Three-dimensional quantization of the electromagnetic field in dispersive and absorbing inhomogeneous dielectrics

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A quantization scheme for the phenomenological Maxwell theory of the full electromagnetic field in an inhomogeneous three-dimensional, dispersive, and absorbing dielectric medium is developed. The classical Maxwell equations with spatially varying and Kramers-Kronig consistent permittivity are regarded as operator-valued field equations, introducing additional current- and charge-density operator fields in order to take into account the noise associated with the dissipation in the medium. It is shown that the equal-time commutation relations between the fundamental electromagnetic fields $\hat{\mathbf{E}}$ and $\hat{\mathbf{B}}$ and the potentials $\hat{\mathbf{A}}$ and $\hat{\varphi}$ in the Coulomb gauge can be expressed in terms of the Green tensor of the classical problem. From the Green tensors for bulk material and an inhomogeneous medium consisting of two bulk dielectrics with a common planar interface it is explicitly proven that the well-known equal-time commutation relations of QED are preserved. [S1050-2947(98)01905-2]

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I. INTRODUCTION

It is well known that the quantum statistical properties of electromagnetic fields including their interactions with atomic systems can be strongly influenced by the presence of dielectric bodies. Typical examples are the Casimir effect [1,2], the modification of the spontaneous emission rate of excited atoms [3–5] in the presence of dielectric media, and the degradation or improvement of nonclassical properties of light propagating through optical devices, such as cavities, beam splitters, wave guides, etc., which typically can be regarded as dielectric bodies [6–8]. Therefore, it has been of considerable interest to formulate QED on a dielectric-matter background. Various quantization schemes have been proposed for dispersionless [9–16], dispersive [17–23], and nonlinear [18–22,24–26] dielectrics. However, most of these quantization schemes run into difficulties when an absorbing medium is attempted to be included in the concept, which is crucial for studying propagation effects and keeping the theory consistent with the causality principle.

The problem has been considered by a number of authors [27–38]. In [29], a fully canonical quantization scheme for the macroscopic electromagnetic field in a linear harmonic-oscillator bulk material is developed that is based on the Hopfield model of a dielectric [39]. The electromagnetic field is coupled to a harmonic-oscillator polarization field that interacts with a continuum of harmonic-oscillator reservoir fields. The resulting Hamiltonian, which is a bilinear form of bosonic fields, is diagonalized in two steps—first the polarization-reservoir part and after that the total Hamiltonian. The scheme is much more involved when it allows the electromagnetic field to be in an inhomogeneous medium, as is the case in practice, and much effort must be made to perform the diagonalization even for simple dielectric-body configurations [37].

Another approach to the problem of including losses in the quantization scheme is the method of Green function expansion [40,41], which can be regarded as a natural extension of the familiar method of mode expansion (which only applies to strictly nonabsorbing media) to arbitrary Kramers-Kronig consistent media. The approach, which resembles, in a sense, the method of (operator) Langevin forces [30,31,42,43], directly starts with the Maxwell equations for the macroscopic electromagnetic field, including the dielectric displacement vector and a (phenomenologically) given permittivity. The quantization of the radiation field is based on the classical Green function representation of the vector potential, identifying the external sources therein with the noise sources that are necessarily associated with the losses in the medium and replacing the c-number sources with operator-valued ones such that the equal-time basic commutation relations of QED are preserved. The advantage of the method is that the calculation of the Green function is—similar to the determination of the mode structure in the standard scheme—a purely classical problem. The Green function is essentially determined by the permittivity of the medium, which is a space-dependent, complex function of frequency. The configuration of the dielectric bodies is described by the dependence on space of the permittivity, and the effects of dispersion and absorption, respectively, are described by its real and imaginary parts. It is worth noting that they are not independent of each other, but they must satisfy Kramers-Kronig relations, because of causality (see, e.g., [44,45]). The Green function method has been proved correct for radiation in 3D bulk material [40,46] and in 1D multilayer structures [40,41,46], and applications to various problems have been studied (e.g., ground-state field fluctuations [41,47], photonic wave packets at dielectric barriers [48], and nonclassical-light propagation in dispersive and absorbing dielectrics [49,50]).

The aim of this paper is to extend the Green function

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method to the quantization of the electromagnetic field in a dispersive and absorbing three-dimensional (3D) inhomogeneous medium. For this purpose, both the transverse and the longitudinal parts of the electromagnetic field vectors must be included in the analysis in a unified manner. Relating the overall noise current to a bosonic basic field, the electromagnetic field operators can be expressed—through the dyadic Green function of the classical problem—in terms of this field, and all the fundamental electromagnetic-field commutation relations can be expressed in terms of the Green function. Using earlier results of the calculation of the classical 3D Green function for multilayer systems [51–54], we explicitly consider an inhomogeneous medium that is composed of two bulk dielectrics with a common planar interface.

The paper is organized as follows. In Sec. II the quantization scheme is developed. The Green function expansions of both the electromagnetic field vectors and the vector and scalar potentials in Coulomb gauge are given, and the fundamental commutation relations of QED are studied. In Sec. III the theory is applied to a bulk dielectric, and it is shown that the equal-time basic commutation relations of QED are preserved. In particular, earlier results for the transverse electromagnetic field are recovered. Quantization of the electromagnetic field in an inhomogeneous medium that consists of two bulk dielectrics with a common interface is studied in detail in Sec. IV. Finally, a summary and conclusions are given in Sec. V. Lengthy mathematical derivations are outlined in the Appendix.

II. QUANTIZATION SCHEME

A. Classical Maxwell equations

Let us start by writing the phenomenological Maxwell equations in the frequency domain as

$$\nabla\cdot\underline{\mathbf{B}}(\mathbf{r},\omega)=0, \tag{1}$$

$$\nabla\cdot\underline{\mathbf{D}}(\mathbf{r},\omega)=0, \tag{2}$$

$$\nabla\times\underline{\mathbf{E}}(\mathbf{r},\omega)=i\omega\underline{\mathbf{B}}(\mathbf{r},\omega), \tag{3}$$

$$\nabla\times\underline{\mathbf{H}}(\mathbf{r},\omega)=-i\omega\underline{\mathbf{D}}(\mathbf{r},\omega), \tag{4}$$

where we have assumed that no ''visible'' charges and currents are embedded in the background medium. The Maxwell equations must be supplemented with constitutive relations, which for linear dielectric media are usually given by

$$\underline{\mathbf{D}}(\mathbf{r},\omega)=\epsilon_0\epsilon(\mathbf{r},\omega)\underline{\mathbf{E}}(\mathbf{r},\omega), \tag{5}$$

$$\underline{\mathbf{B}}(\mathbf{r},\omega)=\mu_0\underline{\mathbf{H}}(\mathbf{r},\omega). \tag{6}$$

Here, the (relative) permittivity, which is a complex function of frequency,

$$\epsilon(\mathbf{r},\omega)=\epsilon_{\mathrm{R}}(\mathbf{r},\omega)+i\epsilon_{\mathrm{I}}(\mathbf{r},\omega), \tag{7}$$

is also allowed to be varying with space in order to model inhomogeneous media. For causality reasons, the real and imaginary parts of the permittivity, which are responsible for dispersion and absorption, respectively, are uniquely related to each other through Kramers-Kronig relations, i.e., dispersion and absorption are intimately linked. It can be shown that $\epsilon(\mathbf{r},\Omega)$ as a function of the complex frequency Ω is analytic and has no zeros in the upper complex half-plane, and $\epsilon(\mathbf{r},\Omega)\to 1$ if $|\Omega|\to\infty$ [44,45]. The fields in the time domain are obtained by Fourier transforming the fields in the frequency domain, e.g.,

$$\mathbf{E}(\mathbf{r},t)=\int_0^\infty d\omega e^{-i\omega t}\underline{\mathbf{E}}(\mathbf{r},\omega)+\text{c.c.}, \tag{8}$$

and $\mathbf{B}(\mathbf{r},t)$, $\mathbf{D}(\mathbf{r},t)$, and $\mathbf{H}(\mathbf{r},t)$ accordingly.

The Maxwell equations (1)–(4) together with the constitutive relations (5) and (6) cannot be transferred to quantum theory by simply regarding the electromagnetic field vectors as operator-valued quantities, otherwise the operators would be damped to zero. This is not surprising, because equations of the form given here violate, in general, the dissipation-fluctuation theorem, which states that damping is always connected with additional noise. In other words, even the classical equations are equations for the field averages but not equations for the ''naked'' fields, and therefore they cannot be used to study the statistics of fluctuating fields, such as thermal fields. Hence, transferring the above given equations to quantum theory can only yield equations for the (now quantum-mechanical) expectation values of the fields (which of course can be damped to zero). The noise that is unavoidably associated with absorption can be described by introducing a corresponding source term in the Maxwell equations [42,55], which can be thought of as arising from a noise polarization in the constitutive relation between the dielectric displacement vector and the vector of the electric field strength (see, e.g., [56]),

$$\underline{\mathbf{D}}(\mathbf{r},\omega)=\epsilon_0\epsilon(\mathbf{r},\omega)\underline{\mathbf{E}}(\mathbf{r},\omega)+\underline{\mathbf{P}}(\mathbf{r},\omega). \tag{9}$$

Before specifying the noise source, let us first turn to quantum theory.

B. Quantum Maxwell equations

The Maxwell equations (1)–(4) together with the constitutive relations (6) and (9) can be transferred to quantum theory, regarding the electromagnetic field vectors and the noise polarization field vector as operators:

$$\nabla\cdot\underline{\hat{\mathbf{B}}}(\mathbf{r},\omega)=0, \tag{10}$$

$$\nabla\cdot[\epsilon_0\epsilon(\mathbf{r},\omega)\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)]=\underline{\hat{\rho}}(\mathbf{r},\omega), \tag{11}$$

$$\nabla\times\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)=i\omega\underline{\hat{\mathbf{B}}}(\mathbf{r},\omega), \tag{12}$$

$$\nabla\times\underline{\hat{\mathbf{B}}}(\mathbf{r},\omega)=-i\omega\mu_0\epsilon_0\epsilon(\mathbf{r},\omega)\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)+\mu_0\underline{\hat{\mathbf{j}}}(\mathbf{r},\omega). \tag{13}$$

Here, the operator noise charge density $\underline{\hat{\rho}}$ and the operator noise current density $\underline{\hat{\mathbf{j}}}$ are introduced, which are related to the operator noise polarization $\underline{\hat{\mathbf{P}}}$ as

$$\underline{\hat{\rho}}(\mathbf{r},\omega)=-\nabla\cdot\underline{\hat{\mathbf{P}}}(\mathbf{r},\omega), \tag{14}$$

$$\underline{\hat{\mathbf{j}}}(\mathbf{r},\omega)=-i\omega\underline{\hat{\mathbf{P}}}(\mathbf{r},\omega). \tag{15}$$

It follows from Eqs. (14) and (15) that $\underline{\hat{\rho}}$ and $\underline{\hat{\mathbf{j}}}$ fulfill the equation of continuity,

$$\nabla\cdot\underline{\hat{\mathbf{j}}}(\mathbf{r},\omega)=i\omega\underline{\hat{\rho}}(\mathbf{r},\omega). \tag{16}$$

The electric-field strength operator $\hat{\mathbf{E}}(\mathbf{r})$ (in the Schrödinger picture) is defined in terms of the Fourier transform $\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)$ as

$$\hat{\mathbf{E}}(\mathbf{r})=\int_0^\infty d\omega\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)+\text{H.c.}, \tag{17}$$

and similar relations hold for $\hat{\mathbf{B}}(\mathbf{r})$, $\hat{\mathbf{D}}(\mathbf{r})$, and $\hat{\mathbf{H}}(\mathbf{r})$.

As already mentioned, the source terms $\underline{\hat{\rho}}$ and $\underline{\hat{\mathbf{j}}}$ are closely related to the noise associated with the losses in the medium, which themselves are described by the imaginary part of the permittivity. Following [40,41], we relate $\underline{\hat{\mathbf{j}}}$ to a bosonic vector field $\hat{\mathbf{f}}$ as

$$\underline{\hat{\mathbf{j}}}(\mathbf{r},\omega)=\frac{\omega}{\mu_0c^2}\sqrt{\frac{\hbar}{\pi\epsilon_0}\epsilon_{\mathrm{I}}(\mathbf{r},\omega)}\hat{\mathbf{f}}(\mathbf{r},\omega), \tag{18}$$

$$[\hat{f}_i(\mathbf{r},\omega),\hat{f}_j^\dagger(\mathbf{r}',\omega')]=\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')\delta(\omega-\omega'), \tag{19}$$

$$[\hat{f}_i(\mathbf{r},\omega),\hat{f}_j(\mathbf{r}',\omega')]=0=[\hat{f}_i^\dagger(\mathbf{r},\omega),\hat{f}_j^\dagger(\mathbf{r}',\omega')]. \tag{20}$$

Obviously, in the Heisenberg picture the basic operator field evolves as $\hat{\mathbf{f}}(\mathbf{r},\omega,t) = \hat{\mathbf{f}}(\mathbf{r},\omega,t')\exp[-i\omega(t-t')]$, which is governed by the Hamiltonian

$$\hat{H}=\int d^3\mathbf{r}\int_0^\infty d\omega\hbar\omega\hat{\mathbf{f}}^\dagger(\mathbf{r},\omega)\cdot\hat{\mathbf{f}}(\mathbf{r},\omega). \tag{21}$$

The system of equations (10)–(13) together with Eqs. (17)–(21) is complete, i.e., further equations are not required. In particular, all the electromagnetic-field commutation relations are uniquely determined from the equations given. It should be pointed out that—in contrast to [40]—the current density $\underline{\hat{\mathbf{j}}}$ is not transverse, because the whole electromagnetic field is considered. Hence, the vector field $\hat{\mathbf{f}}$ introduced here is not transverse as well, and the spatial δ function in Eq. (19) is an ordinary δ function instead of a transverse one.

C. Integral representation of $\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)$ and $\underline{\hat{\mathbf{B}}}(\mathbf{r},\omega)$

Equations (12) and (13) imply that the electric field $\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)$ obeys the partial differential equation

$$\nabla\times\nabla\times\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)-\frac{\omega^2}{c^2}\epsilon(\mathbf{r},\omega)\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)=i\omega\mu_0\underline{\hat{\mathbf{j}}}(\mathbf{r},\omega), \tag{22}$$

whose solution can be represented as

$$\underline{\hat{E}}_i(\mathbf{r},\omega)=i\omega\mu_0\int d^3\mathbf{s}G_{ij}(\mathbf{r},\mathbf{s},\omega)\underline{\hat{j}}_j(\mathbf{s},\omega), \tag{23}$$

where $\underline{\hat{\mathbf{j}}}$ is given by Eq. (18), and $G_{ij}(\mathbf{r},\mathbf{s},\omega)$ is the dyadic Green function (Green tensor) of the classical problem. It satisfies the equation

$$\left[\partial_i^r\partial_m^r-\delta_{im}\left(\Delta^r+\frac{\omega^2}{c^2}\epsilon(\mathbf{r},\omega)\right)\right]G_{mj}(\mathbf{r},\mathbf{s},\omega)=\delta_{ij}\delta(\mathbf{r}-\mathbf{s}), \tag{24}$$

together with appropriate boundary conditions. In particular, it must vanish at infinity. The notation ∂_i^r means $\partial/\partial x_i$, and $\Delta^r=\partial_i^r\partial_i^r$ (here and in the following the summation convention is used). When the electric field $\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)$ is known, then the magnetic field $\underline{\hat{\mathbf{B}}}(\mathbf{r},\omega)$ can be obtained as, on recalling Eq. (12),

$$\underline{\hat{\mathbf{B}}}(\mathbf{r},\omega)=(i\omega)^{-1}\nabla\times\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega). \tag{25}$$

In this way, the electromagnetic field operators can be expressed in terms of the bosonic field $\hat{\mathbf{f}}(\mathrm{r},\omega)$.

D. Commutation relations

Let us consider the (equal-time) commutation relations between the fundamental fields $\hat{\mathbf{E}}(\mathbf{r})$ and $\hat{\mathbf{B}}(\mathbf{r})$. Recalling the definitions of $\hat{\mathbf{E}}(\mathbf{r})$ and $\hat{\mathbf{B}}(\mathbf{r})$ [see Eq. (17)], using Eqs. (23) and (25) together with Eq. (18), and applying the commutation relations (19) and (20), we find that

$$[\hat{E}_i(\mathbf{r}),\hat{B}_k(\mathbf{r}')]=\frac{i\hbar}{\pi\epsilon_0}\epsilon_{kmj}\partial_m^{r'}\int_0^\infty d\omega\,\frac{\omega^3}{c^4}\int d^3\mathbf{s}\epsilon_{\mathrm{I}}(\mathbf{s},\omega)\times G_{il}(\mathbf{r},\mathbf{s},\omega)G_{jl}^*(\mathbf{r}',\mathbf{s},\omega)-\text{c.c.}, \tag{26}$$

where ϵ_{kmj} is the Levi-Civita tensor. In order to simplify Eq. (26), we first note that the relation

$$\frac{\omega^2}{c^2}\int d^3\mathbf{s}\epsilon_{\mathrm{I}}(\mathbf{s},\omega)G_{il}(\mathbf{r},\mathbf{s},\omega)G_{jl}^*(\mathbf{r}',\mathbf{s},\omega)=\frac{1}{2i}[G_{ij}(\mathbf{r},\mathbf{r}',\omega)-G_{ij}^*(\mathbf{r},\mathbf{r}',\omega)] \tag{27}$$

is valid (see Appendix A). Further, from Eq. (24) and the relation $\epsilon^*(\mathbf{r},\omega) = \epsilon(\mathbf{r},-\omega)$ it follows that

$$G_{ij}^*(\mathbf{r},\mathbf{r}',\omega)=G_{ij}(\mathbf{r},\mathbf{r}',-\omega). \tag{28}$$

Combining Eqs. (26)–(28), we derive

$$[\hat{E}_i(\mathbf{r}),\hat{B}_k(\mathbf{r}')]=\frac{\hbar}{\pi\epsilon_0}\epsilon_{kmj}\partial_m^{r'}\int_{-\infty}^\infty d\omega\frac{\omega}{c^2}G_{ij}(\mathbf{r},\mathbf{r}',\omega). \tag{29}$$

Similarly, we find that

$$[\hat{E}_i(\mathbf{r}),\hat{E}_k(\mathbf{r}')]=0=[\hat{B}_i(\mathbf{r}),\hat{B}_k(\mathbf{r}')], \tag{30}$$

which is in full agreement with QED (see, e.g., [57]). Equation (29) reveals that the commutator between the electric and magnetic fields can be expressed in terms of a single frequency integral of the Green function multiplied by the frequency. In order to calculate this integral, knowledge of the Green function is required. Note that a single pole at $\omega=0$ has to be treated as a principal value.

E. Vector potential and scalar potential

1. Potential equations and integral representations

It is often necessary to use electromagnetic potentials. In the frequency domain, the vector and scalar potentials $\underline{\hat{\mathbf{A}}}$ and $\underline{\hat{\varphi}}$, respectively, are related to the fields as

$$\underline{\hat{\mathbf{B}}}(\mathbf{r},\omega)=\nabla\times\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega), \tag{31}$$

$$\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)=i\omega\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega)-\nabla\underline{\hat{\varphi}}(\mathbf{r},\omega). \tag{32}$$

Substituting in Eqs. (11) and (13) for the fields the potentials according to Eqs. (31) and (32) then yields

$$\nabla\cdot[\epsilon(\mathbf{r},\omega)\nabla\underline{\hat{\varphi}}(\mathbf{r},\omega)]=-\frac{\hat{\underline{\rho}}}{\epsilon_0}+i\omega\nabla\cdot[\epsilon(\mathbf{r},\omega)\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega)], \tag{33}$$

$$\nabla\times\nabla\times\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega)-\frac{\omega^2}{c^2}\epsilon(\mathbf{r},\omega)\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega)$$
$$=\mu_0\underline{\hat{\mathbf{j}}}(\mathbf{r},\omega)+\frac{i\omega}{c^2}\epsilon(\mathbf{r},\omega)\nabla\underline{\hat{\varphi}}(\mathbf{r},\omega). \tag{34}$$

In Coulomb gauge,

$$\nabla\cdot\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega)=0, \tag{35}$$

Eq. (32) corresponds—in the sense of the Helmholtz theorem—to a unique decomposition of the electric field $\underline{\hat{\mathbf{E}}}$ into a transverse part $i\omega\underline{\hat{\mathbf{A}}}$ and a longitudinal part $-\nabla\underline{\hat{\varphi}}$. Hence we may write

$$\underline{\hat{A}}_i(\mathbf{r},\omega)=(i\omega)^{-1}\int d^3\mathbf{s}\,\delta^{\perp}_{ij}(\mathbf{r}-\mathbf{s})\underline{\hat{E}}_j(\mathbf{s},\omega), \tag{36}$$

$$\partial^r_i\underline{\hat{\varphi}}(\mathbf{r},\omega)=-\int d^3\mathbf{s}\,\delta^{\parallel}_{ij}(\mathbf{r}-\mathbf{s})\underline{\hat{E}}_j(\mathbf{s},\omega), \tag{37}$$

where $\delta^{\perp}(\mathbf{r})$ and $\delta^{\parallel}(\mathbf{r})$, respectively, are the transverse and longitudinal δ functions (see, e.g., [2]),

$$\delta^{\perp}_{ij}(\mathbf{r})=\delta_{ij}\delta(\mathbf{r})+\partial^r_i\partial^r_j(4\pi r)^{-1}, \tag{38}$$

$$\delta^{\parallel}_{ij}(\mathbf{r})=-\partial^r_i\partial^r_j(4\pi r)^{-1}. \tag{39}$$

We insert Eq. (23) into Eq. (36) and obtain the following integral representation of the vector potential:

$$\underline{\hat{A}}_i(\mathbf{r},\omega)=\mu_0\int d^3\mathbf{s}\,G^{\perp}_{im}(\mathbf{r},\mathbf{s},\omega)\underline{\hat{j}}_m(\mathbf{s},\omega), \tag{40}$$

where

$$G^{\perp}_{im}(\mathbf{r},\mathbf{s},\omega)=\int d^3\mathbf{s}'\,\delta^{\perp}_{ij}(\mathbf{r}-\mathbf{s}')G_{jm}(\mathbf{s}',\mathbf{s},\omega) \tag{41}$$

is the (from the left) one-sided transverse Green tensor. Next we substitute in Eq. (37) for the longitudinal δ function the expression (39) and find that

$$\underline{\hat{\varphi}}(\mathbf{r},\omega)=\partial^r_j\int d^3\mathbf{s}\frac{\underline{\hat{E}}_j(\mathbf{s},\omega)}{4\pi|\mathbf{r}-\mathbf{s}|}. \tag{42}$$

The integral representation of the scalar potential can then be found, substituting in Eq. (42) for the electric field the integral representation (23).

2. Commutation relations

It is well known that $\hat{\mathbf{A}}(\mathbf{r})$ and $\epsilon_0\dot{\hat{\mathbf{A}}}(\mathbf{r})$ are canonically conjugated field variables. In order to calculate the commutation relation between them and the scalar potential $\hat{\varphi}(\mathbf{r})$, we first note that $\hat{\mathbf{A}}(\mathbf{r})$, $\dot{\hat{\mathbf{A}}}(\mathbf{r})$, and $\hat{\varphi}(\mathbf{r})$ are given by integrals of the type (17), but with $\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega)$, $-i\omega\underline{\hat{\mathbf{A}}}(\mathbf{r},\omega)$, and $\underline{\hat{\varphi}}(\mathbf{r},\omega)$, respectively, in place of $\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)$. Using Eq. (40) and following the lines outlined for calculating the commutation relations (29) and (30), we obtain

$$[\hat{A}_i(\mathbf{r}),\dot{\hat{A}}_j(\mathbf{r}')]=\frac{\hbar}{\pi\epsilon_0}\int_{-\infty}^{\infty}d\omega\frac{\omega}{c^2}G^{\perp\perp}_{ij}(\mathbf{r},\mathbf{r}',\omega) \tag{43}$$

and

$$[\hat{A}_i(\mathbf{r}),\hat{A}_j(\mathbf{r}')]=0=[\dot{\hat{A}}_i(\mathbf{r}),\dot{\hat{A}}_j(\mathbf{r}')], \tag{44}$$

where

$$G^{\perp\perp}_{ij}(\mathbf{r},\mathbf{r}',\omega)=\int d^3\mathbf{s}\int d^3\mathbf{s}'\,\delta^{\perp}_{im}(\mathbf{r}-\mathbf{s})\times G_{mn}(\mathbf{s},\mathbf{s}',\omega)\,\delta^{\perp}_{nj}(\mathbf{s}'-\mathbf{r}') \tag{45}$$

is the two-sided transverse Green tensor. Similarly, the commutation relation between the vector potential and the scalar potential can be given by

$$[\hat{\varphi}(\mathbf{r}),\hat{A}_j(\mathbf{r}')]=\frac{\hbar}{\pi\epsilon_0}\partial^r_m\int d^3\mathbf{s}\int_{-\infty}^{\infty}d\omega\frac{\omega}{c^2}\frac{G^{\ \perp}_{mj}(\mathbf{s},\mathbf{r}',\omega)}{4\pi|\mathbf{r}-\mathbf{s}|}, \tag{46}$$

where

$$G^{\ \perp}_{mj}(\mathbf{s},\mathbf{r}',\omega)=\int d^3\mathbf{s}'\,G_{mn}(\mathbf{s},\mathbf{s}',\omega)\,\delta^{\perp}_{nj}(\mathbf{s}'-\mathbf{r}') \tag{47}$$

is the (from the right) one-sided transverse Green tensor, and finally

$$[\hat{\varphi}(\mathbf{r}),\hat{\varphi}(\mathbf{r}')]=0=[\hat{\varphi}(\mathbf{r}),\hat{A}_i(\mathbf{r}')]. \tag{48}$$

In order to further calculate the commutators (43) and (46), the Green function multiplied by the frequency must be integrated over frequency, which is quite similar to the commutation relation (29).

III. HOMOGENEOUS DIELECTRICS

Let us first consider the electromagnetic field in an absorbing bulk material such that the permittivity can be assumed to be independent of space: $\epsilon(\mathbf{r},\omega)=\epsilon(\omega)=\epsilon_R(\omega)+i\epsilon_I(\omega)$ for all $\mathbf{r}$. In this case, the solution of Eq. (24) that satisfies the boundary conditions at infinity is [54]

$$G_{ij}(\mathbf{r},\mathbf{r}',\omega)=[\partial_i^r\partial_j^r+\delta_{ij}q^2(\omega)]q^{-2}(\omega)g(|\mathbf{r}-\mathbf{r}'|,\omega), \tag{49}$$

where the notation $q^2(\omega)=(\omega^2/c^2)\epsilon(\omega)$ is used, and

$$g(|\mathbf{r}-\mathbf{r}'|,\omega)=\frac{e^{iq(\omega)|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}=\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{e^{i\mathbf{k}\cdot(\mathbf{r}-\mathbf{r}')}}{k^2-q^2(\omega)}. \tag{50}$$

Substituting in Eq. (23) for the Green function the function given in Eq. (49) and integrating over frequency yields, together with Eq. (18), the Green function expansion of the operator of the electric field strength of the electromagnetic field in a dispersive and absorbing bulk dielectric. The Green function expansions of the magnetic field and the potentials can be obtained accordingly.

A. Commutation relations

We insert the Green function (49) into Eq. (29) and find that

$$[\hat{E}_i(\mathbf{r}),\hat{B}_k(\mathbf{r}')]=\frac{\hbar}{\pi\epsilon_0}\epsilon_{kmi}\partial_m^{r'}\int_{-\infty}^{\infty}d\omega\frac{\omega}{c^2}g(|\mathbf{r}-\mathbf{r}'|,\omega) \tag{51}$$

[note that $\epsilon_{kmj}\partial_m^{r'}\partial_j^{r'}(\;)=0$, because of the antisymmetry of the Levi-Civita tensor]. We now substitute in Eq. (51) for $g(|\mathbf{r}-\mathbf{r}'|,\omega)$ the Fourier expansion according to Eq. (50), which enables us to calculate the ω integral by means of contour integral techniques. Recalling the properties of $\epsilon(\Omega)$ as a function of the complex frequency Ω, we obtain after some straightforward calculation (cf. [40,41])

$$[\hat{E}_i(\mathbf{r}),\hat{B}_k(\mathbf{r}')]=-\frac{i\hbar}{\epsilon_0}\epsilon_{ikm}\partial_m^r\delta(\mathbf{r}-\mathbf{r}'). \tag{52}$$

From Eqs. (52) and (30) we see that the quantization scheme yields exactly the equal-time electromagnetic-field commutation relations that are well established in QED. Quite similarly, it can be proved that the commutation relations

$$[\hat{A}_i(\mathbf{r}),\dot{\hat{A}}_j(\mathbf{r}')]=\frac{i\hbar}{\epsilon_0}\delta_{ij}^{\perp}(\mathbf{r}-\mathbf{r}') \tag{53}$$

and

$$[\hat{\varphi}(\mathbf{r}),\hat{A}_j(\mathbf{r}')]=0 \tag{54}$$

are also preserved.

B. Relation to earlier work

To make contact with earlier work, we first note that, according to the Helmholtz theorem, the noise current $\underline{\hat{\mathbf{j}}}$ can be decomposed in a unique way into a transverse and a longitudinal part,

$$\underline{\hat{\mathbf{j}}}(\mathbf{r},\omega)=\underline{\hat{\mathbf{j}}}^{\perp}(\mathbf{r},\omega)+\underline{\hat{\mathbf{j}}}^{\parallel}(\mathbf{r},\omega), \tag{55}$$

where $\underline{\hat{\mathbf{j}}}^{\perp(\parallel)}$ can be related to $\hat{\mathbf{f}}^{\perp(\parallel)}$ according to Eq. (18) with $\hat{\mathbf{f}}^{\perp(\parallel)}(\mathbf{r},\omega)$ in place of $\hat{\mathbf{f}}(\mathbf{r},\omega)$, where

$$\hat{f}_i^{\perp(\parallel)}(\mathbf{r},\omega)=\int d^3\mathbf{s}\,\delta_{ij}^{\perp(\parallel)}(\mathbf{r}-\mathbf{s})\hat{f}_j(\mathbf{s},\omega). \tag{56}$$

The commutation relations (19) and (20) obviously imply that

$$[\hat{f}_i^{\perp(\parallel)}(\mathbf{r},\omega),(\hat{f}_j^{\perp(\parallel)}(\mathbf{r}',\omega'))^{\dagger}]=\delta_{ij}^{\perp(\parallel)}(\mathbf{r}-\mathbf{r}')\delta(\omega-\omega'), \tag{57}$$

$$[\hat{f}_i^{\perp(\parallel)}(\mathbf{r},\omega),\hat{f}_j^{\perp(\parallel)}(\mathbf{r}',\omega')]=[\hat{f}_i^{\perp}(\mathbf{r},\omega),(\hat{f}_j^{\parallel}(\mathbf{r}',\omega'))^{\dagger}]=0. \tag{58}$$

Obviously, the motion of the transverse and longitudinal degrees of freedom is governed by their own Hamiltonians, as can be readily seen from the Hamiltonian (21), which can be rewritten as $\hat{H}=\hat{H}^{\perp}+\hat{H}^{\parallel}$, where $\hat{H}^{\perp(\parallel)}$ is given according to Eq. (21) but with $\hat{\mathbf{f}}^{\perp(\parallel)}(\mathbf{r},\omega)$ in place of $\hat{\mathbf{f}}(\mathbf{r},\omega)$.

From Eqs. (40), (41), and (49) it can be seen that, after partial integration, the derivatives in Eq. (49) do not contribute to the vector potential, because of the vanishing divergence of the transverse δ function. Therefore, we may write

$$\underline{\hat{A}}_i(\mathbf{r},\omega)=\mu_0\int d^3\mathbf{r}'g(|\mathbf{r}-\mathbf{r}'|,\omega)\underline{\hat{j}}_i^{\perp}(\mathbf{r}',\omega), \tag{59}$$

which is nothing but the representation of the transverse vector potential given in [40] (if we identify $\mu_0\underline{\hat{\mathbf{j}}}^{\perp}$ with $\hat{\mathbf{j}}_n$ in [40]). Similarly, from Eqs. (23) and (42) and the Green function (49) [together with Eq. (50)] it can be derived that

$$\underline{\hat{\varphi}}(\mathbf{r},\omega)=\frac{1}{4\pi\epsilon_0\epsilon(\omega)}\int d^3\mathbf{s}\frac{\underline{\hat{\rho}}(\mathbf{s},\omega)}{|\mathbf{r}-\mathbf{s}|}, \tag{60}$$

where $\underline{\hat{\rho}}(\mathbf{r},\omega)=(i\omega)^{-1}\nabla\cdot\underline{\hat{\mathbf{j}}}^{\parallel}(\mathbf{r},\omega)$ [cf. Eq. (16)]. Note that from the commutation relations (58) and Eqs. (59) and (60) it is immediately seen that vector and scalar potentials are commuting quantities, i.e., the commutation relation (54) is fulfilled.

IV. DIELECTRIC INTERFACE

A. The Green function

The determination of the dyadic Green function for three-dimensional configurations of dielectric bodies is a very involved problem in general, and only for rather simple configurations has the Green function been calculated explicitly. Such a configuration, which can be thought of as being the basic element of multilayer dielectric structures, is composed of two infinite half-spaces (V_1 and V_2) with a common planar interface such that

$$\epsilon(\mathbf{r},\omega)=\begin{cases}\epsilon_1(\omega) & \text{if } \mathbf{r}\in V_1, \quad \text{i.e., } z<0,\\ \epsilon_2(\omega) & \text{if } \mathbf{r}\in V_2, \quad \text{i.e., } z>0.\end{cases} \tag{61}$$

Following [40,48,54], we write the solution of Eq. (24) in the form

$$G_{ij}(\mathbf{r},\mathbf{r}',\omega) = \begin{cases} G^{\alpha}_{ij}(\mathbf{r},\mathbf{r}',\omega)+R^{\alpha}_{ij}(\mathbf{r},\mathbf{r}',\omega) & \text{if } \mathbf{r},\mathbf{r}'\in V_\alpha,\\ T^{\alpha\alpha'}_{ij}(\mathbf{r},\mathbf{r}',\omega) & \text{if } \mathbf{r}\in V_\alpha, \quad \mathbf{r}'\in V_{\alpha'} \quad (\alpha\neq\alpha')\end{cases} \tag{62}$$

($\alpha,\alpha'=1,2$), where $G^{\alpha}_{ij}(\mathbf{r},\mathbf{r}',\omega)$ is the Green function (49) [together with Eq. (50)] for the bulk material with $\epsilon_\alpha(\omega)$,

$$G^{\alpha}_{ij}(\mathbf{r},\mathbf{r}',\omega)=[\partial^r_i\partial^r_j+\delta_{ij}q^2_\alpha(\omega)]q_\alpha^{-2}(\omega)g^\alpha(|\mathbf{r}-\mathbf{r}'|,\omega), \tag{63}$$

with

$$g^\alpha(|\mathbf{r}-\mathbf{r}'|,\omega)=\frac{e^{iq_\alpha(\omega)|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|} \tag{64}$$

$[q^2_\alpha(\omega)=(\omega^2/c^2)\epsilon_\alpha(\omega)]$. The functions $R^{\alpha}_{ij}(\mathbf{r},\mathbf{r}',\omega)$ and $T^{\alpha\alpha'}_{ij}(\mathbf{r},\mathbf{r}',\omega)$ describe the effects of reflection and transmission at the interface and obey the homogeneous equations

$$\{\partial^r_i\partial^r_m-\delta_{im}[\Delta^r+q^2_\alpha(\omega)]\}R^\alpha_{mj}(\mathbf{r},\mathbf{r}',\omega)=0 \tag{65}$$

($\mathbf{r},\mathbf{r}'\in V_\alpha$) and

$$\{\partial^r_i\partial^r_m-\delta_{im}[\Delta^r+q^2_\alpha(\omega)]\}T^{\alpha\alpha'}_{mj}(\mathbf{r},\mathbf{r}',\omega)=0 \tag{66}$$

($\mathbf{r}\in V_\alpha$, $\mathbf{r}'\in V_{\alpha'}$, $\alpha\neq\alpha'$) together with the boundary conditions such that they vanish at infinity and the tangential components of the $\hat{\mathbf{E}}$ and the $\hat{\mathbf{H}}$ field are continuous at the surface of discontinuity. In order to determine the functions $R^\alpha_{ij}(\mathbf{r},\mathbf{r}',\omega)$ and $T^{\alpha\alpha'}_{ij}(\mathbf{r},\mathbf{r}',\omega)$, it is convenient to take advantage of the translational invariance of the system in the (x,y) plane, which enables us to expand the Green function (and G^α_{ij}, R^α_{ij}, and $T^{\alpha\alpha'}_{ij}$) as follows:

$$G_{ij}(\mathbf{r},\mathbf{r}',\omega)=\int\frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}G_{ij}(\mathbf{k}_\parallel,\omega;z,z')e^{i\mathbf{k}_\parallel\cdot(\mathbf{r}_\parallel-\mathbf{r}'_\parallel)}, \tag{67}$$

where $\mathbf{k}_\parallel=(k_x,k_y,0)$ and $\mathbf{r}_\parallel=(x,y,0)$ are two-dimensional vectors in the (x,y) plane. Using the general formulas for multilayer structures given in [54] (see also [51]), after some manipulations we obtain, for the reflection functions $R^\alpha_{ij}\equiv R^\alpha_{ij}(\mathbf{k}_\parallel,\omega;z,z')$,

$$R^\alpha_{xx}=\frac{i}{2\beta_\alpha}e^{i\beta_\alpha(|z|+|z'|)}\left[\frac{r^p_{\alpha\alpha'}}{q^2_\alpha}\left(-\beta^2_\alpha\frac{k^2_x}{k^2_\parallel}\right)+r^s_{\alpha\alpha'}\frac{k^2_y}{k^2_\parallel}\right], \tag{68}$$

$$R^\alpha_{xy}=\frac{i}{2\beta_\alpha}e^{i\beta_\alpha(|z|+|z'|)}\left[\frac{r^p_{\alpha\alpha'}}{q^2_\alpha}\left(-\beta^2_\alpha\frac{k_xk_y}{k^2_\parallel}\right)-r^s_{\alpha\alpha'}\frac{k_xk_y}{k^2_\parallel}\right], \tag{69}$$

$$R^\alpha_{xz}=\frac{i}{2\beta_\alpha}e^{i\beta_\alpha(|z|+|z'|)}\frac{r^p_{\alpha\alpha'}}{q^2_\alpha}[-\mathrm{sgn}(z')\beta_\alpha k_x], \tag{70}$$

$$R^\alpha_{yx}=R^\alpha_{xy}, \tag{71}$$

$$R^\alpha_{yy}=R^\alpha_{xx}(k_x\leftrightarrow k_y), \quad R^\alpha_{yz}=R^\alpha_{xz}(k_x\leftrightarrow k_y), \tag{72}$$

$$R^\alpha_{zx}=-R^\alpha_{xz}, \quad R^\alpha_{zy}=-R^\alpha_{yz}, \tag{73}$$

$$R^\alpha_{zz}=\frac{i}{2\beta_\alpha}e^{i\beta_\alpha(|z|+|z'|)}\frac{r^p_{\alpha\alpha'}}{q^2_\alpha}k^2_\parallel, \tag{74}$$

and accordingly for the transmission functions $T^{\alpha\alpha'}_{ij}\equiv T^{\alpha\alpha'}_{ij}(\mathbf{k}_\parallel,\omega;z,z')$,

$$T^{\alpha\alpha'}_{xx}=\frac{i}{2\beta_\alpha}e^{i\beta_\alpha|z|+i\beta_{\alpha'}|z'|}\times\left(\frac{t^p_{\alpha\alpha'}}{q_\alpha q_{\alpha'}}\beta_\alpha\beta_{\alpha'}\frac{k^2_x}{k^2_\parallel}+t^s_{\alpha\alpha'}\frac{k^2_y}{k^2_\parallel}\right), \tag{75}$$

$$T^{\alpha\alpha'}_{xy}=\frac{i}{2\beta_\alpha}e^{i\beta_\alpha|z|+i\beta_{\alpha'}|z'|}\times\left(\frac{t^p_{\alpha\alpha'}}{q_\alpha q_{\alpha'}}\beta_\alpha\beta_{\alpha'}\frac{k_xk_y}{k^2_\parallel}-t^s_{\alpha\alpha'}\frac{k_xk_y}{k^2_\parallel}\right), \tag{76}$$

$$T^{\alpha\alpha'}_{xz}=\frac{i}{2\beta_\alpha}e^{i\beta_\alpha|z|+i\beta_{\alpha'}|z'|}\frac{t^p_{\alpha\alpha'}}{q_\alpha q_{\alpha'}}\mathrm{sgn}(z')\beta_\alpha k_x, \tag{77}$$

$$T^{\alpha\alpha'}_{yx}=T^{\alpha\alpha'}_{xy}, \tag{78}$$

$$T^{\alpha\alpha'}_{yy}=T^{\alpha\alpha'}_{xx}(k_x\leftrightarrow k_y), \quad T^{\alpha\alpha'}_{yz}=T^{\alpha\alpha'}_{xz}(k_x\leftrightarrow k_y), \tag{79}$$

$$T^{\alpha\alpha'}_{zx}=\frac{i}{2\beta_\alpha}e^{i\beta_\alpha|z|+i\beta_{\alpha'}|z'|}\frac{t^p_{\alpha\alpha'}}{q_\alpha q_{\alpha'}}\mathrm{sgn}(z')\beta_{\alpha'}k_x, \tag{80}$$

$$T^{\alpha\alpha'}_{zy}=T^{\alpha\alpha'}_{zx}(k_x\leftrightarrow k_y), \tag{81}$$

$$T^{\alpha\alpha'}_{zz}=\frac{i}{2\beta_\alpha}e^{i\beta_\alpha|z|+i\beta_{\alpha'}|z'|}\frac{t^p_{\alpha\alpha'}}{q_\alpha q_{\alpha'}}k^2_\parallel, \tag{82}$$

where $\alpha'=1(2)$ for $\alpha=2(1)$, and

$$q_\alpha\equiv q_\alpha(\omega),$$

$$\beta_\alpha\equiv\beta_\alpha(\omega)=\sqrt{q^2_\alpha(\omega)-k^2_\parallel}, \quad \mathrm{Re}\,\beta_\alpha\geqslant0, \quad \mathrm{Im}\,\beta_\alpha\geqslant0, \tag{83}$$

with $r^q_{\alpha\alpha'}\equiv r^q_{\alpha\alpha'}(\omega)$ and $t^q_{\alpha\alpha'}\equiv t^q_{\alpha\alpha'}(\omega)$, $q=p,s$, being the generalized reflection and transmission coefficients for the p- and s-polarized components of the electromagnetic field, which are defined by

$$r^q_{\alpha\alpha'}=\frac{\beta_\alpha-\gamma^q_{\alpha\alpha'}\beta_{\alpha'}}{\beta_\alpha+\gamma^q_{\alpha\alpha'}\beta_{\alpha'}}=-r^q_{\alpha'\alpha}, \tag{84}$$

$$\gamma^p_{\alpha\alpha'}=\frac{\epsilon_\alpha}{\epsilon_{\alpha'}},\qquad \gamma^s_{\alpha\alpha'}=1, \tag{85}$$

$$t^q_{\alpha\alpha'}=\sqrt{\gamma^q_{\alpha\alpha'}}(1+r^q_{\alpha\alpha'})=\frac{\beta_\alpha}{\beta_{\alpha'}}t^q_{\alpha'\alpha} \tag{86}$$

(for details, see [54]).

It should be pointed out that—in contrast to the usually considered external current—the noise current $\underline{\hat{\mathbf{j}}}$ as given in Eq. (18) jumps at the interface, i.e., at $z\,=\,0$, which obviously implies the existence of a surface noise charge density, and hence the normal component of $\epsilon\underline{\hat{\mathbf{E}}}$ is not continuous at $z=0$. For more details and a derivation of the fields $\underline{\hat{\mathbf{E}}}$ and $\underline{\hat{\mathbf{B}}}$ by direct solution of the Maxwell equations, the reader is referred to Appendix B.

B. Commutation relations

As shown in Appendix C, the functions $R^\alpha_{ij}(\mathbf{r},\mathbf{r}',\omega)$ and $T^{\alpha\alpha'}_{ij}(\mathbf{r},\mathbf{r}',\omega)$ [Eqs. (65)–(82)] obey the relations

$$\int_{-\infty}^{\infty}d\omega\frac{\omega}{c^2}R^\alpha_{ij}(\mathbf{r},\mathbf{r}',\omega)=\partial^r_i\partial^{r'}_j\widetilde{R}^\alpha(\mathbf{r},\mathbf{r}'), \tag{87}$$

$$\int_{-\infty}^{\infty}d\omega\frac{\omega}{c^2}T^{\alpha\alpha'}_{ij}(\mathbf{r},\mathbf{r}',\omega)=\partial^r_i\partial^{r'}_j\widetilde{T}^{\alpha\alpha'}(\mathbf{r},\mathbf{r}'), \tag{88}$$

where

$$\widetilde{R}^\alpha(\mathbf{r},\mathbf{r}')=\int\frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}\,e^{i\mathbf{k}_\parallel\cdot(\mathbf{r}_\parallel-\mathbf{r}'_\parallel)}\times\int_{-\infty}^{\infty}d\omega\frac{\omega}{c^2}\,\frac{i}{2\beta_\alpha}e^{i\beta_\alpha(|z|+|z'|)}\frac{r^p_{\alpha\alpha'}}{q^2_\alpha}, \tag{89}$$

$$\widetilde{T}^{\alpha\alpha'}(\mathbf{r},\mathbf{r}')=\int\frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}\,e^{i\mathbf{k}_\parallel\cdot(\mathbf{r}_\parallel-\mathbf{r}'_\parallel)}\times\int_{-\infty}^{\infty}d\omega\frac{\omega}{c^2}\,\frac{i}{2\beta_\alpha}e^{i\beta_\alpha|z|+i\beta_{\alpha'}|z'|}\left(-\frac{t^p_{\alpha\alpha'}}{q_\alpha q_{\alpha'}}\right). \tag{90}$$

Further, it can be shown that

$$\int_{-\infty}^{\infty}d\omega\frac{\omega}{c^2}G^\alpha_{ij}(\mathbf{r},\mathbf{r}',\omega)=i\pi\delta_{ij}\delta(\mathbf{r}-\mathbf{r}')-\partial^r_i\partial^{r'}_j\widetilde{G}^\alpha(\mathbf{r},\mathbf{r}'), \tag{91}$$

where we have used the Weyl expansion of $g^\alpha(|\mathbf{r}-\mathbf{r}'|,\omega)$ [54] to obtain

$$\widetilde{G}^\alpha(\mathbf{r},\mathbf{r}')=\int\frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}e^{i\mathbf{k}_\parallel\cdot(\mathbf{r}_\parallel-\mathbf{r}'_\parallel)}\int_{-\infty}^{\infty}d\omega\frac{\omega}{c^2}\frac{ie^{i\beta_\alpha|z-z'|}}{2\beta_\alpha q^2_\alpha}. \tag{92}$$

When we substitute in Eq. (29) for $G_{ij}(\mathbf{r},\mathbf{r}',\omega)$ the actual Green function (62), take advantage of the relations (87), (88), and (91), and recall that $\epsilon_{kmj}\partial^{r'}_m\partial^{r'}_j(\,)=0$, then we can readily prove that the fundamental QED commutation relation

$$[\hat{E}_i(\mathbf{r}),\hat{B}_k(\mathbf{r}')]=-\frac{i\hbar}{\epsilon_0}\epsilon_{ikm}\partial^r_m\delta(\mathbf{r}-\mathbf{r}') \tag{93}$$

is preserved.

In order to prove that the commutation relations between the potentials are correct, we first write Eqs. (43) and (46) [together with Eqs. (45) and (47)] as

$$[\hat{A}_i(\mathbf{r}),\hat{\dot{A}}_j(\mathbf{r}')]=I^{(1)}_{ij}(\mathbf{r},\mathbf{r}'), \tag{94}$$

$$[\hat{\varphi}(\mathbf{r}),\hat{A}_j(\mathbf{r}')]=\partial^r_iI^{(2)}_{ij}(\mathbf{r},\mathbf{r}'), \tag{95}$$

where

$$I^{(k)}_{ij}(\mathbf{r},\mathbf{r}')=\frac{\hbar}{\pi\epsilon_0}\int d^3\mathbf{s}\int d^3\mathbf{s}'\,\chi^{(k)}_{im}(\mathbf{r}-\mathbf{s})\times\int_{-\infty}^{\infty}d\omega\frac{\omega}{c^2}G_{mn}(\mathbf{s},\mathbf{s}',\omega)\,\delta^\perp_{nj}(\mathbf{s}'-\mathbf{r}'), \tag{96}$$

with

$$\chi^{(1)}_{im}(\mathbf{r}-\mathbf{s})=\delta^\perp_{im}(\mathbf{r}-\mathbf{s}) \tag{97}$$

and

$$\chi^{(2)}_{im}(\mathbf{r}-\mathbf{s})=\frac{\delta_{im}}{4\pi|\mathbf{r}-\mathbf{s}|}. \tag{98}$$

We use the Green function (62) and again take advantage of Eqs. (87), (88), and (91) to derive

$$\begin{aligned}I^{(k)}_{ij}(\mathbf{r},\mathbf{r}')&=\frac{\hbar}{\pi\epsilon_0}\sum_{\alpha=1}^{2}\int_{V_\alpha}d^3\mathbf{s}\chi^{(k)}_{im}(\mathbf{r}-\mathbf{s})\bigg\{\int_{V_\alpha}d^3\mathbf{s}'\{i\pi\delta_{mn}\\&\quad\times\delta(\mathbf{s}-\mathbf{s}')+\partial^s_m\partial^{s'}_n[-\widetilde{G}^\alpha(\mathbf{s},\mathbf{s}')\\&\quad+\widetilde{R}^\alpha(\mathbf{s},\mathbf{s}')]\}\delta^\perp_{nj}(\mathbf{s}'-\mathbf{r}')\\&\quad+\int_{V_{\alpha'}}d^3\mathbf{s}'[\partial^s_m\partial^{s'}_n\widetilde{T}^{\alpha\alpha'}(\mathbf{s},\mathbf{s}')]\delta^\perp_{nj}(\mathbf{s}'-\mathbf{r}')\bigg\}\\&=\frac{i\hbar}{\epsilon_0}\sum_{\alpha=1}^{2}\int_{V_\alpha}d^3\mathbf{s}\chi^{(k)}_{im}(\mathbf{r}-\mathbf{s})\delta^\perp_{mj}(\mathbf{s}-\mathbf{r}')+\widetilde{I}^{(k)}_{ij}(\mathbf{r},\mathbf{r}')\\&=\frac{i\hbar}{\epsilon_0}\int d^3\mathbf{s}\chi^{(k)}_{im}(\mathbf{r}-s)\delta^\perp_{mj}(s-\mathbf{r}')\end{aligned} \tag{99}$$

[$\alpha'=2(1)$ for $\alpha=1(2)$]. The last line follows from the line before last, because of

$$\tilde{I}^{(k)}_{ij}(\mathbf{r},\mathbf{r}')=\frac{\hbar}{\pi\epsilon_0}\sum_\alpha \int_{V_\alpha} d^3\mathbf{s}\chi^{(k)}_{im}(\mathbf{r}-\mathbf{s})\Bigg\{\int_{V_\alpha} d^3\mathbf{s}'\{\partial^s_m\partial^{s'}_n \times[-\tilde{G}^\alpha(\mathbf{s},\mathbf{s}')+\tilde{R}^\alpha(\mathbf{s},\mathbf{s}')]\}\delta^\perp_{nj}(\mathbf{s}'-\mathbf{r}') + \int_{V_{\alpha'}} d^3\mathbf{s}'[\partial^s_m\partial^{s'}_n\tilde{T}^{\alpha\alpha'}(\mathbf{s},\mathbf{s}')]\delta^\perp_{nj}(\mathbf{s}'-\mathbf{r}')\Bigg\}=0 \tag{100}$$

(Appendix D). Recalling the definitions (97) and (98) of $\chi^{(1)}_{im}$ and $\chi^{(2)}_{im}$, respectively, from Eq. (99) it is easily seen that

$$I^{(1)}_{ij}(\mathbf{r},\mathbf{r}')=\frac{i\hbar}{\epsilon_0}\int d^3\mathbf{s}\delta^\perp_{im}(\mathbf{r}-\mathbf{s})\delta^\perp_{mj}(\mathbf{s}-\mathbf{r}')=\frac{i\hbar}{\epsilon_0}\delta^\perp_{ij}(\mathbf{r}-\mathbf{r}') \tag{101}$$

and

$$\partial^r_i I^{(2)}_{ij}(\mathbf{r},\mathbf{r}')=\frac{i\hbar}{\epsilon_0}\int d^3\mathbf{s}\partial^r_m\frac{\delta^\perp_{mj}(\mathbf{s}-\mathbf{r}')}{4\pi|\mathbf{r}-\mathbf{s}|}=0, \tag{102}$$

i.e.,

$$[\hat{A}_i(\mathbf{r}),\hat{A}_j(\mathbf{r}')]=\frac{i\hbar}{\epsilon_0}\delta^\perp_{ij}(\mathbf{r}-\mathbf{r}') \tag{103}$$

and

$$[\hat{\varphi}(\mathbf{r}),\hat{A}_j(\mathbf{r}')]=0. \tag{104}$$

Hence the theory yields the correct equal-time commutation relations for both the fields and the potentials.

V. CONCLUSIONS

We have developed a quantization scheme for the electromagnetic field in a spatially varying three-dimensional linear dielectric which gives rise to both dispersion and absorption. Based on the classical phenomenological Maxwell equations, the dielectric is described in terms of a complex frequency- and space-dependent permittivity, which satisfies the Kramers-Kronig relations, and fluctuating current and charge densities are introduced in order to be consistent with the dissipation-fluctuation theorem. The noise current and charge densities can be thought of as arising from an additional noise polarization in the constitutive equation between the dielectric displacement vector and the vector of the electric field strength. The resulting inhomogeneous Maxwell equations are then transferred to quantum theory, and the noise polarization is specified such that the fundamental equal-time commutation relations of QED are preserved.

From the inhomogeneous Maxwell equations together with the boundary conditions at infinity, it follows that the electromagnetic field operators can be related, through the dyadic Green function of the classical problem, to a bosonic field that represents the elementary (energy) excitations of the overall system. This integral representation can be regarded as a natural extension of the familiar mode expansion in free space or in cavitylike systems with perfectly reflecting walls. Vector and scalar potentials are introduced in the usual way, and their integral representations are derived, which can be used in order to couple the electromagnetic field to additional atomic sources embedded in the medium. The fundamental equal-time commutation relations are studied, and it is found that some of them can be calculated without kowledge of the explicit form of the Green function in order to prove the consistence of the quantization scheme with QED. Others require this knowledge, because single frequency integrals of the Green function remain to be calculated.

The determination of the Green function is—similar to the determination of the mode functions in a mode-expansion approach—a purely classical problem. Its solution is very difficult in general, and only for simple dielectric-body configurations has the Green function been calculated so far. For a homogeneous dielectric, the Green function is well known. We have used it and explicitly shown that the quantization scheme outlined here yields exactly the fundamental equal-time commutation relations of QED. We have further shown that earlier results derived in [40,46] for the transverse part of the electromagnetic field are contained in our theory.

An example of an inhomogeneous medium for which the Green function is known is a configuration of two infinitely extended dielectric bodies with a common planar interface. We have also used this Green function and explicitly proved that the quantization scheme is consistent with QED. Again, recent results given in [40,41] for paraxial light propagation are recognized. It is worth noting that the contributions to the Green function that result from the reflections and transmissions at the interface do not contribute to the equal-time commutation relations. Since this is expected to be true also for more complicated configurations, such as multilayer structures, the equal-time commutation relations are expected to be preserved also for these configurations.

The quantization scheme developed in this paper can be regarded as the basis for studying the interaction between radiation and atomic systems in the presence of three-dimensional configurations of dielectric bodies with dispersion and absorption. In this case, additional atomic sources must be introduced into the theory, and the coupled equations of motion for the atomic variables and the electromagnetic-field variables in the Green function expansion (similar to the photonic variables in a mode expansion) must be tried to be solved.

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APPENDIX A: PROOF OF EQ. (27)

From Eq. (24), the equation for $G_{ni}(\mathbf{s},\mathbf{r},\omega)$ reads as

$$\left[\partial^s_l\partial^s_n-\delta_{ln}\left(\Delta^s+\frac{\omega^2}{c^2}\epsilon(\mathbf{s},\omega)\right)\right]G_{ni}(\mathbf{s},\mathbf{r},\omega)=\delta_{li}\delta(\mathbf{s}-\mathbf{r}). \tag{A1}$$

We multiply Eq. (A1) from the right by $G^{*}_{lj}(\mathbf{s},\mathbf{r}',\omega)$ and integrate over $\mathbf{s}$. We derive, on integrating by parts and taking into account that the Green function vanishes at infinity,

$$\frac{\omega^2}{c^2}\int d^3\mathbf{s}\epsilon(\mathbf{s},\omega)G_{li}(\mathbf{s},\mathbf{r},\omega)G^{*}_{lj}(\mathbf{s},\mathbf{r}',\omega)$$

$$=\int d^3\mathbf{s}[(\partial^s_l\partial^s_n-\delta_{ln}\Delta^s)G_{ni}(\mathbf{s},\mathbf{r},\omega)]G^{*}_{lj}(\mathbf{s},\mathbf{r}',\omega)$$

$$-G^{*}_{ij}(\mathbf{r},\mathbf{r}',\omega)$$

$$=-\int d^3\mathbf{s}[\partial^s_n G_{ni}(\mathbf{s},\mathbf{r},\omega)][\partial^s_l G^{*}_{lj}(\mathbf{s},\mathbf{r}',\omega)]$$

$$+\int d^3\mathbf{s}[\partial^s_k G_{li}(\mathbf{s},\mathbf{r},\omega)][\partial^s_k G^{*}_{lj}(\mathbf{s},\mathbf{r}',\omega)]-G^{*}_{ij}(\mathbf{r},\mathbf{r}',\omega). \tag{A2}$$

Now we take the complex conjugate of Eq. (A2), make the interchanges $\mathbf{r}\leftrightarrow\mathbf{r}'$ and $i\leftrightarrow j$, and subtract the resulting equation from Eq. (A2). In this way we arrive at

$$\frac{\omega^2}{c^2}\int d^3\mathbf{s}\epsilon_{\mathrm{I}}(\mathbf{s},\omega)G_{li}(\mathbf{s},\mathbf{r},\omega)G^{*}_{lj}(\mathbf{s},\mathbf{r}',\omega)$$

$$=\frac{1}{2i}[G_{ji}(\mathbf{r}',\mathbf{r},\omega)-G^{*}_{ij}(\mathbf{r},\mathbf{r}',\omega)]. \tag{A3}$$

Recalling the symmetry property [52]

$$G_{ji}(\mathbf{r}',\mathbf{r},\omega)=G_{ij}(\mathbf{r},\mathbf{r}',\omega) \tag{A4}$$

and combining Eqs. (A3) and (A4) then yields Eq. (27).

APPENDIX B: DIRECT SOLUTION OF THE MAXWELL EQUATIONS

In order to directly solve the Maxwell equations (10)–(13) together with the permittivity in Eq. (61), i.e., without using the Green function, we expand $\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)$ as

$$\underline{\hat{\mathbf{E}}}(\mathbf{r},\omega)=\int\frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}\underline{\hat{\mathbf{E}}}(\mathbf{k}_\parallel,\omega;z)e^{i\mathbf{k}_\parallel\cdot\mathbf{r}_\parallel} \tag{B1}$$

[cf. Eq. (67) and the comment made there]. Obviously, similar expressions hold for $\underline{\hat{\mathbf{B}}}$ and $\underline{\hat{\mathbf{j}}}$. Choosing a reference system in which $\mathbf{k}_\parallel$ is parallel to the x axis, the Maxwell equations (12) and (13) then yield

$$-\partial_z\underline{\hat{E}}_y=i\omega\underline{\hat{B}}_x, \tag{B2}$$

$$\partial_z\underline{\hat{E}}_x-ik_\parallel\underline{\hat{E}}_z=i\omega\underline{\hat{B}}_y, \tag{B3}$$

$$ik_\parallel\underline{\hat{E}}_y=i\omega\underline{\hat{B}}_z, \tag{B4}$$

$$-\partial_z\underline{\hat{B}}_y=-\frac{i\omega}{c^2}\epsilon\underline{\hat{E}}_x+\mu_0\underline{\hat{j}}_x, \tag{B5}$$

$$\partial_z\underline{\hat{B}}_x-ik_\parallel\underline{\hat{B}}_z=-\frac{i\omega}{c^2}\epsilon\underline{\hat{E}}_y+\mu_0\underline{\hat{j}}_y, \tag{B6}$$

$$ik_\parallel\underline{\hat{B}}_y=-\frac{i\omega}{c^2}\epsilon\underline{\hat{E}}_z+\mu_0\underline{\hat{j}}_z, \tag{B7}$$

where, for notational convenience, we have omitted the arguments $\mathbf{k}_\parallel$, ω, and z of the fields $\underline{\hat{\mathbf{E}}}(\mathbf{k}_\parallel,\omega;z)$, $\underline{\hat{\mathbf{B}}}(\mathbf{k}_\parallel,\omega;z)$, and $\underline{\hat{\mathbf{j}}}(\mathbf{k}_\parallel,\omega;z)$, and we have used the same notation for the rotated reference system as for the original one. According to the Maxwell equations (12) and (13), the tangential components of $\underline{\hat{\mathbf{E}}}$ and $\underline{\hat{\mathbf{H}}}$ must be continuous at the surface of discontinuity [58].

From Eqs. (B2), (B4), and (B6) together with the boundary conditions we find that $\underline{\hat{E}}_y$ obeys the equation

$$\partial_z^2\underline{\hat{E}}_y+\left(\frac{\omega^2}{c^2}\epsilon-k_\parallel^2\right)\underline{\hat{E}}_y=-i\omega\mu_0\underline{\hat{j}}_y, \tag{B8}$$

and $\underline{\hat{E}}_y$ and $\partial_z\underline{\hat{E}}_y$ are continuous at the plane $z=0$. $\underline{\hat{B}}_x$ and $\underline{\hat{B}}_z$ can then be obtained from $\underline{\hat{E}}_y$, using Eqs. (B2) and (B4). Similarly, from Eqs. (B3), (B5), and (B7) together with the boundary conditions it follows that $\underline{\hat{B}}_y$ satisfies

$$\partial_z^2\underline{\hat{B}}_y+\left(\frac{\omega^2}{c^2}\epsilon-k_\parallel^2\right)\underline{\hat{B}}_y=ik_\parallel\mu_0\underline{\hat{j}}_z-\mu_0\partial_z\underline{\hat{j}}_x, \tag{B9}$$

and $\underline{\hat{B}}_y$ and $\epsilon^{-1}(\partial_z\underline{\hat{B}}_y+\mu_0\underline{\hat{j}}_x)$ are continuous at the plane $z=0$. Knowing $\underline{\hat{B}}_y$, we can find $\underline{\hat{E}}_x$ and $\underline{\hat{E}}_z$ from Eqs. (B5) and (B7).

Thus, Eqs. (B2)–(B7) together with the boundary conditions at infinity and the conditions of continuity of the tangential components of $\underline{\hat{\mathbf{E}}}$ and $\underline{\hat{\mathbf{H}}}$ at the interface provide us with unique solutions for $\underline{\hat{\mathbf{E}}}$ and $\underline{\hat{\mathbf{B}}}$. Solving Eqs. (B8) and (B9) under the conditions mentioned (and going back to the original reference system) yields the fields $\underline{\hat{\mathbf{E}}}$ and $\underline{\hat{\mathbf{B}}}$ in full agreement with Eqs. (23) and (25) and the Green function from Sec. IV A. It should be emphasized that, as can be seen from Eq. (B7), the normal component of $\epsilon\underline{\hat{\mathbf{E}}}$ is not continuous at the plane $z=0$, because $\underline{\hat{B}}_y$ is continuous and

$\underline{\hat{j}}_z$ makes a jump at $z=0$ [see Eq. (18) together with Eq. (61)].

In the derivation of the Green function in [51] it is stated that the continuity of the normal component of $\epsilon\underline{\hat{\mathbf{E}}}$ at the plane $z = 0$ is chosen as a boundary condition. However, one can verify that it is the continuity of ϵG_{zi}, $i = x,y,z$, that actually matters, and this can be deduced from the continuity of the tangential components of $\underline{\hat{\mathbf{E}}}$ and $\underline{\hat{\mathbf{H}}}$. From the continuity of ϵG_{zi} it does not follow that $\epsilon\underline{\hat{E}}_z$ is continuous, though the reverse is true. For example, when (as in our case) ϵG_{zz} contains a term proportional to $\delta(z - z')$, which is continuous at the plane $z=0$, then $\epsilon\underline{\hat{E}}_z$ jumps there, because it contains a term proportional to $\underline{\hat{j}}_z$, which makes a jump at $z=0$ [see Eq. (18) together with Eq. (61)].

APPENDIX C: PROOF OF EQS. (87) AND (88)

In order to prove Eqs. (87) and (88) for $R^{\alpha}_{ij}(\mathbf{r},\mathbf{r}',\omega)$ and $T^{\alpha\alpha'}_{ij}(\mathbf{r},\mathbf{r}',\omega)$, it is sufficient to prove them for the Fourier components $R^{\alpha}_{ij}(\mathbf{k}_\parallel,\omega;z,z')$ and $T^{\alpha\alpha'}_{ij}(\mathbf{k}_\parallel,\omega;z,z')$. From Eqs. (68)–(74) and Eqs. (75)–(82), respectively, the functions $R^{\alpha}_{ij}(\mathbf{k}_\parallel,\omega;z,z')$ and $T^{\alpha\alpha'}_{ij}(\mathbf{k}_\parallel,\omega;z,z')$ are seen to consist, in general, of two parts. One part is associated with the p-polarized and the other part is associated with the s-polarized electromagnetic field. Using contour integral techniques (cf. [40,41]), it can be proved that the latter does not contribute to the integrals on the left-hand sides in Eqs. (87) and (88), so that we are left with the contributions from the p-polarized field only.

Let us consider, e.g., $R^{\alpha}_{xx}(\mathbf{k}_\parallel,\omega;z,z')$, Eq. (68). From

$$\int_{-\infty}^{\infty} d\omega \frac{\omega}{c^2}\frac{i}{2\beta_\alpha} e^{i\beta_\alpha(|z|+|z'|)}\frac{r^p_{\alpha\alpha'}}{q^2_\alpha}(-\beta^2_\alpha)\frac{k_x^2}{k_\parallel^2} = \int_{-\infty}^{\infty} d\omega \frac{\omega}{c^2}\frac{i}{2\beta_\alpha} e^{i\beta_\alpha(|z|+|z'|)}\frac{r^p_{\alpha\alpha'}}{q^2_\alpha}(-q^2_\alpha+k^2_\parallel)\frac{k_x^2}{k_\parallel^2} = \int_{-\infty}^{\infty} d\omega \frac{\omega}{c^2}\frac{i}{2\beta_\alpha} e^{i\beta_\alpha(|z|+|z'|)}\frac{r^p_{\alpha\alpha'}}{q^2_\alpha}k_x^2, \tag{C1}$$

Eq. (87) can readily be proved correct for $R^{\alpha}_{xx}(\mathbf{r},\mathbf{r}',\omega)$. In exactly the same way, one can show that Eq. (87) also holds for the remaining functions $R^{\alpha}_{ij}(\mathbf{r},\mathbf{r}',\omega)$. In order to show that Eq. (88) is valid, it is helpful to perform the integration over ω in Eq. (90) explicitly. Using the relation

$$e^{i\beta_\alpha|z|} = \int_{-\infty}^{\infty}\frac{dk}{2\pi}e^{ikz}\frac{2i\beta_\alpha}{\beta^2_\alpha - k^2}, \tag{C2}$$

we have

$$\int_{-\infty}^{\infty} d\omega \frac{\omega}{c^2}\frac{i}{2\beta_\alpha} e^{i\beta_\alpha|z|+i\beta_{\alpha'}|z'|}\left(-\frac{t^p_{\alpha\alpha'}}{q_\alpha q_{\alpha'}}\right) = \int_{-\infty}^{\infty}\frac{dk}{2\pi}e^{ikz}\int_{-\infty}^{\infty}\frac{dk'}{2\pi}e^{ik'z'}\int_{-\infty}^{\infty} d\omega \frac{\omega}{c^2}\frac{i}{2\beta_\alpha}\frac{2i\beta_\alpha}{\beta^2_\alpha-k^2}\frac{2i\beta_{\alpha'}}{\beta^2_{\alpha'}-k'^2}\left(-\frac{t^p_{\alpha\alpha'}}{q_\alpha q_{\alpha'}}\right). \tag{C3}$$

Since the equation $\beta^2_\alpha - k^2 = 0$ has no solutions in the upper complex frequency half-plane [35,41], the integrand in the ω integral in Eq. (C3) has no poles there. Treating the single pole at $\omega = 0$ as a principal value, we obtain

$$\int_{-\infty}^{\infty} d\omega \frac{\omega}{c^2}\frac{i}{2\beta_\alpha} e^{i\beta_\alpha|z|+i\beta_{\alpha'}|z'|}\left(-\frac{t^p_{\alpha\alpha'}}{q_\alpha q_{\alpha'}}\right) = \int_{-\infty}^{\infty}\frac{dk}{2\pi}\frac{e^{ikz}}{k^2_\parallel+k^2}\int_{-\infty}^{\infty}\frac{dk'}{2\pi}\frac{e^{ik'z'}}{k^2_\parallel+k'^2}2i\pi k_\parallel\left(-\frac{t^p_{\alpha\alpha'}(0)}{\sqrt{\epsilon_\alpha(0)\epsilon_{\alpha'}(0)}}\right). \tag{C4}$$

Let us now turn to the left-hand side in Eq. (88) and consider, e.g., $T^{\alpha\alpha'}_{xx}(\mathbf{k}_\parallel,\omega;z,z')$, Eq. (75). We derive

$$\int_{-\infty}^{\infty} d\omega \frac{\omega}{c^2}\frac{i}{2\beta_\alpha} e^{i\beta_\alpha|z|+i\beta_{\alpha'}|z'|}\frac{t^p_{\alpha\alpha'}}{q_\alpha q_{\alpha'}}\frac{\beta_\alpha\beta_{\alpha'}}{k^2_\parallel} = \int_{-\infty}^{\infty}\frac{dk}{2\pi}e^{ikz}\int_{-\infty}^{\infty}\frac{dk'}{2\pi}e^{ik'z'}\int_{-\infty}^{\infty} d\omega \frac{\omega}{c^2}\frac{i}{2\beta_\alpha}\frac{2i\beta_\alpha}{\beta^2_\alpha-k^2}\frac{2i\beta_{\alpha'}}{\beta^2_{\alpha'}-k'^2}\frac{t^p_{\alpha\alpha'}}{q_\alpha q_{\alpha'}}\frac{\beta_\alpha\beta_{\alpha'}}{k^2_\parallel}$$

$$= \int_{-\infty}^{\infty}\frac{dk}{2\pi}\frac{e^{ikz}}{k^2_\parallel+k^2}\int_{-\infty}^{\infty}\frac{dk'}{2\pi}\frac{e^{ik'z'}}{k^2_\parallel+k'^2}2i\pi k_\parallel\left(-\frac{t^p_{\alpha\alpha'}(0)}{\sqrt{\epsilon_\alpha(0)\epsilon_{\alpha'}(0)}}\right), \tag{C5}$$

which exactly agrees with Eq. (C4), i.e., Eq. (88) is proved correct for $T_{xx}^{\alpha\alpha'}(\mathbf{r},\mathbf{r}',\omega)$. Equation (C5) for the other functions $T_{ij}^{\alpha\alpha'}(\mathbf{r},\mathbf{r}',\omega)$ can be given in the same way.

APPENDIX D: PROOF OF EQ. (100)

We perform in Eq. (100) a partial integration over s' and obtain, on recalling that $\partial_n^{s'}\ \delta_{nj}^{\perp}(\mathbf{s}'-\mathbf{r}')=0$,

$$\tilde{I}_{ij}^{(k)}(\mathbf{r},\mathbf{r}')=-\frac{\hbar}{\pi\epsilon_0}\sum_{\alpha}\int_{V_\alpha}d^3\mathbf{s}\chi_{im}^{(k)}(\mathbf{r}-\mathbf{s})\int ds'_x ds'_y \times\partial_m^s[-\tilde{G}^{\alpha}(\mathbf{s},\mathbf{s}')+\tilde{R}^{\alpha}(\mathbf{s},\mathbf{s}')-\tilde{T}^{\alpha\alpha'}(\mathbf{s},\mathbf{s}')]\delta_{nj}^{\perp}(\mathbf{s}'-\mathbf{r}')\Big|_{s'_z=0}. \tag{D1}$$

Using Eqs. (89), (90), and (92), we derive

$$\begin{aligned}
&[-\tilde{G}^{\alpha}(\mathbf{s},\mathbf{s}')+\tilde{R}^{\alpha}(\mathbf{s},\mathbf{s}')-\tilde{T}^{\alpha\alpha'}(\mathbf{s},\mathbf{s}')]\Big|_{s'_z=0}\\
&=\int\frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}e^{i\mathbf{k}_\parallel\cdot(\mathbf{s}_\parallel-\mathbf{s}'_\parallel)}\int_{-\infty}^{\infty}d\omega\frac{\omega}{c^2}\frac{i}{2\beta_\alpha}\times e^{i\beta_\alpha|s_z|}\left(-\frac{1}{q_\alpha^2}+\frac{r_{\alpha\alpha'}^p}{q_\alpha^2}+\frac{t_{\alpha\alpha'}^p}{q_\alpha q_{\alpha'}}\right)\\
&=\int\frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}e^{i\mathbf{k}_\parallel\cdot(\mathbf{s}_\parallel-\mathbf{s}'_\parallel)}\int_{-\infty}^{\infty}d\omega\frac{\omega}{c^2}\frac{i}{2\beta_\alpha}\times e^{i\beta_\alpha|s_z|}\left(-\frac{1}{q_\alpha^2}+\frac{r_{\alpha\alpha'}^p}{q_\alpha^2}+\frac{\beta_\alpha}{\beta_{\alpha'}}\sqrt{\frac{\epsilon_{\alpha'}}{\epsilon_\alpha}}\frac{1-r_{\alpha\alpha'}^p}{q_\alpha q_{\alpha'}}\right)\\
&=\int\frac{d^2\mathbf{k}_\parallel}{(2\pi)^2}e^{i\mathbf{k}_\parallel\cdot(\mathbf{s}_\parallel-\mathbf{s}'_\parallel)}\int_{-\infty}^{\infty}d\omega\frac{\omega}{c^2}\frac{i}{2\beta_\alpha}\times e^{i\beta_\alpha|s_z|}\frac{1-r_{\alpha\alpha'}^p}{q_\alpha^2}\left(-1+\frac{\beta_\alpha}{\beta_{\alpha'}}\right)=0.
\end{aligned} \tag{D2}$$

To obtain the second equation from the first one, we have used Eq. (86) for $t_{\alpha\alpha'}^p$. The third equation gives zero because of the integration over ω. Combining Eqs. (D2) and Eq. (D1) yields Eq. (100).

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