Echo in optical lattices: Stimulated revival of breathing oscillations

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We analyze a stimulated revival (echo) effect for the breathing modes of atomic oscillations in optical lattices. The effect arises from the dephasing due to the weak anharmonicity being partly reversed in time by means of additional parametric excitation of the optical lattice. The shape of the echo response is obtained by numerically simulating the equation of motion for the atoms with subsequent averaging over the thermal initial conditions. A qualitative analysis of the phenomenon shows that the suggested echo mechanism combines the features of both spin and phonon echoes. [S1050-2947(98)08305-X]

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I. INTRODUCTION

The coherent manipulation of the atomic center-of-mass motion in optical lattices [1] by means of the nonstationary off-resonance dipole potentials has provided experimental capabilities to study dynamical systems with time-dependent potentials [2–5]. In the off-resonant coherent regime, the parametric nonadiabatic excitations of the optical lattice give rise to oscillations of the atomic momentum and coordinate distribution dispersions (breathing modes) [3–5] and may be used for the manipulation of the coordinate or momentum dispersions of the atomic distribution by means of squeezing in phase space [4].

The squeezed states of matter in various systems have been extensively discussed in connection with the possibility of overcoming the standard quantum limit for noise imposed by vacuum fluctuations (see, e.g., [6,7]). In an optical lattice, the squeezing effect can be significant in the classical regime producing classical squeezed states [8] and resulting in the reduction of thermal fluctuations.

Recently, the breathing modes of atomic oscillations in optical lattices have been observed experimentally with the use of the Bragg scattering techniques based on the fact that the cross section of the Bragg diffraction scales with the Debye-Waller factor, which is related to the dispersion of the atomic coordinate distribution [3,5]. The observed decay of the oscillations may be due to both the dissipative and the dephasing effects.

In classical oscillating systems with anharmonicity, the frequencies have a continuous distribution due to the energy-dependent corrections. This leads to the dephasing of small oscillations [6]. Together with possible irreversible dissipation, the dephasing determines an apparent damping of the oscillations. The dephasing, caused by the nonlinearity effects, can lead to partial revivals under certain circumstances. Examples of such revivals are the phonon (see [9]), spin, and photon echo effects [10,11].

One has to distinguish between the classical stimulated revival (echo) caused by external perturbations applied to the oscillating system and the quantum revival related to the discrete structure of the energy levels of the system that does not require any external disturbances [6,12]. As will be shown below, these two revivals can have different time scales and therefore can be separated.

In this paper we show that an analog of the phonon echo effect exists for the breathing modes of atomic oscillations in optical lattices. However, the proposed echo mechanism possesses features different from those of the conventional phonon echo mechanism. First, the proposed mechanism is related to the oscillations of the dispersion of the atomic distribution as opposed to the average coordinate and momentum oscillations involved in the phonon echo model [9]. In contrast to the conventional model of phonon echo [9], the excitation of the system in our model is achieved by means of the parametric modulation of the frequency of the atomic oscillations as opposed to applying a time-dependent external force to the atoms [9]. As shown below, the suggested echo mechanism incorporates the features of both the spin (photon) [10,11] and the phonon echo mechanisms due to the specific nature of the parametric excitation of the oscillations. This effect can be employed to study the revival and damping mechanisms of the breathing modes in optical lattices.

II. MODEL AND NUMERICAL RESULTS

An atom subjected to an off-resonance laser field experiences an energy shift of the ground state proportional to the intensity of the field. If the field is formed by a standing wave with large detuning, the effective potential for the atoms in the ground state is given by [13]

$$U(x)=V[1-\cos(2qx)], \tag{1}$$

where x denotes the atomic center-of-mass coordinate, V is the amplitude of the dipole potential (proportional to the intensity of the laser field), and q is the wave vector of the laser field. Normally, the initial state of the system is formed by Doppler cooling, so that the trapped atoms are in thermal equilibrium near the minima of the optical potential and form an optical lattice. In this paper we will consider the case of temperatures T much larger than the energy of the atomic oscillations at the bottom of the potential. Consequently, no quantum effects will be taken into account. On the other hand, the temperature should be low enough to ensure that the atoms do not escape from the potential well. If the magnitude of the optical lattice potential is time dependent, then

$$V=V_0[1+k(t)], \tag{2}$$

where $k(t)$ describes the parametric excitation of the lattice ($V_0=$ const). Therefore, the effective potential can be approximated as an oscillator with weak anharmonicity and time-dependent frequency. The harmonic frequency corresponding to the potential (1) is given by

$$\omega_0=2q\sqrt{\frac{V_0}{m}}, \tag{3}$$

where m is the atomic mass. We assume that the parametric excitation of the system occurs in the form of two short pulses

$$k(t)=s_1\delta(t-t_1)+s_2\delta(t-t_2), \tag{4}$$

where $s_{1,2}$ and $t_{1,2}$ characterize the intensities and the instants when the pulses are applied to the optical lattice, respectively. In what follows we assume that $t_1=0$ and $t_2=t_p$. The equation of motion is given by

$$\ddot{\tilde{x}}=-\frac{1}{2}\sin(2\tilde{x})[1+2\xi_1\delta(\tau)+2\xi_2\delta(\tau-\tau_p)], \tag{5}$$

where we introduced the dimensionless variables

$$\tilde{x}(\tau)=qx(\tau), \qquad \tau=\omega_0 t, \tag{6}$$

with $\xi_1=\omega_0 s_1/2$, $\xi_2=\omega_0 s_2/2$, $\tau_1=\omega_0 t_1=0$, and $\tau_2=\tau_p=\omega_0 t_p$.

The physics of the echo effect in the optical lattice can be understood as follows. We assume that initially the atomic system is in thermal equilibrium characterized by the values of coordinate and momentum dispersions $m\omega_0^2\langle x^2\rangle_T=\langle p^2\rangle_T/m=k_BT$. The first pulse initiates the squeezing of the atomic phase-space distribution creating the classical squeezed states [8]. In harmonic systems, such squeezing results in oscillations of $\langle x(t)^2\rangle$ and $\langle p(t)^2\rangle$ with the frequency $2\omega_0$. The nonlinearity leads to dephasing and therefore to the apparent damping of these oscillations. Since the dephasing is reversible in time, one expects to observe at least a partial revival of the oscillations after the application of the second pulse to the system. A qualitative physical picture of the phenomenon will be discussed in the next section. Here we concentrate on the numerical results.

In order to study the echo effect, we performed a numerical simulation using the equation of motion (5) with subsequent averaging of $\tilde{x}^2(\tau)$ over the initial conditions $\tilde{x}(0),\dot{\tilde{x}}(0)$ distributed in accordance with the initial thermal ensemble. The results for the normalized average dispersion $\langle\tilde{x}^2(\tau)\rangle/\langle\tilde{x}^2\rangle_T$ are presented in Fig. 1. One can see the expected damping of the squeezing oscillations after each pulse due to the dephasing effect. The characteristic damping rate increases with an increase of the temperature. The most remarkable feature of the time evolution of $\langle\tilde{x}^2(\tau)\rangle$ is the pronounced echo effect at $\tau\approx 2\tau_p$.

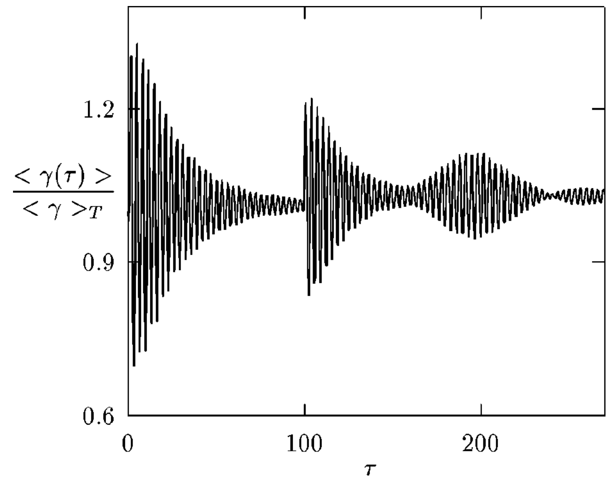

FIG. 1. Numerical simulation of the echo effect discussed in the text, $\gamma(\tau)=x^2(\tau)$. The parameters for the simulations are $\tau_p=105$, $\xi_1=0.2$, $\xi_2=0.1$, and $\theta=T/4V_0=0.03$.

The revival time is approximately $2\tau_p$ according to the general considerations involved in the two-pulse phonon and the spin echo models. Note, however, that the revival of $\langle\tilde{x}^2(\tau)\rangle$ is only partial (approximately 0.25 of the initial amplitude).

In Fig. 2 we present the dependence of the echo amplitude on the intensity of the second pulse. The echo amplitude is very small for small values of ξ_2; then it increases sharply in the intermediate region of ξ_2 and saturates in the nonlinear regime.

III. QUALITATIVE ANALYSIS: COMPARISON WITH THE PHONON AND SPIN ECHO

In order to clarify the mechanism of the proposed echo effect and compare it with the other echo mechanisms, we consider the approximate analytical solution of Eq. (5) valid for the low temperature (small anharmonicity). In this case, it is sufficient to take into account only the first (quartic) correction to the effective harmonic potential. Equation (5) reduces to

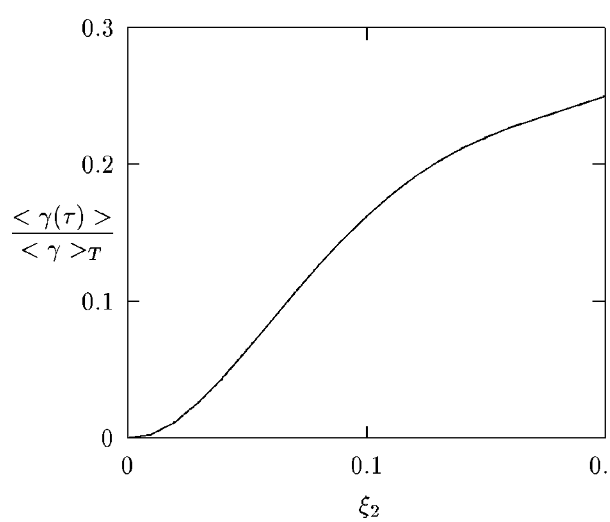

FIG. 2. Maximum echo amplitude as a function of the intensity ξ_2 of the second pulse.

$$\ddot{\tilde{x}} = -\tilde{x}[1+2\xi_1\delta(\tau)+2\xi_2\delta(\tau-\tau_p)]+\frac{2}{3}\tilde{x}^3, \qquad (7)$$

where we neglected the modulation of the nonlinear term, assuming that the amplitudes of the pulses ξ_1 and ξ_2 are sufficiently small. For $\tau\neq 0,\tau_p$, Eq. (7) has an integral of motion corresponding to the energy of the atomic system $\tilde{E} = E/4V_0$ (i.e., there are three different values of energy E_k: $k=0$, 1, and 2 for times $\tau<0$, $0<\tau<\tau_p$, and $\tau>\tau_p$, respectively).

For small anharmonicity and amplitudes of the external pulses, the solution of Eq. (7) for $\gamma(\tau)=\tilde{x}^2(\tau)$ up to the second-order terms in $\tilde{E}_k$ can be obtained in the form

$$\gamma(\tau)=C_k+\{A_k\exp[2i\omega(\tilde{E}_k)\tau]+B_k\exp[4i\omega(\tilde{E}_k)\tau]+\text{c.c.}\},$$

$$C_k=\tilde{E}_k+\frac{3}{4}\tilde{E}_k^2, \qquad (8)$$

where $k=0,1,2$. From Eq. (7) it follows that $A_k=\tilde{E}_k(1+\frac{2}{3}\tilde{E}_k)\exp(2i\phi_k)/2$ and $B_k=-A_k^2/6$, where ϕ_k corresponds to the phase of the atomic oscillations in different time intervals. Note that A_k are the complex constants characterizing both the amplitude and phase of the atomic oscillations and $\omega(\tilde{E})$ stands for the effective frequency. This frequency is determined in the limit $\tilde{E}\to 0$ as

$$\omega(\tilde{E})=1-\frac{1}{2}\tilde{E}. \qquad (9)$$

The solution given by Eqs. (8) and (9) corresponds to the first term of the asymptotic expansion for the anharmonic oscillator in the limit of weak anharmonicity and large evolution time [14].

Each pulse in Eq. (7) results in a jump of $\dot{\gamma}$ and $\tilde{E}$, while γ remains continuous. We obtain from Eq. (7)

$$\dot{\gamma}(\tau_s^+)-\dot{\gamma}(\tau_s^-)=-4\xi_k\gamma(\tau_s),$$

$$\tilde{E}(\tau_s^+)-\tilde{E}(\tau_s^-)=2\xi_k^2\gamma(\tau_s)-\xi_k\dot{\gamma}(\tau_s^-), \qquad (10)$$

where $\tau_s=0,\tau_p$; τ_s^+ and τ_s^- denote the moment of time just before and just after each pulse, and $k=1,2$. After the application of the first pulse, we obtain from Eqs. (8) and (10), for small ξ_1,

$$A_1=A_0+i\xi_1(\tilde{E}_0+2A_0) \qquad (11)$$

and

$$\tilde{E}_1=\tilde{E}_0-2i\xi_1(A_0-A_0^*). \qquad (12)$$

Combining Eqs. (11) and (9) and averaging over the random initial phase and the thermally distributed initial energy in the limit of low temperatures ($\Theta\equiv T/4V_0\ll 1$), we obtain the time evolution of the coordinate dispersion in the form

$$\langle\gamma(\tau)\rangle=\Theta+\Theta\left(i\xi_1\frac{\exp(2i\tau)}{(1+i\tau/\tau_c)^2}+\text{c.c.}\right), \quad \tau_p>\tau>0, \qquad (13)$$

with the dephasing rate of oscillations defined by

$$\tau_c^{-1}=\Theta. \qquad (14)$$

Equation (13) indicates that the amplitude of the oscillations of the coordinate dispersion exhibits an apparent damping. As discussed above, this damping is caused by the anharmonicity, which results in the dependence of the effective oscillation frequency on energy, and the thermal energy distribution leading to an averaging out of the oscillations. Therefore, this damping is reversible in time. The dephasing rate given by Eq. (14) is in good qualitative agreement with the numerical results discussed above. Fitting the data presented in Fig. 1 for $0<\tau<\tau_p$ by Eq. (13) gives the value $\tau_c^{-1}\approx 1.1\Theta$ for the dephasing rate in our numerical simulations.

Now we will analyze the effect of the second pulse. Applying Eq. (10) for $\tau_s=0$ and $\tau_s=\tau_p$ consecutively, one obtains the amplitude A_2 in Eq. (8) after the second pulse ($t>\tau_p$)in the limit of small ξ_2,

$$\tilde{A}_2(\tau_p)\approx\tilde{A}_1(\tau_p)(1+2i\xi_2)+i\xi_2\tilde{E}_1-\frac{1}{3}i\xi_2\tilde{E}_1\tilde{A}_1^*(\tau_p), \qquad (15)$$

where $\tilde{A}_{1,2}(\tau)=A_{1,2}e^{2i\omega(\tilde{E}_{1,2})\tau}$ correspond to the envelope solution for $\gamma(\tau)$ given by Eq. (8). The $\tilde{A}$ vectors rotate with the frequencies $2\omega(\tilde{E}_{1,2})$ in the complex plane. The energy after the second pulse is given by

$$\tilde{E}_2=\tilde{E}_1-2i\xi_2(A_1e^{2i\omega(\tilde{E}_1)\tau_p}-\text{c.c.}), \qquad (16)$$

with A_1 and $\tilde{E}_1$ defined by Eqs. (11) and (12).

In order to obtain a qualitative picture of the echo effect, we disregard the energy change due to both pulses upon the frequencies $\omega(\tilde{E}_{1,2})$. Equation (15) indicates that the amplitude $\tilde{A}_2(\tau_p)$ after the second pulse is a linear combination of the amplitude before the pulse $\sim\tilde{A}_1(\tau_p)$ and the amplitude $\tilde{A}_{2e}(\tau_p)=-i\xi_2\tilde{E}_1\tilde{A}_1^*(\tau_p)/3$. Note that $\tilde{A}_{2e}(\tau_p)$ is proportional to the complex conjugate of $\tilde{A}_1(\tau_p)$.

In Fig. 3, we present an illustration of the echo mechanism described above. Two vector amplitudes $\tilde{A}_1$ and $\tilde{A}_1'$ correspond to the two different values of the initial atomic energies and rotate with the frequencies $\omega_1>\omega_1'$ in a complex plane. If the two vectors have the same phase $\phi=0$ at $\tau=0$ (the horizontal dotted line in Fig. 3), then the phase difference between $\tilde{A}_1$ and $\tilde{A}_1'$ is accumulated over the time interval τ_p. The second pulse at $\tau=\tau_p$ mixes the amplitudes with their complex conjugates so that $\tilde{A}_1$ and $\tilde{A}_1'$ acquire the components $\delta\tilde{A}_1\sim\tilde{A}_1^*$ and $\delta\tilde{A}_1^*\sim\tilde{A}_1'^*$, respectively. These components are shown as the mirror reflections with respect to the line of zero phase. Since for $\tau>\tau_p$ the new vectors continue rotating with the same frequencies and in the same direction, the phase difference between the vectors $\tilde{A}_1$ and $\tilde{A}_1'$ increases, contributing to the dephasing effect. In contrast, the phase difference between $\delta\tilde{A}_1$ and $\delta\tilde{A}_1'$ vanishes at the time $\tau=2\tau_p$, giving rise to the echo effect analogous to the well-known spin and photon echo [10,11]. Taking into

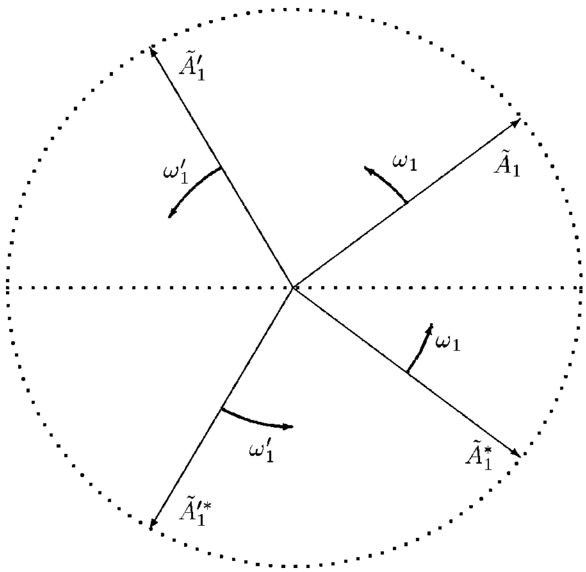

FIG. 3. Illustration of the echo mechanism.

account Eqs. (8)–(10) and (15), one arrives at the echo contribution after thermal averaging in the lowest order with respect to ξ_1, ξ_2, and Θ,

$$\langle \gamma(\tau)\rangle_e = \Theta^2 \quad \xi_1 \xi_2 \frac{\exp[2i(\tau - 2\tau_p)]}{[1 + i(\tau - 2\tau_p)/\tau_c]^3} + \text{c.c.}, \qquad \tau > \tau_p . \tag{17}$$

One can see that the amplitude of the echo is maximal at the time $\tau = 2\tau_p$. Note that the echo amplitude given by Eq. (17) is proportional to the product $\xi_1\xi_2$ in contrast to the dependence $\langle\gamma\rangle_e \sim \xi_1\xi_2^2$ typical for the phonon echo mechanism (for small ξ_1, ξ_2) as a response to the external resonant forces [9]. The mechanism described above also leads to the terms $\xi_1\xi_2^2$ omitted in Eq. (15). According to the conventional model, the phonon echo originates from the change of the rotation frequency of the $\widetilde{A}$ vectors due to the external pulses. This mechanism is also present in our model if we take into account the change of the energy $\widetilde{E}$ in each time interval. In this case, the main contribution comes from the second term on the right-hand side of Eq. (15). However, this effect is proportional to $\xi_1\xi_2^2$ and vanishes for small pulse intensities. Analytical calculations in this case become rather cumbersome and we do not present them here. Since we disregard the change of the rotating frequency due to the second pulse, the approximation leading to Eq. (17) is valid if $\xi_2 \widetilde{E}_0 \tau_p \ll 1$. This condition can only be satisfied in the limit $\xi_2 \ll 1$ for large evolution time ($\tau_p \gg 1$). This is a consequence of the asymptotic nature of the proposed analytical solutions [14]. The extension of our results to the quantum limit ($T \sim h\omega_0$) is straightforward and can be achieved by replacing the continuous Boltzmann distribution with a quantum one on the stage of thermal averaging.

Note that the echo effect analyzed above is a stimulated classical revival of breathing oscillations, in contrast to the effect of spontaneous quantum revival [12] caused by the discrete nature of the energy levels in quantum systems. We estimated a characteristic time of the quantum revival t_r (or $\tau_r = \omega t_r$) by extending the method of Ref. [12] to finite temperatures. The ratio τ_c/τ_r is $4\hbar\omega_0/3\pi T$ and therefore $\tau_r \gg \tau_c$ in the classical limit. Since the classical echo mechanism described above takes place in the time domain $\tau \approx 2\tau_p$, the quantum revival will not contribute to the effect if $\tau_p < \tau_r$. For the value of $\Theta = 0.03$ used in our numerical simulations and assuming that the number of levels in the potential well (1) is $N \approx 100$, we obtain $\tau_c \approx 30$ and $\tau_r \approx 300$. Since $2\tau_p \approx 200$, the above-mentioned condition is satisfied.

IV. CONCLUSION

We investigated the dephasing and stimulated revival (echo) of the breathing oscillations in optical lattices subject to two-pulse parametric excitation created by the abrupt change of off-resonance dipole optical potential induced by the laser field. The dephasing originates from the energy dependent corrections to the oscillation frequency and due to the thermal distribution of energy. The shape of the echo response was obtained by numerical simulation using the equation of motion for the atoms with subsequent averaging over the initial thermal distribution. An approximate analytical analysis demonstrates that the suggested echo mechanism incorporates the essential physics features of the spin (photon) echo, where the echo effect is caused by the change of the phase of the oscillations due to the second pulse, as well as the phonon echo, where the effect is caused by the change of the oscillation frequency. An optimal control analysis would be desirable (e.g., as proposed for the spin systems [15]) for optimizing the echo response in view of the rapid development of experimental techniques (as in, e.g., [2,5,16,17]) for the coherent manipulation of the atomic motion.

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