Bound-free heavy-lepton pair production for photon and ion impact on atomic nuclei

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Finite nuclear size is found to cause reduction in photoproduction of μ and τ pairs with bound negative lepton. The reduction is less dramatic for muons than claimed previously but very severe for experimental production of tauonic atoms. For both types of leptons, L-shell production exceeds K-shell production in heavy targets, and light targets are more favorable for ground-state τ-atom production. Implications for heavy-ion collisions are outlined. [S1050-2947(98)10205-6]

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I. INTRODUCTION

The large electromagnetic fields encountered in relativistic heavy-ion collisionse are expected to result in a substantial production of pairs of heavy leptons [1–3]. It has been predicted that in nongrazing collisions with small impact parameters, the produced negative lepton has a non-negligible probability to end up bound to one of the bare ions. This latter process is potentially a new and unique way to produce exotic atoms of short lived species such as tauonic atoms. For the heaviest leptons, the earliest estimate for the number of produced bound-free pairs at The Relativisitc Heavy-Ion Collider at Brookhaven National Laboratory (RHIC) has been given by Gould [2], who assumes a RHIC design of counter-rotating beams of fully stripped uranium at 100 GeV/amu. With a luminosity of 10^{27} cm^{-2} s^{-1} his estimate yields a number of one τ pair per second, which, undoubtedly, is an upper limit, since effects of the finite nuclear size have been ignored. Such effects will reduce the production cross sections and will be extremely important for the heavy τ, as we shall elaborate in this paper.

Traditionally exotic atoms are produced through slowing down of negative particles (μ^-, π^-, or K^-) in matter where they are first captured into high orbits of the atomic target and then cascade down into deeper states by emitting Auger electrons or x rays [4]. However, the time it takes to produce, transport, slow down, capture, and cascade the negative particles into deeply bound atomic states is typically of the order of nanoseconds. As a consequence, this method is not suited for the production of atoms with very short lived species such as τ's, D-mesons, B-mesons or, even, pions and kaons in deeply bound states. An alternative and potentially more efficient method for creating these exotic atoms is by using high-energy photons, which may convert in a target of high atomic number Z to a heavy particle-antiparticle pair with the negative partner created directly in an atomic state of a heavy nucleus. In the present paper we report on a study of such production of tauonic and muonic atoms for photon impact on a fixed target. Implications for bound-free heavy lepton pair production in heavy-ion collisions are discussed in the last part of the paper. Throughout, we restrict the study to cases where the capturing nucleus remains intact during the lepton production.

Electron-positron pair production upon photon impact on atomic nuclei is well studied; for the case of production of pairs of unbound particles, we may refer the reader to the classic textbook by Heitler [5], and for the case of so-called bound-free pair production on bare nuclei where the electron ends up being bound to the nucleus, we may refer to [6] as well as to papers cited there. The situation for photoproduction of pairs of heavy leptons with the negative particle bound to the nucleus is quite different. Here little has been published. As compared to the electron-positron case, cross sections for point nuclei are reduced by the square of the ratio of the mass of the heavy lepton to that of the electron. In addition, finite nuclear size causes further depletion. When using the Compton wavelength for the created leptons as the unit of measure, the nuclear size is much smaller than unity for electrons (that is, the nucleus appears essentially pointlike), of the order of unity for μ's, and several tens of units for τ's. Below, we shall study the effect of the nuclear size on the cross section for production of a bound-free lepton pair as well as its dependence on the atomic number of the target, the final state of the produced exotic atom, and also the dependence of the cross section on photon energy. In particular, we shall demonstrate that reduction due to finite nuclear size is basically fatal for bound-free τ production, at least as long as we require the nucleus to remain intact. For production of μ's, on the other hand, we find less reduction than has been claimed previously by others, cf. [7].

In the calculations presented below, we represent the positive lepton by a plane wave. This is an obvious approximation in case of τ production where the depth of the nuclear potential is only a few percent of the τ mass. We also expect the approximation to be reasonable for μ production, at least not too close to threshold. In any case, for μ and τ production, corrections due to nonperturbative effects are much smaller than those due to finite nuclear size.

II. GENERAL EXPRESSIONS

The cross section for bound-free pair production for photon impact may be written as

$$\frac{d\sigma}{d\Omega}=\frac{\alpha p_+E_+}{2\pi k_0}\frac{1}{2}\sum_{\lambda,i,b}|M|^2. \tag{1}$$

Here α is the fine-structure constant, p_+ and E_+ are the momentum and energy of the positively charged lepton, and k_0 is the momentum of the incoming photon (momenta and energies are given in natural units, that is, in units of mc and mc^2, where m is the lepton mass). The sums are over spin directions of the positive lepton (Σ_i), over magnetic quantum number for the bound lepton (Σ_b), and average over direction of photon polarization ($\frac{1}{2}\Sigma_\lambda$). The matrix element is given as

$$M=\int d^3r\,\psi_b^\dagger(\mathbf{r})\alpha_\lambda e^{i\mathbf{k}_0\cdot\mathbf{r}}\psi_i(\mathbf{r}), \tag{2}$$

where ψ_b denotes the Dirac spinor for the bound, negatively charged lepton and ψ_i is the initial Dirac spinor belonging to the negative-energy continuum. The quantity α_λ is the projection of the Dirac α matrices on the direction of photon polarization. Throughout, the unit of length is the Compton wavelength $\lambda\!\!\!^{-}_C\equiv\hbar/mc$ and the unit of cross section is $\lambda\!\!\!^{-2}_C$. See, e.g., [5] or [6] for details.

With free particle solutions substituted for ψ_i in Eq. (2), the expression (1) for the cross section reduces to

$$\frac{d\sigma}{d\Omega}=\pi\alpha\frac{p_+(E_++1)}{k_0}\left\{|\kappa|\left[g^2(q)+\left(\frac{p_+}{E_++1}\right)^2f^2(q)\right]\right.$$
$$\left.-2\kappa\frac{p_+}{E_++1}\,\frac{\cos\theta(k_0-p_+\cos\theta)}{q}g(q)f(q)\right\}. \tag{3}$$

The momentum transfer q is given in terms of the minimum value $q_{\min}=k_0-p_+=k_0-\sqrt{E_+^2-1}$ and the emission angle θ of the positive lepton relative to the direction of the incoming photon as

$$q^2=q_{\min}^2+2p_+k_0(1-\cos\theta)=q_{\min}^2+4p_+k_0\sin^2\theta/2, \tag{4}$$

and $d\Omega=\sin\theta d\theta d\phi$ refers to the emission of the positive lepton. The energy of the latter is of course fixed by the energies of the incoming photon and the bound, negatively charged lepton as $E_+=k_0-E_-$, where the energy E_- of the bound lepton is slightly less than unity. The quantity κ relates to the total angular momentum quantum number j of the populated bound state as $\kappa=\mp(j+\frac{1}{2})$, where the upper sign applies for states where the orbital angular momentum quantum number l of the large component equals $j-\frac{1}{2}$ while the lower applies for $l=j+\frac{1}{2}$. The quantities $g(q)$ and $f(q)$ are given in terms of the radial part of the large and small component, $g(r)$ and $f(r)$, of $\psi_b(\mathbf{r})$ as

$$g(q)=(2/\pi)^{1/2}\int_0^\infty dr\,r^2g(r)j_l(qr),$$

$$f(q)=(2/\pi)^{1/2}\int_0^\infty dr\,r^2f(r)j_{\bar{l}}(qr), \tag{5}$$

where j_l denotes a spherical Bessel function of order l and $\bar{l}\equiv l\pm1=j\pm\frac{1}{2}$ (again, upper and lower sign correspond to upper and lower sign in the relation $l=j\mp\frac{1}{2}$).

As a reference, we shall use the so-called Sauter cross section

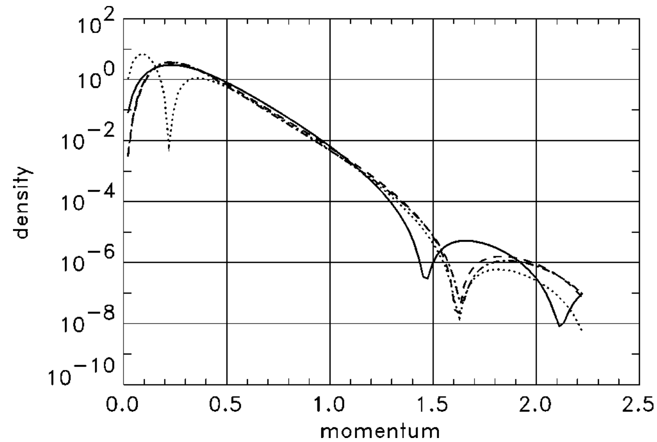

FIG. 1. Normalized momentum-space density $q^2[g^2(q)+f^2(q)]$ as a function of momentum q for a negative μ bound in the K or the L shell of an otherwise bare uranium ion. The full-drawn curve is for the $1s_{1/2}$ state, the dotted for $2s_{1/2}$, the dashed for $2p_{1/2}$, while the chained curve displays the density for the $2p_{3/2}$ state. The uranium nucleus is assumed to be a homogeneously charged sphere with a radius of 7.31 fm, which yields a potential depth of 27.17 MeV, and the binding energies of the four states are 12.12 MeV, 4.30 MeV, 5.92 MeV, and 5.69 MeV, respectively. Momenta are given in units of mc.

$$\sigma_0=4\pi\alpha(\alpha Z)^5/k_0, \tag{6}$$

which applies for pair creation with K-shell capture for pointlike, low-Z nuclei at high energies ($\alpha Z\ll1$, $k_0\gg1$), cf. [6]. The expression (6) may be verified directly from Eq. (3) by application of the analytical expressions given in [8] for the ground-state momentum waves. Note that in the perturbative, high-energy Coulomb case, the g^2 term in the differential cross section (3) gives a contribution to the total cross section of $8\sigma_0/3$, the f^2 term similarly brings a total of σ_0, while the last term in curly brackets in Eq. (3) contributes $-8\sigma_0/3$ when integrated over angles.

III. RESULTS

A. μ production

Consider first μ production. In order to compute cross sections we solve the Dirac equation numerically to obtain $\psi_b(\mathbf{r})$ for bound μ states in the field of an extended nucleus whose charge is distributed homogeneously within a radius of 1.18 fm$\times A^{1/3}$, where A is the mass number [9]. The momentum waves $g(q)$ and $f(q)$ to be applied in Eq. (3) are obtained subsequently by numerical transformation according to Eq. (5). As an example, Fig. 1 shows the momentum-space density $q^2(g^2+f^2)$ for a negative μ bound in the K or the L shell of a bare heavy ion. For all states the density peaks at rather low momenta and then, beyond roughly 0.5 units, falls off rapidly by several orders of magnitude over one momentum unit. This rapid fall-off is an effect of the finite nuclear size. It may be noted that for the μ, the level of the momentum content is essentially the same if the Fermi distribution of the nuclear charge is chosen. It may also be noted that the momentum-space densities obtained in a nonrelativistic calculation are close to those displayed in Fig. 1: At momenta below 0.5, the nonrelativistic densities would

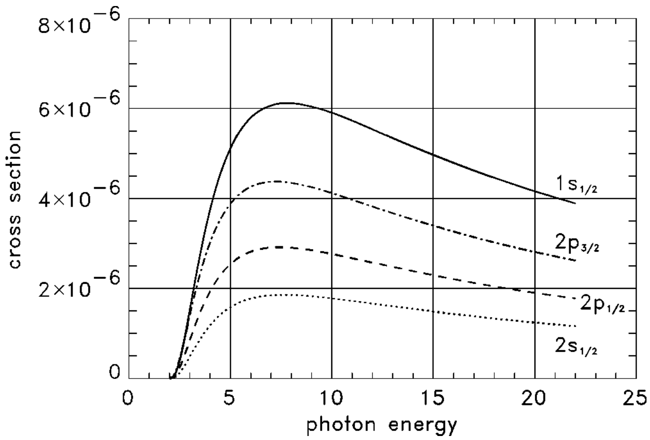

FIG. 2. Cross section for photon induced μ pair production with the negative μ bound in the K or the L shell of a bare uranium ion as a function of the photon impact energy. Curves are labeled as in Fig. 1 and the nucleus is again assumed to be a homogeneously charged sphere. The photon energy is given in units of the μ rest energy mc^2 and the cross section is given in units of 3.488 $\times 10^{-26}$ cm^2, the square of the μ Compton wavelength.

hardly be distinguishable from the relativistic ones if plotted on the figure. In the region between 1 and 2 units, where the small component becomes comparable to the large, the relativistic result is higher than the nonrelativistic only by at most a factor of 2–3.

With the rapid fall-off of the momentum density, the cross section is essentially set by the momentum content of the populated bound state near the minimum momentum transfer, which varies only moderately with photon energy: it decreases monotonously from $1+E_-$ at threshold to E_- at infinity, that is, $q_{\min}$ varies roughly between 2 and 1 (in natural units). From the last expression in Eq. (4) we then find that only angles where $4p_+k_0\sin^2\theta/2 \lesssim K^2 q_{\min}^2$ contribute to the total cross section, the quantity K attaining a value in the vicinity of 1/2. This reduces to

$$\theta \lesssim K/k_0 \tag{7}$$

beyond one or two units above threshold; hence characteristic emission angles show the usual $1/k_0$ scaling.

Figure 2 shows total cross sections corresponding to the cases selected in Fig. 1. The structure is quite similar to that found for pointlike nuclei, cf. [6], with a peak a few units above threshold followed by a fall-off essentially inversely proportional to the impact energy. Well above the peak, which appears somewhat later than for the pointlike case, the cross section may be estimated within a factor of 2 or so, as

$$\sigma \sim K^2 2\pi^2 \alpha g^2(q_{\min})/k_0 \tag{8}$$

(in natural units). See also [10].

For pointlike nuclei the perturbative result for pair production with K-shell capture by high energy photons is given by the Sauter cross section (6). However, for the range of energies displayed in Fig. 2, σ/σ_0 does not reach full saturation. If instead cross sections are plotted in units of the general perturbation result for the ground state as listed in (52) in [6], saturation is essentially reached already at a photon energy of 10 mc^2. For the ground state we hence infer that the asymptotic value of the cross section is $0.0090\sigma_0$. This reduction of two orders of magnitude is due to the finite nuclear size. Nonperturbative effects examined in [6] for pointlike nuclei (electron production) were found to cause a reduction by only a factor of 5 for uranium. In the actual situation the nonperturbative correction is expected to be even smaller as the finite depth of the interaction potential (due to finite nuclear size) is accounted for.

Figure 2 further illustrates that in high-Z targets, μ pair production with L-shell capture, when adding contributions from all substates, becomes at least as probable as production with K-shell capture. This is in contrast to the electron case, where the capture to the 2s state is close to $1/2^3$ for all Z while the total L-shell cross section is roughly 0.20 times the K-shell cross section for high Z [6]. Inclusion of even higher states is expected to contribute significantly to the total cross section for bound-free μ pair production.

We have computed the momentum content of bound μ states for a wide range of different target ions in order to investigate the dependence of the total cross section on Z. For the pointlike case, perturbation theory predicts a scaling of the cross section with Z^5 for the ground state, cf. σ_0 given above. With the extended nucleus, we find a much slower scaling for heavy elements. The ratio of the nonrelativistic momentum densities at one unit for the ground states of neon and boron equals the ratio of the nuclear charges to the power 4.6, for xenon and neon the corresponding power is 3.2, and for uranium and xenon the power is only 1.2. For comparison, we mention that the ratio of the high-energy cross sections for uranium and xenon for the nonperturbative electron case corresponds to a power as large as 4.2, cf. [6].

It is of interest to compare the findings discussed above with those published previously by Mikhailov and Fomichev [7]. Table I in [7] lists cross sections for pair production with capture to the K shell for photon impact at 10 mc^2 on various targets. For uranium, the listing is 1.2 nbarn, which may be compared to our result of 206 nbarn, that is, our number is two orders of magnitude higher. See Ref. [11]. Also the scaling with Z is different from ours; the ratio of cross sections listed in [7] for $Z=60$ and 92 corresponds to a power as low as 0.23. On the other hand, for high Z, the relative contributions from the various states, cf. Table II in [7], seem to be about the same; according to [7] the K and the L shell bring about equal cross sections at $Z=80$ [12].

B. τ production

Let us now turn to τ production. For this case we shall stick to a nonrelativisitic calculation since we currently have higher precision here and since there is no need to determine cross sections better than within a factor of 2 or so. With the depth of the potential attaining at maximum 2% of the τ mass, the bound-state problem is basically nonrelativistic. Fourier transformation of the wave function for the tauonic ground state obtained by numerical solution of the Schrödinger equation for a homogeneously charged uranium nucleus yields a momentum density as displayed in Fig. 3. As is immediately apparent, the density is reduced by $\sim$15 orders of magnitude at one momentum unit as compared to the value at maximum. This implies a reduction of the cross section (as measured in units of the square of the Compton

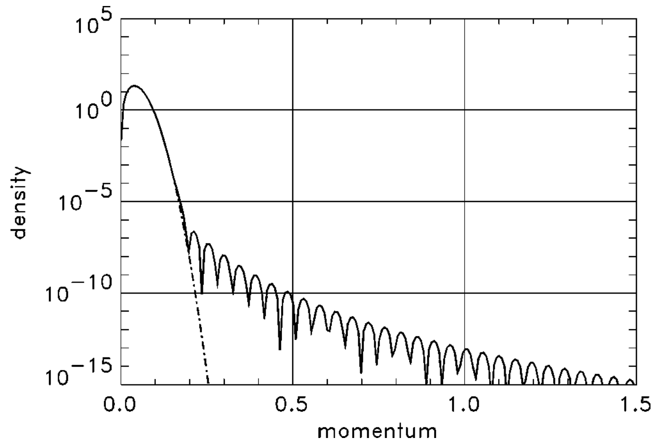

FIG. 3. Nonrelativistic momentum-space density for a negative τ bound in the $1s$ state of an otherwise bare uranium ion. The uranium nucleus is assumed to be a homogeneously charged sphere with a radius of 7.31 fm. The chained curve is the analytical momentum density for the ground state in a harmonic oscillator potential, which at all distances varies as does the actual nuclear potential inside the nucleus. Momenta are given in units of mc.

wavelength, that is, in units of 1.233×10^{-28} cm^2) of similar magnitude. As an example, by substitution of the nonrelativistic momentum wave for the large spinor component and neglect of terms beyond the first in the curly brackets of Eq. (3) we get a maximum cross section for tauonic pair production on a xenon nucleus with population of the $1s$ ground state of 4.3×10^{-44} cm^2 or 4.3×10^{-8} pbarn, a result that cannot change by more than a factor of 2 in a relativistic calculation. This finding is of course fatal as far as measurements are concerned. Note, however, that incoherent production may well ease the situation, cf. Sec. V.

The lowest states are confined inside the nucleus where the potential for a homogeneous charge distribution is harmonic. It is interesting to note that although the harmonic oscillator energies (-23.087 MeV for the uranium K shell) approximate the actual binding energies closely (-23.089 MeV for the same state), the tails of the momentum waves fall off less dramatically than those of the pure oscillator. This is also demonstrated in Fig. 3, which further shows that at low momenta (corresponding to typical kinetic energies in the bound state) the numerical result and the analytical oscillator Gaussian are very close.

The further away from the potential minimum the bound-state energy comes, the less is the suppression of the momentum tails relative to the pointlike case. This implies that, for the heavy τ, pair production with K-shell capture on a high-Z nucleus is less probable than production with L-shell capture on the same nucleus, easily by an order of magnitude, and, also less probable than production with K-shell capture on a light nucleus, again easily by an order of magnitude. However, total bound-free pair production cross sections for the τ still seem to be well below 10^{-41} cm^2 for any nucleus. We may end the discussion of the photoproduction of τ pairs by noting that for a case like that displayed in Fig. 3, application of the Fermi distribution causes a reduction of the momentum density in the relevant region by roughly an order of magnitude relative to the level obtained for the homogeneous charge distribution.

IV. HEAVY-ION COLLISIONS

The photo-cross-sections obtained above may be applied in a Weizsäcker-Williams construction to estimate the cross section for bound-free heavy-lepton pair production in heavy-ion collisions. The relativistic projectile ion of atomic number Z_p is assumed to move on a rectilinear path at a constant velocity v throughout the collision, and the electromagnetic field that it generates is ascribed to an equivalent bunch of photons, which then interacts with the target nucleus through the previously determined photo-cross-sections, cf. [13,14]. This leads to a cross section for ion impact of

$$\sigma_{\rm ion} = \int d\omega \sigma(\omega)\omega^{-1} dI/d\omega, \tag{9}$$

where $\sigma(\omega)$ is the cross section for photon impact. The photon power spectrum is given as [14]

$$\frac{dI}{d\omega} = \frac{2\alpha Z_p^2}{\pi v^2}\left[xK_0(x)K_1(x) - \frac{v^2x^2}{2}[K_1^2(x) - K_0^2(x)]\right],$$

$$x = \frac{\omega b_{\rm min}}{\gamma v}, \tag{10}$$

where K_0 and K_1 denote modified Bessel functions of the second kind and the quantity γ is the usual Lorentz factor $\gamma = 1/\sqrt{1-v^2}$; $c\equiv 1$. The total spectrum (10) has been obtained by integrating the spectrum pertaining to a given impact parameter b over impact parameters beyond the minimum value $b_{\rm min}$. To keep the colliding nuclei intact, the latter quantity cannot be smaller than the sum of the nuclear radii $R_1 + R_2$. This sum exceeds the Compton wavelength of both the μ and the τ as well as the spatial extent of the deeply bound lepton states for the τ, and for heavy nuclei also for the μ. Hence the entire cross section for heavy ion impact is in general well estimated by the virtual photon method with the choice $b_{\rm min} = R_1 + R_2$.

Since K_0 and K_1 fall off exponentially for arguments larger than 1, and since the photon energy has to be in excess of roughly 2 mass units, Eq. (10) implies an effective threshold for bound-free pair production in heavy-ion collisions at, roughly, $\gamma_{\rm th} = (A_1^{1/3} + A_2^{1/3})m_r/164$, where m_r denotes the ratio of the lepton to the electron mass. Typically $\alpha_{\rm th}$ amounts to something like 15 for μ's and 250 for τ, see also [3]. From Eq. (10) we further infer that well above the effective threshold the cross section for heavy-ion impact exceeds the (maximum) cross section for photon impact by a factor of order $(2/\pi)\alpha Z_p^2 \ln(\gamma/\gamma_{\rm th})$, whose number easily is beyond 100.

Figure 4 shows our cross sections for bound-free μ pair production in uranium-uranium collisions. The photo-cross-sections applied are those of Fig. 2. The only published results we are aware of for this process are those presented in [15] and [16]. As to the former, the results listed there are computed at an energy that is essentially below threshold, that is, $\gamma < \gamma_{\rm th}$. Here the intensity (10) is exponentially suppressed and thereby quite sensitive to the exact choice of

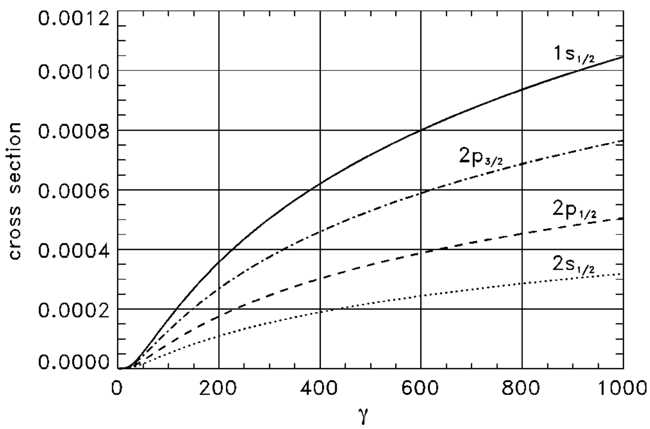

FIG. 4. Cross section for heavy-ion induced μ pair production with the negative μ bound in the K or the L shell of a bare uranium ion as a function of the Lorentz-γ of the projectile. The cross section is proportional to the square of the charge of the projectile ion and is shown here for impact of a bare uranium ion. It is given in units of 3.488×10^{-26} cm^2, the square of the μ Compton wavelength. Curves are labeled as in Fig. 1.

$b_{\min}$. This makes the construction inaccurate and we refrain from a detailed comparison. As to the latter, we refer the reader to our comment listed with Ref. [16].

V. CONCLUDING REMARKS

In summary, we used a perturbative approach to calculate the cross section of bound-free heavy lepton pair production for photon impact. Within this approximation we found that the reduction of the cross section due to nuclear size effects for a uranium target is about two orders of magnitude for muons and fifteen orders of magnitude for tauons. We also argued that the use of a nonperturbative approach will not change the above reductions in a substantial way. Our calculated bound-free cross section for μ's is two orders of magnitude larger than those reported by Mikhailov and Fomichev [7]. We have not been able to track down the reason for this difference. Our numerical cross sections and analytical expressions have been checked in various ways and, in particular, in the limit of the high-energy perturbative Coulomb case, our expressions reproduce the Sauter cross section.

The calculated bound-free τ pair production is extremely small, putting it out of reach of any experimental measurement. Alternative production mechanisms for bound-free heavy lepton pairs may hence be considered. One possibility is photoproduction of free pairs in a solid target with subsequent moderation and/or capture of the negative lepton possibly terminated by a cascade to the ground state. In general, the moderation is slow compared to the natural lifetime in the case of the τ. An exception is the case where the negative τ is created with very low kinetic energy, typically of order $10^{-3}\ mc^2$, cf. [4] and [17]. For a photon of energy 10 mc^2, where the total tauonic pair production cross section for a pointlike uranium nucleus would amount to 8×10^{-31} cm^2, this occurs in less than 10^{-4} of all tauonic pair production events. Furthermore, finite nuclear size enters also in the creation of free pairs, this time through the square of the nuclear form factor evaluated at the momentum taken by the nucleus. For production of the negative τ with low kinetic energy, this reduction factor amounts for a uranium nucleus to $\sim10^{-7}$. In total, the cross section for production of a τ^- with sufficiently low energy that it may be captured before it decays is lower than 10^{-41} cm^2. This number is similar to the limit quoted at the end of Sec. III B.

The very low cross sections for bound-free τ pair production found in Sec. III B (cross sections are low on the scale of the square of the Compton wavelength for the τ) reflect the difficulty of the bound-state wave function to accommodate a momentum of order unity, $q\sim mc$. If this excess momentum instead is transferred to a third partner, the suppression of the cross section may come out less severe than, e.g., the 15 orders of magnitude discussed in connection with Fig. 3 — even though the process will be of higher order. Alternative production mechanisms hence could involve an additional scattering event (rescattering, for example) or production on individual nuclear constituents. In incoherent production the excess momentum is taken by a single nucleon that leaves the nucleus, and the negative τ is bound in a state around the new nucleus. These processes await further consideration. Note that the probability for an energetic τ^+ created inside a heavy nucleus to transfer a momentum of order unity (or larger) to a single proton on its way out through the nucleus is of order 10^{-6} if the proton is assumed pointlike. This is much larger than the suppression factor 10^{-15} mentioned above, but the transfer is mainly transverse and not longitudinal as required, cf. Eq. (4). Note also that in incoherent production on a proton the threshold is nearly 6 mc^2 and nuclear structure still plays a role (radius of proton exceeds Compton wavelength by more than a factor of 10), while in incoherent production on a light, pointlike quark, the threshold is above 200 mc^2.

A final remark regards the possibility of having a τ moving deeply inside an atomic nucleus without nuclear reaction. According to [18], the lifetime against nuclear capture is orders of magnitude longer than the natural lifetime of the τ. Hence the latter, which amounts to 2.9×10^{-13} s, remains a measure of the lifetime of the τ when deeply immersed into nuclear matter. See also [19].

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[1] M. Fatyga, M. J. Rhoades-Brown, and M. J. Tannenbaum, Brookhaven National Laboratory Report No. BNL 52247, UC-414, 1990 (unpublished).

[2] H. Gould, Lawrence Berkeley Laboratory Technical Information, Lawrence Berkeley Laboratory Report No. 18593, UC-28, 1984 (unpublished).

[3] C. A. Berturlani and G. Baur, Phys. Rep. **163**, 299 (1988).

[4] Y. N. Kim, *Mesic Atoms and Nuclear Structure* (North-

Holland Publishing Company, Amsterdam, 1971); C. S. Wu and L. Wilets, Annu. Rev. Nucl. Sci. **19**, 527 (1969).

[5] W. Heitler, *The Quantum Theory of Radiation* (Dover, New York, 1984).

[6] C. K. Agger and A. H. Sørensen, Phys. Rev. A **55**, 402 (1997).

[7] A. I. Mikhailov and V. I. Fomichev, Yad. Fiz. **45**, 445 (1987) [Sov. J. Nucl. Phys. **45**, 281 (1987)].

[8] A. H. Sørensen and A. Belkacem, Phys. Rev. A **49**, 81 (1994).

[9] We have used the same numerical method and computer code to obtain $\psi_b(\mathbf{r})$ as in K. Momberger and A. Belkacem, Lawrence Berkeley Laboratory Report No. LBL-38116, UC-401, 1996 (unpublished), which report therefore may be consulted for details.

[10] For later reference note that the expression on the right-hand-side of Eq. (8) holds exactly for the Sauter cross section. By application of the analytical result for the Coulomb $1s$ wave in the limit of high energies and low Z, that is, by writing $g^2(1)=32(\alpha Z)^5/\pi$ in place of g^2, the constant is seen to assume the value $K=1/4$ in this limit.

[11] We have not been able to track down the reason for this difference. Our numerical cross sections have been checked against the simple estimate (8), which in itself, as explained above, conforms to the Sauter cross section in the high-energy perturbative Coulomb case. Furthermore the relativistic momentum density which enters Eq. (8) has been checked against the corresponding nonrelativistic density, which was obtained independently from the relativistic result in any respect. For the case of $Z=20$, Mikhailov and Fomichev checked their numerical results for electron impact at an energy of 10 units against results based on their analytical formula (11), which they claim to be valid for photon impact on low-Z targets. This formula contains the square of the nuclear form factor. However, at the same time it is based on an analytical approximation for the bound-state wave function. Both cannot appear, and in particular the square of the form factor should only be present if both initial and final states are plane waves, as this factor derives from the Fourier transform of the scattering potential.

[12] There are some differences between the relative contributions from the various states. For instance, Mikhailov and Fomichev find a much smaller difference between the contributions from the two $2p$ states than we do.

[13] E. J. Williams, Mat. Fys. Medd. K. Dan. Vidensk. Selsk. **XIII** (4), 1 (1935).

[14] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975).

[15] K. Momberger *et al.*, J. Phys. B **20**, L281 (1987).

[16] J. C. Wells *et al.*, Phys. Rev. A **45**, 6296 (1992). It may be noted that the cross sections for muon production in heavy-ion collisions given here are too high by orders of magnitude, cf. J. C. Wells *et al.*, Phys. Rev. A **53**, 1498 (1996).

[17] E. Fermi and E. Teller, Phys. Rev. **72**, 399 (1947).

[18] H. C. Chiang and E. Oset, Phys. Lett. B **259**, 239 (1991).

[19] For the μ the situation is somewhat different. Here the nuclear capture rate from the $1s$ muonic ground state exceeds the natural decay rate for Z above, roughly, 10 and reaches values one-and-a-half orders of magnitude higher than the latter at high Z; see H. C. Chiang *et al.*, Nucl. Phys. A **510**, 591 (1990).