Reply to "Validity of the Aharonov Bergmann-Lebowitz rule"

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It is argued that the proofs [W. D. Sharp and N. Shanks, Philos. Sci. **60**, 488 (1993); O. Cohen, Phys. Rev. A **51**, 4373 (1995); D. J. Miller, Phys. Lett. A **222**, 31 (1996)] of the general nonvalidity of the counterfactual interpretation of the Aharonov-Bergmann-Lebowitz rule [Y. Aharonov, P. G. Bergmann, and J. L. Lebowitz, Phys. Rev. **134**, B1410 (1964)] are perfectly valid, and that Vaidman's rejection of these proofs [L. Vaidman, Phys. Rev. A (Comment for which this paper is the Reply)] is unsustainable. It is demonstrated that Vaidman's proposed formulation of "counterfactual probability," on which his rejection of these proofs depends, is problematic and inconsistent in several respects. [S1050-2947(98)03603-8]

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I should first make clear that there was no suggestion in [1] that the Aharonov-Bergmann-Lebowitz (ABL) rule, when applied in the way that was originally intended [2], is ever not valid. In other words, if the intermediate measurements on a preselected and postselected quantum system are actually carried out, then the ABL rule will always yield the correct (i.e., in agreement with standard quantum mechanics) probabilities for the outcomes of those measurements. The analysis in [1] was concerned with a *counterfactual* interpretation of the ABL rule, which relates to examples where the intermediate measurements are *not* carried out, and where the ABL rule is used to calculate the probabilities for what the outcomes of these measurements *would* have been *if* they had been carried out. That such an interpretation was being considered was stated very explicitly in [1]. [See, for example, the first paragraph of Sec. IV and the sentences immediately following Eq. (11).]

The general nonvalidity of the counterfactual interpretation of the ABL rule was first shown by Sharp and Shanks [3], and has also been shown by Miller [4]. In the gedanken experiment in [1], a modified Mach-Zehnder apparatus is used to show that the counterfactual interpretation of the ABL rule can yield results that contradict the predictions of quantum theory. Although it is true that there is a numerical error in the calculation leading to this contradiction, this error has no bearing whatsoever on the conclusion (i.e., the contradiction remains when the error is corrected) or on any of the other results and arguments in [1]. The error is simply this: the probability for a detection at D_3, given a detection at D_1, is, according to the ABL rule, $\frac{1}{6}$, and not $\frac{1}{4}$ as stated in [1]. This leads to an overall probability Prob(D_3) that D_3 would have fired of $\frac{1}{3}$, and not $\frac{3}{8}$ as stated in [1]. However, the corrected value of $\frac{1}{3}$ still disagrees with the quantum mechanical prediction of $\frac{1}{4}$.

That the contradiction still holds when the numerical error is corrected is acknowledged in Vaidman's Comment [5]: Vaidman derives the above value of $\frac{1}{3}$ for Prob(D_3) in the unmodified gedanken experiment and concludes that "Cohen's contradiction still holds." He also considers a modified version of the gedanken experiment in [1], which again enables the proof to go through, leading to a contradiction with quantum theory.

So the above-mentioned numerical error is essentially a red herring. The real issue in question is whether the *structure* of the proofs in [1,3,4] is valid. Vaidman claims that the structure of these proofs is not valid. He does not present his argument for their nonvalidity in [5], but refers instead to an unpublished paper [6]. In [5] he simply applies the ABL rule to the gedanken experiment of [1], *on the assumption that the intermediate measurements actually take place*, and shows that the results obtained are in agreement with quantum theory. This is not surprising. As I have already mentioned, there was no suggestion in [1] that the ABL rule, when applied as in its original derivation according to which the intermediate measurements are assumed to be actually carried out, is ever not valid.

But the whole point of the proofs of [1,3,4] is that they relate to cases where the intermediate measurements are *not* carried out, so that the ABL rule is used in a counterfactual interpretation to determine the probabilities that particular results *would* have been obtained *if* the intermediate measurements had been carried out. Given that the intermediate measurements do not take place, the proofs are perfectly valid. It is clearly not legitimate simply to rerun one of the proofs on the assumption that the intermediate measurements *do* take place, show that there is agreement with quantum theory, and then claim that the proof is not valid. But that is exactly what Vaidman has done in [5].

In [1] it is argued that the examples considered [7–9] where the ABL rule has yielded surprising results, *necessarily* require the assumption that the intermediate measurements are not performed. For instance, consider the example from [7], where a quantum system is preselected according to $|A=a\rangle=(1/\sqrt{2})(|x_1\rangle+|x_2\rangle)$ and postselected according to $|B=b\rangle=(1/\sqrt{2})(|x_2\rangle+|x_3\rangle)$. Intermediate measurements of A, B, X, and Q are considered, where the eigenstates of X are $|x_i\rangle$, $i=1,2,3$, and the eigenstates of Q are $(1/\sqrt{2})(|x_1\rangle+|x_3\rangle)$, $|x_2\rangle$, and $(1/\sqrt{2})(|x_1\rangle-|x_3\rangle)$. Not only is the fact that the intermediate measurements are being considered only hypothetically stated explicitly in [7], but it has been shown [10] that, if we assume that the intermediate measurements in this example *are* actually carried out, then the vari-

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ous subensembles yielded by the different intermediate measurements will be distinct, so that it is not legitimate to combine results obtained by applying the ABL rule to the different subensembles, and consequently the surprising results of [7] are not valid on this assumption.

Nevertheless, Vaidman wants us to assume that the intermediate measurements *are* actually carried out, even when the ABL rule is applied in a counterfactual sense. Indeed, this is the whole basis of his criticism of [1] in [5]. He argues that, in order to calculate probabilities when considering a counterfactual interpretation of the ABL rule, we must assume that the apparatus relating to the intermediate measurement (in this case the detector D_3) is in place and that the measurement is actually carried out. Consequently, he argues, the proof of [1], where it is assumed that the intermediate measurement is not carried out, is not valid.

Now, the idea that we can have actually occurring intermediate measurements in a ''counterfactual'' interpretation of the ABL rule seems like a contradiction in terms, so how are we to make sense of it? Vaidman's proposed justification is presented in the unpublished manuscript [6]. Here he concedes that a counterfactual interpretation of the ABL rule is necessary in order to justify some of the surprising results obtained from it. However, he proposes a new definition of ''counterfactual probability'' which, he claims, enables us to maintain the assumption that the intermediate measurements are carried out, even in the counterfactual cases. According to Vaidman's proposal, counterfactual probability is given by [6] ''the probability for the results of a measurement if it has been performed in the world 'closest' to the actual world.'' The world ''closest'' to the actual world is defined by Vaidman as ''the world in which all measurements [except the measurement at the (intermediate) time t if performed] have the same outcomes as in the actual world.'' Vaidman makes clear in [6] that his proposal does not require one to adopt the many worlds interpretation of quantum mechanics; so the implication is that we can assume that the closest world, in which a counterfactual measurement may be carried out, does not actually exist.

In order to establish whether Vaidman's proposal for counterfactual probability (on which, as we have seen, his criticism of [1] in [5] is wholly dependent) is a viable one, it is necessary to consider its implications in greater detail. The physical implications of this proposal can be brought out by considering the bearing that it has on the surprising examples analyzed in [1]. Each of these examples concerns a situation where outcomes relating to different intermediate measurements are *combined* to obtain unexpected results. In the first example [7] it appears that noncommuting observables can be simultaneously well defined and that quantum mechanics is contextual, in the second [8] that we can find a single particle with certainty in any of N different boxes, and in the third [9] that the ''product rule'' is violated. But if each of the individual outcomes that contributes to each surprising result refers to a *different world*, with at most one of these worlds corresponding to the actual world, then the ''surprising results'' cease to be at all surprising or even interesting. It is very surprising if a single particle can be found with certainty in N boxes; but considerably less so if in $N-1$ cases the certain outcome refers only to an imaginary world. So Vaidman's notion of counterfactual probability effectively renders irrelevant and uninteresting the ''surprising'' results which motivated the analysis in [1] in the first place, and which he is apparently trying to defend (as evidenced by the last sentence in [5]).

Furthermore, it can be shown [11] that Vaidman's proposal is seriously problematic in several other respects. In particular, his proposed ''closest world'' cannot, in general, exist, even in principle. This can be seen by considering the proportions of a given ensemble that yield particular preselected and postselected outcomes, for different intermediate measurements; in general these proportions differ, implying that the proposed ''closest world'' does not exist, even as a hypothetical entity.

On the other hand, the usual understanding of counterfactual probability, adopted by myself and others [1,3,4,12,13] who have considered the counterfactual interpretation of the ABL rule, and according to which the intermediate measurements do not take place, is perfectly meaningful and consistent. With this understanding, the proofs of [1,3,4], that the counterfactual interpretation of the ABL rule can lead to contradiction with quantum theory, are perfectly valid.

Finally I should point out that my intention in [1] was not simply to reiterate Sharp and Shanks's argument that the counterfactual interpretation of the ABL rule is not valid in general. Rather, it was to suggest that there is a special class of situations where applying the ABL rule in a counterfactual sense *can* be justified. These situations are identified through a correspondence between the counterfactual interpretation of the ABL rule and the consistent histories interpretation of quantum mechanics [14]. Thus the conclusion of [1] was that, although the counterfactual interpretation of the ABL rule is not valid in general, some of the surprising results it has generated can nevertheless be defended.

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