Squeezing-induced complete transparency in two-level systems

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The modification of the statistical properties of vacuum fluctuations, via quadrature squeezing, can dramatically reduce the absorptive and dispersive properties of two-level atoms. We show that for some range of parameter values the system exhibits zero absorption accompanied by zero dispersion of the probe field. This complete transparency is attributed to the coherent population oscillations induced by the squeezed vacuum. [S1050-2947(98)00403-X]

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I. INTRODUCTION

The absorptive and dispersive properties of a driven system of atoms have been a subject of extensive research in quantum and nonlinear optics [1]. In early theoretical studies it has been predicted that near atomic resonances at which the dispersion is large, absorption by the medium is also large. Recently, several schemes have been proposed [2] for obtaining large dispersion (large refractive index) accompanied by vanishing absorption. This is an area of increasing interest as such materials could have many practical applications [2]. An enhancement of the refractive index accompanied by vanishing absorption recently has been observed experimentally [3] in coherently prepared Rb atoms.

Another area of interest, especially in atomic spectroscopy and optical communication, is to produce an optical medium that would be completely transparent for an optical beam in the sense that both dispersion and absorption of the medium are *simultaneously zero*. In a nonlinear medium this phenomenon is usually achieved by probing the atomic system very far from resonance. In the area of spectroscopy with three- or four-level systems the phenomenon of electromagnetically induced transparency [4–7] can also lead to a complete transparency. While the main interest in this area is focused on obtaining a large refractive index accompanied by vanishing absorption and lasing without inversion, it is possible, for specific values of the parameters, to reduce the susceptibility of a given multilevel system to zero. For example, a cascade three-level atom in which one of the two possible transitions is driven by a strong coherent field can show a complete transparency for a weak laser beam probing the undriven transition [5–7]. This phenomenon can be interpreted in terms of the dressed states of the system. In this system the effect appears in a region of frequencies far from the dressed atomic transition frequencies.

Here we show that the effect of the complete transparency can be obtained in a system of two-level atoms at the resonant frequency. The two-level system is the simplest configuration to provide the complete transparency and does not require the presence of dressing fields. In particular, we show that this effect can be obtained in a weakly driven two-level system damped by a squeezed vacuum.

The role played by the squeezed vacuum fluctuations in the atomic dynamics has attracted considerable attention in recent years [8]. An investigation of atom-squeezed vacuum interactions was made by Gardiner [9], who demonstrated that a two-level atom interacting with a broadband squeezed vacuum field can decay with two rates of greatly different magnitude. The presence of two decay rates was shown to introduce many unusual features not observed in the normal vacuum spectroscopy [8]. In the following, we show that the squeezed fluctuations play a crucial role in obtaining the complete transparency in a two-level medium.

II. LINEAR SUSCEPTIBILITY

The dispersion and absorption properties of a system of atoms can be studied by using the Kramers-Kronig relations, which determine the real and imaginary parts of the susceptibility of a weak field probing the system. The dispersion, which determines the index of refraction of a system, is given by the real part, whereas the absorption is given by the imaginary part of the susceptibility. We consider a collection of two-level atoms of the transition frequency ω_A driven by a coherent laser field and damped by a squeezed vacuum. The master equation for the reduced atomic density operator of the system in a frame rotating at the laser frequency ω_L is [8]

$$\begin{aligned}\dot{\rho}=&-i\Delta[S^Z,\rho]+\frac{1}{2}\,i\Omega[S^+ + S^-,\rho]+\frac{\gamma_p}{2}\,[S^Z\rho S^Z-\rho]\\&+\frac{\gamma[\,\eta N(\omega_L)+1]}{2}\,(2S^-\rho S^+ - S^+S^-\rho-\rho S^+S^-)\\&+\frac{\gamma\eta N(\omega_L)}{2}\,(2S^+\rho S^- - S^-S^+\rho-\rho S^-S^+)\\&-\frac{\gamma\eta|M(\omega_L)|e^{-i\Phi}}{2}\,(2S^+\rho S^+ - S^+S^+\rho\\&-\rho S^+S^+)e^{2i(\omega_s-\omega_L)t}-\frac{\gamma\eta|M(\omega_L)|e^{i\Phi}}{2}\,(2S^-\rho S^-\\&-S^-S^-\rho-\rho S^-S^-)e^{-2i(\omega_s-\omega_L)t},\end{aligned} \quad (1)$$

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where $\Delta=\omega_A-\omega_L$, $S^{\pm},S^Z$ are the usual collective atomic operators, γ is the Einstein A coefficient, and γ_p denotes the nonradiative dephasing, e.g., due to the collisions between the atoms. The Rabi frequency is Ω and the phase difference of the laser and squeezed vacuum is $\Phi=2\phi_L-\phi$, where ϕ_L is the phase of the laser. The parameters $N(\omega_L)$ and $M(\omega_L)=|M(\omega_L)|e^{i\phi}$ characterize squeezing such that $|M(\omega_L)|^2\leqslant N(\omega_L)[N(2\omega_s-\omega_L)+1]$, where the equality holds for the minimum uncertainty squeezed state, ω_s is the carrier frequency of the squeezed vacuum, and ϕ is its phase. We assume that the squeezed vacuum field is produced by a degenerate parametric amplifier, whose output is characterized by the correlation functions [10]

$$\langle a(\omega_i)a^+(\omega_j)\rangle=[N(\omega_i)+1]\delta(\omega_i-\omega_j),$$

$$\langle a^+(\omega_i)a(\omega_j)\rangle=N(\omega_i)\delta(\omega_i-\omega_j), \tag{2}$$

$$\langle a^+(\omega_i)a^+(\omega_j)\rangle=|M(\omega_i)|e^{i\phi}\delta(2\omega_s-\omega_i-\omega_j),$$

where

$$N(\omega_i)=\frac{\lambda^2-\mu^2}{4}\left\{\frac{1}{(\omega_s-\omega_i)^2+\mu^2}-\frac{1}{(\omega_s-\omega_i)^2+\lambda^2}\right\}, \tag{3}$$

$$M(\omega_i)=\frac{\lambda^2-\mu^2}{4}\left\{\frac{1}{(\omega_s-\omega_i)^2+\mu^2}+\frac{1}{(\omega_s-\omega_i)^2+\lambda^2}\right\}, \tag{4}$$

with λ and μ denoting the bandwidths of the squeezed vacuum field.

The parameter η, appearing in Eq. (1), describes the matching of the incident squeezed vacuum modes to the vacuum modes coupled to the atoms. For perfect matching, $\eta=1$, whereas $\eta<1$ for an imperfect matching. The perfect matching can be achieved when all the modes coupled to the atoms are squeezed. This is a major practical problem as this requires a ''three-dimensional'' squeezed vacuum that couples to the atoms through a full 4π solid angle such that the atoms are damped only by the squeezed vacuum. Such a requirement of 4π solid-angle coupling does not suit the present sources of squeezed vacuum fields, the outputs of which are generally in the form of Gaussian beams that can couple only to part of the 4π solid angle enveloping the atoms. A possible solution to this problem has been proposed [11,12] and tested experimentally [13], involving an optical cavity. Inside a cavity the atoms couple strongly to those modes propagating in a small solid angle about the cavity axis. By squeezing those modes we can obtain the perfect ($\eta=1$) coupling between the atoms and the squeezed vacuum field. Therefore, it seems likely that the effects we describe here should be observable in a cavity environment.

The other important assumption here is that all atoms ''see'' the same intensity and the mode correlations of the squeezed vacuum. It should be noted here that in an extended medium the phase Φ can generally be spatially dependent. This problem can be overcome by copropagating the driving field and the squeezed vacuum in the same direction [8]. If the squeezed vacuum and the driving field counterpropagate, Φ will contain the spatially dependent factor $4\vec{k}\cdot\vec{r}$.

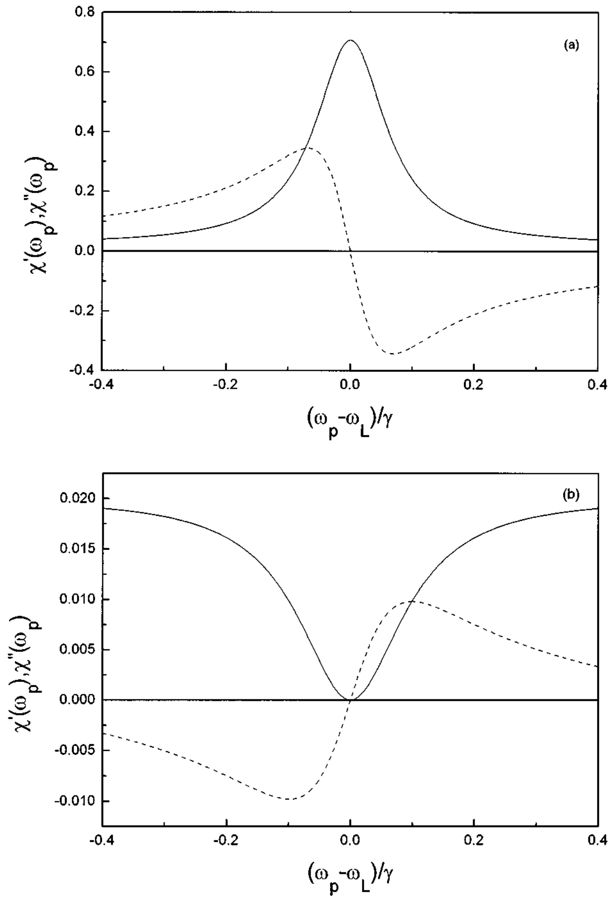

FIG. 1. Absorption $\chi''(\omega_p)$ (solid line) and dispersion $\chi'(\omega_p)$ (dashed line) as a function of $(\omega_p-\omega_L)/\gamma$ for $N=2$, $\eta=1$, $\omega_s=\omega_L$, $\Delta=\Phi=\gamma_p=0$, and (a) $\Omega/\gamma=0.3$ and (b) $\Omega/\gamma=0.5077$.

In order to study the susceptibility of the system, we suppose that after the system (coherently driven atoms plus squeezed vacuum) has attained equilibrium conditions, the atoms are perturbed by a weak probe field of frequency ω_p and amplitude $\mathbf{E}_p$ in whose dispersion and absorption we are interested. The linear susceptibility $\chi(\omega_p)$ of the probe beam at frequency ω_p is given in terms of the Fourier transform of the expectation value of the two-time commutator of the atomic operators as [14]

$$\chi(\omega_p)=i\Omega_p\overline{N}\int_0^\infty d\tau\lim_{t\to\infty}\langle[S^-(t+\tau),S^+(t)]\rangle e^{i\omega_p\tau}, \tag{5}$$

where Ω_p is the Rabi frequency of the probe field and $\overline{N}$ is the atomic density. Equation (5) can be written as

$$\chi(\omega_p)=\Omega_p\overline{N}[\chi'(\omega_p)+i\chi''(\omega_p)], \tag{6}$$

where the real part $\chi'(\omega_p)$ determines the index of refraction of the probe field, whereas the imaginary part $\chi''(\omega_p)$ determines the absorption coefficient. Using Eqs. (1), (5), and (6), we now illustrate the completely transparent effect for two different atomic systems.

First we consider the case of coherently driven two-level atoms damped by a squeezed vacuum in the absence of fast collisions (i.e., $\gamma_p=0$) and collective atomic effects. Hence-

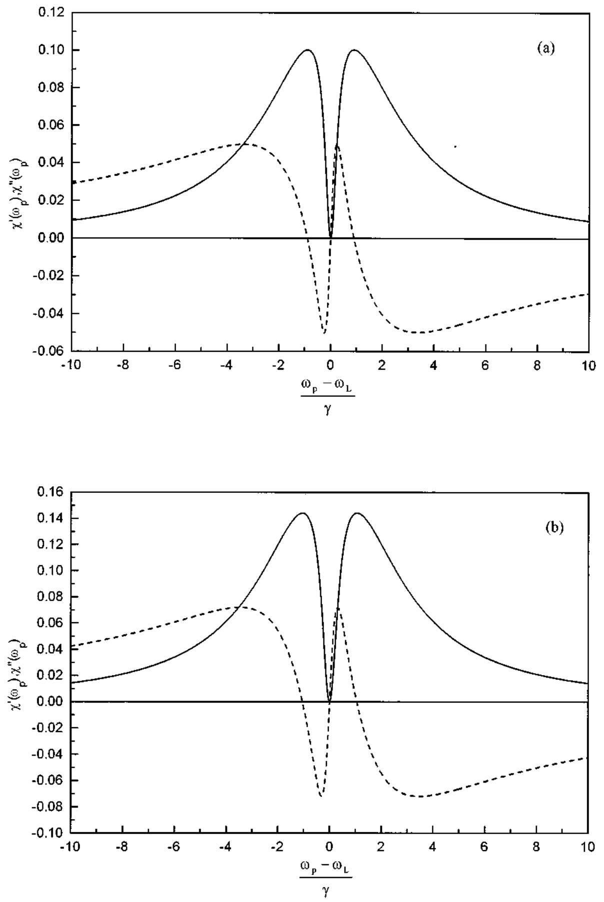

FIG. 2. Absorption $\chi''(\omega_p)$ (solid line) and dispersion $\chi'(\omega_p)$ (dashed line) as a function of $(\omega_p-\omega_L)/\gamma$ for a collective system of atoms with $N=1$, $\eta=1$, $\omega_s=\omega_L$, and $\Delta=\Phi=\gamma_p=0$: (a) two-atom system where $\Omega/\gamma=1.024\,82$ and (b) three-atom system where $\Omega/\gamma=1.526\,268$.

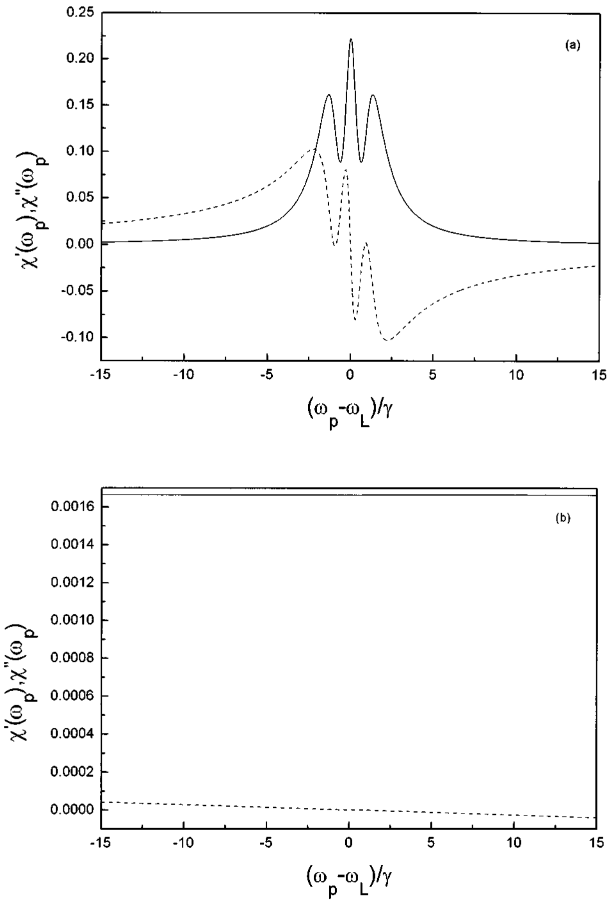

FIG. 3. (a) Same as in Fig. 1(a), but now $\Omega/\gamma=1$ and $N=|M|=0$, i.e., for normal vacuum. (b) Same as (a) but now $\gamma_p/\gamma=600$.

forth, we choose N and $|M|$ to mean $N(\omega_L)$ and $|M(\omega_L)|$, respectively. Figure 1(a) illustrates maximum absorption accompanied by zero dispersion at the central frequency plotted for $\Omega/\gamma=0.3$, $\eta=1$, $\omega_s=\omega_L$, $N=2$, $\Delta=0$, and $\Phi=0$. By increasing the Rabi frequency to $\Omega/\gamma=0.5077$ we find, for the same parameters as in Fig. 1(a), that absorption can be suppressed at the central frequency. Thus the complete transparency is obtained at resonance, as seen in Fig. 1(b). When $\Omega/\gamma>0.5077$ a negative absorption can appear, indicating an amplification of the probe beam at the expense of the squeezed vacuum and driving fields. This phenomenon has been discussed in Ref. [15] and interpreted as an amplification without population inversion. Next we examine the case where collective atomic effects are important. In this case the atomic operators $S^{\pm}$ and S^{Z}, appearing in the master equation (1), are the collective atomic operators. Figures 2(a) and 2(b) show the real and imaginary parts of the susceptibility in the case, respectively, two and three atoms acting collectively [16]. Clearly, we again get the total transparency at the central frequency, which indicates that the effect is not destroyed by the collective interaction of the atoms.

It should be noted here that the significant reduction of the absorption accompanied by vanishing dispersion can be obtained in a two-level medium without squeezing. This can happen in a medium driven by two laser fields of different frequencies [17–20] or in a medium with a large nonradiative dephasing ($\gamma_p\gg\gamma$). Boyd *et al.* [21] have shown that a spectral hole due to fast collisions can appear in the absorption spectrum at the central frequency, which has been confirmed experimentally by Hillman *et al.* [22]. We show in Fig. 3(a) the absorption-dispersion relation for $\gamma_p=0$, $\Delta=0$, $N=|M|=0$, and $\Omega/\gamma=1$. Clearly, for vanishing collisions we see maximum absorption accompanied by vanishing dispersion at the atomic frequency. However, when we increase γ_p [Fig. 3(b)] up to $\gamma_p/\gamma=600$ the absorption can significantly be reduced. We note that the absorption spectrum is virtually a horizontal straight line representing almost vanishing absorption. This corresponds to the dead zone as discussed by several authors [23]. The dispersion is always zero at the central frequency and the absorption is almost reduced to zero. Nevertheless, if we look closer at this particular dead zone [see Fig. 3(b)] we find that the absorption is not exactly zero in the dead zone and hence does not represent the effect of complete transparency.

We should emphasize here that the presence of a squeezed vacuum is essential in obtaining the complete transparency in a two-level medium. The origin of the complete transparency is attributed to the so-called coherent population oscillations [15,21,24]. These are induced by the driving and probe fields beating together at the difference frequency $\delta=\omega_p-\omega_L$. In order to show this, we include a probe beam in the dynamics of a two-level system described by the master equation (1). Assuming that $\Omega_p \ll \gamma$ and $\delta \ll \gamma$, we find that for long times the atomic population difference is given by

$$\langle S^Z\rangle = \langle S^Z\rangle_0 + \langle S^Z\rangle_\delta \exp(i\delta t) + \langle S^Z\rangle_{-\delta}\exp(-i\delta t), \quad (7)$$

where $\langle S^Z\rangle_0$ is the steady-state saturated population inversion induced by the driving field and $\langle S^Z\rangle_{\pm\delta}$ are the amplitudes of the coherent population oscillations induced by the probe beam. To see how the population oscillations affect the absorption spectrum of the probe beam, we find from the master equation (1) the steady-state dipole moment

$$\langle S^-\rangle = \frac{in}{D}\left[\Omega_p - 2\Omega\,\frac{u}{n}\,(n+2|M|e^{-i\Phi})\langle S^Z\rangle_{-\delta}\right], \quad (8)$$

where $n = 2N+1$, $D = n + 2\Omega^2(n+2|M|\cos\Phi)$, and $u = D/(n^2 - 4|M|^2)$.

We see that the dipole moment is composed of two parts: The first part, proportional to Ω_p, is driven directly by the probe field and the second is proportional to Ω. The second term resulting from the population oscillations is significantly enhanced by the squeezing and has a negative amplitude that leads to a reduction of the dipole moment of the atom. The imaginary part of the susceptibility $\chi(\omega_p)$ is proportional to the imaginary part of the dipole moment, which means that the absorption $\chi''(\omega_p)$ can be significantly reduced by the coherent population oscillation. It is easy to show that for the parameter values of Fig. 1(b), the imaginary part of $\langle S^-\rangle$ vanishes, indicating that Eq. (8) predicts accurately the vanishing absorption.

III. SUMMARY

We have shown that two-level atoms driven by relatively weak coherent driving fields and damped by a squeezed vacuum can exhibit complete transparency in the sense that we get zero absorption accompanied by zero dispersion at resonance. This effect can exist even in the presence of collective atomic effects. We attribute this effect to coherent population oscillations, which are significantly enhanced by the squeezed fluctuations.

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