Pumping two dilute-gas Bose-Einstein condensates with Raman light scattering

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We propose an optical method for increasing the number of atoms in a pair of dilute-gas Bose-Einstein condensates. The method uses laser-driven Raman transitions which scatter atoms between the condensate and noncondensate atom fractions. For a range of condensate phase differences there is destructive quantum interference of the amplitudes for scattering atoms out of the condensates. Because the total atom scattering rate into the condensates is unaffected, the condensates grow. [S1050-2947(98)09104-5]

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In the recent experiments demonstrating Bose-Einstein condensation of alkali-metal vapors the first stages of cooling are optical [1]. The final stage utilizes evaporation of the hottest atoms out of the trap. Despite the great success of evaporation it has the disadvantage of removing atoms from the system. Consequently alternative final stage cooling methods are being investigated. Velocity selective coherent population trapping is one optical method potentially capable of cooling to the Bose-Einstein transition point [2].

We propose an optical method for increasing the number of atoms in a pair of overlapping Bose-Einstein condensates (BEC's). This may provide not only a new means for growing condensates, but also a possible pumping mechanism for atom lasers [3]. From a fundamental perspective the method is interesting because it is based on destructive quantum interference between two scattering channels to the same final state. This interference is a consequence of the macroscopic quantum coherence of BEC's.

Two overlapping condensates of $|F=1,\ m=-1\rangle$ and $|F=2,\ m=2\rangle$ states of ^{87}Rb have been produced in the laboratory using sympathetic cooling [4]. The stability of this pair is due to an unexpectedly small inelastic collision rate between these states [5]. Recently, an even more interpenetrating condensate pair of $|1,-1\rangle$ and $|2,1\rangle$ states of ^{87}Rb has been realized at JILA [6]. In general, inelastic collisions will make it difficult to magnetically trap two condensates. However, BEC's have been confined in an optical dipole trap [7] which uses optical forces to trap atoms. The dipole traps have a major advantage over magnetic traps since they can stably trap atoms in arbitrary hyperfine states.

According to conventional spontaneous symmetry-breaking arguments BEC's are in coherent states with a definite global phase [8]. A pair of BEC's therefore has a definite phase difference, which can be measured by a variety of techniques [9–13]. We utilize this phase difference to suppress transitions of atoms out of the condensates while driving transitions into the condensates, producing a net condensate growth. The transitions are driven by spontaneous Raman scattering of laser light by an atomic Λ level scheme. For a certain range of condensate phase differences transitions out of the condensates are suppressed by destructive quantum interference. The interference is between the amplitudes for transitions from each condensate into the same noncondensate state.

This is analogous to the suppression of absorption that occurs in optical lasing without inversion (LWI) [14]. In the Λ atomic level scheme of LWI the photon absorption probability is the squared sum of the amplitudes for absorption from the two lower levels. These amplitudes can interfere destructively, suppressing absorption. On the other hand, the photon emission probability is the sum of the squares of the amplitudes for emission into each lower level. Interference is impossible because of the different final states for each transition. LWI may occur when the emission probability exceeds the suppressed absorption probability. Similarly the final states for transitions into each of the two condensates are different, whereas transitions out of the condensates can have the same noncondensate final state, and hence can destructively interfere.

Spontaneous Raman scattering of laser light from two BEC's has been analyzed by Ruostekoski and Walls [11], whose analysis we shall follow. They showed that the scattered light spectrum depends on the phase difference between the two condensates. The spectrum has two peaks, corresponding to transitions of atoms into and out of the condensates. For certain values of the condensate phase difference the second peak disappears due to the destructive quantum interference previously described. Although this suggests the possibility of condensate growth, only short time behavior was considered. In fact, Raman lasers induce Josephson oscillations between the condensates [15]. We show that growth occurs and can persist over a complete period of this dynamics. However, destructive interference only occurs for a particular range of condensate phase differences. Consequently, condensate growth only occurs for a subensemble of condensate pairs. Suitable pairs might be chosen after a measurement of the phase difference [9–12]. This measurement can be performed without significant loss of condensate atoms using coherent, spontaneous Raman scattering [13]. With this method atoms are scattered between the two condensates and the dominant photon scattering is in the forward direction; so photon collection is feasible.

For definiteness we consider a particular setup first introduced by Javanainen [9]: a quantum degenerate gas with a Λ level scheme of two ground states and a common excited state. We only consider the case in which the magnetic quan-

tum numbers of the ground states differ by two units. However, all that is fundamentally important for the destructive interference is that both ground states have transitions to the same final state. In principle the common excited state could even be a coherent superposition of different atomic states, with the coherence induced by extra optical or rf fields.

All the ground state atoms are confined in the same trap. We consider a spatially overlapping pair of condensates in two different Zeeman sublevels $|b\rangle=|g,m\rangle$ and $|c\rangle=|g,m-2\rangle$. The state $|c\rangle$ is optically coupled to the electronically excited state $|e\rangle=|e,m-1\rangle$ by the field $\mathbf{E}_2$ having a polarization σ_+ and frequency Ω_2. Similarly, the state $|b\rangle$ is coupled to $|e\rangle$ by the field $\mathbf{E}_1$ with a polarization σ_- and frequency Ω_1. Following Ref. [16] the Hamiltonian density for the system is

$$\mathcal{H}=\psi_b^\dagger H\psi_b+\psi_c^\dagger(H+\hbar\omega_{cb})\psi_c+\psi_e^\dagger(H+\hbar\omega_{eb})\psi_e+\mathcal{H}_F-(\mathbf{d}_b\cdot\mathbf{E}_1\psi_b^\dagger\psi_e+\mathbf{d}_c\cdot\mathbf{E}_2\psi_c^\dagger\psi_e+\text{H.c.}). \quad (1)$$

The first three terms reflect the center-of-mass energy, H, and the internal energies of the atoms in the absence of electromagnetic fields. The frequencies for the optical transitions $e\leftrightarrow b$ and $e\leftrightarrow c$ are ω_{eb} and ω_{ec} ($\omega_{cb}=\omega_{eb}-\omega_{ec}$), respectively. $\mathcal{H}_F$ is the Hamiltonian density for the free electromagnetic field. The final, bracketed, terms are for the atom–light-dipole interaction. The dipole matrix element for the atomic transition $e\leftrightarrow b$ ($e\leftrightarrow c$) is given by $\mathbf{d}_b$ ($\mathbf{d}_c$).

We assume that the BEC's are optically thin and that the driving light fields $\mathbf{E}_{di}^+$ are in coherent states and detuned far from single photon resonance so that multiple scattering can be ignored [16]. Various mechanisms may produce significant reabsorption of the scattered light in dense atomic gases. These include resonant dipole-dipole interactions [17] and Raman resonant reabsorption [18]. It has been shown that even moderate size condensates may in practice turn out to be optically thick. Nonetheless, there are still quite basic unsettled issues in the theory of the optical response of dense atomic gases. Indeed a fully quantum field-theoretical analysis of light matter interactions is a pathological problem [19]. However, the optical thickness should not be a problem provided at least one condensate dimension is smaller than an optical wavelength. This is the case in a magneto-optical surface trap, for example. This combines a magneto-optical trap with an evanescent wave mirror, resulting in an effectively two-dimensional atomic gas [20]. An emitted photon can then easily escape from the thin dimension.

We describe the driving light fields as plane waves propagating in the positive z direction with wave vectors $\boldsymbol{\kappa}_i$,

$$\tilde{\mathbf{E}}_{di}^+(\mathbf{r})=\frac{1}{2}\mathcal{E}_i\hat{\mathbf{e}}_i\exp(i\boldsymbol{\kappa}_i\cdot\mathbf{r}), \quad (2)$$

where $i=1,2$ and the $\hat{\mathbf{e}}_i$ are the unit circular polarization vectors. We have defined slowly varying fields by $\tilde{\mathbf{E}}_i^+=e^{i\Omega_i t}\mathbf{E}_i^+$. We also define $\tilde{\psi}_c=e^{i(\Omega_1-\Omega_2)t}\psi_c$.

In the limit of large detuning the excited state field operator ψ_e may be eliminated adiabatically. Following Ref. [16] the scattered electric fields may then be expressed in terms of the driving fields as $\tilde{\mathbf{E}}_s^+=\tilde{\mathbf{E}}_{s1}^+ +\tilde{\mathbf{E}}_{s2}^+$ where $\tilde{\mathbf{E}}_{s1}^+$ is radiated by decays into state $|b\rangle$,

$$\tilde{\mathbf{E}}_{s1}^+(\mathbf{r},t)=\int d^3r'\mathbf{K}(\mathbf{d}_b)\psi_b^\dagger\psi_e=\frac{1}{\hbar\Delta_1}\int d^3r'\mathbf{K}(\mathbf{d}_b)\times\{\mathbf{d}_b\cdot\tilde{\mathbf{E}}_{d1}^+\psi_b^\dagger\psi_b+\mathbf{d}_c\cdot\tilde{\mathbf{E}}_{d2}^+\psi_b^\dagger\tilde{\psi}_c\}. \quad (3)$$

The driving fields and atom fields are all functions of $\mathbf{r}'$ and t. $\Delta_1=\Omega_1-\omega_{eb}$ is the atom-field detuning of field 1. The first line represents the radiation from the atomic dipole density, and the second follows after adiabatic elimination of the excited state. $\tilde{\mathbf{E}}_{s2}^+$, which is radiated by decays into state $|c\rangle$, is found by swapping subscripts b and c and swapping the driving fields $\tilde{\mathbf{E}}_{d1}^+$ and $\tilde{\mathbf{E}}_{d2}^+$. We have used the first Born approximation based on the assumption that the incoming fields dominate inside the sample, as multiple scattering is negligible. The kernel $\mathbf{K}(\mathbf{d})$ is the familiar expression [21] for the positive-frequency component of the electric field at $\mathbf{r}$ from a monochromatic dipole with the complex amplitude $\mathbf{d}$, located at $\mathbf{r}'$.

After adiabatic elimination of the excited state from the Hamiltonian density, Eq. (1), and approximation of the electric fields by the driving fields, the following Hamiltonian density is found to first order in the inverse atom-field detuning [11]:

$$\mathcal{H}_M=\psi_b^\dagger(H-\hbar\delta_1)\psi_b+\tilde{\psi}_c^\dagger(H-\hbar\delta_{cb}-\hbar\delta_2)\tilde{\psi}_c+\hbar\kappa[\psi_b^\dagger\tilde{\psi}_c\exp(-i\boldsymbol{\kappa}_{12}\cdot\mathbf{r})+\tilde{\psi}_c^\dagger\psi_b\exp(i\boldsymbol{\kappa}_{12}\cdot\mathbf{r})], \quad (4)$$

where $\boldsymbol{\kappa}_{12}=\boldsymbol{\kappa}_1-\boldsymbol{\kappa}_2$ is the wave vector difference of the driving light fields. We have introduced the light-induced level shifts δ_i, the detuning from two-photon resonance $\delta_{cb}=\Omega_1-\Omega_2-\omega_{cb}$, and the Raman coupling coefficient κ,

$$\delta_1=\frac{|\mathcal{E}_1|^2d_b^2}{4\hbar^2\Delta_1},\quad \delta_2=\frac{|\mathcal{E}_2|^2d_c^2}{4\hbar^2\Delta_1},\quad \kappa=\frac{\mathcal{E}_1^*\mathcal{E}_2d_bd_c}{4\hbar^2\Delta_1}. \quad (5)$$

The dipole matrix elements d_b and d_c contain the reduced dipole matrix elements and the corresponding nonvanishing Clebsch-Gordan coefficients. To simplify the algebra, we assume κ to be real.

The intensity of the scattered light at position $\mathbf{r}$ is given by

$$I(\mathbf{r})=2c_L\epsilon_0\langle\tilde{\mathbf{E}}_s^-\cdot\tilde{\mathbf{E}}_s^+\rangle, \quad (6)$$

where c_L is the speed of light. Substituting in the expressions for the scattered fields in terms of the atom fields, Eq. (3), generates a sum of terms for the intensity of the form

$$2c_L\epsilon_0\left(\frac{1}{\hbar\Delta_1}\right)^2\int d^3r'd^3r''[\mathbf{K}(\mathbf{d}_b)']^*\cdot\mathbf{K}(\mathbf{d}_b)''(\mathbf{d}_b^*\cdot\tilde{\mathbf{E}}_{d1}^-)\times(\mathbf{d}_b\cdot\tilde{\mathbf{E}}_{d1}^+)\langle\psi_b^{\dagger\prime}\psi_b'\psi_b^{\dagger\prime\prime}\psi_b''\rangle. \quad (7)$$

The primes and double primes, respectively, denote functional dependence on $\mathbf{r}'$ and $\mathbf{r}''$. We now assume that the driving fields have the same wave vectors, so that $\boldsymbol{\kappa}_{12}=\mathbf{0}$. This simplifies the analysis, and may in principle be exactly true, provided the effective two-photon detuning is sufficiently small, as discussed later.

The dynamics of the ground state fields $\tilde{\psi}_c$ and ψ_b follows from the Hamiltonian, Eq. (4). We assume a translationally invariant and noninteracting Bose gas. The matter-field operators are given by the familiar plane wave representations $\psi_b(\mathbf{r}t)=V^{-1/2}\Sigma_{\mathbf{k}}\ (i\mathbf{k}\cdot\mathbf{r})b_{\mathbf{k}}(t)$ and $\tilde{\psi}_c(\mathbf{r}t)=V^{-1/2}\Sigma_{\mathbf{k}}\exp(i\mathbf{k}\cdot\mathbf{r})\tilde{c}_{\mathbf{k}}(t)$, where V is the mode volume. In the absence of light, the center-of-mass motion in both ground states satisfies the dispersion relation $\epsilon_{\mathbf{k}}=\hbar|\mathbf{k}|^2/2m$, with m the atomic mass. Defining the effective two-photon detuning $2\bar{\delta}=\delta_{cb}-\delta_1+\delta_2$ and the condensate oscillation frequency $\Omega_R=(\bar{\delta}^2+\kappa^2)^{1/2}$, the mode operators at time t are given in terms of the operators at time $t=0$ by

$$\tilde{c}_{\mathbf{k}}(t)=e^{i\alpha t}\{A\tilde{c}_{\mathbf{k}}(0)-Bb_{\mathbf{k}}(0)\}, \tag{8a}$$

$$b_{\mathbf{k}}(t)=e^{i\alpha t}\{A^*b_{\mathbf{k}}(0)-B\tilde{c}_{\mathbf{k}}(0)\}, \tag{8b}$$

$$\alpha=\bar{\delta}+\delta_1-\epsilon_{\mathbf{k}}, \tag{8c}$$

$$A=\cos\Omega_R t+\frac{i\bar{\delta}}{\Omega_R}\sin\Omega_R t,\quad B=i\frac{\kappa}{\Omega_R}\sin\Omega_R t. \tag{8d}$$

Before the light is switched on, the atoms in the states $|b\rangle$ and $|c\rangle$ are assumed to be uncorrelated. The Raman fields induce Josephson oscillations between the condensates [9,15]. The expectation values of products of four atom-field operators, such as occurs in Eq. (7), may be evaluated after substituting in expressions (8). For example,

$$\langle\psi_b^{\dagger\prime}\psi_{b\prime}\psi_b^{\dagger\prime\prime}\psi_{b\prime\prime}\rangle|_t=\langle[A\psi_b^{\dagger\prime}+B\psi_c^{\dagger\prime}][A^*\psi_{b\prime}-B\psi_{c\prime}]\times[A\psi_b^{\dagger\prime\prime}+B\psi_c^{\dagger\prime\prime}][A^*\psi_{b\prime\prime}-B\psi_{c\prime\prime}]\rangle, \tag{9}$$

where all the field operators on the right-hand side are evaluated at time zero.

The field operators are sums over the condensate and noncondensate modes. Since we are *only* interested in the change in the number of condensate atoms due to light scattering, we need *only* evaluate terms corresponding to scattering of atoms into or out of the condensate. We ignore scattering of atoms between noncondensate modes. Together with momentum conservation this leads to a considerable simplification of the terms like Eq. (9). Once a particular plane wave mode is chosen for the first factor in Eq. (9), the requirement for a nonzero expectation value determines the modes occurring in all the remaining factors. For example, the part of Eq. (9) relevant to condensate depletion and growth is

$$\langle D_0^\dagger D_-D_-^\dagger D_0+D_+^\dagger D_0D_0^\dagger D_+\rangle, \tag{10}$$

where $D_i=A^*(t)b_i(0)-B(t)c_i(0)$ and the subscripts 0 and $\pm$, respectively, refer to the condensate mode and the noncondensate modes having momenta $\pm\hbar\Delta\boldsymbol{\kappa}$. Here $\Delta\boldsymbol{\kappa}=\Omega\hat{\mathbf{n}}/c_L-\boldsymbol{\kappa}$ is the wave vector change of the scattered photon, and $\hat{\mathbf{n}}=\mathbf{r}/|\mathbf{r}|$ is the unit vector in the light-scattering direction under consideration. The two noncondensate modes $+/-$, respectively, arise from scattering of atoms into/out of the condensate. Note that the particular atomic mode denoted depends on the light-scattering direction $\hat{\mathbf{n}}$, as does the polarization of the scattered light. In general, the polarizations of the emitted photons from the two different atomic transitions are not orthogonal. However, although the resulting interference terms $\langle\tilde{\mathbf{E}}_{s1}^-\cdot\tilde{\mathbf{E}}_{s2}^+\rangle$ and $\langle\tilde{\mathbf{E}}_{s2}^-\cdot\tilde{\mathbf{E}}_{s1}^+\rangle$ are nonvanishing in a particular direction [11], their contribution to the total intensity of the scattered light vanishes after integration over the polar scattering angle ϕ about the laser direction. This is because these terms are proportional to $\exp(\pm 2i\phi)$, whose integral from 0 to 2π vanishes.

For brevity we assume that the numbers of atoms in the ground noncondensate states are the same, $n_\pm=\langle b_\pm^\dagger b_\pm\rangle=\langle c_\pm^\dagger c_\pm\rangle$, and that $n_+=n_-=n$ due to isotropy. Further simplification occurs if we assume that there are equal numbers of atoms N in each condensate and that the laser intensities are chosen so that the level shifts are equal $\delta=\delta_1=\delta_2$. Evaluating all the relevant terms in Eq. (6) we find the following expressions for the intensity due to scattering of atoms into and out of the condensates:

$$I_{\text{in}}=2Cn\delta d_s^2\{N+2\text{Re}[A^*B\langle c_0^\dagger b_0\rangle]d_d^2\}, \tag{11}$$

$$I_{\text{out}}=2C(n+1)\delta d_s^2\{N+\text{Re}[(A^{*2}-B^2)\langle c_0^\dagger b_0\rangle]\}, \tag{12}$$

$$C=\frac{c_L|\boldsymbol{\kappa}|^4}{8\pi^2\epsilon_0\Delta_1}\frac{1}{|\mathbf{r}|^2}\left(1-\frac{1}{2}\sin^2\theta\right), \tag{13}$$

where $d_s^2=d_b^2+d_c^2$ and $d_d^2=(d_b^2-d_c^2)/d_s^2$. We next integrate these intensities over the Josephson oscillation period $P=2\pi/\Omega_R$. We find the time-averaged intensities

$$\frac{1}{P}\int_0^P I_{\text{in}}dt=C'n\left\{1+\frac{\bar{\delta}\kappa}{\Omega_R^2}d_d^2\cos\Theta\right\}, \tag{14}$$

$$\frac{1}{P}\int_0^P I_{\text{out}}dt=C'(n+1)\left\{1+\frac{\kappa^2}{\Omega_R^2}\cos\Theta\right\}, \tag{15}$$

where $C'=2C\delta d_s^2N$ and Θ is the condensate phase difference. The noncondensate populations n are functions of $\Delta\boldsymbol{\kappa}$ and hence of the scattering angle θ between $\hat{\mathbf{n}}$ and the laser propagation direction. Our final step is integration over all scattering directions. This yields the total scattered light intensity and hence the total atom transition rates. The angular integration has the effect of replacing n by

$$\frac{8\pi}{3}\tilde{n}\equiv 2\pi\int_0^\pi\left(1-\frac{1}{2}\sin^2\theta\right)n(\theta)\sin\theta d\theta, \tag{16}$$

and $n+1$ by $8\pi(\tilde{n}+1)/3$. This integral may be interpreted as the number of noncondensate atoms available for scattering into the condensates. In an infinite homogeneous system the integral is divergent at the low-energy end. However, for a finite, trapped system a low-energy cutoff is provided by the first excited state. We assume a Bose-Einstein distribution at $T=400$ nK and a trap frequency of 100 Hz, corresponding to a low-energy cutoff of $\hbar(2\pi\times 100\ \text{s}^{-1})$. With these parameters a numerical integration of Eq. (16) gives $(8\pi/3)\tilde{n}\approx 65$ for rubidium. However, this is a crude estimate since realistic systems are not expected to be in thermal equilibrium.

Assuming, for simplicity, equal dipole moments $d_d=0$ the net rate of scattering of atoms into the condensates (atoms per second) is then, from Eqs. (14)–(16),

$$R=-6\pi\left(\frac{\gamma}{\Delta_1}\right)^2\left(\frac{I_d}{\hbar c_L|\boldsymbol{\kappa}|^3}\right)N\left\{1+\frac{\kappa^2}{\Omega_R^2}\cos\Theta[1+\tilde{n}]\right\}, \tag{17}$$

where $\gamma=d_b^2|\boldsymbol{\kappa}|^3/(3\pi\epsilon_0\hbar)$ is the free space spontaneous emission rate and I_d is the intensity of the lasers. Condensate growth corresponds to a positive rate R. Assuming that $\tilde{n}\gg 1$ this is equivalent to the following requirement on the condensate phase difference:

$$\cos\Theta<-\frac{1}{\tilde{n}}\frac{\Omega_R^2}{\kappa^2}. \tag{18}$$

This inequality is fulfilled by particular negative values of $\cos\Theta$ provided that $\tilde{n}$ is sufficiently large and that the effective two-photon detuning $2\bar{\delta}$ is sufficiently small. The latter may be chosen small by manipulating the light-induced level shifts or the relative frequency of the driving light beams. The prior assumption of equal wave numbers for the driving light fields is not very restrictive for the relative frequency, because an atom trap introduces an uncertainty for the momentum conservation. In the limit of small two-photon detuning the effective linewidth of the transition $c\rightarrow b$ may have an effect. However, it may be shown to be proportional to Δ_1^{-2} or smaller.

With a 1μm wavelength, $1\,\mathrm{mW\,cm^{-2}}$ laser intensity, and $(\kappa^2/\Omega_R^2)\cos\Theta=-1/2$, the growth rate Eq. (17) is

$$R\approx N\tilde{n}\left(\frac{\gamma}{\Delta_1}\right)^2(10^7\ \mathrm{s}^{-1}). \tag{19}$$

A laser detuning of $\Delta_1/\gamma=10^3$, condensates with $N=10^3$, and $(8\pi/3)\tilde{n}=65$ give a condensate growth rate of $R\approx 7\times 10^4$ atoms per second. This rate is large enough to be useful for both atom laser pumping and for condensate growth. However, sustained growth will require repopulation of the relevant noncondensate atom modes by atom-scattering processes. Other limitations on growth include diffusion of the condensate phase difference due to atom-atom interactions [22,23] and heating of the noncondensate atom fraction by Raman transitions.

Our conventional argument that BEC's are in coherent states is by no means necessary. In fact the relative phase between the two condensates has been established in stochastic simulations of the measurements of spontaneously scattered photons, even though the condensates are initially in pure number states [13]. Without any measurements of the condensate phase difference the macroscopic quantum coherence is expected to undergo collapses and revivals [23]. Because the detections of spontaneously scattered photons establish the relative phase, they could also stabilize the phase against the collapse of the macroscopic wave function.

We have shown that destructive quantum interference enables the growth of two Raman-driven Bose condensates. The mechanism is analogous to that for gain in the laser without inversion. Although we have considered a particular configuration, the interference mechanism might be generalized to other atomic level schemes.

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