Dressed states for a multilevel atom and localized field in a photonic band-gap crystal

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An excited atom in a photonic band-gap crystal can emit a photon that is then reabsorbed and reemitted by the atom, to form a dressed atom-field state. We discuss the existence conditions for such a trapped photon dressed state with a general multilevel atom driven by coherent fields and show that more than one such trapped state can exist. The field characteristics of the trapped field and the population trapped in the excited state are studied for various atomic systems. It is found that the driving field can determine the spread and energy of the trapped field, as well as the number of possible trapped dressed states. [S1050-2947(98)11506-8]

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I. INTRODUCTION

The photonic band-gap structure (PBS) is a man-made periodic structure in which some electromagnetic field frequency band can propagate and some cannot [1]. The potential of obtaining a piece of material with an effective index of refraction profile variable through adjustment of structure constants has great appeal to many applicational purposes such as communication and micro-optical devices. Recent efforts have been directed to the realization of a photonic crystal with passing and forbidden bands in different frequency bands from microwave to optical frequency [2].

From the very first days after such a structure was proposed it was known that the existence of forbidden bands and the changes in the electromagnetic (em) field mode density profile lead to distinctive quantum electrodynamic effects, such as the prohibition of spontaneous emission of an atom in a PBS crystal, when the emission frequency is located in one of the forbidden bands [3]. However, the simplistic picture of a forbidden spontaneous emission due to the absence of field modes at the transition frequency is not an accurate one, as demonstrated by John and collaborators [4,5]. In a continuous, source-free, and infinite PBS crystal, finite radiation field in the forbidden band cannot exist. On the other hand, the forbidden band field can exist in the form of an attenuating wave propagating from a source point; the source role is played by the atom's dipole moment. In a detailed analysis John and Tran [6] showed that when the transition frequency is close to the band edge the atom can retain part of its energy in the excited state. In a recent paper on a more complex system with two upper levels emitting photons into the same continuum [7], we showed that the energy of the initial excited state is transferred into a part of a propagating wave, a part in the form of attenuating wave, and a part remains in the excited state. The latter two are trapped in the vicinity of the atom. This suggests a physical picture of a photon trapped in a ''cavity'' made of a medium in which it has a complex wave vector, and a atom resonant with the photon frequency. The terminal field-atom state is a stationary state in the form of a ''dressed state,'' the atom dressed by the photon it helps to trap. We also showed the presence of energy trapping and oscillations due to the interference between the two decaying channels. Therefore a dressed state by a trapped photon is not restricted to two-level atoms.

In the current report we study the trapped photon dressed atom-field state in a PBS crystal for general atomic systems with one transition near the forbidden band-gap edge. The density of states at the band-gap edge was singular in previous discussions. In practice, crystals are of finite size and not perfect, thus density of states is not singular. We demonstrate that a trapped state can exist in such a nonideal situation. It is shown that when there are several upper sublevels there may be more than one trapped states. These trapped states have different localized field patterns and frequencies. On the other hand, systems with several lower sublevels do not possess multiple trapped dressed states. Of particular interest is the case of an upper level coupled by a coherent field to another level. We show that the trapped states and localized field can be influenced by the strength and detuning of the driving field.

Our paper is organized as follows: in Sec. II we use a two-level atom model to demonstrate the method of analysis and use the result obtained to discuss the general properties of trapped dressed states, followed by a discussion on the existence conditions for a general driven multilevel system. We show that generally more than one trapped dressed state may exist. In Sec. III the realistic two-level atom with sublevels is examined. In Sec. IV we consider the coupling of two levels to the same continuum and interference effects. The dressed states of a driven atom are discussed in Sec. V, and in the Summary we discuss the limits and possible extensions of the present work.

II. BASIC PICTURE OF TRAPPED PHOTON DRESSED STATE

We start from the simplest case of a two-level atom in a photonic band-gap crystal of infinite size (Fig. 1). The assumptions are: (a) there is no other relaxation process apart from spontaneous emission; (b) the crystal is treated as iso-

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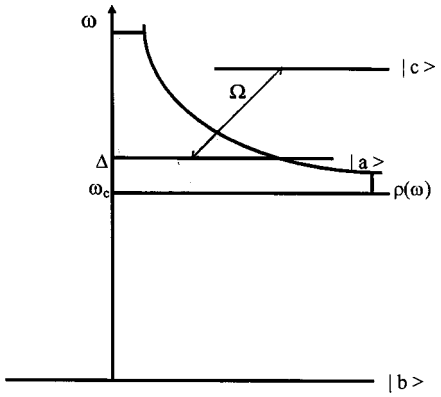

FIG. 1. Energy-level diagram of a transition near the band-gap edge. The level a may be coupled to other levels via coherent fields.

tropic; (c) the detuning of the transition frequency ω_{ab} from the band-gap edge Δ is much smaller than the optical frequency and the forbidden bandwidth, therefore the density of field modes can be taken as zero from 0 to ω_c. At first we omit any coupling to levels other than a,b. Here we follow the model of photonic crystals in Ref. [6]. The effective dispersion relation is

$$\omega_k=\omega_c+A(k-k_0)^2, \quad A=\omega_c/k_0^2, \tag{1}$$

therefore in the frequency domain the density of field state has a singularity at ω_c. The atom-field Hamiltonian after the rotating wave approximation (RWA) is

$$H=\sum_k \hbar(\omega_k-\omega_c)a_k^\dagger a_k+\hbar\left[\sum_k g_k a_k^\dagger|b\rangle\langle a|+\text{H.c.}\right] + \sum_{i=a,b}\hbar\Delta_i|i\rangle\langle i|. \tag{2}$$

Here $\Delta_a=\Delta$, $\Delta_b=0$. The summation Σ_k represents the summation over photon modes in the three-dimensional crystal. In the frequency domain the summation is in the passing band only.

The spontaneous decay of the initial excited state $|a\rangle_A|\text{vac}\rangle_F$ was discussed in detail in Ref. [6], where a nondecaying state was found. Now we look at the model from the eigenvalue problem angle. Consider the expansion of the state vector

$$|\psi(t)\rangle=A(t)|a\rangle_A|\text{vac}\rangle_F+\sum_k B_k(t)|b\rangle_A|1_k\rangle_F. \tag{3}$$

The time-dependent amplitudes satisfy the Schrödinger's equations:

$$\dot{A}=-i\Delta A-i\sum_k g_k^* B_k, \qquad \dot{B}_k=-i(\omega_k-\omega_c)B_k-ig_k A. \tag{4}$$

We search for the eigenvalue and eigenstate $[A(t),B_k(t)]\sim(A,B_k)e^{-i(\omega-\omega_c)t}$:

$$(\omega-\omega_c-\Delta)A=\sum_k g_k^* B_k, \qquad (\omega-\omega_k)B_k=g_k A. \tag{5}$$

The eigenvalue $\omega-\omega_c$ is determined by the eigenequation:

$$\omega-\omega_c-\Delta-\sum_k |g_k|^2/(\omega-\omega_k)=0. \tag{6}$$

Notice that the summation over frequency is in the passing band only. The eigenfrequency we are looking for corresponds to a localized photon; i.e., it is in the forbidden band, otherwise the summation over photon modes would have both a real part (frequency shift) and an imaginary part (decaying). Hence the last term on the left is a positive number. We can identify the above equation with the equation

$$s+i\Delta=-\sum_k |g_k|^2/(s+i\omega_k), \quad s+i\Delta-i\beta^{3/2}/\sqrt{s}=0 \tag{7}$$

with the replacement $-i(\omega-\omega_c)\rightarrow s$. The eigenequation is then reduced to

$$\omega-\omega_c-\Delta+\beta^{3/2}/\sqrt{|\omega-\omega_c|}=0. \tag{8}$$

Here $\beta=[\omega_{ab}^{7/2}d_{ab}^2/(6\pi\epsilon_0\hbar c^3)]^{2/3}$ is the characteristic strength of coupling between the continuum and the level a.

It is helpful to draw a sketch of the left-hand side (lhs) of Eq. (8) in the allowed range of $\omega-\omega_c$, $(-\infty,0]$. The function is monotonically increasing and has different signs at two ends of the interval. Therefore it has only one negative solution. In particular, for negative Δ the solution is on the left of Δ. We will come back to this point later.

The eigenstate represents a dressed atom-field state; it contains a vacuum field-excited atom component and a one-photon ground-state atom component. The one-photon component has a localized field distribution with its energy trapped around the atom. Hence we call such states *trapped dressed states*. The field distribution at a point in the direction $\hat{k}$ and a distance $r\gg\lambda$ away from the atom is the sum over contributions from modes in all frequencies and directions:

$$\begin{aligned}E(r,t)&=\langle 0|\hat{E}^{(+)}(r,t)|\psi(t)\rangle\\ &=\sum_k\left(\frac{\hbar\omega_k}{2\epsilon_0 V}\right)^{1/2}e^{-i[\omega_k t+kr]}B_k(t)\\ &\sim\frac{e^{-i\omega t}}{k_c r}g_k\int_{-\infty}^{\infty}dk\,\frac{[e^{ikr}-e^{-ikr}]}{\omega_k-\omega}\\ &\sim\frac{e^{-i\omega t}}{k_c r}g_k e^{-\kappa r}\sin k_c r.\end{aligned} \tag{9}$$

Here k_c is the wave vector corresponding to the gap-edge frequency ω_c, and we have factorized out g_k by assuming that g_k varies little in the narrow frequency range in question and can be assumed to be a constant $g_k|_{k=k_c}$. The field distribution is a decaying standing wave. The decaying constant

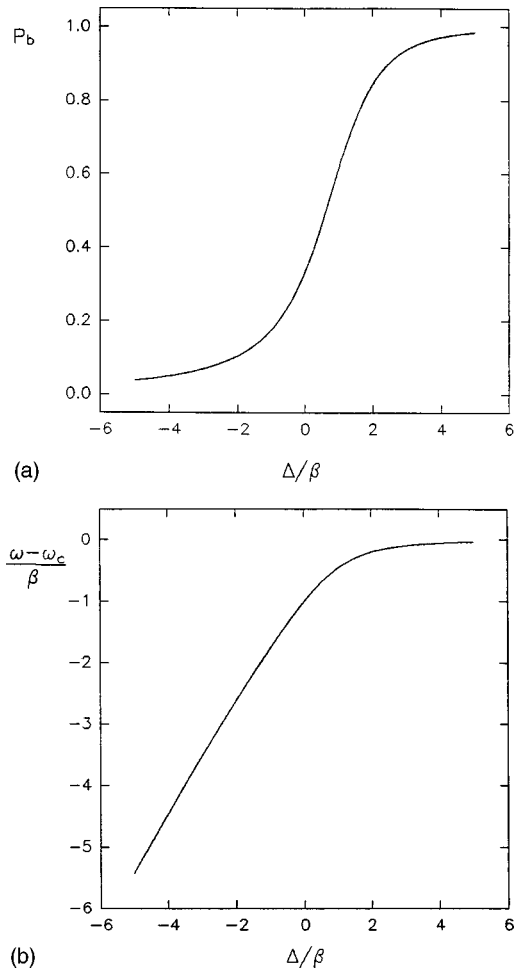

FIG. 2. (a) The portion of the one-photon component P_b in the dressed state. (b) The eigenvalue of the dressed state for a two-level atom in PBS as a function of the detuning $\Delta=\omega_{ab}-\omega_c$.

in space is $\kappa=\sqrt{|\omega-\omega_c|/A}/c$. It represents a field oscillating at a frequency in the forbidden band and being bounced back in the medium (similar to cavity modes in a distributed feed back cavity). We see that the size of field confinement is directly related to the depth of the dressed state inside the forbidden band.

The ratio between the one-photon component and the zero-photon component populations can be calculated in principle:

$$\frac{P_b}{P_a}=\sum_k |g_k|^2/(\omega-\omega_k)^2=-\frac{\partial}{\partial x}\sum_k |g_k|^2/(x-\omega_k)|_{x=\omega}$$

$$=\frac{1}{2}\left(\frac{\beta}{|\omega-\omega_c|}\right)^{3/2}. \tag{10}$$

In Fig. 2(a) the proportion of energy trapped in photon form (P_b) is plotted against the detuning Δ. In Fig. 2(b) we show the dependence of the eigenvalue $\omega-\omega_c$ on Δ. Here as well in subsequent figures frequency-dimension quantities such as $\omega-\omega_c$ and Δ are all in units of β. As the eigenvalue drops deeper into the forbidden band, more energy is carried by the atom, and less energy is contained in the field.

The paragraphs above show the general method of treating the single-photon trapped state problem and the basic properties of the trapped state. Now we extend the approach to treat more complex atomic systems. In particular, we are interested in when a laser field is used to couple some transition. This offers a practical way both to prepare and to probe the system in the crystal.

Consider a multilevel atom with the transition frequency between levels a and b near the band-gap edge. The level a and other levels $(c,d,\ldots)$ are coupled together by coherent driving fields (Fig. 1). Here we assume that relaxations between these levels can be ignored; the validity of this assumption depends on the ratio of the corresponding relaxation rates and the characteristic rate involving the photon modes near the gap edge, i.e., β. These relaxations destroy the phase relation between atom levels and thus take the system out of the trapped dressed state. After taking steps to render the Schrödinger's equations time independent and to eliminate the B_k coefficients we arrive at

$$(\omega-\omega_c-\Delta_a)C_a+\sum_{i=c,\ldots}\Omega_{ai}C_i+U(\omega)C_a=0,$$
$$(\omega-\omega_c-\Delta_i)C_i+\sum_{j\neq i}\Omega_{ij}C_j=0 \quad (i=c,d,\ldots). \tag{11}$$

Here C_i is the amplitude of the state i component in an energy eigenstate, Ω_{ij} is the Rabi frequency for the coherent coupling between i and j. The coefficient Δ_i is determined in such a way: Δ_a is defined as before; starting from level a and from the value Δ_a, follow the coherent couplings to level i, add the difference of atomic levels, and subtract the coherent photon energy for one photon absorbed from the coupling field and do the opposite for one photon emitted to the coupling field, one can define a detuning for the level i. The coupling matrix $\mathbf{\Omega}=(\Omega_{ij})$ and the detuning matrix $\mathbf{D}=(\Delta_i\delta_{ij})$ have the dimension equal to the number of levels linked by coherent driving fields (N). They characterize the driven system $a,c,\ldots$ without the coupling to b. The function $U(\omega)$ now is defined for a photonic band-gap material with an arbitrary density of states function in the passing band:

$$U(\omega)=-\sum_k \frac{|g_k|^2}{\omega-\omega_k}. \tag{12}$$

Notice here we do not presume a singularity at the gap edge as before.

The eigenequation for the frequency ω can be written as

$$\det[(\omega-\omega_c)\mathbf{I}+\mathbf{\Omega}-\mathbf{D}]+\det[(\omega-\omega_c)\mathbf{I}'+\mathbf{\Omega}'-\mathbf{D}']U(\omega)=0. \tag{13}$$

Here $\mathbf{I}'$, $\mathbf{\Omega}'$, $\mathbf{D}'$ are matrices defined in the Hilbert space with the levels a,b eliminated. Let us assume that the two determinants have real roots $\{\omega_m'\}$, $\{\omega_n''\}$. The numbers in these two sets are N and $(N-1)$, respectively. Equation (13) can be rewritten as

$$\frac{\Pi_m(\omega-\omega_m')}{\Pi_n(\omega-\omega_n'')}+U(\omega)=0. \tag{14}$$

As $U(\omega)$ is a monotonically increasing function, and the two products are polynomials of orders N and $N-1$, the maximum number of roots for Eq. (13) is also N.

Equation (14) does not always have negative roots for $\omega-\omega_c$ as required for trapped photon dressed states, as shown in the following discussions. (a) If both polynomials in Eq. (14) have negative roots, it is always possible to find roots ω' and ω'' so that within (ω',ω'') there are no other roots. It is easy to verify that Eq. (14) has a negative root $\omega-\omega_c$ in (ω',ω''). (b) The first term in Eq. (14) $\rightarrow\omega$ as $\omega\rightarrow-\infty$, $U(\omega)\rightarrow 0$. Therefore it has at least one negative root to the left of the left most root among the roots ω_m', if this root is negative. (c) If all ω_m' are positive there is no definite conclusion for an arbitrary $U(\omega)$. However, when $U(\omega)$ has a singularity at the band-gap edge $\omega=\omega_c$ a negative root always exists. This is the case discussed in Ref. [6] with $N=1$ and $U(\omega)=\beta^{3/2}/\sqrt{|\omega-\omega_c|}$ where John *et al.* pointed out that energy trapping occurs even for an upper level in the passing band.

III. REALISTIC "TWO-LEVEL" ATOMS

The true two-level atom actually has sublevels either at the top or at the bottom. Here we first look at an $S\rightarrow P$ transition, assuming the three lower levels b_j $(j=0,\pm1)$ are split by an external magnetic field. In the previous section we assumed the time dependence $|\psi(t)\rangle\sim e^{-i(\omega_b+\omega_c)t}$, while here we assume that $|\psi(t)\rangle\sim e^{-i\omega_a t}$. The modified Schrödinger's equations are

$$\dot{A}=-i\sum_{k,j} g_{kj}^{*}B_{kj},\quad j=0,\pm1,$$
$$\dot{B}_{kj}=-i(\omega_k-\omega_c+\Delta_j)B_{kj}-ig_{kj}A. \tag{15}$$

Here the sum over field modes inexplicitly includes polarizations. $\Delta_j=\omega_{bj}+\omega_c-\omega_a$ is the lower level energy after rotation. The eigenequation becomes

$$\omega-\omega_c+\sum_j \beta^{3/2}/\sqrt{|\omega-\omega_c-\Delta_j|}=0. \tag{16}$$

When we retain only one term out of the sum of three, we obtain an equation and its solution (ω_j) due to the coupling of only one sublevel. After eliminating B_{kj}'s we find an equation containing the sum of three integrals. The integrals contain the factor $1/[\omega-(\Delta_j+\omega_k)]$ and will pass singularities if for any $j,\omega-\omega_c-\Delta_j<0$, which in turn will yield an imaginary part to the exponent. Therefore the solution $\omega-\omega_c$ is to the left of all three Δ_j. Again here the left-hand side of Eq. (16) is monotonic and goes to $-\infty$ as $\omega\rightarrow-\infty$. The solution is unique and located further to the left of all three solutions due to individual lower levels, as shown in Fig. 3(a). This may be the result of three independent coupling channels, which corresponds to a greater positive $U(\omega)$ and pushes the root further to the left.

The one-photon component contains three parts corresponding to the decay to the three different lower levels with different angular momenta. Each part behaves similarily to the previous case but has different wave-packet sizes because they have different frequencies inside the band gap. However, the oscillation frequency of the three parts is the same. The ratios between their amplitudes are determined by the eigensolution, but they do not interfere since the corresponding atom states are different. In Fig. 3(b), we see that the excited-atom component P_a is between the results when only one of the lower levels is present.

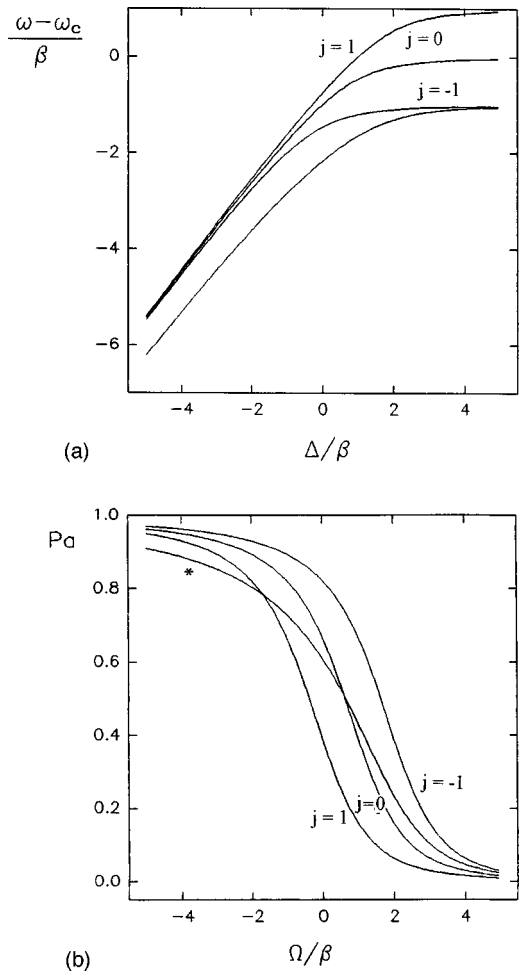

FIG. 3. (a) The eigenvalue of an S-P transition with the lower level split $\Delta_j=\Delta_0+\Delta_{j,j}=0,\pm1$ (bottom curve) and the three roots for the three separate two-level problems (top three curves). (b) The portion of excited atom state in the dressed state. Curve j: the case with only one lower level at $\Delta_0+\Delta_j$; curve $*$: the case with all three lower levels.

Next let us look at a $P\rightarrow S$ transition with the upper level split by an external magnetic field. The Schrödinger's equations are

$$\dot{A}_j=-i\Delta_j A_j-i\sum_k g_{kj}^{*}B_k,\quad j=0,\pm1,$$
$$\dot{B}_k=-i(\omega_k-\omega_c)B_k-i\sum_j g_{kj}A_j. \tag{17}$$

Since the photons emitted in transitions from the three upper levels to the common lower level have different spin states, $\Sigma_{k,\sigma}g_{kj}g_{kj'}^{*}\sim\delta_{jj'}$, by eliminating B_k we obtain three decoupled equations:

$$\omega-\omega_c-\Delta_j+\beta^{3/2}/\sqrt{|\omega-\omega_c|}=0. \tag{18}$$

Therefore this is equivalent to three separate transitions. Starting from an arbitrary initial state, the final field state also has three localized fields with different oscillation frequencies and amplitudes, and they do interfere since the lower atom level is the same. The ratios between these components are determined by the initial condition.

IV. TWO TRANSITIONS WITH PARALLEL DIPOLE MOMENTS

In the last section transitions between the sublevels a_j and b involve photons of different angular momentum quantum numbers. There are situations where two transitions may involve photons of the same angular quantum number. In other words, the transition dipole moments are parallel. First consider two upper levels that can emit photons into the same continuum. This is the case discussed in our earlier paper [7]. The Schrödinger's equations are similar to the previous ones, but by eliminating B_k we find

$$\begin{aligned}(\omega-\omega_c-\Delta_1)A_1&=-[\beta_1^{3/2}A_1+\beta_{12}^{3/2}A_2]/\sqrt{|\omega-\omega_c|},\\(\omega-\omega_c-\Delta_2)A_2&=-[\beta_2^{3/2}A_2+\beta_{12}^{3/2}A_1]/\sqrt{|\omega-\omega_c|},\end{aligned}\tag{19}$$

here $\beta_{12}=\sqrt{\beta_1\beta_2}$. The eigenequation is now

$$\begin{aligned}&[\omega-\omega_c-\Delta_1+\beta_1^{3/2}/\sqrt{|\omega-\omega_c|}]\\&\quad\times[\omega-\omega_c-\Delta_2+\beta_2^{3/2}/\sqrt{|\omega-\omega_c|}]-\beta_{12}^3/|\omega-\omega_c|=0.\end{aligned}\tag{20}$$

For $\omega\to-\infty$ the lhs is positive; between the two real roots corresponding to only one transition ($a_1\to b$ or $a_2\to b$) (we call these two roots ω' and ω'') it is negative; near $\omega=\omega_c$ its sign is determined by $(\Delta_1+\Delta_2)$. Therefore for $\Delta_1+\Delta_2>0$ there is only one root, for $\Delta_1+\Delta_2<0$ there are two roots. For $\Delta_1+\Delta_2=0$, result in Ref. [7] shows there is only one root and one eigenstate, not two degenerate eigenstates. In the first case, the solution is to the left of both ω',ω'', and in the second case, the two solutions are either to the left or to the right of ω' and ω''. In Fig. 4(a), the left half shows the single eigenvalue, and the right half shows two eigenvalue solutions.

Given an eigenvalue ω we determine the ratio between A_1 and A_2. The portion of the one-photon component is $P_b=|\beta_1/(\omega-\omega_c|^{3/2}A_1^2+|\beta_2/(\omega-\omega_c)|^{3/2}A_2^2+2|\beta_{12}/(\omega-\omega_c)|^{3/2}A_1A_2$. The total population in the two excited states for the dressed state is plotted for each of the two eigenstates in Fig. 4(b).

The case of a transition from an upper level to two lower levels with parallel dipole moments is not different from the transition with perpendicular dipoles since the final atom states are different, and there is no quantum interference. The eigenequation is similar to Eq. (16) rather than Eq. (20). There is only one solution.

V. TRAPPED DRESSED STATES OF A DRIVEN SYSTEM

In this section we consider a three-level atom where two levels are coupled together by a coherent field. Our focus is how such coupling will alter the number of possible trapped dressed state, and how the energy distribution between the

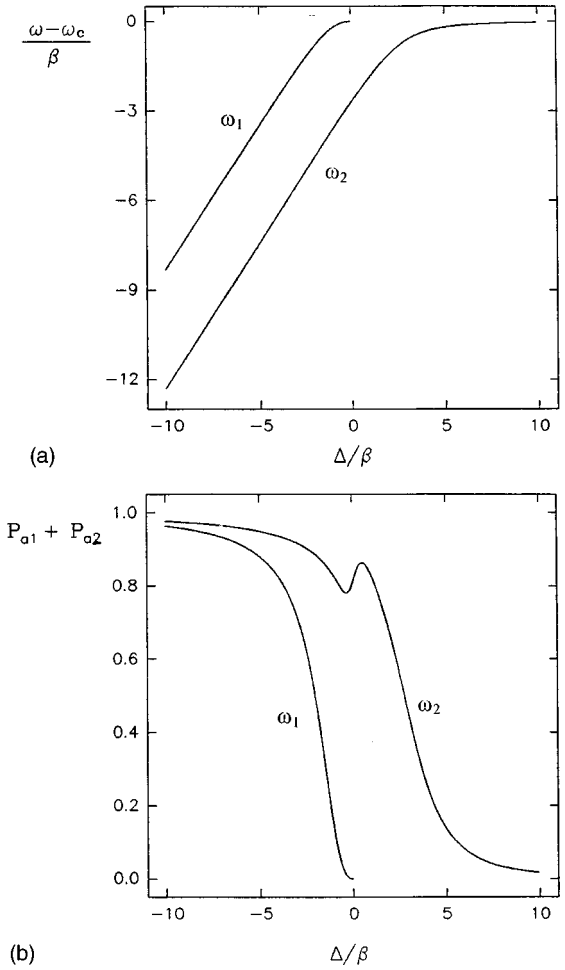

FIG. 4. (a) The eigenvalues of Eq. (20) with $\Delta_1=\Delta+2\beta$, $\Delta_2=\Delta-2\beta$. (b) The excited atom component weights $P_{a_1}+P_{a_2}$ for the two dressed states in (a).

atom and the field will be changed. In the following discussion we ignore the decoherent factors in the channel coupled by the coherent field. Here we discuss two possibilities. First, the upper level is coupled to a third level c by a c-number field. The decay from a to c (or vice-versa) is ignored. Under such an assumption it makes no difference whether c is higher or lower than a. We assume that c is higher henceforth. After RWA we have the following coupled equations:

$$\begin{aligned}(\omega-\omega_c)C&=\Delta_cC+\Omega A,\\(\omega-\omega_c)A&=\Delta_aA+\sum_k g_k^*B_k+\Omega C,\\\omega B_k&=\omega_kB_k+g_kA.\end{aligned}\tag{21}$$

Eliminating B_k one finds

$$\begin{aligned}(\omega-\omega_c-\Delta_c)C&=\Omega A\\[\omega-\omega_c-\Delta_a+\beta^{3/2}/\sqrt{|\omega-\omega_c|}]A&=\Omega C,\end{aligned}\tag{22}$$

i.e., the eigenequation becomes

$$(\omega-\omega_c-\Delta_c)[\omega-\omega_c-\Delta_a+\beta^{3/2}/\sqrt{|\omega-\omega_c|}]-\Omega^2=0.\tag{23}$$

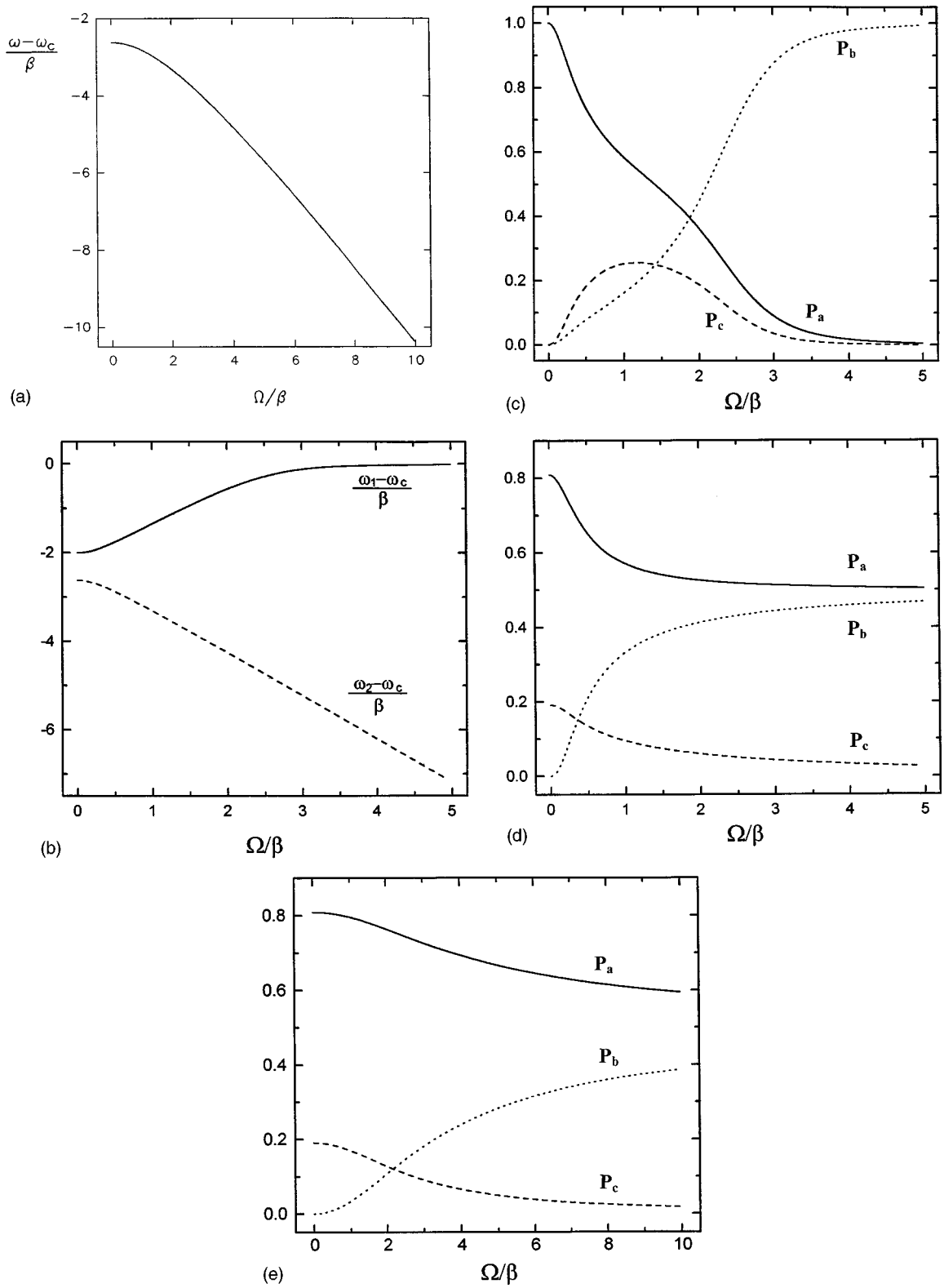

FIG. 5. (a) The single eigenvalue ω for a three-level system in PBS described by Eq. (21), $\Delta_a=-2\beta$, $\Delta_c=2\beta$ as a function of the Rabi frequency Ω. (b) The two eigenvalues ω_1 and ω_2 for $\Delta_a=-2$, $\Delta_c=-2$. (c) The portions of a, c, and one-photon component in the dressed state of ω, parameters are the same as (b), (d), (e). The portions of the three components in the dressed state of ω_1 and ω_2.

The portions in a, c, and the one-photon component are easily determined using the generalized result from Eq. (10) once the eigenvalue is solved.

There are two different situations. (1) $\Delta_c>0$. The first term then is monotonic and runs from $-\infty$ to $+\infty$. There is one solution, again to the left of the solution when $\Omega=0$. When Ω increases this root is pushed further into the depth of the forbidden band. Figure 5(a) shows the single eigenvalue as a function of the Rabi frequency Ω for a positive Δ_c. The proportion of the one-photon component (i.e., the

B_k part) will decrease with larger Ω [Fig. 5(c)]. (2) $\Delta_c<0$. Here the first term goes to $+\infty$ for both $\omega\rightarrow-\infty$ and $\omega\rightarrow 0$. It dips below zero between ω' and $\Delta_c+\omega_c$ (here ω' stands for the solution when $\Delta=0$). Therefore there are two negative roots ω_1,ω_2 for ω, which move toward to $-\infty$ and 0, respectively, as Ω increases. The two eigenvalues are shown in Fig. 5(b) as functions of Ω, and the portions of the a, c, and one-photon components are shown in Figs. 5(d) and 5(e), respectively.

The second possibility is with the lower level b coupled to c. Here the essential states are $|a\rangle$, $|b\rangle|1_k\rangle$, and $|c\rangle|1_k\rangle$. The coupled equations are

$$(\omega-\omega_c)A=\Delta_a A+\sum_k g_k^* B_k,$$

$$\omega B_k=\omega_k B_k+g_k A+\Omega C_k, \tag{24}$$

$$\omega C_k=(\omega_k+\Delta_c)C_k+\Omega B_k.$$

We can first obtain the dressed states B_{1k},B_{2k} after diagonalizing the Hamiltonian containing the b-c coupling. The result is a set of equations identical to two transitions to different lower levels with parallel dipoles. Again we see that the multiple lower sublevel problem has only one trapped dressed state.

VI. SUMMARY

We have demonstrated that an atom in a photonic band-gap crystal can trap a localized field wave packet in a wide range of cases. Such a trapped photon dressed state is produced when the atom is first prepared in an excited state and then left for spontaneous emission. A portion of the ensemble will go to the trapped energy state, the portion being equal to the initial state's projection onto the trapped state. Generally the deeper the excited level lies in the forbidden band, the higher the trapped energy portion is. When there exists more than one trapped state (at different eigenfrequencies) the localized field will exhibit beating both in space and in time.

In our analysis, the crystal has been assumed to be infinite in size. For a realistic crystal of finite size, photons can tunnel from the atom through the barrier of the crystal. Thus the trapped states have finite lifetime, and they will decay to emit photons at the eigenfrequencies. This will allow the observation of the narrow lines of trapped states and the beating of the trapped state inside the crystal.

Our discussion has been limited to one atom and one quantum in the field. The spontaneous decay problem for single excitation of many atoms in a PBS has been examined, where the feature of photon exchange between atoms was discovered [8]. On the other hand, the many-excitation trapped state problem in the context of polaritons in the medium was treated by Rupasov *et al.* [9] although it does not address the issue of a dispersive medium, which has no resonances near the atom transition frequency. We expect the many-photon trapped state problem to be an area of good prospect for future research.

ACKNOWLEDGMENT

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