Direct measurement of the transverse excess noise factor in a geometrically stable laser resonator

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The excess noise factor due to the nonorthogonality of the transverse modes of a geometrically stable cavity subject to large diffraction losses is measured. This transverse excess noise factor is isolated from other sources of modification of the laser linewidth due to a differential measurement method, leading to the experimental evidence of values as large as 13 in cylindrical symmetry. Moreover, it is shown experimentally that the introduction of a well-chosen second diffracting aperture inside the cavity permits one to drastically reduce the overall laser excess noise factor, without altering the laser losses and intracavity power.
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I. INTRODUCTION

The linewidth of a monomode laser is fundamentally limited by spontaneous emission into the laser mode, leading to the well-known Schawlow-Townes linewidth [1]. This theory predicts a Lorentzian line shape for the laser power spectral density, which has been found to be in agreement with experiments [2–4]. However, more recently, it has been shown that the non-orthogonality of the different laser modes can give rise to an enhancement of this fundamental laser linewidth. This so-called excess noise factor has been discussed [5,6] and observed [7,8] in the case of gain-guided semiconductor lasers and amplifiers. Independently, Siegman [9–11] has shown that the non-Hermitian nature of the Huygens-Fresnel operator for one round-trip in *unstable* resonators leads to peculiar nonorthogonality properties of the *transverse* modes of such resonators and hence to large excess noise factors. In such geometrically unstable cavities, excess noise factors as large as a few 10^3 have been predicted [11–17] and observed [18–21]. Physically, this effect has been attributed to the adjoint coupling of the vacuum fluctuations into the laser resonator [22,23]. Concerning geometrically *stable* laser cavities, it has recently been shown that large output coupling could lead to a nonorthogonality of the *longitudinal* modes of the cavity and thus to an enhancement of the Schawlow-Townes linewidth of the laser [24–27]. Moreover, since even in the case of a geometrically stable cavity the Huygens-Fresnel kernel is in general non-Hermitian [10], the *transverse modes of a geometrically stable resonator* must be nonorthogonal. This has led to the observation of a peculiar behavior of the losses in such cavities [28] and to the prediction of the existence of a non-negligible *transverse* excess noise factor in stable resonators [29]. The aim of this paper is consequently to measure this transverse excess noise factor in a non-Hermitian stable resonator. Section II is thus devoted to the description of an experimental method designed to isolate this transverse excess noise factor in a high-gain gas laser with a stable cavity containing a single aperture. In Sec. III we discuss whether the oscillating behavior of the excess noise factor in the presence of a second diffracting aperture can allow us to control the laser noise without modifying its losses or its intracavity power. We conclude in Sec. IV.

II. SINGLE-APERTURE STABLE CAVITY

A. Description of the experiment

Let us consider the stable cavity schematized in Fig. 1. It consists of two spherical mirrors M_1 (radius of curvature $R_1=0.5$ m) and M_2 (radius of curvature $R_2=1.2$ m), separated by a distance L. In this section we consider only a single circular aperture of diameter ϕ_1 located near mirror M_1. In these conditions, we have seen in Ref. [29] that the transverse eigenmodes of the cavity become nonorthogonal, leading to the prediction of a non-negligible transverse excess noise factor K_T. However, K_T becomes significantly important only for small diameters ϕ_1, i.e., in the presence of large diffraction losses. This is why we choose the high-gain $\lambda=3.51$ μm laser transition of xenon. Moreover, for the laser linewidth to be relatively important and hence easily measurable, we choose a rather short cavity ($L=0.27$ m). The active medium is consequently a 17-cm-long discharge tube closed with two quasiperpendicular silica windows. Its bore diameter is 6 mm and it is filled with a 15:1 ^{3}He-^{136}Xe mixture at a total pressure of 1.1 Torr. It is excited with a 7-mA continuous discharge current. The mirrors M_1 and M_2

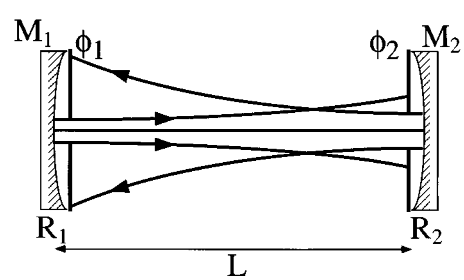

FIG. 1. Geometrically stable cavity of length L built with a spherical mirror M_1 (radius of curvature R_1) and a spherical mirror M_2 (radius of curvature R_2). The cavity contains a circular diffracting aperture of diameter ϕ_1 located near M_1. A second aperture of diameter ϕ_2 can also be introduced near mirror M_2.

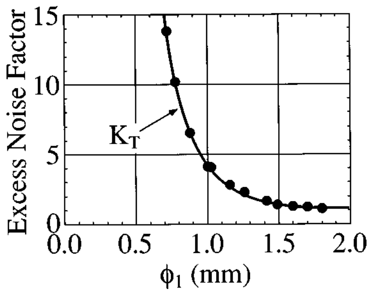

FIG. 2. ●, measured evolution of the transverse excess noise factor K_T versus ϕ_1; full line, corresponding computation of K_T with $\phi_2=\infty$, $R_1=0.5$ m, $R_2=1.2$ m, $L=0.27$ m, and $\lambda=3.51$ μm.

have intensity reflection coefficients $r_1^2=0.9$ and $r_2^2=0.6$, respectively. Without any aperture inside the cavity, we compute the TEM_{00} fundamental Gaussian mode diameters to be $w_1=699$ μm and $w_2=538$ μm on mirrors M_1 and M_2, respectively.

Then, as described in Ref. [29], we calculate the transverse excess noise factor K_T of the fundamental mode versus ϕ_1. Following Siegman [11,14,15], the exact field distribution of the fundamental mode is computed due to a Fox-Li iterative method and the value of K_T is obtained by comparing the fields in both directions of propagation. The result of this theoretical calculation is reproduced in Fig. 2 (full line). We observe experimentally that our laser reaches threshold for aperture diameters ϕ_1 as small as 0.72 mm. In this case, we expect to observe a transverse excess noise factor K_T as large as 13.4 (see the full line in Fig. 2). To observe the laser linewidth, we carefully compensate for the residual intracavity phase and loss anisotropies and apply a 0.5-G longitudinal magnetic field on the active medium. Then the Faraday effect raises the degeneracy between the frequencies of the σ^+ and σ^- polarizations [30] and leads to a frequency difference of the order of 800 kHz between these two circularly polarized eigenstates. Then, as shown in Ref. [31], the observation through a linear polarizer of the spectral linewidth of the beat note between these two polarizations leads to a direct self-heterodyne determination of the fundamental laser linewidth, as is now going to be performed in our laser.

B. Experimental results

In such a He-Xe laser, many effects must be taken into account to predict the exact value of the fundamental laser linewidth. Indeed, as summarized in the Schawlow-Townes formula [1], this linewidth first depends on the cold cavity linewidth and on the power inside the cavity. Moreover, in such lasers, one must add corrections to take into account the influence of the bad cavity regime [32], the inhomogeneous transition linewidth [33], and the incomplete inversion [34]. This leads to the appearance of extra factors in the laser linewidth that depend on the discharge current, the intracavity losses, and the intracavity intensity. Finally, since in our laser the diffraction losses, the mirror transmissions, and the losses due to the tube windows (window intensity transmission coefficient $T=0.9$) are not uniformly distributed, a longitudinal excess noise factor [24,25,32] K_L must appear.

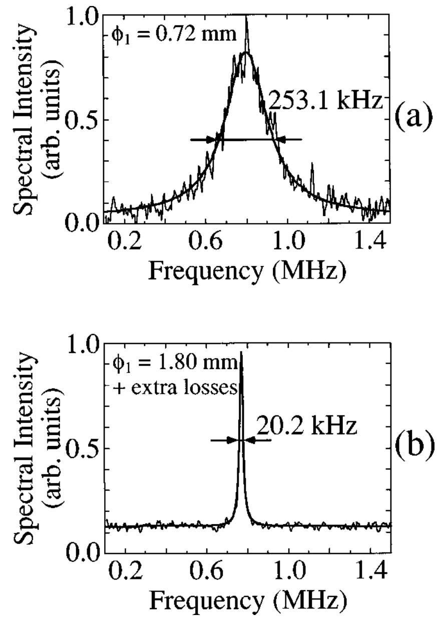

FIG. 3. Experimental spectra (linear scale) of the beat notes between the σ^+ and σ^- eigenstates, together with their Lorentzian fits obtained for (a) $\phi_1=0.72$ mm and (b) $\phi_1=1.80$ mm with the same intracavity losses and the same output power.

Here we wish to measure the transverse excess noise factor K_T alone. To isolate this term, we adopt the following differential procedure. Let us first choose a value of the aperture diameter ϕ_1 for which we wish to measure K_T. We then measure the width of the beat signal at atomic line center by monitoring the output power spectral density on a spectrum analyzer and fitting the resulting spectrum with a Lorentzian line shape. Figure 3(a) shows such an experimental spectrum obtained for $\phi_1=0.72$ mm, together with its Lorentzian fit. Notice that to improve the precision of our fit, we perform it using the logarithm of the spectrum. The resulting width of the Lorentzian (full width at half maximum) is 253.1 kHz in the case of Fig. 3(a). We then measure, due to a piezoelectric transducer carrying mirror M_1, the maximum cavity frequency detuning for which the laser oscillates. We then remove the intracavity aperture and insert another aperture with diameter $\phi_1=1.8$ mm near M_1. This value $\phi_1=1.8$ mm is the largest aperture diameter for which the laser is transversally monomode at line center. We then introduce extra intracavity losses using a sufficient number of glass plates located near the aperture, so that the maximum cavity detuning for which the laser reaches threshold is equal to the one measured with the preceding aperture. We are thus sure that the laser losses are unchanged and we can check experimentally that the laser output powers are identical in both cases. Moreover, the introduction of the extra intracavity losses near M_1 permits us to make sure that K_L is identical in both cases since the spatial distribution of the losses along the cavity axis remains unchanged. With this new aperture diameter, we obtain the spectrum reproduced in Fig. 3(b), whose width is equal to 20.2 kHz. Consequently, the ratio $253.1/20.2=12.5$ of the widths of the two Lorentzians of Fig. 3 must be exactly equal to the ratio of the transverse excess noise factors in both cases, all other parameters remaining unchanged. If we admit that the value of K_T computed for $\phi_1 = 1.8$ mm is correct ($K_T = 1.1$ for $\phi_1 = 1.8$ mm; see the full line in Fig. 2), we obtain $K_T = 12.5 \times 1.1 = 13.8$ for $\phi_1 = 0.72$ mm, in very good agreement with the expected value $K_T = 13.4$ (see Fig. 2). By repeating this differential experiment for several values of ϕ_1 between 0.72 and 1.8 mm while keeping constant losses and by multiplying the obtained linewidth ratios by the value of K_T computed for $\phi_1 = 1.8$ mm ($K_T = 1.1$ for $\phi_1 = 1.8$ mm), we obtain the measurements reproduced as closed circles in Fig. 2. These measurements are in very good agreement with the theoretically computed values of K_T (full line). Consequently, the results of Fig. 2, obtained due to our careful compensation of the variations of the losses and of K_L, are an unambiguous evidence of the existence of the transverse excess noise factor in a stable cavity, with a value as large as $K_T = 13.4$ obtained for $\phi_1 = 0.72$ mm.

III. REDUCTION OF THE EXCESS NOISE DUE TO A SECOND APERTURE

Since now our stable cavity contains an aperture of diameter ϕ_1 near M_1, we know that it is non-Hermitian. In addition to the transverse excess noise factor K_T measured in Sec. II, we have already mentioned above that the linewidth of the laser must also be enhanced by the longitudinal excess noise factor K_L, given by [32]

$$K_L = \left[\frac{(r_{1e} + r_{2e})(1 - r_{1e} r_{2e})}{2 r_{1e} r_{2e} \ln(r_{1e} r_{2e})} \right]^2, \tag{1}$$

where the effective amplitude reflectivities of the mirrors are given by

$$r_{1e} = T \gamma_1 r_1 \tag{2}$$

and

$$r_{2e} = T r_2, \tag{3}$$

where γ_1 is the amplitude transmission of the intracavity aperture of diameter ϕ_1. The computed evolutions of K_T, K_L, and their product $K_T K_L$ versus ϕ_1 are reproduced in Fig. 4(a), together with the evolution of the associated diffraction losses [Fig. 4(b)]. As predicted in Ref. [29], the introduction of a second aperture near M_2 (diameter ϕ_2; see Fig. 1) in our non-Hermitian cavity must lead to a behavior to some extent similar to the one of an unstable cavity. Let us choose, for example, $\phi_1 = 0.77$ mm, which has been shown experimentally to lead to a transverse excess noise factor $K_T = 10.6$ (see Fig. 2) and for which we have computed diffraction losses equal to $\Gamma = 90.4\%$ [see Fig. 4(b)]. With the second aperture, Fig. 5(a) then reproduces the computed evolutions of the different excess noise factors K_T and K_L, of their product $K_L K_T$, and of the total diffraction losses Γ, versus ϕ_2. Notice that Eq. (3) must now be replaced by

$$r_{2e} = T \gamma_2 r_2, \tag{4}$$

where γ_2 is the amplitude transmission of the second intracavity aperture of diameter ϕ_2. We can observe in Fig. 5(a) that these quantities oscillate. In particular, we can predict that there exists a value of ϕ_2 for which the diffraction losses Γ are unchanged with respect to the case where only the first aperture is present in the cavity ($\phi_2 = \infty$). Indeed, Fig. 5(a) shows that $\Gamma = 90.4\%$ for $\phi_2 = 1.0$ mm, which coincides with $\Gamma = 90.4\%$ obtained for $\phi_2 = \infty$ [see Fig. 4(b) with $\phi_1 = 0.77$ mm]. Consequently, by comparing these two situations, we expect the laser linewidth to be modified only because of the changes of K_T and K_L because all other laser parameters must remain unchanged. By introducing the sec-

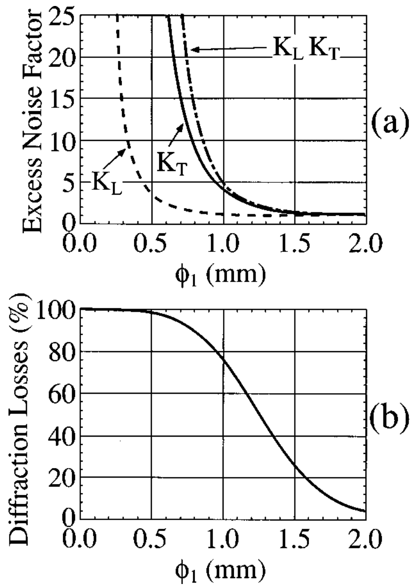

FIG. 4. (a) Computed evolutions of the transverse excess noise factor K_T (full line), the longitudinal excess noise factor K_L (dashed line), and their product (dot-dashed line) versus ϕ_1 with $\phi_2 = \infty$, $R_1 = 0.5$ m, $R_2 = 1.2$ m, $L = 0.27$ m, $\lambda = 3.51\ \mu$m, $r_1^2 = 0.9$, $r_2^2 = 0.6$, and $T = 0.9$. (b) Corresponding theoretical evolution of the diffraction losses per round-trip Γ versus ϕ_1.

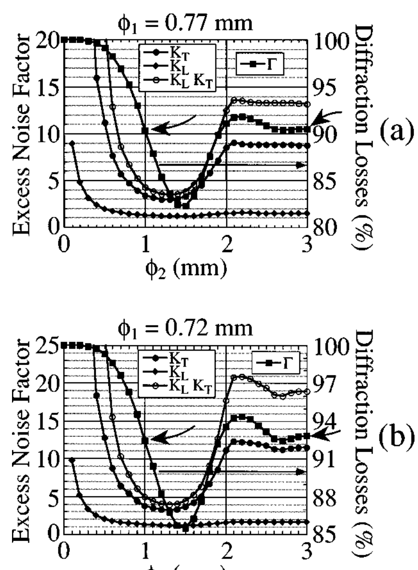

FIG. 5. Computed evolutions of the diffraction losses per round-trip Γ (■), the transverse excess noise factor K_T (●), the longitudinal excess noise factor K_L (♦), and their product $K_L K_T$ (○) for the fundamental mode of the cavity of Fig. 1 versus ϕ_2, with the parameters used in Fig. 4 and with (a) $\phi_1 = 0.77$ mm and (b) $\phi_1 = 0.72$ mm.

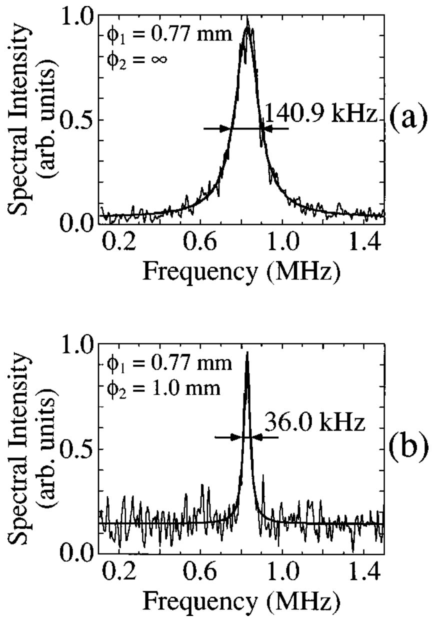

FIG. 6. Experimental spectra (linear scale) of the beat notes between the σ^+ and σ^- eigenstates, together with their Lorentzian fits obtained for (a) $\phi_1 = 0.77$ mm and $\phi_2 = \infty$ and (b) $\phi_1 = 0.77$ mm and $\phi_2 = 1.00$ mm with the same intracavity losses.

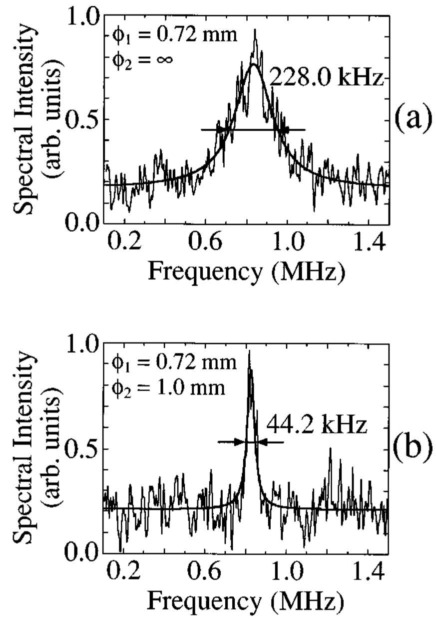

FIG. 7. Same as Fig. 6 for (a) $\phi_1 = 0.72$ mm and $\phi_2 = \infty$ and (b) $\phi_1 = 0.72$ mm and $\phi_2 = 1.00$ mm with the same intracavity losses.

ond aperture with diameter $\phi_2 = 1.0$ mm in front of M_2 in our cavity containing the first aperture of diameter $\phi_1 = 0.77$ mm in front of M_1, we consequently hope to reduce the laser linewidth by a factor given by

$$\frac{K_T(\phi_2=\infty)K_L(\phi_2=\infty)}{K_T(\phi_2=1.0\ \ \text{mm})K_L(\phi_2=1.0\ \ \text{mm})} \approx \frac{10.6\times 1.58}{3.4\times 1.27} \approx 3.9, \tag{5}$$

where the numerical data are taken from the computations reproduced in Figs. 4(a) and 5(a).

The corresponding experimental spectra are reproduced in Fig. 6. Figure 6(a) corresponds to the presence of the first aperture ($\phi_1 = 0.77$ mm) alone and leads to a width equal to 140.9 kHz. We then introduce the second aperture with diameter $\phi_2 = 1.0$ mm in front of mirror M_2. Then we observe that the cavity detuning range for which the laser oscillates is unchanged, showing that the cavity losses, and consequently the intracavity power produced by stimulated emission are unchanged. Obviously, since now the equivalent transmission of mirror M_2 is diminished by the factor $\gamma_2 < 1$, the laser output power becomes much lower. However, the fundamental linewidth of the laser depends on the intracavity power produced by stimulated emission and not on the output power [10,35]. The corresponding beat note spectrum is reproduced in Fig. 6(b), exhibiting a width equal to 36.0 kHz. Consequently, the measurement of the ratio of the widths of the spectra of Figs. 6(a) and 6(b) leads to 140.9/36.0≈3.9, in very good agreement with the expected value [see Eq. (5)].

We have performed a similar experiment in the case of a smaller aperture diameter ϕ_1, namely, $\phi_1 = 0.72$ mm. Figure 5(b) reproduces the corresponding theoretical predictions. For $\phi_2 = \infty$, we start from $K_L K_T = 23.63$ and $\Gamma = 92.7\%$ (see Fig. 4). Figure 5(b) then shows that a second aperture of diameter $\phi_2 = 1.0$ mm must here also lead to almost unchanged diffraction losses ($\Gamma = 92.4\%$) and to the following reduction factor for the laser linewidth:

$$\frac{K_T(\phi_2=\infty)K_L(\phi_2=\infty)}{K_T(\phi_2=1.0\ \ \text{mm})K_L(\phi_2=1.0\ \ \text{mm})} \approx \frac{13.4\times 1.76}{3.8\times 1.31} \approx 4.7. \tag{6}$$

The corresponding experimental results are reproduced in Fig. 7. Figure 7(a) corresponds to the presence of the first aperture ($\phi_1 = 0.72$ mm) alone and leads to a width equal to 228.0 kHz. We then introduce the second aperture with diameter $\phi_2 = 1.0$ mm in front of mirror M_2. The corresponding beat note spectrum is reproduced in Fig. 7(b), exhibiting a width equal to 44.2 kHz. Consequently, the measurement of the ratio of these linewidths leads to 228.0/44.2≈5.2. Although the laser output power is now very weak and the signal-to-noise ratio is poorer, the experimental result is still in good agreement with the prediction of Eq. (6).

IV. CONCLUSION

In conclusion, we have experimentally isolated the transverse excess noise factor K_T in a geometrically stable laser due to a differential method that permits us to ignore the influence of any other factor on the fundamental laser linewidth. In the case of our high-gain gas laser, values of this excess noise factor K_T as large as 13 have been isolated and the dependence of K_T on the aperture diameter has been found to be in very good agreement with the theoretical predictions. Moreover, we have shown that the non-Hermitian nature of an apertured geometrically stable resonator permits us to drastically reduce the overall excess noise factor by introduction of a second intracavity aperture. This illustrates the fact that, to some extent, one can ''restore the orthogonality'' between the transverse modes due to the second aperture. We have indeed shown experimentally that the laser linewidth can be reduced by a factor of approximately 5, without altering the laser losses, gain, or intracavity power. This effect could probably find applications in the reduction of the linewidth of gain-guided semiconductor lasers, where an excess noise factor exists in the absence of any intracavity aperture [5–8]. Moreover, now that the excess noise factors induced by the nonorthogonality of the longitudinal and/or the transverse modes of a stable cavity have been explored, we can expect similar effects due to the nonorthogonality of the two polarization eigenstates of the laser [36].

Note added in proof. Recently, an independent work on related aspects of the present problem has been published [37].

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