Matched pulse propagation in a three-level system

Q-Han Park* and H. J. Shin†

Department of Physics and Research Institute of Basic Sciences, Kyunghee University, Seoul, 130-701, Korea

(Received 19 August 1997)

The Bäcklund transformation for the three-level coupled Schrödinger-Maxwell equation is presented in the matrix potential formalism. By applying the Bäcklund transformation to a constant-electric-field background, we obtain a general solution for matched pulses (a pair of solitary waves) that can emit or absorb a light velocity solitary pulse but otherwise propagate with their shapes invariant. In the special case, this solution describes a steady-state pulse without emission or absorption, and becomes the matched pulse solution recently obtained by Hioe and Grobe [Phys. Rev. Lett. **73**, 2559 (1994)]. A nonlinear superposition rule is derived from the Bäcklund transformation and used for the explicit construction of two solitons as well as non-Abelian breathers. Various features of these solutions are addressed. In particular, we analyze in detail the scattering of ''binary solitons,'' a specific pair of different wavelength solitons, one of which moves with the velocity of light. Unlike the usual case of soliton scattering, the broader soliton changes its sign after the scattering, thus exhibiting a binary behavior. Surprisingly, the light velocity soliton receives a time advance through the scattering, thereby moving faster than light, which, however, does not violate causality. [S1050-2947(98)09206-3]

PACS number(s): 42.50.Md, 42.65.Tg, 42.55.−f

I. INTRODUCTION

The nonlinear interaction between radiation and a multi-level optical medium has received considerable interest for many years. Recently, this topic has attracted more attention in the context of lasing without inversion [1,2] and electromagnetically induced transparency (EIT) [3]. EIT is a technique for rendering an otherwise optically thick medium transparent to a weak probe laser by coupling the upper level coherently to a third level by a strong laser field. The transparency for pulses propagating through an optically thick medium has been known earlier, particularly for a soliton (2π pulse) in the shape of the hyperbolic-secant type [4] and through a three-level medium [5]. More recently, there appeared exact analytic solutions for a pair of solitary waves, so-called matched-solitary-wave pairs (MSPs), propagating through a three-level medium whose invariant shapes are more general than the hyperbolic-secant type [6].

In this paper, we present an analysis of the three-level coupled Schödinger-Maxwell equation based on the matrix potential formalism. In particular, we find the Bäcklund transformation of the coupled Schödinger-Maxwell equation in terms of the matrix potential variable and apply the Bäcklund transformation to the constant electric field background to obtain MSP solutions that generalize the result in Ref. [6]. These solutions in general describe the breakup of a MSP into another MSP with slower velocity and a soliton pulse moving with light velocity, or the reverse process of fusing a MSP and a soliton into another MSP. With a specific choice of parameters, these solutions reduce to the MSP solution in Ref. [6], which describes steady-state propagation of MSP without breakup. The generality in the shape of a MSP solution is explained through the SU(2)×U(1) group symmetry of the three-level Λ system with equal oscillator strengths.

————
*Electronic address: qpark@nms.kyunghee.ac.kr
†Electronic address: thjshin@nms.kyunghee.ac.kr

We show that the Bäcklund transformation also allows a nonlinear superposition rule for solitons as well as MSP solutions. An explicit formula for the nonlinear superposition is given in terms of matrix potentials and used to generate two-soliton and non-Abelian breather solutions. We consider in detail the scattering of a specific type of soliton that exhibits a binary behavior during the scattering process. They are a pair of different wavelength solitons, one of which is moving with the velocity of light and the other with slower velocity. Unlike the usual case of soliton scattering, the broader soliton changes its sign during the scattering process. Surprisingly, the light velocity soliton receives a time advance through the scattering, thereby moving faster than light. We show, however, that causality is not violated. A typical non-Abelian breather describes a breathing 0π pulse $E_1(v<c)$, which afterwards transfers to the nonbreathing E_2 pulse moving with light velocity.

II. MATRIX POTENTIAL FORMALISM OF THE Λ SYSTEM

Consider a Λ configuration where level three is higher than levels one and two. The system of equations governing the propagation of pulses is given by the Schrödinger equation

$$\frac{\partial c_1}{\partial t} = i\Omega_1 c_3,$$

$$\frac{\partial c_2}{\partial t} = i\Omega_2 c_3, \tag{1}$$

$$\frac{\partial c_3}{\partial t} = i(\Omega_1^* c_1 + \Omega_2^* c_2),$$

and the Maxwell equation

$$i\left(\frac{\partial}{\partial x}+\frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_1=l_1c_1c_3^*,$$

$$i\left(\frac{\partial}{\partial x}+\frac{1}{c}\frac{\partial}{\partial t}\right)\Omega_2=l_2c_2c_3^*. \tag{2}$$

Here, $l_i=2\pi N\mu_i^2\omega_i/\hbar$, $i=1,2$, and c_k, $k=1,2,3$, are slowly varying probability amplitudes for the level occupations, $\Omega_i=\mu_iE_i/2\hbar$ are the Rabi frequencies for the transitions $i\to3$, E_1 and E_2 are the slowly varying electromagnetic field amplitudes, μ_i is the dipole matrix element for the relevant transition, ω_i is the corresponding laser frequency, and N is the density of resonant three-level atoms. For brevity, we introduce a coordinate $z=t-x/c$, $\bar{z}=x/c$, so that $\partial\equiv\partial/\partial z$ $=\partial/\partial t$, $\bar{\partial}\equiv\partial/\partial\bar{z}=\partial/\partial t+c\partial/\partial x$.

As in our earlier papers [7,8], the main tool of our analysis will be using a matrix potential g instead of the probability amplitudes c_i in the following way: let g be a 3×3 unitary matrix whose second row is the complex conjugation of probability amplitudes, i.e.,

$$g=\begin{pmatrix} * & * & * \\ c_1^* & c_2^* & c_3^* \\ * & * & * \end{pmatrix}, \tag{3}$$

where the first and the third rows are to be determined later. In terms of g, the density matrix ρ whose components are $\rho_{mk}=c_mc_k^*$ takes a simple form

$$\rho=\frac{i}{l_1}g^{-1}\bar{T}g, \quad \bar{T}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & -il_1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{4}$$

The specific choice of the matrix $\bar{T}$ is not essential. One may consider an arbitrary diagonal matrix $\bar{T}$ to handle density matrices in a more general context. Note that the first and the third rows of the matrix g do not affect the density matrix ρ. In other words, the density matrix ρ is invariant under the left multiplication of g by any matrix h,

$$g\to g'=hg, \tag{5}$$

which commutes with $\bar{T}$ and is thus of the form

$$h=\begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}. \tag{6}$$

At first sight, introducing the matrix g with more redundant components than c_i's may seem an unnecessary complication. However, this is not so. In fact, it not only manifests the symmetry group structure of the system, but it also simplifies the problem of solving differential equations. Later, we show that the Bäcklund transformation of the system, a solution generating technique, also takes a simple form in terms of g. The main advantage of using g is that g solves the Schrödinger equation identically. In order to see this, we fix the redundancy introduced by Eqs. (5) and (6). Adopting the notation for the following matrix decomposition:

$$\Sigma=\Sigma_M+\Sigma_H, \quad \Sigma_M=\begin{pmatrix} 0 & 0 & * \\ 0 & 0 & * \\ * & * & 0 \end{pmatrix},$$

$$\Sigma_H=\begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}, \tag{7}$$

we fix the redundancy by imposing the constraint condition on g,

$$(g^{-1}\partial g)_H=0, \quad (\bar{\partial}gg^{-1})_H=0. \tag{8}$$

One can always solve the constraint by finding an h that makes g satisfy the constraint via the transform in Eq. (5).[1] We also parametrize the remaining components of $g^{-1}\partial g$ such that[2]

$$g^{-1}\partial g=\begin{pmatrix} 0 & 0 & -i\Omega_1 \\ 0 & 0 & -i\Omega_2 \\ -i\Omega_1^* & -i\Omega_2^* & 0 \end{pmatrix}. \tag{9}$$

The nonzero components in Eq. (9) express the Rabi frequencies in terms of g. In this parametrization, one can readily check that the Schrödinger equation (1) arises from the simple identity

$$\partial g^\dagger=\partial g^{-1}=-g^{-1}\partial gg^{-1}=-g^{-1}\partial gg^\dagger. \tag{10}$$

This situation may be compared with ordinary electromagnetism where the static electric field $\vec{E}$ in terms of a scalar potential ϕ, $\vec{E}=-\vec{\nabla}\phi$, solves the curl-free condition $\vec{\nabla}\times\vec{E}=\vec{0}$. Likewise, we solve the Schrödinger equation in terms of a matrix potential g and express the electric field components Ω_i in terms of g as in Eq. (9). Originally, the Schrödinger and the Maxwell equations are made of five complex, component equations in total. Three of them (the Schrödinger part) are now solved identically in terms of g, where g is partially constrained by Eq. (8). Then, the remaining degree of g can be parametrized by two unknown complex functions and the Maxwell equation changes into two component equations for these two unknown functions only.[3] In this

_____

[1]The existence of such an h can be proved by adopting the field theory formulation of the problem as in [8]. However, we do not need the explicit expression of h.

[2]Note that $g^{-1}\partial g$ is anti-Hermitian and g is unitary.

[3]In mathematical terms, we have associated the density matrix ρ with the coset $G/H=[SU(3)/SU(2)\times U(1)]$ and introduced the matrix g for the parametrization of G/H. The constraint equation on g makes a specific choice for each equivalence class, and variables that parametrize equivalence classes are determined by the Maxwell equation. Since the constraint restricts the subgroup $H[=SU(2)\times U(1)]$ part of the variable g, the remaining unconstrained part of g can be expressed in terms of two unknown complex functions φ_1 and φ_2. In the gauged sigma model formulation, the Maxwell equation is vector gauge invariant so that it decouples from the H degree of freedom, i.e., it reduces to two complex equations only in φ_1 and φ_2.

way, the Maxwell equation resembles the Poisson equation in electrostatics. However, we do not need explicit component expressions in this paper so we suppress them. What we need is the expression of the Maxwell equation in terms of g such that

$$\bar{\partial}(g^{-1}\partial g) = Q[T, g^{-1}\bar{T}g]Q, \qquad (11)$$

where

$$T = \begin{pmatrix} -\dfrac{i}{2} & 0 & 0 \\ 0 & -\dfrac{i}{2} & 0 \\ 0 & 0 & \dfrac{i}{2} \end{pmatrix}, \quad Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & l_2/l_1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (12)$$

Note that this is indeed consistent with Eq. (9). The Maxwell equation in Eq. (11) possesses a symmetry under the change; $g \rightarrow g' = gh$ where h is an arbitrary constant diagonal matrix. That is, g' again satisfies Eqs. (9) and (11). This symmetry becomes enhanced to a larger one when the oscillator strengths are equal ($l_1 = l_2 = s$) so that Q is the identity matrix. For the Λ system, the oscillator strength s is positive, which we assume throughout the paper. With equal oscillator strengths, $\tilde{h}$ can be a constant unitary matrix of the form

$$\tilde{h} = \begin{pmatrix} h_{11} & h_{12} & 0 \\ h_{21} & h_{22} & 0 \\ 0 & 0 & h_{33} \end{pmatrix}. \qquad (13)$$

In other words, $\tilde{h}$ is a constant matrix belonging to the group SU(2)×U(1). In terms of physical variables, this symmetry amounts to the transform,

$$\begin{pmatrix} \Omega_1' \\ \Omega_2' \end{pmatrix} = \begin{pmatrix} h_{11}^* & h_{21}^* \\ h_{12}^* & h_{22}^* \end{pmatrix} \begin{pmatrix} \Omega_1 \\ \Omega_2 \end{pmatrix},$$

$$\begin{pmatrix} c_1' \\ c_2' \\ c_3' \end{pmatrix} = \begin{pmatrix} h_{11}^* & h_{21}^* & 0 \\ h_{12}^* & h_{22}^* & 0 \\ 0 & 0 & h_{33}^* \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}, \qquad (14)$$

where the primed variables are solutions of Eqs. (9) and (11) provided that unprimed variables are. In particular, if the unprimed solution is a single 2π pulse [$\Omega_1 = 0$, $\Omega_2 \sim \mathrm{sech}(t - x/v)/t_p$], the primed solution represents a simulton solution [10]. When the oscillator strengths are equal ($Q = 1$), the theory becomes integrable and exact analytic solutions can be found. However, even for $Q \neq 1$, the Maxwell equation admits a Lax pair representation and infinite

sequences of conserved integrals can be found [12]. In this paper, we assume the equal oscillator strengths so that the Maxwell equation becomes

$$\bar{\partial}(g^{-1}\partial g) = [T, g^{-1}\bar{T}g]. \qquad (15)$$

III. BÄCKLUND TRANSFORMATION

The Maxwell equation in Eq. (15) is equivalent to the consistency condition $[L_1, L_2] = 0$ of the overdetermined linear equations:

$$L_1 \Psi = (\partial + g^{-1}\partial g + \lambda T)\Psi = 0,$$

$$L_2 \Psi = \left(\bar{\partial} + \frac{1}{\lambda} g^{-1}\bar{T}g \right) \Psi = 0, \qquad (16)$$

where λ is a spectral parameter. We may apply the inverse scattering method to Eq. (16) and obtain exact solutions as in [8]. Instead, we present in this paper an alternative, simpler method—the Bäcklund transformation (BT)—which allows a more direct construction of exact solutions. Moreover, the advantage of using the matrix potential g becomes clear when the BT is used to establish a nonlinear superposition rule for a couple of single pulses. This is similar to the linear case where the application of the usual superposition rule is easier in terms of the scalar potential rather than the electric field itself. Let g_0 and Ψ_0 be a particular solution of Eqs. (15) and (16), then g_1 is also a solution of Eq. (15) if it satisfies the Bäcklund transformation of type I (type-I BT):
   type-I BT:

$$g_1^{-1}\partial g_1 - g_0^{-1}\partial g_0 - i\eta[g_1^{-1}g_0, T] = 0,$$

$$i\eta\bar{\partial}(g_1^{-1}g_0) + g_1^{-1}\bar{T}g_1 - g_0^{-1}\bar{T}g_0 = 0. \qquad (17)$$

Here, η is an arbitrary parameter of the transformation. Once again, the type-I BT is a set of overdetermined first-order partial differential equations whose consistency requires that g_0 and g_1 should be both solutions of Eq. (15). An equivalent expression of the BT is in terms of the linear function Ψ as in [13]. We define the Bäcklund transformation of type II (type-II BT):
   type-II BT:

$$\Psi_1 = \frac{\lambda}{\lambda - i\eta} \left(1 + \frac{i\eta}{\lambda} g_1^{-1}g_0 \right) \Psi_0. \qquad (18)$$

It can be readily checked that g_1 and Ψ_1 satisfy Eqs. (15) and (16) provided Eq. (17) holds and vice versa. The type-II BT is particularly useful for establishing a nonlinear superposition rule. Assume that (g_a, Ψ_a) and (g_b, Ψ_b) are two sets of solutions with BT parameters η_a and η_b, respectively, solving the BT for a particular solution, (g_0, Ψ_0). If we apply the BT once more to (g_a, Ψ_a) with $\eta = \eta_b$ and also to (g_b, Ψ_b) with $\eta = \eta_a$, and require that they result in the same solution (this amounts to the commutability of the diagram in Fig. 1), then we obtain from the type-II BT,
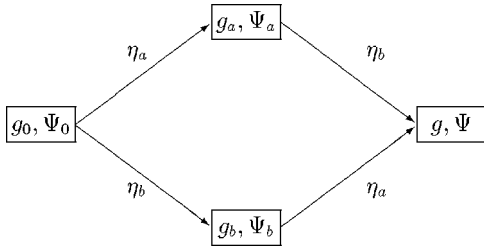
FIG. 1. Commutability diagram for the nonlinear superposition rule.

$$\Psi = \frac{\lambda}{(\lambda - i\eta_b)} \frac{\lambda}{(\lambda - i\eta_a)} \left(1 + \frac{i\eta_b}{\lambda} g^{-1} g_a \right)$$

$$\times \left(1 + \frac{i\eta_a}{\lambda} g_a^{-1} g_0 \right) \Psi_0$$

$$= \frac{\lambda}{(\lambda - i\eta_a)} \frac{\lambda}{(\lambda - i\eta_b)} \left(1 + \frac{i\eta_a}{\lambda} g^{-1} g_b \right)$$

$$\times \left(1 + \frac{i\eta_b}{\lambda} g_b^{-1} g_0 \right) \Psi_0. \qquad (19)$$

Equivalently, we have the nonlinear superposition of two solutions,

$$g = (\eta_b g_a - \eta_a g_b) g_0^{-1} (\eta_b g_b^{-1} - \eta_a g_a^{-1})^{-1}. \qquad (20)$$

Combining this expression with the type-I BT, we obtain a useful formula for the nonlinear superposition of the Rabi frequencies,

$$g^{-1} \partial g = \frac{1}{2} (g_a^{-1} \partial g_a + g_b^{-1} \partial g_b) + \frac{i}{2} [g^{-1} (\eta_a g_b + \eta_b g_a), T], \qquad (21)$$

or,

$$\Omega_1 = \frac{1}{2} (\Omega_1^a + \Omega_1^b) - \frac{i}{2} F_{13},$$

$$\Omega_2 = \frac{1}{2} (\Omega_2^a + \Omega_2^b) - \frac{i}{2} F_{23}, \qquad (22)$$

$$F \equiv (\eta_b g_b^{-1} - \eta_a g_a^{-1}) g_0 (\eta_b g_a - \eta_a g_b)^{-1} (\eta_a g_b + \eta_b g_a).$$

IV. MATCHED PULSES

Now, we construct solutions by integrating the type-I BT directly. We choose the particular solution g_0 for a constant electric field.

$$g_0^{-1} \partial g_0 = \begin{pmatrix} 0 & 0 & -i\Omega_1 \\ 0 & 0 & -i\Omega_2 \\ -i\Omega_1^* & -i\Omega_2^* & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & -i\Omega_0 \\ 0 & 0 & 0 \\ -i\Omega_0 & 0 & 0 \end{pmatrix} \equiv \Lambda \qquad (23)$$

for a real constant Ω_0, so that

$$g_0 = e^{\Lambda z} = \begin{pmatrix} \cos(\Omega_0 z) & 0 & -i\sin(\Omega_0 z) \\ 0 & 1 & 0 \\ -i\sin(\Omega_0 z) & 0 & \cos(\Omega_0 z) \end{pmatrix}. \qquad (24)$$

If we set $f \equiv e^{-\Lambda z} g_1$, the type-I BT becomes

$$f^{-1} \partial f + f^{-1} \Lambda f - \Lambda - i\eta [f^{-1}, T] = 0, \qquad (25)$$

$$i\eta \bar{\partial} f^{-1} + f^{-1} \bar{T} f - \bar{T} = 0. \qquad (26)$$

Since $f^{-1} \partial f$ is anti-Hermitian, Eq. (25) requires that

$$[f^{-1} - f, T] = 0, \qquad (27)$$

which we solve by taking

$$f^{-1} - f = 2i\sin\theta \qquad (28)$$

for an arbitrary real parameter $\sin\theta$. If we rewrite f in terms of another matrix P,

$$f \equiv e^{-\Lambda z} g_1 = \cos\theta(2P - 1) - i\sin\theta, \qquad (29)$$

then Eqs. (28) and (29) imply that P is a Hermitian projection operator, i.e.,

$$P^2 = P, \quad P^\dagger = P. \qquad (30)$$

In terms of P, Eqs. (25) and (26) can be written by

$$(1 - P)(\partial + \Lambda - i\tilde{\eta} T) P = 0,$$
$$(1 - P)(i\tilde{\eta} \bar{\partial} - \bar{T}) P = 0, \qquad (31)$$

where $\tilde{\eta} \equiv e^{i\theta} \eta$. Since P is a projection operator acting on the three-dimensional space, we may express P using a three-dimensional vector $\vec{s} = (s_1, s_2, s_3)$ by

$$P_{ij} = s_i s_j^* \left/ \left(\sum_{k=1}^{3} s_k s_k^* \right) \right., \qquad (32)$$

which transforms Eq. (31) into a linear one,

$$(\partial + \Lambda - i\tilde{\eta} T) \vec{s} = 0,$$
$$(i\tilde{\eta} \bar{\partial} - \bar{T}) \vec{s} = 0. \qquad (33)$$

Since Λ and $\bar{T}$ commute, this may be integrated immediately resulting in

$$s_i = \sum_{k=1}^{3} [\exp\Delta]_{ik} u_k \quad \text{for } \Delta = (i\tilde{\eta} T - \Lambda) z - \frac{i}{\tilde{\eta}} \bar{T} \bar{z}, \qquad (34)$$

where $\vec{u} = (u_1, u_2, u_3)$ is an arbitrary complex constant vector. Explicitly,

$$s_1 = \left(\cosh\sqrt{K}z + \frac{\tilde{\eta}}{2\sqrt{K}}\sinh\sqrt{K}z \right) u_1 + \frac{i\Omega_0}{\sqrt{K}}(\sinh\sqrt{K}z)u_3,$$

$$s_2 = \exp\left(\frac{\tilde{\eta}z}{2} - \frac{s\bar{z}}{\tilde{\eta}} \right) u_2, \tag{35}$$

$$s_3 = \frac{i\Omega_0}{\sqrt{K}}(\sinh\sqrt{K}z)u_1 + \left(\cosh\sqrt{K}z - \frac{\tilde{\eta}}{2\sqrt{K}}\sinh\sqrt{K}z \right) u_3,$$

where $K = \tilde{\eta}^2/4 - \Omega_0^2$. Finally, Rabi frequencies are given by

$$\Omega_1 = \Omega_0 - 2i\,\eta(\cos\theta)s_1 s_3^* \left/ \left(\sum_{k=1}^{3} s_k s_k^* \right) \right., \tag{36}$$

$$\Omega_2 = -2i\,\eta(\cos\theta)s_2 s_3^* \left/ \left(\sum_{k=1}^{3} s_k s_k^* \right) \right.$$

and probability amplitudes are obtained through Eqs. (3) and (29),

$$c_1 = 2(\cos\theta)s_1 s_2^* \left/ \left(\sum_{k=1}^{3} s_k s_k^* \right) \right.,$$

$$c_2 = (\cos\theta)\left[2s_2 s_2^* \left/ \left(\sum_{k=1}^{3} s_k s_k^* \right) - 1 \right] \right. + i\sin\theta,$$

$$c_3 = 2(\cos\theta)s_3 s_2^* \left/ \left(\sum_{k=1}^{3} s_k s_k^* \right) \right.. \tag{37}$$

Note that this solution, combined with the symmetry transformation in Eq. (14), represents a rich family of single pulse solutions. For the vanishing ($\Omega_0 = 0$), it becomes

$$\Omega_1 = \frac{1}{N}[-2i\,\eta(\cos\theta)u_1 u_3^* \exp(i\,\eta z\sin\theta)], \tag{38}$$

$$\Omega_2 = \frac{1}{N}\left\{ -2i\,\eta(\cos\theta)u_2 u_3^* \right.$$
$$\left. \times \exp\left[i\left(\eta z + \frac{s}{\eta}\bar{z} \right)\sin\theta - \frac{s}{\eta}\bar{z}\cos\theta \right] \right\}$$

and

$$c_1 = \frac{1}{N}\left\{ 2(\cos\theta)u_1 u_2^* \exp\left[\left(\eta z - \frac{s}{\eta}\bar{z} \right)\cos\theta - i\frac{s}{\eta}\bar{z}\sin\theta \right] \right\},$$

$$c_2 = \frac{1}{N}\left\{ -|u_1|^2\exp[\,\eta z(\cos\theta) - i\theta] + |u_2|^2\exp\left[\left(\eta z - 2\frac{s}{\eta}\bar{z} \right)\cos\theta + i\theta \right] - |u_3|^2\exp[-\eta z\cos\theta - i\theta] \right\}, \tag{39}$$

$$c_3 = \frac{1}{N}\left\{ 2(\cos\theta)u_3 u_2^* \exp\left[-\frac{s}{\eta}\bar{z}\cos\theta - i\left(\eta z + \frac{s}{\eta}\bar{z} \right)\sin\theta \right] \right\},$$

where

$$N \equiv |u_1|^2\exp[\,\eta z\cos\theta] + |u_2|^2\exp\left[\eta z\cos\theta - \frac{2s}{\eta}\bar{z}\cos\theta \right] + |u_3|^2\exp[-\eta z\cos\theta]. \tag{40}$$

This solution for $u_1 \neq 0$, $u_2 \neq 0$, $\eta\cos\theta > 0$ describes the transfer of the 2π pump pulse in the limit $x \to -\infty$:

$$\Omega_1 \to 0, \quad \Omega_2 \to -i\,\eta(\cos\theta)(\mathrm{sech}\Sigma_1)e^{i\Sigma_2},$$

$$c_1 \to 0, \quad c_2 \to \frac{e^{\Sigma_1 + i\theta} - e^{-\Sigma_1 - i\theta}}{e^{\Sigma_1} + e^{-\Sigma_1}},$$

$$c_3 \to \cos\theta e^{-i\Sigma_2}\,\mathrm{sech}\Sigma_1, \tag{41}$$

$$\Sigma_1 = (\cos\theta)\left(\eta t - \frac{(\eta^2 + s)}{c\,\eta}x + \phi_1 \right),$$

$$\Sigma_2 = (\sin\theta)\left(\eta t - \frac{(\eta^2 - s)}{c\,\eta}x + \phi_2 \right)$$

to the 2π-Stokes pulse moving with the velocity of light in the limit $x \to \infty$:

$$\Omega_1 \to -i\,\eta(\cos\theta)\,\mathrm{sech}\left[(\cos\theta)\,\eta\left(t - \frac{x}{c} \right) \right]$$
$$\times \exp\left[i(\sin\theta)\,\eta\left(t - \frac{x}{c} \right) \right],$$

$$\Omega_2 \to 0, \quad c_1 \to 0, \quad c_2 \to -e^{-i\theta}, \quad c_3 \to 0, \tag{42}$$

where the arbitrary constants ϕ_i, $i = 1,2$ determine soliton positions in time and space. In the case of $\theta = 0$, this transfer of 2π pulse has been given in [14]. For $u_1 = 0$ or $u_2 = 0$, the solution remains as the steady-state 2π pulse given in Eqs. (41) or (42), respectively, without causing any transfer of

pulses. This steady pulse, in connection with the symmetry transform in Eq. (14), is the simulton solution [10]. The free parameter θ measures the amount of self-detuning of a pulse from the carrier frequency. The 2π Stokes pulse in Eq. (42) is the same as the usual 2π pulse ($\theta=0$) but with the carrier frequency shifted by the amount $\delta w = \eta \sin\theta$. On the other hand, Eq. (41) shows that the 2π pump pulse receives a time-independent phase factor $\exp[i(s\sin\theta/c\eta)x]$ in addition to the shift of carrier frequency. We emphasize that this detuning has nothing to do with the frequency detuning of electromagnetic fields from the resonance line. In fact, our system is on resonance and thus the parameter θ measures the self-generated detuning of each pulse. The effect of θ to a single 2π pulse is to broaden the pulse shape maintaining the 2π area of the envelope, which is adjusted by the shift of the carrier frequency. Recall that due to the symmetry in Eq. (14), a more general expression for a single pulse arises as a linear mixture of Ω_1 and Ω_2 in Eq. (38), which possess a wide range of free parameters.

If $\Omega_0 \neq 0$, the solution describes pulses more general than the hyperbolic-secant type. For the simplicity of analysis, we assume that $\theta=0$ and $|\eta| \geq 2|\Omega_0|$. We also restrict to the parameters $u_1=r_1$, $u_2=r_2$, $u_3=ir_3$ for real r_i and rewrite Eq. (35) for the notational convenience as follows:

$$s_1 = A\exp(\sqrt{K}z) + B\exp(-\sqrt{K}z),$$

$$s_2 = \sqrt{A^2+C^2}\exp\left(\frac{\eta z}{2} - \frac{s\bar{z}}{\eta}\right),$$ (43)

$$s_3 = iC\exp(\sqrt{K}z) + iD\exp(-\sqrt{K}z),$$

where r_2 is chosen to be $\sqrt{A^2+C^2}$ by an appropriate choice of the coordinate origin. The coefficients are defined by

$$A = \frac{1}{2}\left(1 + \frac{\eta}{2\sqrt{K}}\right)r_1 - \frac{\Omega_0}{2\sqrt{K}}r_3,$$

$$B = \frac{1}{2}\left(1 - \frac{\eta}{2\sqrt{K}}\right)r_1 + \frac{\Omega_0}{2\sqrt{K}}r_3,$$

$$C = \frac{\Omega_0}{2\sqrt{K}}r_1 + \frac{1}{2}\left(1 - \frac{\eta}{2\sqrt{K}}\right)r_3,$$ (44)

$$D = -\frac{\Omega_0}{2\sqrt{K}}r_1 + \frac{1}{2}\left(1 + \frac{\eta}{2\sqrt{K}}\right)r_3,$$

$$K = \frac{1}{4}\eta^2 - \Omega_0^2.$$

In the limit where $x \to -\infty$, the solution takes an asymptotic form:

$$\Omega_1 = -\Omega_0 \tanh\Sigma_I, \quad \Omega_2 = \frac{\eta C}{\sqrt{A^2+C^2}}\text{sech}\Sigma_I$$ (45)

and

$$c_1 = -\frac{\eta C}{\sqrt{A^2+C^2}}\text{sech}\Sigma_I, \quad c_2 = -\tanh\Sigma_I,$$

$$c_3 = \frac{iC}{\sqrt{A^2+C^2}}\text{sech}\Sigma_I,$$ (46)

$$\Sigma_I = \frac{2s + (\eta^2 - 2\eta\sqrt{K})}{2\eta c}(x - v_I t),$$

where the velocity v_I is

$$v_I = \frac{\eta^2 - 2\eta\sqrt{K}}{2s + \eta^2 - 2\eta\sqrt{K}}c,$$ (47)

which is less than the light velocity c. Comparison of electric fields in Eq. (45) with initial populations in Eq. (46) shows that two laser pulses are arranged in the so-called counterintuitive order [9]. In the $x \to \infty$ limit, the asymptotic form of the solution is

$$\Omega_1 = -\Omega_0 \tanh\Sigma_F + \Omega_1^S, \quad \Omega_2 = \frac{\eta C}{\sqrt{A^2+C^2}}\text{sech}\Sigma_F$$ (48)

and

$$c_1 = \frac{B}{\sqrt{B^2+D^2}}\text{sech}\Sigma_F, \quad c_2 = -\tanh\Sigma_F,$$

$$c_3 = \frac{iD}{\sqrt{B^2+D^2}}\text{sech}\Sigma_F,$$ (49)

where

$$\Sigma_F = \frac{2s + (\eta^2 + 2\eta\sqrt{K})}{2\eta c}(x - v_F t) - \Delta_0,$$

$$v_F = \frac{\eta^2 + 2\eta\sqrt{K}}{2s + \eta^2 + 2\eta\sqrt{K}}c,$$ (50)

$$\Delta_0 = \frac{1}{2}\ln\frac{B^2+D^2}{A^2+C^2},$$

$$\Omega_1^S \equiv \frac{\Omega_0(r_1^2+r_3^2) - \eta r_1 r_3}{(A^2+C^2)e^{\Delta_0}\cosh[2\sqrt{K}(t-x/c)-\Delta_0]+AB+CD}.$$

In the far past, this solution represents a matched-solitary-wave pair (MSP) moving with velocity v_I whose invariant shape is not the hyperbolic-secant type. Equation (48) shows that this MSP in general breaks up into another MSP with slower velocity ($v_F < v_I$) and a soliton pulse moving with light velocity. Figure 2 shows explicitly this breakup behavior with parameters, $s=1$, $r_1=0.7$, $r_3=1$, $\Omega_0=0.5$, and $\eta=1.5$.

By changing the sign of η, one could equally consider the reverse process of fusing a MSP and a soliton into another MSP. In the specific case where $K = \eta^2/4 - \Omega_0^2$ vanishes, this
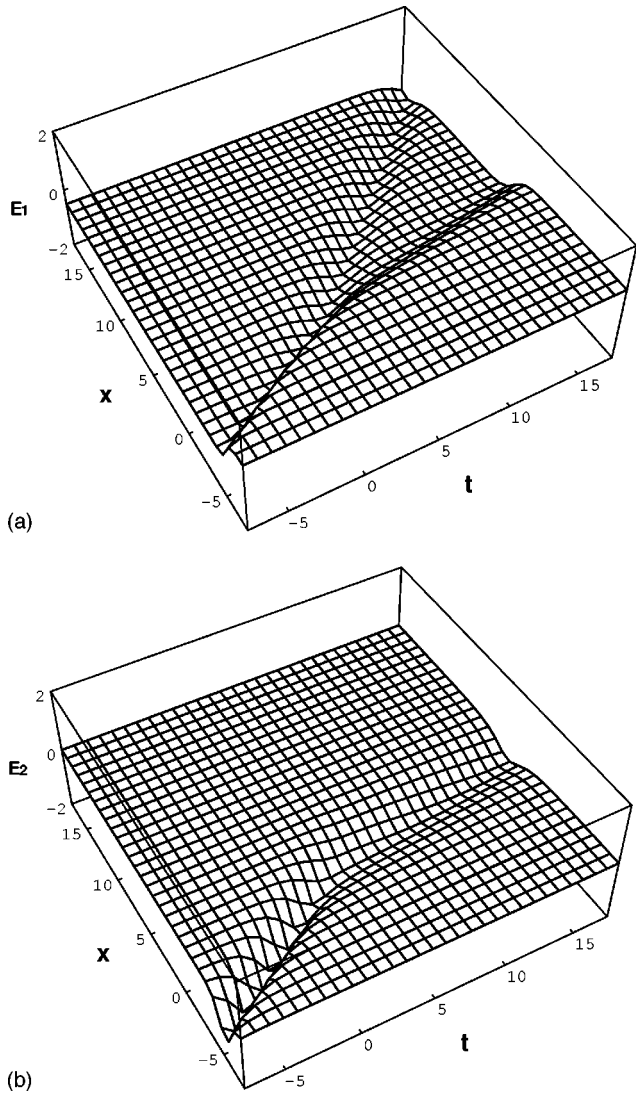
(a)

(b)

FIG. 2. Rabi frequencies $E_1 = \Omega_1$ (a) and $E_2 = \Omega_2$ (b) of the MSP.

solution reduces to the steady state pulse, which agrees with the MSP solution by Hioe and Grobe [6],

$$\Omega_1 = -\Omega_0 \tanh \Sigma_S, \quad \Omega_2 = \sqrt{2}\Omega_0 \operatorname{sech}\Sigma_S \qquad (51)$$

and

$$c_1 = \frac{1}{\sqrt{2}} \operatorname{sech}\Sigma_S, \quad c_2 = -\tanh \Sigma_S, \quad c_3 = \frac{i}{\sqrt{2}} \operatorname{sech}\Sigma_S,$$
$$(52)$$

where

$$\Sigma_S = \frac{s + 2\Omega_0^2}{2\Omega_0 c}(x - v_S t), \quad v_S = 2\Omega_0^2/(s + 2\Omega_0^2). \qquad (53)$$

Thus, our solution generalizes the result of Hioe and Grobe and shows that the MSP propagates steadily only when the velocity v_S of the MSP is specifically given by the parameter Ω_0 as in Eq. (53). Otherwise, it breaks up as explained before. In other words, in the case of MSPs where $K \neq 0$, the hyperbolic-tangent pulse Ω_1 induces a partial transfer of the other soliton pulse Ω_2 into the light velocity one. This is similar to the behavior of adiabaton moving with a slower velocity which also generates a light velocity solitary pulse during its formation [11]. The most general expression of our MSP solution arises when we incorporate the SU(2)×U(1) symmetry transform as in Eq. (14) as well as the self-detuning effect ($\theta \neq 0$). This solution contains a large number of free parameters that account for the variety of pulse shapes and initial conditions required for the atoms. The stability of this MSP solution against small perturbations is an important issue [15] that we plan to consider elsewhere.

V. BINARY SOLITONS AND BREATHERS

The nonlinear superposition rule given in Eqs. (20)–(22) allows the superposition of two MSP solutions g_a and g_b whenever they are obtained from g_0 with BT parameters η_a and η_b, respectively. In general, g_a and g_b could possess different set of free parameters and the superposition rule requires only that they are obtained from the same g_0 through BT. If g_a and g_b have the same set of parameters except for BT parameters, we could analytically continue the BT parameter in Eq. (20) to the complex case in such a way that

$$\eta_a = \exp[i\theta_B], \quad \eta_b = \exp[-i\theta_B] \qquad (54)$$

for a certain real parameter θ_B, and that g is still unitary and becomes a non-Abelian breather solution. As an example, we take the one-soliton to be the MSP in the previous section with a set of parameters

$$\Omega_0 = 0, \quad u_1 = 1, \quad u_2 = 1, \quad u_3 = -i. \qquad (55)$$

Then, the non-Abelian breather solution has an asymptotic form for $x \to -\infty$,

$$\Omega_1 = 0, \qquad (56)$$

$$\Omega_2 = \frac{(\sin 2\theta_B)\{\exp[(2x/c - t)\cos\theta_B]\sin(\theta_B + t\sin\theta_B) + \exp[(t - 2x/c)\cos\theta_B]\sin(\theta_B - t\sin\theta_B)\}}{-1 + \cos^2\theta_B\cos(2t\sin\theta_B) - \cosh[(4x/c - 2t)\cos\theta_B]\sin^2\theta_B}$$

and for $x \rightarrow \infty$,

$$\Omega_1 = \frac{(\sin 2\theta)_B \{ e^{Z_C} \sin(\theta_B - Z_S) + e^{-Z_C} \sin(\theta_B + Z_S) \}}{-1 + \cos^2 \theta_B \cos(2Z_S) - \cosh(2Z_C) \sin^2 \theta_B},$$

$$\Omega_2 = 0,$$

$$Z_C = (t - x/c) \cos \theta_B, \quad Z_S = (t - x/c) \sin \theta_B. \quad (57)$$

This describes a breathing 0π pulse $\Omega_2(v < c)$, which transfers to the non-breathing Ω_1 pulse moving with the velocity of light. This is a typical motion of a non-Abelian breather. Figure 3 shows a breathing motion with $\theta_B = 1.2$. Different values of parameters u_i in general distort the shape of the 0π pulse so that the time area of the pulse is nonzero, but they lead to the essentially same type of transferring motion except for the case $u_1 = 0$ or $u_2 = 0$ where it becomes a steady state breather.

For real BT parameters η_a and η_b, the superposed solution g in general describes the scattering of two MSP solutions. Here, we concentrate only on a particular case of two soliton solutions that exhibit a binary behavior during the scattering process. Consider one-solitons g_a and g_b given by

$$g_a = \begin{pmatrix} -1 & 0 & 0 \\ 0 & \cos\phi_a & \sin\phi_a \\ 0 & \sin\phi_a & -\cos\phi_a \end{pmatrix},$$

$$\cos\phi_a = \tanh\left[\eta_a t - \frac{\eta_a^2 + s}{c\, \eta_a} x \right], \quad (58)$$

$$\sin\phi_a = \mathrm{sech}\left[\eta_a t - \frac{\eta_a^2 + s}{c\, \eta_a} x \right],$$

and

$$g_b = \begin{pmatrix} \cos\phi_b & 0 & \sin\phi_b \\ 0 & -1 & 0 \\ \sin\phi_b & 0 & -\cos\phi_b \end{pmatrix},$$

$$\cos\phi_b = \tanh[\, \eta_b(t - x/c)\,], \quad (59)$$

$$\sin\phi_b = \mathrm{sech}[\, \eta_b(t - x/c)\,].$$

In terms of Rabi frequencies, these correspond to

$$\Omega_1^{(a)} = 0, \quad \Omega_2^{(a)} = -i\, \eta_a\, \mathrm{sech}\left[\eta_a t - \frac{\eta_a^2 + s}{c\, \eta_a} x \right] \quad (60)$$
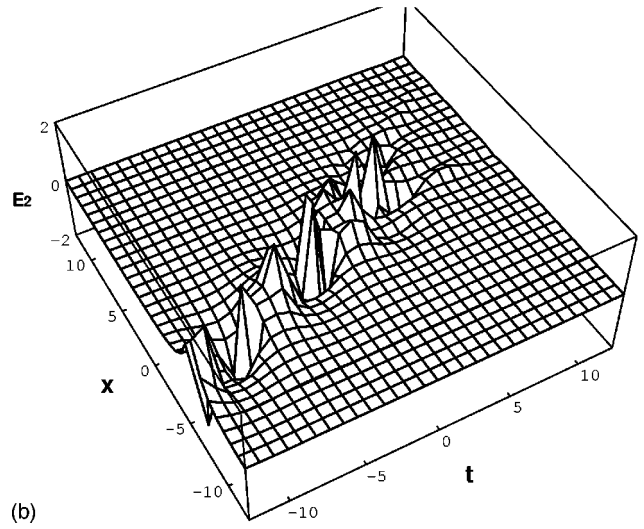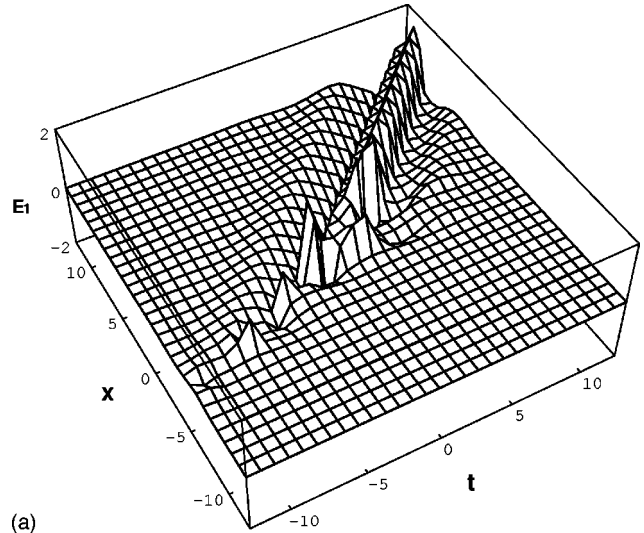

(a)

(b)

FIG. 3. Rabi frequencies $E_1 = \Omega_1$ (a) and $E_2 = \Omega_2$ (b) of the breather.

and

$$\Omega_1^{(b)} = -i\, \eta_b\, \mathrm{sech}[\, \eta_b(t - x/c)\,], \quad \Omega_2^{(b)} = 0, \quad (61)$$

that is, they represent two 2π pulses with different resonance frequencies, one moving with light velocity and the other moving with slower velocity.

The nonlinear superposition rule gives rise to the superposed Rabi frequencies and probabilities such that

$$\Omega_1 = -i\, \eta_b \sin\phi_b\, \frac{\eta_a^2 \cos\phi_a + \eta_a \eta_b(1 + \cos\phi_a) + \eta_b^2}{\eta_a^2 + \eta_b^2 + \eta_a \eta_b(1 + \cos\phi_a + \cos\phi_b - \cos\phi_a \cos\phi_b)},$$

$$\tag{62}$$

$$\Omega_2 = -i\, \eta_a \sin\phi_a\, \frac{\eta_a^2 + \eta_a \eta_b(1 + \cos\phi_b) + \eta_b^2 \cos\phi_b}{\eta_a^2 + \eta_b^2 + \eta_a \eta_b(1 + \cos\phi_a + \cos\phi_b - \cos\phi_a \cos\phi_b)}$$

and

$$c_1 = \frac{(\eta_a + \eta_b)\eta_b \sin\phi_a \sin\phi_b}{\eta_a^2 + \eta_b^2 + \eta_a \eta_b(1 + \cos\phi_a + \cos\phi_b - \cos\phi_a \cos\phi_b)},$$

$$c_2 = \frac{-(\eta_a^2 + \eta_b^2)\cos\phi_a - \eta_a \eta_b(1 + \cos\phi_a - \cos\phi_b + \cos\phi_a \cos\phi_b)}{\eta_a^2 + \eta_b^2 + \eta_a \eta_b(1 + \cos\phi_a + \cos\phi_b - \cos\phi_a \cos\phi_b)},$$

$$c_3 = \frac{-[\eta_a^2 + \eta_a \eta_b(1 + \cos\phi_b) + \eta_b^2 \cos\phi_b]\sin\phi_a}{\eta_a^2 + \eta_b^2 + \eta_a \eta_b(1 + \cos\phi_a + \cos\phi_b - \cos\phi_a \cos\phi_b)}. \tag{63}$$

This solution describes the scattering of two solitons where the fast moving soliton I_F passes through the slow moving one I_S. Before the collision, their asymptotic forms are given by the following configurations:

soliton I_F:

$$\cos\phi_a = 1,$$

$$c_1 = 0, \quad c_2 = -1, \quad c_3 = 0, \tag{64}$$

$$\Omega_1 = -i\eta_b \, \text{sech}[\eta_b(t - x/c)], \quad \Omega_2 = 0$$

and soliton I_S:

$$\cos\phi_b = 1,$$

$$c_1 = 0, \quad c_2 = -\tanh\left[\eta_a t - \frac{\eta_a^2 + s}{c\,\eta_a}x\right],$$

$$c_3 = -\,\text{sech}\left[\eta_a t - \frac{\eta_a^2 + s}{c\,\eta_a}x\right], \tag{65}$$

$$\Omega_1 = 0, \quad \Omega_2 = -i\eta_a \, \text{sech}\left[\eta_a t - \frac{\eta_a^2 + s}{c\,\eta_a}x\right].$$

After the collision, their asymptotic forms are as follows:

soliton I_F:

$$\cos\phi_a = -1,$$

$$c_1 = 0, \quad c_2 = 1, \quad c_3 = 0, \tag{66}$$

$$\Omega_1 = i\eta_b \, \text{sgn}\left(\frac{\eta_a + \eta_b}{\eta_a - \eta_b}\right)\text{sech}[\eta_b(t - x/c) + \delta],$$

$$\Omega_2 = 0$$

and soliton I_S:

$$\cos\phi_b = -1,$$

$$c_1 = 0, \quad c_2 = -\tanh\left[\eta_a t - \frac{\eta_a^2 + s}{c\,\eta_a}x + \delta\right], \tag{67}$$

$$c_3 = -\,\text{sgn}\left(\frac{\eta_a + \eta_b}{\eta_a - \eta_b}\right)\text{sech}\left[\eta_a t - \frac{\eta_a^2 + s}{c\,\eta_a}x + \delta\right],$$

$$\Omega_1 = 0, \quad \Omega_2 = -i\eta_a \, \text{sgn}\left(\frac{\eta_a + \eta_b}{\eta_a - \eta_b}\right)\text{sech}\left[\eta_a t - \frac{\eta_a^2 + s}{c\,\eta_a}x + \delta\right],$$

where sgn denotes the sign function and the phase shift parameter δ is

$$\delta = \ln\left|\frac{\eta_a + \eta_b}{\eta_a - \eta_b}\right|. \tag{68}$$
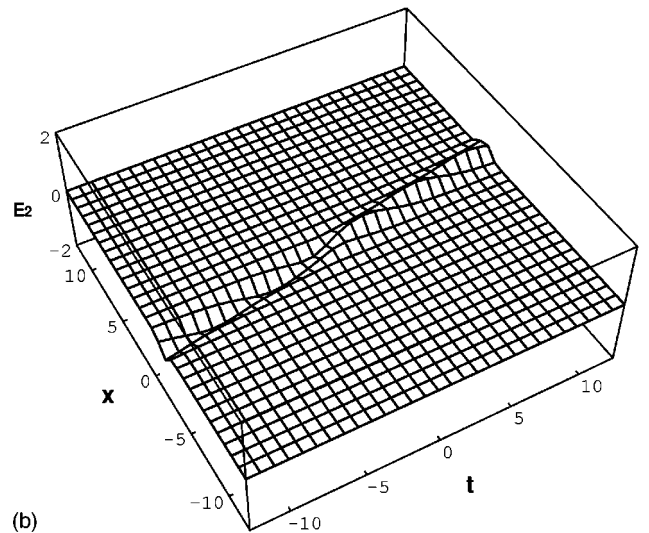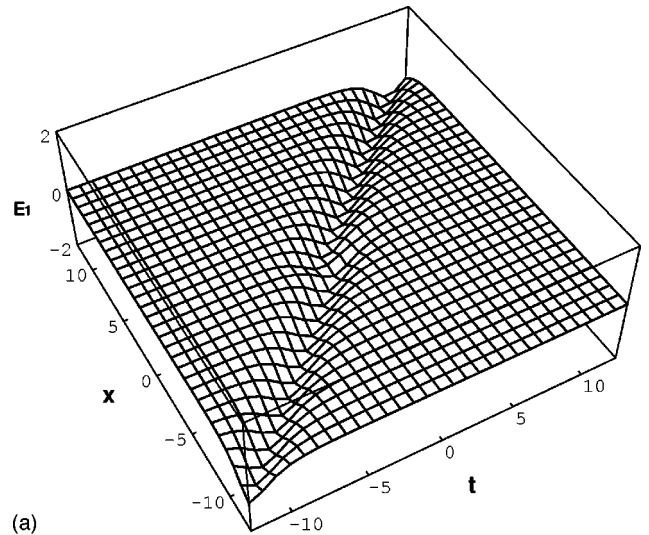

(a)

(b)

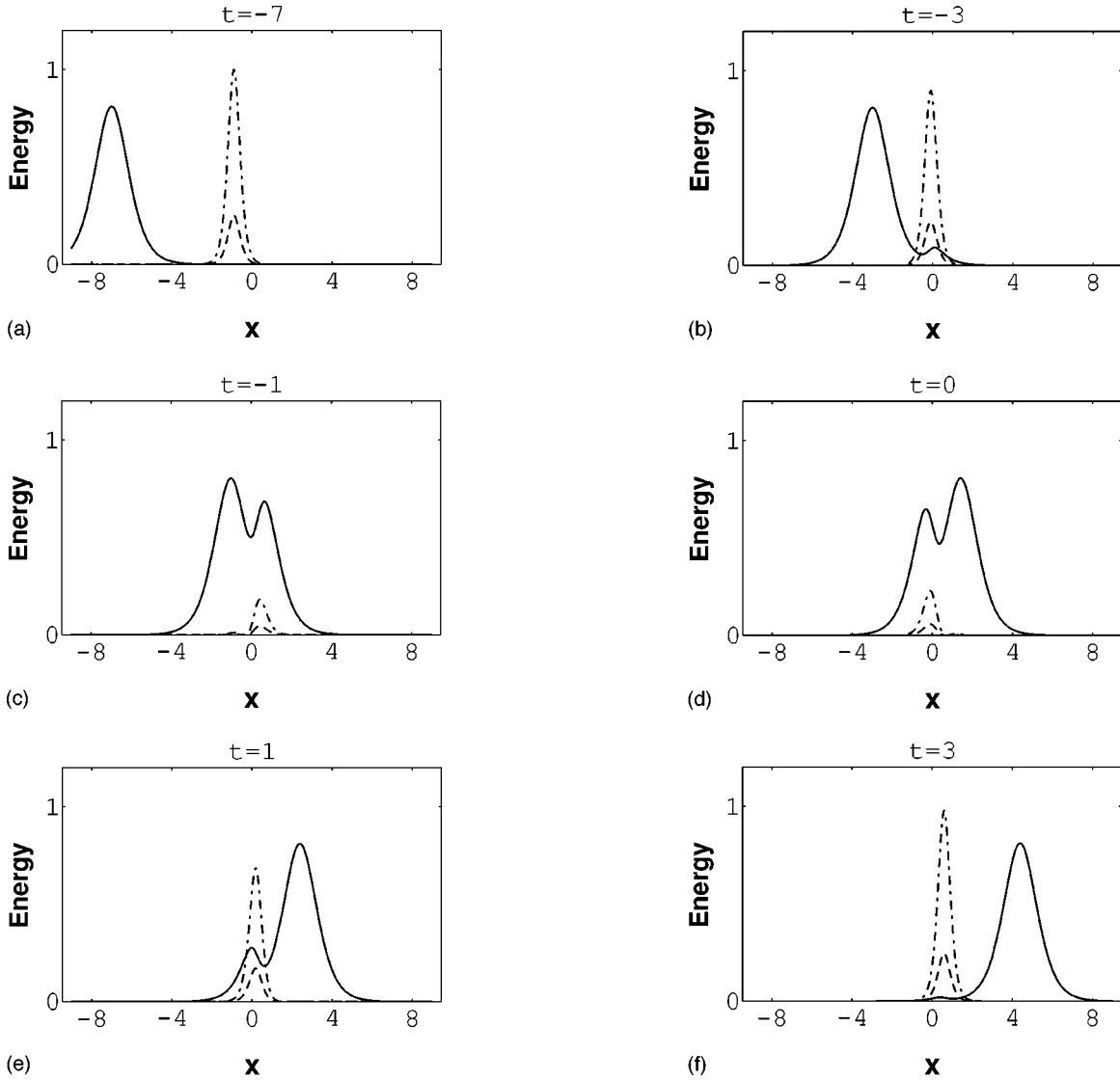FIG. 4. Rabi frequencies $E_1 = -i\Omega_1$ (a) and $E_2 = -i\Omega_2$ (b) of the soliton.

FIG. 5. Energy transfer process between the binary solitons. The energy of E_1 field ($|E_1|^2$) is shown by thick lines, while those of E_2 and matter ($|E_2|^2$ and $|c|^2$) are shown by dashed and dot-dashed lines. Time is at $t = -7, -3, -1, 0, 1, 3$.

The appearance of the signum function is a feature of soliton scatterings in a three-level Λ system that does not arise in the two-level atom case. A careful analysis of the above asymptotic solutions shows that the broader pulse changes its sign after the collision while the sharper pulse remains the same. Also, the phase shift always arises in such a way that the slow soliton receives time retardation while the fast one receives time advance. This implies that the fast soliton moving with light velocity moves faster than light through the scattering process. However, this does not violate causality. In fact, the fast soliton has infinitely stretched tails that trigger the slow soliton to transfer its energy to the fast one.

Figure 4 shows the scattering of the binary solitons with parameters $\eta_a = 0.5$, $\eta_b = 0.9$, and $s = 1$. In order to see that causality is not violated, we consider the energy conservation law given by

$$\frac{\partial}{\partial t}\rho = -\frac{\partial}{\partial x}j, \qquad (69)$$

where

$$\rho = |\Omega_1|^2 + |\Omega_2|^2 + |c_1|^2 + |c_2|^2 = |\Omega_1|^2 + |\Omega_2|^2 + 1 - |c_3|^2,$$

$$(70)$$

$$j = c(|\Omega_1|^2 + |\Omega_2|^2).$$

Figure 5 shows the energy transfer process between the two binary solitons. When the right end tail of the fast soliton I_F reaches the slow soliton I_S, the energy is transferred from I_S to I_F and I_F receives phase advance. After the center of I_F reaches I_S, the energy of I_F is absorbed into the I_S, which results in a retarded phase shift of I_S. Thus, no energy is really transferred faster than light. The scattering process is indeed a nicely balanced exchange process where the scattered solitons maintain their shapes invariant except for the change of sign. The change of polarizations of simultons in the scattering process has been known earlier [16] and the scattering of two single-frequency 2π pulses with different resonance frequencies is also considered in [14]. However, the binary, sign changing behavior of solitons is an aspect that did not receive attention in earlier works, and hopefully will find some applications in the future.

VI. DISCUSSION

In this paper, we have introduced a formalism for the three-level coupled Schödinger-Maxwell equation based on a matrix potential variable, and through the Bäcklund transformation we obtained various solutions that generalize previously known solutions. We found MSP solutions, more general than the steady-state MSP found by Hioe and Grobe, which account for the generic breakup behavior of a MSP into another MSP and a light velocity soliton. Two-soliton solutions and non-Abelian breathers are obtained through the nonlinear superposition rule and scattering of solitons are analyzed in detail. In this formalism, we have demonstrated the SU(2)×U(1) group symmetry of the three-level system with equal oscillator strengths and using that we have found a general expression of solutions which mixes pulses with different velocities. This group symmetry also accounts for the variety of the steady-state MSP solution by Hioe and Grobe which linearly superpose different Jacobi elliptic functions. Another feature of our formalism is the introduction of a self-detuning parameter θ to the solutions. For each 2π pulse, these θ values are conserved in time and survive from the scattering process. That is, it is a conserved charge that can be used to label each pulse in addition to the area of pulses. We also emphasize that our formalism is not restricted to the three-level case but also can be extended to other multilevel cases [8] with the same analytic power.

Finally, we comment on matched pulse propagation through absorbing media. In this case, the time rate equation for the probability amplitude c_3 in Eq. (1) is replaced by $\partial c_3/\partial t \rightarrow \partial c_3/\partial t + \gamma c_3$, which implies the decaying of the excited state $|3\rangle$ at a rate γ to states other than $|1\rangle$ and $|2\rangle$. This obviously breaks the probability conservation law; $|c_1|^2+|c_2|^2+|c_3|^2=1$ so that our matrix potential formalism does not apply to this case. Nevertheless, it is important to note that the SU(2)×U(1) group symmetry of the three-level system with equal oscillator strengths persists even in the case of an absorbing medium. This shows that recent analytic and numerical studies on matched pulse propagation through absorbing media [17–21] can be extended to more general initial conditions. For instance, most studies assume the standard initial condition of the population being in the ground state. The SU(2)×U(1) group symmetry as indicated in Eq. (14) maps the standard initial condition to a coherently prepared one which linearly superposes $|1\rangle$ and $|2\rangle$. This also maps two Rabi frequencies Ω_1 and Ω_2 via the linear transform as in Eq. (14) so that one obtains a more general description of pulse propagation. Thus, for example, when a matched pulse of equal amplitude is found, the above symmetry generates a set of new solutions with unequal amplitudes. This agrees with results in the recent numerical study [21], which showed the existence of stable, shape invariant matched pulses of unequal amplitudes depending on the coherent preparation of initial conditions.

[1] O. Kocharovskaya and Ya. I. Khanin, JETP Lett. **48**, 581 (1988); S. E. Harris, Phys. Rev. Lett. **62**, 1033 (1989); M. O. Scully, S.-Y. Zhu, and A. Gavrielides, *ibid.* **62**, 2813 (1989).

[2] A. S. Zibriv *et al.*, Phys. Rev. Lett. **75**, 1499 (1995); G. G. Padmadandu *et al.*, *ibid.* **76**, 2053 (1996).

[3] K. J. Boller, A. Imamoglu, and S. E. Harris, Phys. Rev. Lett. **66**, 2593 (1991).

[4] For a review of self-induced transparency, see, A. I. Maimistov, A. M. Basharov, S. O. Elyutin, and Yu. M. Sklyarov, Phys. Rep. **191**, 1 (1990).

[5] L. A. Bol'shov and V. V. Likhanskii, Sov. J. Quantum Electron. **15**, 889 (1985).

[6] F. T. Hioe and R. Grobe, Phys. Rev. Lett. **73**, 2559 (1994).

[7] Q.-H. Park and H. J. Shin, J. Korean Phys. Soc. **30**, 336 (1997).

[8] Q.-H. Park and H. J. Shin, preceding paper, Phys. Rev. A **57**, 4621 (1998).

[9] J. R. Kuklinski, U. Gaubatz, F. T. Hioe, and K. Bergmann, Phys. Rev. A **40**, 6741 (1989); C. E. Carroll and F. T. Hioe, Phys. Rev. Lett. **68**, 3523 (1992).

[10] M. J. Konopnicki and J. H. Eberly, Phys. Rev. A **24**, 2567 (1981); M. J. Konopnicki, P. D. Drummond, and J. H. Eberly, Opt. Commun. **36**, 313 (1981).

[11] R. Grobe, F. T. Hioe, and J. H. Eberly, Phys. Rev. Lett. **73**, 3183 (1994).

[12] Q.-H. Park and H. J. Shin (unpublished).

[13] Q.-H. Park and H. J. Shin, Nucl. Phys. B **458**, 327 (1996).

[14] L. A. Bol'shov, N. N. Elkin, V. V. Likhanskii, and M. I. Persiantsev, Zh. Eksp. Teor. Fiz. **88**, 47 (1985) [Sov. Phys. JETP **61**, 27 (1985)].

[15] E. Infeld, Phys. Rev. Lett. **75**, 1419 (1995).

[16] A. I. Maimistov, Sov. J. Quantum Electron. **14**, 385 (1984).

[17] S. E. Harris, Phys. Rev. Lett. **72**, 52 (1994).

[18] J. H. Eberly, M. L. Pons, and H. R. Haq, Phys. Rev. Lett. **72**, 56 (1994).

[19] S. E. Harris and Z. Luo, Phys. Rev. A **52**, R928 (1995).

[20] G. Vemuri, K. V. Vasavada, G. S. Agarwal, and Q. Zhang, Phys. Rev. A **54**, 3394 (1996).

[21] G. Vemuri and K. W. Vasavada, Opt. Commun. **129**, 379 (1996).