sum_{k,\lambda} \kappa_{k,\lambda} a_{k,\lambda} e^{-i\omega_k t}.$$
(4)

A broadband squeezed bath centered around the atomic transition frequency  $\omega_A$  is characterized by the correlation functions

$$\langle \Gamma^{\dagger}(t)\Gamma(t')\rangle = \gamma N \,\delta(t-t'),$$

$$\langle \Gamma(t)\Gamma^{\dagger}(t')\rangle = \gamma (N+1)\,\delta(t-t'),$$

$$\langle \Gamma(t)\Gamma(t')\rangle = \gamma M e^{-i\phi} e^{-2i\omega_{A}t}\,\delta(t-t'),$$

$$\langle \Gamma^{\dagger}(t)\Gamma^{\dagger}(t')\rangle = \gamma M e^{i\phi} e^{2i\omega_{A}t}\,\delta(t-t'),$$

$$\langle \Gamma^{\dagger}(t)\Gamma^{\dagger}(t')\rangle = \gamma M e^{i\phi} e^{2i\omega_{A}t}\,\delta(t-t'),$$

$$\langle \Gamma^{\dagger}(t)\Gamma^{\dagger}(t')\rangle = \gamma M e^{i\phi} e^{2i\omega_{A}t}\,\delta(t-t'),$$

with effective photon number *N* and the (real) squeezing parameter *M*. These parameters are restricted by the inequality  $M^2 \leq N(N+1)$ , where the equal sign holds for maximal squeezing. The squeezing phase is denoted by  $\phi$ . Here "broadband squeezed bath" refers to the assumption that the squeezing bandwidth is larger than the other frequency scales in the problem (excluding the optical frequency), such as the spontaneous decay rate  $\gamma$ .

Knowledge of the correlation functions (5) allows one to derive the Master equation for the atomic dynamics in the Born-Markov approximation. In a rotating frame the Master equation is [2,3]

TABLE I. Linewidth in the regime of strong driving. The notations  $\gamma_x$ ,  $\gamma_y$ ,  $\gamma_z$  refers to the notation of Eqs. (2).

	$\phi \!=\! 0$	$\phi = \pi$
Central peak	$\gamma_x = \gamma(N + \frac{1}{2} - M)$	$\gamma_y = \gamma(N + \frac{1}{2} + M)$
Sidebands	$(\gamma_y + \gamma_z)/2 = \frac{\gamma}{4}(6N+3+2M)$	$(\gamma_x + \gamma_z)/2 = \frac{\gamma}{4}(6N + 3 - 2M)$

$$\frac{d}{dt}\rho = \frac{1}{2}\gamma(N+1)(2\sigma\rho\sigma^{\dagger} - \sigma^{\dagger}\sigma\rho - \rho\sigma^{\dagger}\sigma) + \frac{1}{2}\gamma N(2\sigma^{\dagger}\rho\sigma - \sigma\sigma^{\dagger}\rho - \rho\sigma\sigma^{\dagger}) - \gamma M e^{i\phi}\sigma^{\dagger}\rho\sigma^{\dagger} - \gamma M e^{-i\phi}\sigma\rho\sigma.$$
(6)

For nonperfect squeezing we define mean photon numbers  $N_1$  and  $N_2$  through  $N_1(N_1+1) = M^2$  and  $N = N_1 + N_2$ , which allows us to rewrite Eq. (6) in the form

$$\frac{d}{dt}\rho = \frac{1}{2}\gamma(2\Sigma\rho\Sigma^{\dagger} - \Sigma^{\dagger}\sigma\Sigma - \rho\Sigma^{\dagger}\Sigma) + \frac{1}{2}\gamma N_{2}(2\sigma\rho\sigma^{\dagger} - \sigma^{\dagger}\sigma\rho - \rho\sigma^{\dagger}\sigma) + \frac{1}{2}\gamma N_{2}(2\sigma^{\dagger}\rho\sigma - \sigma\sigma^{\dagger}\rho - \rho\sigma\sigma^{\dagger}),$$
(7)

with

$$\Sigma = \sqrt{N_1 + 1} \, \sigma + e^{i\phi} \sqrt{N_1} \, \sigma^{\dagger}.$$

The first line in Eq. (7) represents damping terms for ideal squeezing, while the second and third lines correspond to a thermal reservoir (background).

In a wave-function simulation of the master equation we would interpret  $\Sigma$  and  $\sigma, \sigma^{\dagger}$  as quantum jump operators for the various damping terms in Eq. (7). In this quantum jump picture the master equation is represented by an ensemble of quantum trajectories of pure system wave functions, where the evolution of states is described by an effective, non-Hermitian Hamiltonian  $H_{\text{eff}}$ , interrupted by quantum jumps where the wave function undergoes jumps according to the action of quantum jump operators on the wave function. For a discussion of a quantum jump picture of two-level atoms coupled to a squeezed bath we refer the reader to [24].

With the notation  $S_x = \langle \sigma^{\dagger} + \sigma \rangle$ ,  $S_y = \langle (\sigma^{\dagger} - \sigma)/i \rangle$ , and  $S_z = \langle P_e - P_g \rangle$  for the Bloch vector, the Master equation (6) with the phase choice  $\phi = 0$  is equivalent to Eqs. (1) given in the Introduction. The corresponding equation for a driven system reads

$$\frac{d}{dt}\langle S_x \rangle = -\gamma \left( N + \frac{1}{2} - M\cos\phi \right) \langle S_x \rangle + \gamma M \sin\phi \langle S_y \rangle,$$
$$\frac{d}{dt}\langle S_y \rangle = -\gamma \left( N + \frac{1}{2} + M\cos\phi \right) \langle S_y \rangle + \gamma M \sin\phi \langle S_x \rangle$$
$$+ \Omega_D \langle S_z \rangle, \tag{8}$$

$$\frac{d}{dt}\langle S_z\rangle = -\gamma(2N+1)\langle S_z\rangle - \Omega_D\langle S_y\rangle - \gamma,$$

where  $\phi = \varphi_S - 2\varphi_D$  with  $\varphi_D$  a phase of the driving field with Rabi frequency  $\Omega_D$  and  $\varphi_S$  a squeezing phase in the same reference frame as  $\varphi_D$ . The Bloch equation is given here for the choice  $\varphi_D = 0$ . A consequence of the broken symmetry in the polarization decay is that the steady state of the driven two-level system becomes dependent on the relative phase between the squeezing and the driving field.

### **B.** Steady state

The steady state of the Bloch equation (8) for the driven two-level system becomes phase dependent if the vacuum is squeezed  $(M \neq 0)$ . This means that propagation effects such as absorption and dispersion become dependent on the the relative phase between the driving field and the squeezing phase. Therefore, the steady state might be interesting to be used as a kind of optical switching where propagating beams are controlled by the squeezing phase and the driving laser. For normal vacuum the steady state is (up to a global phase) independent of the phase of the driving laser.

#### C. Spectrum of resonance fluorescence

The spectrum of resonance fluorescence spectrum of a strongly driven two-level atom is determined by the Fourier transform  $S(\omega)$  of the stationary two-time correlation function of the atomic dipole:

$$S_2(\omega) = \Phi^{\rm FT} \langle \sigma^{\dagger}(0) \sigma(\tau) \rangle. \tag{9}$$

Equation (9) implies that the free field corresponding to the particular mode for which the spectrum is observed is not squeezed.

The spectrum of resonance fluorescence for a squeezed bath has been calculated in Ref. [3]. According to the quantum regression theorem, the evolution of the correlation function is governed by the Bloch equations. Therefore, the spectral lines of the resonance fluorescence spectrum are determined by the eigenvalues of the coefficient matrix in Eq. (8). For strong driving  $\Omega_D \gg \gamma_x, \gamma_y$  one finds a Mollow triplet with phase-dependent linewidths and intensities (see Table I). For strong squeezing  $(N \rightarrow \infty)$  the center line can become arbitrarily close to zero linewidth for  $\phi = 0$ .

#### **D.** Atomic absorption spectrum

The absorption spectrum of a weak probe field in the presence of a strong driving field is related to the Fourier transform of the two-time correlation function

$$W(\boldsymbol{\omega}) = \Phi^{\mathrm{FT}} \langle [\sigma(\tau), \sigma^{\dagger}(0)] \rangle.$$
 (10)

According to Ref. [5],  $W(\omega)$  shows a strong dependence on the value of  $\phi$ . For  $\phi = 0$  it shows a strong peak at the center and dispersionlike sidebands. The width of the central absorption peak is again dependent on *N* and narrows to zero as *N* grows, since its linewidth is the same as in the resonance fluorescence. Between the center line and the sidebands is a region with gain. For  $\phi = \pi$  these detuning regions with gain are stronger and one finds even for the center line stimulated emission. The center line is broadened as well, in accordance with the center linewidth of the resonance fluorescence.

# III. DERIVATION OF A SQUEEZED-BATH MASTER EQUATION FOR AN EFFECTIVE TWO-LEVEL SYSTEM

The signature of the squeezed-bath coupling in the master equation (7) is the appearance of a quantum jump operator of the form  $\sqrt{N_1+1}\sigma + e^{i\phi}\sqrt{N_1}\sigma^{\dagger}$ . We interpret the structure of this operator as arising from interference in the atomic transition from the lower to the upper atomic level ( $\sigma^{\dagger}$ ) and the transition from the upper to the lower state ( $\sigma$ ). An analogous interference will occur in optical pumping processes between the two ground states in a four-level system in the emission of  $\pi$ -polarized photons.

Let us consider the four-level atom according to Fig. 1. The two ground states  $|g_{m=\pm 1/2}\rangle$  correspond to the two levels  $|g\rangle$ ,  $|e\rangle$  of the preceding section. The two upper levels  $|e_{m=\pm 1/2}\rangle$  will be used as auxiliary levels. The upper and lower levels are connected by spontaneous decay due to the interaction with the normal vacuum reservoir. The two decay channels are assumed to give the same decay rate  $\Gamma$  and to give rise to emitted photons of the same polarization and frequency, so that the two spontaneous emission processes are indistinguishable. We add two weak resonant laser fields to connect coherently the levels  $|g_{-1/2}\rangle$  to  $|e_{+1/2}\rangle$  with a Rabi frequency  $\epsilon_{-}\Omega \ (\ll\Gamma)$  and  $|g_{+1/2}\rangle$  to  $|e_{-1/2}\rangle$  with a Rabi frequency  $\epsilon_{+}\Omega \ (\ll\Gamma) \ (\epsilon_{+}^{2}+\epsilon_{-}^{2}=1)$ . Thus the two ground states are connected by optical pumping. The first process starts at level  $|g_{-1/2}\rangle$  and transfers via the weak laser field with Rabi frequency  $\epsilon_{-}\Omega$  to level  $|e_{+1/2}\rangle$  from where it decays to level  $|g_{\pm 1/2}\rangle$ . The second process will connect  $|g_{\pm 1/2}\rangle$  to  $|g_{\pm 1/2}\rangle$ , due to absorption from the other laser with Rabi frequency  $\epsilon_+\Omega$  ( $\ll\Gamma$ ). As a result, we end up with a Master equation for the effective two-level system  $|g_{+1/2}\rangle, |g_{-1/2}\rangle$ , where the damping terms have a structure analogous to a squeezed-bath coupling. The cross-decay terms corresponding to emission of a  $\sigma_+$  or  $\sigma_-$  photon will not interfere and thus give rise to a phase-insensitive background (deviation from the ideal squeezed-bath-type couplings).

# A. Adiabatic elimination in the four-level system Master equation

We will now derive the Master equation for the effective two-level system starting with the master equation for the four-level system, including the cross decay (Fig. 1). It is given by

$$\frac{d}{dt}\rho = -i(H_{\rm eff}\rho - \rho H_{\rm eff}^{\dagger}) + g_l^2 \Gamma(\sigma_1 + \sigma_2)\rho(\sigma_1^{\dagger} + \sigma_2^{\dagger}) + g_c^2 \Gamma \sigma_- \rho \sigma_-^{\dagger} + g_c^2 \Gamma \sigma_+ \rho \sigma_+^{\dagger}, \qquad (11)$$

with  $H_{\rm eff}$  an effective Hamiltonian

$$H_{\rm eff} = -i\frac{\Gamma}{2}P_e + \epsilon_{-}\frac{\Omega}{2}(\sigma_{-}^{\dagger} + \sigma_{-}) + \epsilon_{+}\frac{\Omega}{2}(e^{-i\phi_L}\sigma_{+}^{\dagger} + e^{i\phi_L}\sigma_{+}), \qquad (12)$$

where  $\phi_L$  is the relative phase between the two lasers. The atomic lowering operators are

$$\sigma_{1} = |g_{-1/2}\rangle \langle e_{-1/2}|, \quad \sigma_{2} = |g_{+1/2}\rangle \langle e_{+1/2}|,$$
  
$$\sigma_{+} = |g_{+1/2}\rangle \langle e_{-1/2}|, \quad \sigma_{-} = |g_{-1/2}\rangle \langle e_{+1/2}|.$$

Furthermore, we define a projection operator for the upper states as

$$P_e = |e_{-1/2}\rangle \langle e_{-1/2}| + |e_{+1/2}\rangle \langle e_{+1/2}|.$$

The Clebsch-Gordan coefficients for the coupling of the upper to the lower levels are denoted by  $g_l$  and  $g_c$ , respectively (compare Fig. 1).

In a parameter regime satisfying  $\epsilon_{\pm} \Omega \ll \Gamma$  we can adiabatically eliminate the upper levels  $|e_{-1/2}\rangle$  and  $|e_{+1/2}\rangle$  [25]. For this we introduce the operator

$$D = \epsilon_{-}\sigma_{-} + \epsilon_{+}e^{i\phi_{L}}\sigma_{+}$$

and use the projections onto the upper states  $P_e$  and the lower states  $P_g$  to represent the master equation in terms of the evolution of the upper states, the lower states, and the coherence between upper and lower states. The corresponding equations are

$$\frac{d}{dt}P_e\rho P_e = \frac{d}{dt}\rho_{ee} = -\Gamma\rho_{ee} - i\frac{\Omega}{2}(D^{\dagger}\rho_{ge} - \rho_{eg}D), \quad (13)$$

$$\frac{d}{dt}P_{g}\rho P_{e} = \frac{d}{dt}\rho_{ge} = -\frac{\Gamma}{2}\rho_{ge} + i\frac{\Omega}{2}(\rho_{gg}D - D\rho_{ee}), \quad (14)$$

$$\frac{d}{dt}P_{g}\rho P_{g} = \frac{d}{dt}\rho_{gg} = -i\frac{\Omega}{2}(D\rho_{eg} - \rho_{ge}D^{\dagger}) + g_{l}^{2}\Gamma(\sigma_{1} + \sigma_{2})\rho_{ee}(\sigma_{1}^{\dagger} + \sigma_{2}^{\dagger}) + g_{c}^{2}\Gamma\sigma_{-}\rho_{ee}\sigma_{-}^{\dagger} + g_{c}^{2}\Gamma\sigma_{+}\rho_{ee}\sigma_{+}^{\dagger}.$$
(15)

In the adiabatic approximation,  $\rho_{ee}$  follows adiabatically the changes in the coherences described by  $\rho_{eg}$ , which in turn follow adiabatically the dynamics of the ground states  $\rho_{gg}$ . In this approximation we find for the density matrix restricted to the lower levels  $\rho_{gg}$  a Master equation that involves two Lindblad terms

TABLE II. Parameters of the two-level system in a squeezed vacuum expressed as a function of the parameters of the mimicking four-level system.

Two-level atom in squeezed	vacuum	Four-level system
Spontaneous emission rate in Normal vacuum	γ	$(\boldsymbol{\epsilon}_{+}^{2}-\boldsymbol{\epsilon}_{-}^{2})rac{\Omega^{2}}{\Gamma}$
Photon-number expectation Value of squeezed vacuum	Ν	$rac{oldsymbol{\epsilon}_{-}^2}{oldsymbol{\epsilon}_{+}^2-oldsymbol{\epsilon}_{-}^2}$
Squeezing parameter	М	$rac{oldsymbol{\epsilon}_{-}oldsymbol{\epsilon}_{+}}{oldsymbol{\epsilon}_{+}^2-oldsymbol{\epsilon}_{-}^2}$
Phase of squeezing	$e^{i\phi}$	$e^{i\phi_L}$

$$\frac{d}{dt}\rho_{gg} = \frac{1}{2} \frac{\Omega^2}{\Gamma} g_l^2 (2\widetilde{\Sigma}\rho\widetilde{\Sigma}^{\dagger} - \widetilde{\Sigma}^{\dagger}\widetilde{\Sigma}\rho - \rho\widetilde{\Sigma}^{\dagger}\widetilde{\Sigma})$$
(16)

$$+\frac{1}{8}g_c^2\frac{\Omega^2}{\Gamma}(2\sigma_z\rho\sigma_z-2\rho),\qquad(17)$$

with the jump operator  $\tilde{\Sigma}$  given in terms of the Raman operator  $\sigma = |g_{-1/2}\rangle \langle g_{+1/2}|$  as

$$\widetilde{\Sigma} = (\epsilon_{+}\sigma + e^{i\phi}\epsilon_{-}\sigma^{\dagger}).$$
(18)

The remaining parts of the four-level density matrix are related to  $\rho_{gg}$  by

$$\rho_{ee} = \frac{\Omega^2}{\Gamma^2} D^{\dagger} \rho_{gg} D \tag{19}$$

and

$$\rho_{eg} = i \frac{\Omega}{\Gamma} D^{\dagger} \rho_{gg} \,. \tag{20}$$

We make the replacements of Table II to write this Master equation as

$$\frac{d}{dt}\rho = -i(H_{\rm eff}\rho - \rho H_{\rm eff}^{\dagger}) + g_l^2 \gamma(\sqrt{N+1}\sigma + e^{i\phi}\sqrt{N}\sigma^{\dagger})\rho(\sqrt{N+1}\sigma^{\dagger} + e^{-i\phi}\sqrt{N}\sigma) + g_c^2 \gamma \left(\frac{N}{2} + \frac{1}{4}\right)\sigma_z \rho \sigma_z,$$
(21)

with the effective Hamiltonian

$$H_{\rm eff} = -i \frac{\gamma}{2} \left[ g_c^2 \left( \frac{N}{2} + \frac{1}{4} \right) + g_l^2 (N \mathbb{1} + P_+) \right].$$
(22)

The additional jump operator damping term involving

$$\sigma_z = P_+ - P_-$$

is characteristic for a process that destroys the decoherence between the two ground-state levels [26]. Now we also consider a laser connecting the Raman transition



FIG. 2. Schematic view of the relation between system and reservoir for the two-level and the four-level situation.

 $|g_{-1/2}\rangle \rightarrow |g_{+1/2}\rangle$ . We will assume that these lasers are far from any atomic resonance so that spontaneous emission can be neglected. (Alternatively, one can consider an ac magnetic field.) The Bloch equation is given by

$$\frac{d}{dt}\langle S_x \rangle = -\gamma \left( N + \frac{1}{2} - g_l^2 M \cos \phi_L \right) \langle S_x \rangle + \gamma g_l^2 M \sin \phi_L \langle S_y \rangle,$$
  
$$\frac{d}{dt} \langle S_y \rangle = -\gamma \left( N + \frac{1}{2} + g_l^2 M \cos \phi_L \right) \langle S_y \rangle + \gamma g_l^2 M \sin \phi_L \langle S_x \rangle$$
  
$$+ \Omega_D \langle S_z \rangle, \qquad (23)$$

$$\frac{d}{dt}\langle S_z\rangle = -\gamma g_l^2 (2N+1)\langle S_z\rangle - \Omega_D \langle S_y\rangle - g_l^2 \gamma,$$

where  $\Omega_D$  is the Rabi frequency of the Raman transition with the phase chosen as  $\phi_R = 0$ .

# **B.** Discussion

For  $g_l = 1$  (no cross decay) Eq. (23) is precisely the Bloch equation (8). We can bring the Master equation (16) into the form (6) or the Bloch form (1) by making the identifications of Table II. The two ground levels thus show a dynamics that is analogous to that of a two-level atom in a squeezed bath. Effectively we "engineer," with the help of the upper levels and the laser fields, a reservoir for the two ground levels such that it looks like the coupling to a squeezed bath (see Fig. 2). This point of view can be brought out more clearly by writing an interaction Hamiltonian for the lower states in the adiabatic elimination as



FIG. 3. Steady state for N=2.1,  $\gamma/\Gamma=1.9\times10^{-5}$ , and  $\Omega_D/\gamma=5.1$  and for the ideal case of  $g_l=1$  (solid line) and three other values with  $g_l=0.99$  (dashed line),  $g_l=0.95$  (dash-dotted line), and  $g_l=0.9$  (dotted line).

$$H_{\rm int} = \sigma_{gg} \widetilde{\Gamma}^{\dagger} + \sigma_{gg}^{\dagger} \widetilde{\Gamma}, \qquad (24)$$

with  $\sigma_{gg} = |g_{1/2}\rangle \langle g_{-1/2}|$  the transition operator between the two ground states and the bath operator  $\tilde{\Gamma}$  given in terms of the coupling constants  $\kappa_{k,\lambda}$  and the photon annihilation operator  $a_{k,\lambda}$  as

$$\widetilde{\Gamma} = \sum_{k,\lambda} \left[ \left( i \frac{\Omega}{\Gamma} \right) (\kappa_{k,\lambda} \epsilon_{-} a_{k,\lambda}^{\dagger} e^{i\omega_{k}t} e^{-i\omega_{L}t} - \kappa_{k,\lambda}^{*} \epsilon_{+} e^{-i\phi} a_{k,\lambda} e^{-i\omega_{k}t} e^{i\omega_{L}t} \right].$$
(25)

The correlation functions of these bath operators are the same as the correlation functions of the operator  $\Gamma$  given in Eq. (5) of the squeezed bath if one uses the identification of Table II and takes into account that for the degenerate ground states the frequency corresponding to  $\omega_A$  vanishes.

In summary, the main elements of the four-level system are that the direct transition between the two levels of the two-level scheme is replaced by a pumping process involving other atomic levels and that the driving between the two ground states is replaced by a Raman coupling.

# IV. DISCUSSION AND RESULTS: THE IDEAL MODEL

We will now present some examples of the ideal  $(g_i=1)$  behavior of the four-level system to illustrate that it indeed mimics a two-level atom coupled to a squeezed bath. The following figures were produced using the full Master equation for the four-level system of Fig. 1. Analytic formulas presented, however, use the adiabatically eliminated equations for the ground states to allow an easier comparison to the two-level system. For the parameters used in the figures, the adiabatically eliminated and the exact calculations are in excellent agreement.

#### A. Steady-state solution

An example of exact correspondence is the steady-state solution for the driven system as investigated by Carmichael, Lane, and Walls [3]. We drive the system by a resonant Raman transition with an effective Rabi frequency  $\Omega_D$ . A typical steady state is shown as a solid line in Fig. 3. The dependence on the phase  $\phi$  allows light fields, causing the two internal transitions and the driving, to interact depending on the relative phase between them. This leads to effects in the propagation of those fields through a cloud of atoms with the effective four-level system as shown above. The results for the four-level system show the  $\phi$  dependence in agreement with a corresponding two-level system.

## **B.** Absorption spectrum

As our first example we study the absorption spectrum of the four-level system. The idea is, in analogy to the investigation concerning the two-level system, to drive the effective two-level system strongly coupling the two ground states in a Raman transition that is tuned on resonance. Then the weakfield absorption spectrum is measured, where the probe field is represented by another stimulated Raman transition. The stationary absorption spectrum is then given by the Fourier transform of a two-time correlation function

$$W(\omega) \sim \Phi^{\text{FT}} \langle \sigma_{gg}(\tau) \sigma_{gg}^{\dagger}(0) - \sigma_{gg}^{\dagger}(0) \sigma_{gg}(\tau) \rangle, \quad (26)$$

which can be calculated using the quantum regression theorem.

The absorption spectrum of the four-level system [see Fig. 4(a)] is identical to that of the two-level system in a squeezed bath and we can see the phase dependence and the sharp peak at the center for  $\phi = 0$  (solid line). This is readily understood since the process involved in measuring the absorption spectrum makes use only of the system level dynamics as described by the Master equation (21). This stands in contrast to the situation of the resonance fluorescence,



FIG. 4. Absorption spectrum (in arbitrary units) for N=1,  $\gamma/\Gamma = \frac{1}{3} \times 10^{-4}$ , and  $\Omega_D/\gamma = 7.1$  and for (a) the ideal case  $(g_l=1)$  and for three values of  $g_l = \{0.9, (\frac{2}{3})^{1/2}, (\frac{1}{3})^{1/2}\}$  [(b), (c), and (d), respectively]. The solid (dotted) lines show the spectra for  $\phi = 0$  ( $\phi = \pi$ ).

which differs slightly from that of the two-level scheme, as will be shown in the next subsection.

$$S(\omega) = \Phi^{\text{FT}} \langle [\epsilon_{+}e^{i\phi}\sigma^{\dagger}_{gg}(0) + \epsilon_{-}\sigma_{gg}(0)] [\epsilon_{+}e^{-i\phi}\sigma_{gg}(\tau) + \epsilon_{-}\sigma^{\dagger}_{gg}(\tau)] \rangle.$$

$$(27)$$

### C. Resonance fluorescence

For resonance fluorescence we drive the system again by a stimulated Raman transition and detect the fluorescence coming from the transitions  $|e_{\pm 1/2}\rangle \xrightarrow{\pi} |g_{\pm 1/2}\rangle$ . This leads, in the adiabatic elimination, to the spectrum given in terms of ground-state correlations as

In contrast, the fluorescence spectrum calculated by [3] is based on the Fourier transform of the correlation function  $S_2(\omega) = \Phi^{\text{FT}} \langle \sigma^{\dagger}(0)\sigma(\tau) \rangle$  (see Sec. II), which is based on the assumption that there is no interference in the detector between the source field and the squeezed vacuum modes. In our case the fluorescence spectrum is due to a kind of atomic



FIG. 5. Resonance fluorescence in arbitrary units for N=0.2,  $\gamma/\Gamma=7.1\times10^{-5}$ , and  $\Omega_D/\gamma=7.1$  and for (a) the ideal case  $(g_l=1)$  and for three values of  $g_l = \{0.9, (\frac{2}{3})^{1/2}, (\frac{1}{3})^{1/2}\}$  [sub-plots (a)–(d)]. The solid (dotted) line shows the spectrum for  $\phi=0$  ( $\phi=\pi$ ).

TABLE III. Linewidth in the regime of strong driving in depending on the effective photon number N and Clebsch-Gordan coefficient  $g_l^2 = 1 - g_c^2$ .

	$\phi = 0$	$\phi = \pi$
Central peak Sidebands	$\gamma [N + \frac{1}{2} - g_l^2 \sqrt{N(N+1)}]$	$\gamma [N + \frac{1}{2} + g_l^2 \sqrt{N(N+1)}]$
	$\frac{\gamma}{4} \{ 2N + 1 + 2g_l^2 [2N + 1 + \sqrt{N(N+1)}] \}$	$\frac{\gamma}{4} \{ 2N + 1 + 2g_l^2 [2N + 1 - \sqrt{N(N+1)}] \}$

quadrature correlation (similar to those appearing in the squeezing spectrum [27]) and there are three additional terms with respect to the two-level scheme if we use the four-level scheme. The effect of the additional terms is to enhance (suppress) the central peak of the Mollow triplet for  $\phi = 0$  $(\phi = \pi)$  relative to the sidebands even more than is the case for the two-level system. This effect becomes more prominent with increasing N and leads for  $N \ge 1$  to a vanishing amplitude for the sidebands ( $\phi=0$ ) or the central line  $(\phi = \pi)$ . However, this does not change the position and width of the lines in the Mollow triplet. Therefore, the line narrowing and the phase dependence of the line intensities can be observed in the four-level system [see Fig. 5(a)]. The resonance fluorescence of the four-level scheme differs from that of the two-level, which is not surprising since its origin can be explained only in the full four-level scheme and not in the effective two-level scheme of the ground states. On this point the resonance fluorescence differs from the absorption spectrum and the steady state, which can be explained just in terms of the master equation for the two ground states.

We continue to speak of the Mollow triplet for the fourlevel system in this parameter regime since only these three lines have non-negligible intensity. As one leaves the regime of validity of the adiabatic elimination, where the effective two-level system and the four-level system are in perfect agreement, one finds, of course, a richer line structure (see Sec. V C).

### V. RESULTS AND DISCUSSION: NONIDEAL EFFECTS

In this section we will address separately two problems connected to the validity of the effective two-level system Master equation with squeezed-bath-type couplings. One is the question of the influence of dephasing terms in the master equation (21) due to cross decay. The other is the question of behavior of the system at the onset of saturation.

# A. Effects of cross decay

For a four-level system  $J_g = \frac{1}{2} \rightarrow J_e = \frac{1}{2}$  the Clebsch-Gordan coefficients are  $g_l^2 = \frac{1}{3}$  and  $g_c^2 = \frac{2}{3}$ , so most of the spontaneous processes go into the (unwanted) dephasing part. This ratio improves by using Zeeman sublevels of higher angular momentum as upper levels where, with the help of the ac Stark effect, only the levels with  $m = \pm \frac{1}{2}$  take part in the dynamics while the other levels are shifted off resonance by a laser field. For  $J_e = \frac{3}{2}$  we then find the slightly more favorable numbers of  $g_l^2 = \frac{2}{3}$  and  $g_c^2 = \frac{1}{3}$ .

The effect of a nonvanishing  $g_c$  on the spectra and the steady state is quite different. The absorption spectrum and the resonance fluorescence spectrum show even for  $g_c^2 = \frac{2}{3}$  a strong dependence on  $\phi$  (see Figs. 4 and 5).

The linewidth of the resonance fluorescence interpolates linearly as a function of  $g_c^2$  between the values valid for the interaction with a squeezed vacuum and those for the normal vacuum as shown in Table III. The steady-state variation with  $\phi$ , especially that of the component  $S_x$ , is more sensitive to the presence of cross decay. To utilize the  $\phi$  dependence of the steady state one has to find a means to suppress this cross decay to an extent as large as possible (see Fig. 3).

## B. Suppressing the effects of cross decay

To suppress the dephasing in favor of the squeezed-bathtype effects, one can use destructive interference of the cross decay in a configuration with more atomic levels, similar to the one proposed in [28]. This scheme employs upper levels Zeeman levels of angular momentum  $J_e = \frac{1}{2}$  and those of angular momentum  $J_a = \frac{3}{2}$  (see Fig. 6). The weak laser fields of frequency  $\omega_L$  are now detuned between the upper levels with  $J_e = \frac{1}{2}$  and those with  $J_a = \frac{3}{2}$  with detunings  $\Delta_e = \omega_L - \omega_{eg}$  and  $\Delta_a = -(\omega_L - \omega_{ag})$ , respectively. The spontaneous decay rate and the Clebsch-Gordan coefficients have a subscript indicating the level they are referring to. We can calculate the rate for processes starting and ending in the same ground level. This rate depends on the detuning and the dipole elements  $d_e$  and  $d_a$ , referring to the transition between the selected ground state and the two intermediate upper levels taking part in the transition, respectively. It is given by

$$\frac{1}{\tau} = \frac{12c^3 \epsilon_0 |E|^2}{\hbar \omega_L^3} \left| \frac{g_c^{(a)2} \Gamma_a}{\Delta_a - i \frac{\Gamma_a}{2}} - \frac{g_c^{(e)2} \Gamma_e}{\Delta_e - i \frac{\Gamma_e}{2}} \right|^2, \quad (28)$$

where we introduced the field strength |E| corresponding to the Rabi frequencies  $\epsilon_{+/-}\Omega$ , respectively. We always can choose the detuning such that  $\Delta_e/\Delta_a = g_c^{(e)^2}\Gamma_e/g_c^{(a)^2}\Gamma_a$ .



FIG. 6. Extended level scheme that is used to suppress spontaneous decay between levels with  $m_J = -\frac{1}{2}$  and  $m_J = +\frac{1}{2}$  (cross decay) due to destructive interference.



FIG. 7. Study of line positions and width for the full four-level system (solid and dash-dotted lines) in comparison with the results of adiabatically eliminated equations (dotted lines). The parameter values are N=0.2,  $\Omega_D/\gamma=7.1$ , and  $g_I=1$ . Compared are the linewidth of the center line (top left), the linewidth of the sideband lines (bottom left), and the positions of the three lines (top right). To demonstrate that the Mollow triplet is clearly distinguishable from other lines, we show the linewidth of the line that is the narrowest after the triplet (bottom right). From this comparison it is clear that for  $\Omega/\Gamma < 0.2$  the approximation made in the adiabatic elimination of the upper states is valid.

This will reduce the cross decay by an order of magnitude in  $\Gamma_{e/a}/\Delta_{e/a}$ . A demonstration of this effect in a similar scheme has been given by Xia, Ye, and Zhu [29]. Any remaining cross decay can be described, within the validity of the adiabatic elimination, by effective Clebsch-Gordan coefficients and an effective decay rate [30].

### C. Validity of the adiabatic elimination

Finally, we investigate the behavior of the four-level system if we increase the Rabi frequencies of the internal transitions. To explore the range of validity of the adiabatic elimination and the effects of onset of saturation, we compare the linewidth and position of the Mollow triplet for the adiabatically eliminated situation and the full four-level system. In the latter we characterize the Mollow triplet as the three narrowest lines in the resonance fluorescence spectrum. We can find their positions and linewidth as eigenvalues of the  $15 \times 15$  matrix appearing in the Bloch equation of the four-level system. The effect of an increased value of  $\Omega/\Gamma$ for the parameters used to plot the resonance fluorescence in Fig. 5 is shown in Fig. 7. Up to  $\Omega/\Gamma \approx 0.2$  the linewidth and positions of the two-level Mollow triplet follow the curve predicted by the adiabatically eliminated theory. For higher values of  $\Omega/\Gamma$  the same position is still predicted by both equations, but all three lines become narrower in the full system than expected by the reduced equations. Other lines of the full four-level system prove to be clearly distinct in their linewidth from the ones of the Mollow triplet. We have checked that the whole appearance of the resonant fluorescence (that is, including the line amplitudes) shows an increasing deviation from the adiabatically eliminated equations as  $\Omega/\Gamma$  increases beyond the value  $\Omega/\Gamma = 0.2$ , just as expected from the behavior of the eigenvalues.

# VI. CONCLUSION

The central result of this paper is that there is a way to engineer an environment for a two-level system. The basic idea is to use the regime of adiabatic elimination to link coherent transitions from system levels to additional atomic levels with spontaneous transitions from these additional levels back to system levels. This leads to jump operators in the Master equation, which can be designed for a specific purpose. Especially interesting jump operators can be achieved if one uses, as in the model presented here, that there are indistinguishable spontaneous processes leading to linear superpositions of jump operators. With this engineering one converts the trivial reservoir of normal vacuum modes into the more sophisticated one of an effective squeezed vacuum filling the whole solid angle of  $4\pi$ .

We have illustrated this procedure for the example of a squeezed-bath-type coupling. Within this example we demonstrated that this reservoir can be engineered by the use of Zeeman sublevels. The predicted effects of the squeezedbath coupling on the steady state, the absorptions spectrum, and, with a slight change, the resonance fluorescence have been recovered in this model. We have shown that one can select spontaneous decay channels in the multilevel scheme and suppress others. Therefore, one can enhance the effect of engineered jump operators in the final master equation and suppress other processes that would partially destroy the desired effects. In a numerical study we have shown the effects of a nonideal realization resulting in decoherence processes that are collisionlike. This study shows that the predicted effects are robust enough to allow their observation. The regime of validity of the adiabatic elimination of the upper levels in our model has been explored.

Our work presents an approach that allows the observa-



FIG. 8. Subsystem relevant to investigate the destructive interference of spontaneous emission along the pumping transitions. The total decay rates of the upper levels are  $\Gamma_e$  and  $\Gamma_a$ . The Clebsch-Gordan coefficients for the spontaneous decay emitting circularly (linearly) polarized photons are denoted by  $g_c^{(e)}$  and  $g_c^{(a)}$  ( $g_l^{(e)}$  and  $g_l^{(a)}$ ). The laser is described by the electric-field amplitude |E| and shown as a solid arrow.

tion of theoretically predicted effects within reach of todays experimental techniques. It encourages the investigation of other reservoir couplings that could be engineered with the ideas presented in this paper.

# ACKNOWLEDGMENTS

This work has been supported by the Austrian Science Foundation and by the EU as part of the TMR-Network "Microlaser and Cavity QED" ERB-4061-P1-95-1021.

### APPENDIX: TRANSITION RATE FOR CROSS DECAY

The value of the detuning that gives destructive interference can be found using second-order perturbation theory. We consider the subsystem as shown in Fig. 8. The Hamiltonian describing the subsystem is given by

$$H = H_A + H_B + H_{AB} + H_{AL}, \qquad (A1)$$

with the system Hamiltonians of the atom, the bath, and the laser mode given by

$$H_{A} = \hbar \,\omega_{e} P_{e_{+1/2}} + \hbar \,\omega_{a} P_{a_{+1/2}}, \tag{A2}$$

$$H_B = \sum_{k,\lambda} \hbar \omega_k a_{k\lambda}^{\dagger} a_{k\lambda} , \qquad (A3)$$

and the interaction Hamiltonian's atom-bath and atom-laser couplings given by

$$H_{AB} = \sum_{k,\lambda} \left[ \hbar \kappa_{k\lambda}^{(e)} a_{k\lambda} | e_{+1/2} \rangle \langle g_{-1/2} | \right. \\ \left. + \hbar \kappa_{k\lambda}^{(a)} a_{k\lambda} | 2a \rangle \langle g_{-1/2} | + \text{H.c.} \right]$$
(A4)

$$+\sum_{k,\lambda} \left[ \hbar \kappa_{k\lambda}^{(e)} a_{k\lambda} | e_{-1/2} \rangle \langle g_{+1/2} | \right. \\ \left. + \hbar \kappa_{k\lambda}^{(a)} a_{k\lambda} | 1a \rangle \langle g_{+1/2} | + \text{H.c.} \right], \tag{A5}$$

$$H_{AL} = \hbar \Omega_{(e)} |e_{+1/2}\rangle \langle g_{-1/2} | + \hbar \Omega_{(a)} |a_{+1/2}\rangle \langle g_{-1/2} | + \text{H.c.}$$
(A6)

Here  $\kappa_{k\lambda}^{(e)}$ ,  $\kappa_{k\lambda}^{(a)}$  are coupling constants defined in terms of dipole vectors  $d_{(e/a)}$ , unit vectors for the electric field  $e_{k\lambda}$ , and a quantization volume V by

$$\kappa_{k\lambda}^{(e/a)} = \sqrt{\frac{\omega_{k\lambda}}{2\hbar\epsilon_0 V}} (e_{k\lambda}d_{(e/a)}). \tag{A7}$$

The Rabi frequencies due to the interaction with the classically treated laser fields with electric field of amplitude |E|and unit vector  $e_L$  are

$$\Omega_{(e/a)} = \frac{2|E|}{\hbar} (e_L d_{(e/a)}). \tag{A8}$$

In second-order perturbation theory the transition rate for processes starting and ending in  $|g_{-1/2}\rangle$  under spontaneous emission of a circular polarized photon is given by

$$\frac{1}{\tau} = \frac{4|E|^2 \omega_L^3}{3 \pi c^3 \hbar^3 \epsilon_0} \left| \frac{|d_a|^2}{\Delta_a} - \frac{|d_e|^2}{\Delta_e} \right|^2$$
(A9)

$$= \frac{12c^{3}\epsilon_{0}|E|^{2}}{\hbar\omega_{L}^{3}} \left| \frac{g_{c}^{(a)2}\Gamma_{a}}{\Delta_{a}} - \frac{g_{c}^{(e)2}\Gamma_{e}}{\Delta_{e}} \right|^{2}.$$
(A10)

Inclusion of the main terms of higher orders by the resolvent method [31] yields the result

$$\frac{1}{\tau} = \frac{12c^3\epsilon_0 |E|^2}{\hbar\omega_L^3} \left| \frac{g_c^{(a)2}\Gamma_a}{\Delta_a - i\frac{\Gamma_a}{2}} - \frac{g_c^{(e)2}\Gamma_e}{\Delta_e - i\frac{\Gamma_e}{2}} \right|^2.$$
(A11)

This leads finally to expression (28). The same result can be obtained using adiabatic elimination in the Master equation for the full system as shown in Fig. 6.

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