

Direct measurement of the quantum state of radiation field from the resonance fluorescence spectrum

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We show that, for a pure state, the quantum state of the radiation field can be directly determined from the resonance fluorescence spectrum when the quantized field serves as the driving field for the two-level atoms. This happens when the vacuum Rabi frequency is much larger than the atomic decay rate. The proposed method is insensitive to the detector efficiency. [S1050-2947(98)09102-1]

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I. INTRODUCTION

One of the conceptually simplest systems in quantum optics is a two-level atom interacting resonantly with a single-mode radiation field, such as in the resonance fluorescence from a two-level atom. In this paper we show that the quantum state of the radiation field can be *directly* recovered from the spectrum of the resonance fluorescence from two-level atoms driven by the quantized field.

A number of schemes has recently been proposed to obtain information about the quantum state of the radiation field. These include methods based on dispersive atom-field coupling in a Ramsey method of separated oscillatory fields [1], atomic beam deflection [2], quantum state tomography [3], conditional measurements on the atoms in a micromaser set-up [4], quantum Rabi oscillations [5], Autler-Townes spectroscopy [6], homodyning [7], photon chopping [8], and photon counting [9]. An interest in this subject stems from the possibility of studying nonclassical states of the radiation field, such as the squeezed state [10] and the so-called Schrödinger cat state (which is a coherent superposition of two coherent states) [11]. These states lead to oscillating photon distribution functions. In this paper we show that a determination of the quantum state, and hence the photon distribution function, from the spectrum of the resonance fluorescence can lead to a direct observation of such nonclassical features.

The present scheme has two major advantages. First, the photon number amplitudes of the field are recovered from the fluorescence spectrum directly, and no major numerical computations are involved. Second, the proposed scheme is insensitive to the detector efficiency, which poses serious problems in observing the nonclassical aspects explicitly in certain other schemes.

The proposed scheme relies on the fact that the atomic levels display dynamic Stark splitting in the presence of the driving field. The splitting is proportional to the associated Rabi frequency. Thus the resonance fluorescence spectrum displays peaks whose heights are independent of the Rabi frequency, and which are displaced from the resonance by the Rabi frequency. If the driving field is quantized, the associated Rabi frequencies are distributed according to the photon distribution function of the field and the resulting spectrum would mimic the photon distribution function of the driving field.

The theoretical analysis of the problem of resonance fluorescence by a quantized driving field is much more complicated as compared to the case when the driving field is taken to be classical. The problem arises due to the fact that the stationarity condition is no longer valid. As the atom interacts with the quantized field, photons are gradually removed from the field at the rate of the atomic decay rate Γ . We therefore need to calculate the nonstationary spectrum of the fluorescent light.

A necessary condition to recover the quantum statistics of the driving field from the resonance fluorescence spectrum is that the vacuum Rabi frequency is much larger than the atomic decay rate. This condition was satisfied in recent experiments on vacuum Rabi splitting in optical [12] and microwave [13] regions. In addition we would require that at most one photon is emitted during the atom-field interaction. This requires that the interaction time T , which should be much larger than the vacuum Rabi frequency g , should be much smaller than the atomic decay rate Γ . We shall therefore consider the situation when the following inequalities are satisfied:

$$g \gg \gamma_d \gg 1/T \gg \Gamma, \quad (1)$$

where γ_d is the half-band-width of the spectrometer which is used to measure the spectrum.

II. RESONANCE FLUORESCENCE SPECTRUM AND QUANTUM STATE MEASUREMENT

We consider a system of two-level atoms with levels $|a\rangle$ and $|b\rangle$ driven by a quantized field inside a high- Q cavity [14]. The driven atoms radiate spontaneously in all directions, and we look at the spectrum of the radiated light.

Before calculating the resonance fluorescence spectrum, we consider the essential features of the spectrum from a dressed-state picture of the atom-field interaction. The interaction picture Hamiltonian of a quantized field mode interacting resonantly with a two-level atom, in the rotating-wave approximation, is

$$H = \hbar g (|a\rangle\langle b| + a^\dagger |b\rangle\langle a|), \quad (2)$$

where a and a^\dagger are the annihilation and creation operators of the driving field, and g is the corresponding vacuum Rabi frequency. The eigenstates of the Hamiltonian are

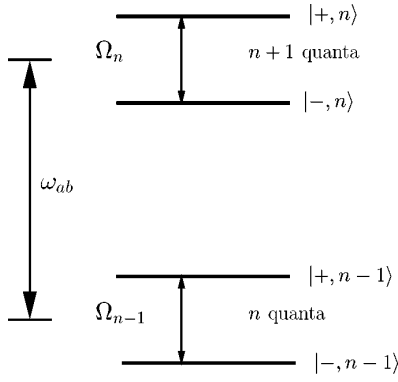


FIG. 1. Splitting of the atomic levels in the dressed-atom-field picture.

$$|\pm, n\rangle = \frac{1}{\sqrt{2}}(|a, n\rangle \pm |b, n+1\rangle), \quad (3)$$

with eigenvalues $+\hbar\Omega_n/2$ and $-\hbar\Omega_n/2$, respectively, where the “generalized” Rabi frequency is defined by $\Omega_n = 2g\sqrt{n+1}$. Therefore, the previously degenerate states $|a, n\rangle$ and $|b, n+1\rangle$ are split into a doublet of dressed states separated by Ω_n as shown in Fig. 1. For a classical field ($n \gg 1$), these dynamical Stark split doublets have almost equal spacing, thus leading to the well-known three-peak spectrum.

For a quantized field, there are four single-photon spontaneous emission lines (see Fig. 1). These lines are located at

$$\begin{aligned} \omega &= \omega_{ab} \pm (\Omega_n \pm \Omega_{n-1})/2, \\ &= \omega_{ab} \pm g(\sqrt{n+1} \pm \sqrt{n}). \end{aligned} \quad (4)$$

Thus a plot of the fluorescence spectrum $S(\omega)$ versus δ/g (with $\delta = \omega - \omega_{ab}$) would yield four peaks for each value of n located at $\pm(\sqrt{n+1} \pm \sqrt{n})$ distributed according to the photon distribution function $p(n)$. However, it may be noted that the peaks for $n=0$ will be degenerate, and will be located at $\delta/g = \pm 1$. We therefore expect the peaks at $\delta/g = \pm 1$ to have double heights as compared to the peaks at other values of δ/g . Thus we should be able to recover the distribution function of the field from the resonance fluorescence spectrum via the relation

$$p_s(n) = \begin{cases} \frac{1}{\mathcal{N}} S(\omega)|_{\delta/g = (\sqrt{n+1} + \sqrt{n})} & \text{when } n > 0 \\ \frac{1}{2\mathcal{N}} S(\omega)|_{\delta/g = 1} & \text{when } n = 0, \end{cases} \quad (5)$$

where

$$\mathcal{N} = \frac{1}{2} S(\omega)|_{\delta/g=1} + \sum_{n=1}^{\infty} S(\omega)|_{\delta/g = (\sqrt{n+1} + \sqrt{n})}. \quad (6)$$

In the dressed-state picture we have not accounted for the atomic decay Γ from level $|a\rangle$ to level $|b\rangle$. A complete calculation of the spontaneous emission spectrum $S(\omega)$ for the quantized driving field can be carried out by first finding the two-time field correlation function of the scattered light and then taking the Fourier transform.

The physical spectrum $S(\omega)$ of the nonstationary fluorescent light at some suitably chosen point \mathbf{r} in the far field is obtained by taking the Fourier transform of the normally ordered correlation function of the field $\langle E^{(-)}(\mathbf{r}, t_1) E^{(+)}(\mathbf{r}, t_2) \rangle$ [15], i.e.,

$$\begin{aligned} S(\omega) &= 2\gamma_d \int_0^T dt_1 \int_0^T dt_2 e^{-(\gamma_d - i\omega)(T-t_1)} e^{-(\gamma_d + i\omega)(T-t_2)} \\ &\quad \times \langle E^{(-)}(\mathbf{r}, t_1) E^{(+)}(\mathbf{r}, t_2) \rangle. \end{aligned} \quad (7)$$

As the field operator $E^{(+)}$ is proportional to the atomic lowering operator σ_- at a retarded time, we obtain

$$\langle E^{(-)}(\mathbf{r}, t_1) E^{(+)}(\mathbf{r}, t_2) \rangle = I_0(\mathbf{r}) \langle \sigma_+(t_1) \sigma_-(t_2) \rangle, \quad (8)$$

where $I_0(\mathbf{r})$ is a constant. The two-time correlation function $\langle \sigma_+(t_1) \sigma_-(t_2) \rangle$ can be calculated by using the quantum regression theorem if we know the appropriate single-time correlation function. We are thus interested in the expectation value of the interaction picture dipole operator $\langle \sigma_-(t) \rangle$.

The equation of motion for the various atom-field matrix elements can be written in a compact matrix form as

$$\dot{R}^{(n,m)} = -M^{(n,m)} R^{(n,m)} + B^{(n,m)}, \quad (9)$$

where $R^{(n,m)}$ is a column vector with elements $R_1^{(n,m)} = \rho_{an,am}$, $R_2^{(n,m)} = \rho_{bn+1,am}$, $R_3^{(n,m)} = \rho_{an,bm+1}$, $R_4^{(n,m)} = \rho_{bn+1,bm+1}$, $B^{(n,m)}$ is a column vector whose only nonvanishing element is $B_4^{(n,m)} = \Gamma \rho_{n+1,m+1}$, and

$$M^{(n,m)} = \begin{bmatrix} \Gamma & ig\sqrt{n+1} & -ig\sqrt{m+1} & 0 \\ ig\sqrt{n+1} & \Gamma/2 & 0 & -ig\sqrt{m+1} \\ -ig\sqrt{m+1} & 0 & \Gamma/2 & ig\sqrt{n+1} \\ 0 & -ig\sqrt{m+1} & ig\sqrt{n+1} & \Gamma \end{bmatrix}. \quad (10)$$

In writing this equation, we used

$$\rho_{an,am} + \rho_{bn,bm} = \rho_{n,m}, \quad (11)$$

where $\rho_{n,m}$ represents the matrix elements for the field operator only. A solution of Eq. (9) can be found as (with $t_2 > t_1$)

$$R^{(n,m)}(t_2) = e^{-M^{(n,m)}(t_2-t_1)} R^{(n,m)}(t_1) + \int_{t_1}^{t_2} e^{-M^{(n,m)}(t_2-t')} B^{(n,m)} dt'. \quad (12)$$

When inequalities (1) are satisfied, we can ignore the second term in Eq. (12), and obtain

$$R^{(n,m)}(t_2) \approx e^{-M^{(n,m)}(t_2-t_1)} R^{(n,m)}(t_1). \quad (13)$$

The interaction picture dipole operator $\langle \sigma_-(t_2) \rangle$ is now given by

$$\begin{aligned} \langle \sigma_-(t_2) \rangle &= \text{Tr}(\rho|b\rangle\langle a|) = \sum_{n=0}^{\infty} \rho_{an,bn}(t_2) e^{-i\omega_{ab}t_2} \\ &= \sum_{n=0}^{\infty} [(e^{-M^{(n,n-1)}(t_2-t_1)})_{34} \rho_{bn+1,bn}(t_1) \\ &\quad + (e^{-M^{(n,n-1)}(t_2-t_1)})_{33} \rho_{an,bn}(t_1) \\ &\quad + (e^{-M^{(n,n-1)}(t_2-t_1)})_{32} \rho_{bn+1,an-1}(t_1) \\ &\quad + (e^{-M^{(n,n-1)}(t_2-t_1)})_{31} \rho_{an,an-1}(t_1)] e^{-i\omega_{ab}t_2}. \end{aligned} \quad (14)$$

Using the quantum regression theorem, we can simply calculate $\langle \sigma_+(t_1)\sigma_-(t_2) \rangle$ by employing $\rho_{\text{atom}}(t_1)\sigma_+(t_1)$ instead of $\rho_{\text{atom}}(t_1)$ in Eq. (14). The resulting expression for $\langle \sigma_+(t_1)\sigma_-(t_2) \rangle$ is

$$\begin{aligned} \langle \sigma_+(t_1)\sigma_-(t_2) \rangle &= \sum_{n=0}^{\infty} [(e^{-M^{(n,n-1)}(t_2-t_1)})_{34} \rho_{bn+1,an}(t_1) \\ &\quad + (e^{-M^{(n,n-1)}(t_2-t_1)})_{33} \rho_{an,an}(t_1)] \\ &\quad \times e^{-i\omega_{ab}(t_2-t_1)}. \end{aligned} \quad (15)$$

We assume that the atom is initially in a coherent superposition of states, i.e.,

$$|\psi_A(0)\rangle = \cos(\theta/2)|a\rangle + \sin(\theta/2)e^{i\phi}|b\rangle, \quad (16)$$

where ϕ is a relative phase and the state of the field is given by $\sum_n w_n |n\rangle$. We then obtain

$$\begin{aligned} \rho_{an,an}(t_1) &= (e^{-M^{(n,n)}t_1})_{11} \cos^2(\theta/2) |w_n|^2 \\ &\quad + (e^{-M^{(n,n)}t_1})_{12} \cos(\theta/2) \sin(\theta/2) w_n w_{n+1}^* e^{i\phi} \\ &\quad + (e^{-M^{(n,n)}t_1})_{13} \cos(\theta/2) \sin(\theta/2) w_{n+1} w_n^* e^{-i\phi} \\ &\quad + (e^{-M^{(n,n)}t_1})_{14} \sin^2(\theta/2) |w_{n+1}|^2, \end{aligned} \quad (17)$$

$$\begin{aligned} \rho_{bn+1,an}(t_1) &= (e^{-M^{(n,n)}t_1})_{21} \cos^2(\theta/2) |w_n|^2 \\ &\quad + (e^{-M^{(n,n)}t_1})_{22} \cos(\theta/2) \sin(\theta/2) w_n w_{n+1}^* e^{i\phi} \\ &\quad + (e^{-M^{(n,n)}t_1})_{23} \cos(\theta/2) \sin(\theta/2) w_{n+1} w_n^* e^{-i\phi} \\ &\quad + (e^{-M^{(n,n)}t_1})_{24} \sin^2(\theta/2) |w_{n+1}|^2. \end{aligned} \quad (18)$$

On substituting for $\rho_{an,an}(t_1)$ and $\rho_{bn+1,an}(t_1)$ from Eqs. (17) and (18) into Eq. (15), we obtain the two-time correlation function $\langle \sigma_+(t_1)\sigma_-(t_2) \rangle$ for $t_2 > t_1$. The correlation function for $t_1 > t_2$ can be obtained by taking the complex conjugate and interchanging t_1 and t_2 .

The matrix elements $[\exp(-M^{(n,m)}t)]_{ij}$ can be determined by expanding in terms of the eigenstates of the matrix $M^{(n,m)}$. For example,

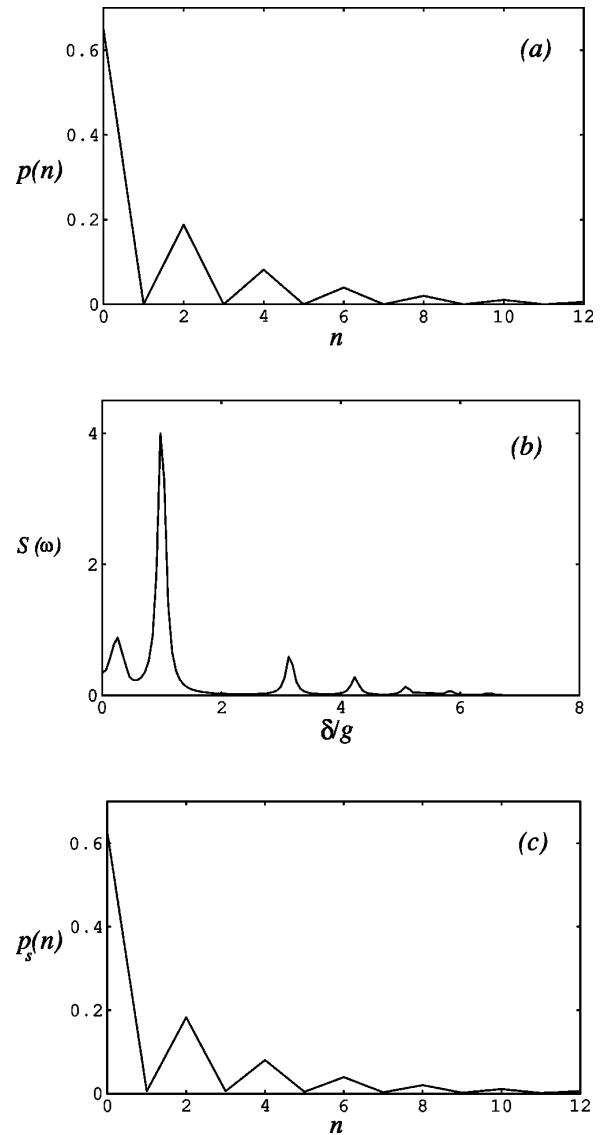


FIG. 2. (a) Photon distribution function $p(n)$ for a squeezed vacuum state with $r=1$. (b) The spontaneous emission spectrum S (in arbitrary units) vs δ/g for $\gamma_d/g=0.1$, $gT=100$, and $\Gamma/g=0.001$. (c) The photon distribution function $p_s(n)$ recovered from (b) according to Eq. (5).

$$\begin{aligned} (e^{-M^{(n,n-1)}t})_{34} &= \sum_{l=1}^4 (e^{-M^{(n,n-1)}t})_{34} E_l^\dagger E_l \\ &= \sum_{l=1}^4 e^{-\lambda_l^{(n,n-1)}t} (\alpha_3^l)^* \alpha_4^l, \end{aligned} \quad (19)$$

where E_l is the eigenstate of the matrix $M^{(n,n-1)}$ with eigenvalue $\lambda_l^{(n,n-1)}$. The eigenstates E_l ($l=1, 2, 3,$ and 4) form a complete set provided all the eigenvalues $\lambda_l^{(n,n-1)}$ are non-degenerate. The elements of E_l are α_j^l ($j=1, 2, 3,$ and 4). All the matrix elements in Eqs. (15), (17), and (18) have nondegenerate eigenvalues except $[\exp(M^{(0,-1)}t)]_{34}$ and $[\exp(M^{(0,-1)}t)]_{33}$ in Eq. (15). So the procedure described above is valid for all elements except these two. Now the matrix $M^{(0,-1)}$ is a simple matrix with 2×2 blocks along the diagonal, and the matrix elements of $\exp(M^{(0,-1)}t)$ can be determined in a straightforward manner.

The power spectrum of the emitted field can now be calculated by taking the Fourier transform of $\langle \sigma_+(t_1) \sigma_-(t_2) \rangle$ according to Eqs. (7) and (8). The resulting expression is rather cumbersome, and will not be reproduced here. It is,

$$S(\omega, \theta, \phi) = \begin{cases} \frac{\gamma_d}{4} \sum_{n=0}^{\infty} \{ \mathcal{L}[\delta + g(\sqrt{n+1} + \sqrt{n})] + \mathcal{L}[\delta + g(\sqrt{n+1} - \sqrt{n})] + \mathcal{L}[\delta - g(\sqrt{n+1} + \sqrt{n})] + \mathcal{L}[\delta - g(\sqrt{n+1} - \sqrt{n})] \} \\ \times [\cos^2(\theta/2) |w_n|^2 + \cos(\theta/2) \sin(\theta/2) (w_n w_{n+1}^* e^{i\phi} + w_{n+1} w_n^* e^{-i\phi}) + \sin^2(\theta/2) |w_{n+1}|^2], & \text{when } n > 0 \\ \frac{\gamma_d}{2} \sum_{n=0}^{\infty} [\mathcal{L}(\delta + g) + \mathcal{L}(\delta - g)] [\cos^2(\theta/2) |w_n|^2 + \cos(\theta/2) \sin(\theta/2) (w_n w_{n+1}^* e^{i\phi} + w_{n+1} w_n^* e^{-i\phi}) + \sin^2(\theta/2) |w_{n+1}|^2] \\ & \text{when } n = 0, \end{cases} \quad (22)$$

where

$$\mathcal{L}(x) = \frac{1}{\gamma_d^2 + x^2}. \quad (23)$$

In the special case when the atoms are injected either in the excited state ($\theta=0$) or in the ground state ($\theta=\pi$), the fluorescence spectrum depends only on the photon statistics and does not depend upon the off-diagonal elements of the density operator.

We now consider an example to illustrate how the photon statistics $p(n)$ can be determined from the spectrum $S(\omega)$ [Eq. (15)] via relation (5). We consider the case when $\theta=0$.

We assume that the driving field is in a squeezed vacuum state whose photon distribution function is given by

$$p(n) = \begin{cases} (\cosh r)^{-1} n! [(n/2)!]^{-2} \left(\frac{1}{2} \tanh r \right)^n & \text{when } n \text{ is even} \\ 0, & \text{when } n \text{ is odd,} \end{cases} \quad (24)$$

however, clear from the expression of the two-time correlation function (15) combined with Eqs. (17) and (18) that the spectrum will depend on the quantum state of the initial field inside the cavity.

The main features of the spectrum can be understood by looking at the eigenvalues of the matrix $M^{(n,n-1)}$, which are given by

$$\lambda_{1,2}^{(n,n-1)} = \frac{3\Gamma}{4} \pm i \sqrt{g^2(\sqrt{n+1} + \sqrt{n})^2 - \Gamma^2/16}, \quad (20)$$

$$\lambda_{3,4}^{(n,n-1)} = \frac{3\Gamma}{4} \pm i \sqrt{g^2(\sqrt{n+1} - \sqrt{n})^2 - \Gamma^2/16}. \quad (21)$$

Thus it follows from Eq. (15) that, for $g \gg \Gamma$, the eigenvalues $\lambda_{1,2}^{(n,n-1)}$ and $\lambda_{3,4}^{(n,n-1)}$ give rise to peaks at $\pm(\sqrt{n+1} + \sqrt{n})$ and $\pm(\sqrt{n+1} - \sqrt{n})$, respectively, when S is plotted against δ/g . The width of all the peaks will, however, be equal to γ_d/g when inequality (1) is satisfied. Under assumption (1), a simplified expression of the spectrum $S(\omega, \theta, \phi)$ can be obtained:

where r is the squeezing parameter. This photon distribution is highly oscillatory as shown in Fig. 2(a). The corresponding spontaneous emission spectrum (S vs δ/g) as given by Eq. (15) is shown in Fig. 2(b) for $\gamma_d/g=0.1$, $gT=100$, and $\Gamma/g=0.001$. It shows peaks located at $\sqrt{n+1} \pm \sqrt{n}$. The only visible peaks corresponding to $(\sqrt{n+1} - \sqrt{n})$ are located at $\delta/g=1$ and 0.41 corresponding to $n=0$ and 1 , respectively, the rest being too close to $\delta/g \approx 0$. The photon distribution $p_s(n)$ can now be recovered according to Eq. (5), and the resulting distribution is shown in Fig. 2(c). The agreement between the original photon distribution and the recovered distribution is excellent. Thus direct evidence of the oscillatory photon distribution can be obtained using this method.

So far we have shown how the diagonal elements of the field density matrix can be determined from the spontaneous emission spectrum when the atom initially in the level $|a\rangle$ interact with the field. However, a complete determination of the field state requires, in addition to the diagonal density-matrix elements, the off-diagonal elements as well. In particular, we need to find the amplitudes w_n for all values of n . This can also be done if the atoms are prepared in a symmetric coherent superposition of states $|a\rangle$ and $|b\rangle$, i.e., for $\theta = \pi/2$. The real and imaginary values of $w_n w_{n+1}^*$ can then be

obtained by determining the spectrum for four choices of the phase ϕ , namely, $\phi=0, \pi/2, \pi$, and $3\pi/4$, and forming the combinations $R(\omega)=[S(\omega, \pi)-S(\omega, 0)]$ and $I(\omega)=[S(\omega, 3\pi/2)-S(\omega, \pi/2)]$. A determination of $R(\omega)$ and $I(\omega)$ yields the values of $a_n=w_n w_{n-1}^*$ in the same way as before. The probability amplitudes w_n can now be determined (apart from an arbitrary and uninteresting phase factor) from the recursion relation

$$w_n = \frac{a_n}{w_{n-1}^*}. \quad (25)$$

Here w_0 can be determined by taking the root of $P(0)$ and the rest of w_n can be found from this recursion relation. Thus the full state is determined to within an arbitrary phase factor. However, this procedure is possible only if none of the amplitudes w_n is zero. Unfortunately, such is not the case for a squeezed vacuum state. In such cases where the amplitudes are zero for alternate n , a two-photon process may be required.

III. CONCLUDING REMARKS

In an experiment to measure the photon statistics using the proposed method, a number of conditions [in addition to inequalities (1)] need to be satisfied. In order to resolve clearly the various peaks associated with different values of n , we also require that $(\sqrt{n+2}-\sqrt{n}) \gg \gamma_d/g$. For large n , this reduces to $n \ll (g/\gamma_d)^2$. Thus the present method would be valid for states with small number of photons. In the cavity configuration, the photon lifetime inside the cavity τ_c should also be much larger than Γ^{-1} . All these conditions can be achieved in recent experiments on single-atom micro-maser [14]. For example, in the microwave region, $g=2\pi \times 17$ kHz, $\tau_c^{-1}=2\pi \times 6$ kHz, and $\Gamma=2\pi \times 5$ Hz has been reported [16]. In recent experiments in the optical region on

the realization of ‘‘one-dimensional’’ atom [17], a ratio of g/Γ of 8 has also been reported.

We also emphasize that the proposed scheme has the advantage of being independent of the detector efficiency. The detector inefficiency places a limit on the observation of non-classical features of light in schemes such as optical homodyne tomography [18]. In such schemes, the phase quadratures are measured in a balanced homodyne detection setup in which the input field is superimposed on the field from a local oscillator at a lossless beam splitter. The two outputs are directed to detectors whose quantum efficiency is less than unity. The photocount mean and variances are different from the photon-number mean and variances. Nonideal detection is therefore a considerable source of noise in homodyne detection. The present scheme, which requires *direct* detection of the scattered field, is independent of the detector efficiency as a no count can be ignored. Thus if no photon is detected due to the nonideal nature of the detectors, we can ignore that particular measurement and proceed with the next measurement.

The present method can be used to determine the photon statistics for arbitrary states, pure or mixed. However it is limited to finding the complete quantum state of the radiation field for a pure state from the recursion relation (25) only if none of the amplitudes w_n is zero. For states where some of the amplitudes are zero, multiphoton process may be used. We also note that the present method can only determine the diagonal and first off-diagonal elements ρ_{nn} and $\rho_{n,n+1}$ of a mixed state as seen from Eqs. (17) and (18). In order to determine the higher off-diagonal elements, multiphoton transitions may be required. This point will be discussed in a later publication.

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