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Trapped photon state along an atomic chain in photonic band-gap crystals

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When a chain of two two-level atoms is placed in a photonic band-gap crystal, trapped photon-atom dressed states can form in the neighborhood of the chain due to emission and reabsorption of the photon by atoms in the chain. For a long chain such states form a quasicontinuum inside the forbidden frequency band. We examine the density of states as a function of the chain length and separation between atoms. The pattern of excitation for atoms in the chain forms a ''lattice wave,'' which may be used to understand the propagation of excitation along the chain. [S1050-2947(98)11606-2]

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It is known that in a photonic band-gap crystal certain unusual phenomena occur due to the presence of one or more than one forbidden electromagnetic wave bands in the crystal [1]. The photonic band crystal is a three-dimensional periodic dielectric structure. This periodicity changes the dispersion characteristics of radiation waves traveling in such a medium. Much literature has been dedicated to such structures both theoretically and experimentally [2], exploring the possibility to construct an artificial medium with controllable properties. Recent efforts have been concentrated on three-dimensional photonic band-gap crystals in the optical frequency range [3].

The dispersion relation in a photonic band crystal is very unsmooth, particularly for the frequency ranges near the edges of the forbidden band(s), where theoretical analysis predicts singularities. Thus the interaction of radiation of such frequency ranges with atoms embedded in the photonic band structure differs from the free-space field case drastically. The crystal acts as a cavity that can trap photon energy that will not dissipate away. The atom-field interaction process is strongly non-Markovian because photons are emitted and reabsorbed by the atoms. The dynamics of both a single atom exchanging energy with the field and more than one atom exchanging energy with the field and among themselves have been studied [4–9], and the interaction channels identified. In an earlier paper [10] we gave a rather general survey on the dressed state with a photon trapped around a single multilevel atom. Recently, it has also been demonstrated that multiple quanta of energy can be trapped in such structured systems [11]. The area contains many directions to be explored.

In the current report we study the possibility of obtaining multiple trapped photon dressed atom-field state when many atoms are embedded in a photonic band-gap structure (PBS) crystal. In particular we study an atom chain geometry in detail. This is the topic discussed in Ref. [8] where the authors illustrated the formalism with which the eigenvalues of the dressed state should be solved. We restrict the discussion to only one quantum of energy in the system, so only one atom can be excited at a time, and examine the details of the dressed state for both the field state part and the atomic state part. The atoms exchange energy with the field as well as with other atoms by radiative dipole-dipole interaction. For such a chain with the spacing between the atoms close to one optical wavelength or smaller, we show that there can be as many dressed states as the number of atoms. These dressed states form a subband in the forbidden band. We examine the distribution and properties of these dressed states and their relationship to the size of the chain. We further point out that it is possible to use the multiple dressed state method to study the propagation of atom excitation along the chain.

The basic method of treating nondecaying states with photons localized around atoms in a photonic crystal has received detailed discussion in the literature [4,6,7,10]. We follow the notation of Ref. [10]. Consider the simple case of N identical atoms arranged along a straight line and with equal spacing d. The location of the ith atom in the chain is labeled by the vector $\mathbf{x}_i$. The Hamiltonian of such an N-atom system is

$$H=\sum_{i=1}^{N} \hbar\omega_a(|a\rangle\langle a|)_i+\sum_k \hbar\omega_k a_k^\dagger a_k +\hbar\sum_{k,i}[g_{k,i}a_k(|a\rangle\langle b|)_i+g_{k,i}^* a_k^\dagger(|b\rangle\langle a|)_i], \tag{1}$$

where

$$g_{k,i}=\frac{\omega_a}{\hbar}\left[\frac{\hbar}{2\epsilon_0\omega_k V}\right]^{1/2}\hat{\mathbf{e}}_k\cdot\mathbf{d}_{ab}e^{i\mathbf{k}\cdot\mathbf{x}_i} \tag{2}$$

is the coupling coefficient between the ith atom and field mode k. We consider the following essential state expansion of the stationary state:

$$|\psi\rangle=\sum_i a_i|e_i\rangle_A|\mathrm{vac}\rangle_F+\sum_k b_k|g\rangle_A|1_k\rangle, \tag{3}$$

where $|e_i\rangle$ represents the ith atom in the excited state $|a\rangle$ with all other atoms in the ground state $|b\rangle$, and $|g\rangle$ represents all atoms in $|b\rangle$.

The eigenvalue problem $H|\psi\rangle=\hbar\omega|\psi\rangle$ can be solved using methods similar to Refs. [6,10]. Eliminating the one-

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*Present address: Rochester Center for Optical Science and Engineering, University of Rochester, Rochester, NY 14627. Electronic address:gjc-dmp@mail.tsinghua.edu.cn photon state coefficients b_k from the Schrödinger's equations after a rotating-wave approximation (RWA), which removes fast oscillation at the band-gap edge frequency ω_c for the no-photon component and fast oscillation at ω_k for the one-photon component with one photon of frequency ω_k we find

$$(\omega-\omega_c-\Delta)a_i=\sum_k g_{k,i}b_k,$$
$$(\omega-\omega_k)b_k=\sum_i g_{k,i}^{*}a_i \tag{4}$$

($\Delta=\omega_a-\omega_c$ is the detuning of the transition from the band-gap edge), we find the coupled equations for the excited atom state coefficients a_i:

$$[\omega-\omega_c-\Delta]a_i=\sum_j\sum_k\frac{g_{k,i}g_{k,j}^{*}}{\omega-\omega_k}a_j,\quad i,j=1,\ldots,N. \tag{5}$$

The expression $\Sigma_k g_{k,i}g_{k,j}^{*}/(\omega-\omega_k)$ can be shown to be equal to $f_{i,j}\beta^{3/2}/\sqrt[1]{|\omega-\omega_c|}$, where

$$\beta=\left[\frac{\omega_a^{7/2}d_{ab}^2}{6\pi\epsilon_0\hbar c^3}\right]^{2/3} \tag{6}$$

is the characteristic coupling strength between the atom and field continuum modes defined in Ref. [6], and the numerical coefficients

$$f_{i,j}=\frac{3}{2}\sum_\sigma\frac{1}{4\pi}\int\int d\hat{\mathbf{k}}^2 e^{ik_c\hat{\mathbf{k}}\cdot\mathbf{x}_{ij}}(\hat{\mathbf{e}}_{k\sigma}\cdot\hat{\mathbf{d}}_{ab})^2 \tag{7}$$

represent the relative coupling strength between the atom pair (i,j) via the emission and reabsorption of a photon. Equation (6) comes from the summation over the frequency, and Eq. (7) comes from the summation over the unit wave vector $\hat{\mathbf{k}}$ and polarization σ, evaluated at the band-gap edge wave vector value $\mathbf{k}_c$. As the distance x_{ij} in question is small we have used the constant value k_c in Eq. (7) rather than the variable k. The self-interaction part $f_{i,i}=1$ is reflected in the single atom problem. $f_{i,j}$ depends on the distance x_{ij} as well as the angle between $\vec{\mathbf{d}}_{ab}$ and $\vec{\mathbf{x}}_{ij}$. Here we only consider two special cases: (1) for $\vec{\mathbf{d}}_{ab}\parallel\vec{\mathbf{x}}_{ij}$,

$$f_{ij}=3\frac{\sin u}{u}+\frac{6(u\cos u-\sin u)}{u^3}; \tag{8}$$

(2) for $\vec{\mathbf{d}}_{ab}\perp\vec{\mathbf{x}}_{ij}$,

$$f_{ij}=\frac{3(\sin u-u\cos u)}{u^3}. \tag{9}$$

Here $u=kx_{ij}$.

Equation (5) can be written as a matrix equation of the form

$$DA=FA, \tag{10}$$

where the vector A consists of the amplitudes a_i, and D and F are $N\times N$ matrices. As D is a number matrix, Eq. (10) can be written in a diagonalized form after a transformation that does not depend on ω. The result is N separated eigenequations of the form

$$\omega-\omega_c-\Delta+\frac{\beta^{3/2}f_m}{\sqrt{|\omega-\omega_c|}}=0,\quad m=1,\ldots,N, \tag{11}$$

where f_m is the mth eigenvalue of the matrix F. Thus the eigenvalue problem is similar to that of the single atom case. The exact form of the dressed state for each f_m is also similar to the single two-level atom counterpart.

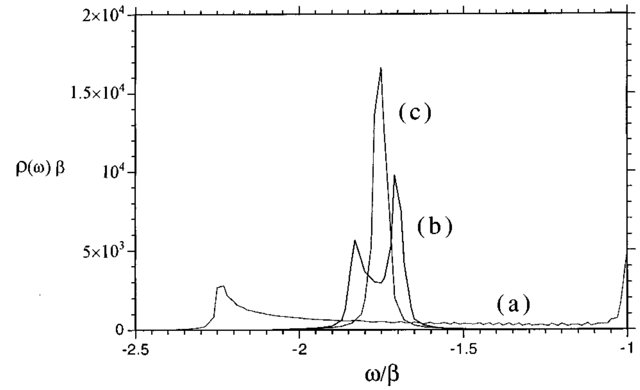

FIG. 1. The smoothed density of dressed states for a chain of 100 atoms. The dipole moments are perpendicular to the direction of the chain, as in subsequent figures. The quantities are dimensionless by setting β to unity. $\Delta=-1$. (a) $d=0.4\lambda$; (b) $d=0.8\lambda$; (c) $d=1.2\lambda$.

We found that all f_m's are positive, thus there are always N localized dressed states. For a large number of atoms in the chain the corresponding eigenvalues form a quasicontinuum in the forbidden band. Two questions arise: (1) What is the density of states in this quasicontinuum? (2) How will the density of states scale with the length of the chain? We have conducted numerical studies for various chain lengths (N) and interatomic spacing d. The results presented in the following figures are for the case with the dipole moment perpendicular to the chain. Changing the angle between the two will alter the form of the matrix F, which will lead to some changes in the details but not the main features described below.

The profile of the density of states shows a strong dependence on the interatomic spacing. In Fig. 1 the density of the dressed states is shown for different atom spacings; a Lorentzian line shape is introduced to make the profile a smooth one. For atoms separated farther than a wavelength (curve c), the exchange of photons between atoms is weak, thus the eigenvalues are essentially the single-atom result with small local corrections. The ensemble of these eigenvalues forms a single-peaked profile. For closely packed atoms the density of states has two peaks (curves a and b). We also notice that dressed states very deep inside the forbidden band emerge. Analysis shows that the depth of the lowest dressed state from the band-gap edge should scale linearly as the linear density of atoms λ/d for $\lambda/d\gg1$. On the other hand, increase of chain length (N) increases the density of states but does not change its profile (Fig. 2). It is equivalent to the sum of contributions from sections of the chain.

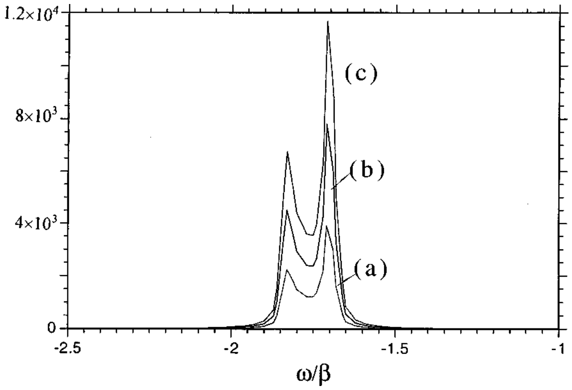

FIG. 2. The smoothed density of dressed states function for a chain of N atoms. $\Delta=-1$, $d=0.8\lambda$. (a) $N=40$; (b) $N=80$; (c) $N=120$.

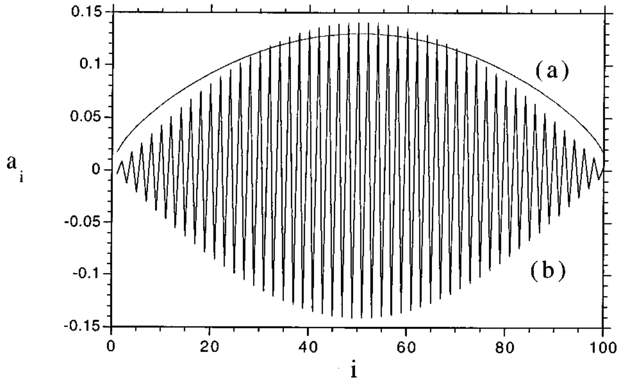

FIG. 3. The profile of the lattice waves for a chain of 100 atoms. The horizontal axis is the atom index i; the vertical axis is the excited-state amplitude of the ith atom a_i. $\Delta=-1$, $d=\lambda$. (a) The dressed state $\omega=-1.5977$ (the closest to the band-gap edge); (b) the dressed state $\omega=-1.8313$ (the one deepest inside the forbidden band).

In each trapped dressed state, the atom excited state component is a coherent superposition of one of the N atoms in the excited state. We stress the word ''coherent'' because the dipole moment of each atom is coherent with each other. In Fig. 3 we plotted the normalized coefficients a_i (i.e., $\Sigma_i|a_i|^2=1$) for each comparison. It should be noted that in the dressed state there are one-photon components, thus this normalization condition is not satisfied. The set of probability amplitudes of finding the atom at a given site is similar to a lattice wave, and we can associate each eigenfrequency $\omega_m(m=1,...,N)$ with a mode of such lattice waves. The mode with the frequency closest to the gap edge (curve a) has no nodes; the one furthest from the gap edge (curve b) has $N-1$ nodes.

While the N mode functions (the sets of normalized upper level coefficients for each eigenvalue) form a complete set for lattice waves defined on the N sites, the set of dressed states (the set of discrete eigenstates of the Hamiltonian [Eq. (1)], including both the upper and lower level components) is not a proper complete set that one can use to expand any initial state with one of the atoms excited. For such purposes the continuous eigenstates with unbounded photons must also be included. The general initial state problem is difficult (the two-atom dynamics problem was treated in Ref. [7]), and interesting questions can be posed. For example, if the atom at one end of the chain is excited initially, will the excitation be passed from one atom to the next in sequence, and will such excitation be reflected once it reaches the other end of the chain? Even without probing into the realm of transient behaviors, we can ask many questions about the dynamics of nondecaying states, which are superposed by the set of dressed states. The relationship between the spatial oscillation of lattice waves and the eigenfrequencies can be interpreted as a dispersive relation. Superpositions of these lattice waves generally represent lattice wave packets that travel along the chain while expanding. Such lattice wave packets probably describe the one-atom excitation at a selected site on the chain more properly, as it is difficult to pinpoint the pumping laser beam on one atom only without exciting the neighboring atoms.

As a final comment, we point out that our model deals with an ideal chain. Non-uniform interatom separations or nonideal alignment of dipole moments will destroy the translational invariance of f_{ij}. We may compare the ideal problem with oscillations on a uniform string, and the nonideal problem with the nonuniform string problem. In a nonideal problem qualitative conclusions such as the existence of a quasicontinuum, and trapped states with coherent superposition of individual atoms in excited state still are valid, but the picture of lattice wave traveling along the chain needs be modified. Problems such as the propagation of excitation along jointed chain sections are very interesting and will be discussed in follow-up studies.

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[1] S. John, Phys. Rev. Lett. **53**, 2169 (1984); P. W. Anderson, Philos. Mag. B **52**, 505 (1985); E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987); S. John, *ibid.* **58**, 2486 (1987).

[2] E. Yablonovitch and T. Gimitter, Phys. Rev. Lett. **63**, 1950 (1989); K. M. Ho, C. T. Chan, and C. M. Soukoulis, *ibid.* **65**, 3152 (1990); E. Yablonovitch, T. J. Gmitter, and K. M. Leung, *ibid.* **67**, 2295 (1991); G. Kweon and N. M. Lawandy, J. Mod. Opt. **41**, 311 (1994); U. Gruning, V. Lehmann, and C. M. Engelhardt, Appl. Phys. Lett. **66**, 3254 (1995); I. I. Tarhan and G. H. Watson, Phys. Rev. Lett. **76**, 315 (1996).

[3] S. Kawakami, O. Hanaizumi, T. Sato, Y. Ohtera, and T. Kawashima, Trans. Inst. Electron. Inf. Commun. Eng. C-I **J80C11**, 222 (1997); V. N. Astratov *et al.*, in *Proceedings of the Workshop on Highlights of Light Spectroscopy on Semiconductors,* Rome 1995 (World Scientific, Singapore), pp. 167; C. C. Cheng, V. Arbet-Engels, A. Scherer, and E. Yablonovitch, Phys. Scr. **T68**, 17 (1997).

[4] S. John and J. Wang, Phys. Rev. Lett. **64**, 2418 (1990); S. John and J. Wang, Phys. Rev. B **43**, 12 772 (1991).

[5] S. John and T. Quang, Phys. Rev. Lett. **76**, 1320 (1996); **74**, 3419 (1995).

[6] S. John and T. Quang, Phys. Rev. A **50**, 1764 (1994).

[7] S. Bay, P. Lambropoulos, and K. Moller, Phys. Rev. A **55**, 1485 (1997).
[8] V. I. Rupasov and M. Singh, Phys. Rev. A **55**, 898 (1997).
[9] S.-Y. Zhu, H. Chen, and H. Huang, Phys. Rev. Lett. **97**, 205 (1997).
[10] H. Huang, X. H. Lu, and S. Y. Zhu, Phys. Rev. A **57**, 4945 (1998).
[11] V. I. Rupasov and M. Singh, Phys. Rev. Lett. **77**, 338 (1996); S. John and V. I. Rupasov, *ibid.* **79**, 821 (1997).