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Polarization of Lyman-α radiation from atomic hydrogen excited by electron impact from near threshold to 1800 eV

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The polarization of Lyman-α radiation, produced by electron-impact excitation of atomic hydrogen, has been measured over the extended energy range from near threshold to 1800 eV. Measurements were obtained in a crossed-beam experiment using a silica-reflection linear polarization analyzer in tandem with a vacuum-ultraviolet monochromator to isolate the emitted line radiation. Comparison with various theoretical calculations shows that the present experimental results are in good agreement with theory over the entire range of electron-impact energies and, in particular, are in excellent agreement with theoretical convergent-close-coupling (CCC) calculations performed in the present work. Our polarization data are significantly different from the previous experimental measurements of Ott, Kauppila, and Fite [Phys. Rev. A **1**, 1089 (1970)]. [S1050-2947(98)09602-4]

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I. INTRODUCTION

Polarization of atomic line radiation has been of general interest since its early discovery in the Zeeman effect, and there is now a relatively large body of data available on polarization of electron impact-induced radiation (Refs. [1, 2]). Polarization measurements in the vacuum-ultraviolet (VUV) present particular difficulties for experimentalists. Since most of the standard birefringent materials do not transmit in the VUV, reflection devices must be used. The problem is further compounded by the instability of some of the commonly used optical materials (such as LiF), the lack of reliable optical data for some materials, and, in some cases, low polarizance. In addition, reflection devices are susceptible to changes in their reflection characteristics from the accumulation of surface films, even in systems employing clean vacuum (Hammond *et al.* [3]).

Much of the available experimental VUV polarization data have been obtained by the Windsor group, and refer to the excitation of the rare gases and various molecules (see, for example, Refs. [4–10]).

Accurate experimental values for the polarization of radiation produced by electron-impact excitation provide a sensitive test for theory by determining the relative populations of the degenerate magnetic sublevels in the excitation process. In addition, since electron-impact excitation cross sections are typically measured in a crossed-beam configuration, with the emitted radiation detected at 90° to the electron-beam axis, polarization measurements are required to correct these data to obtain values for the integral cross section.

The first experimental measurement of the polarization of Lyman-α radiation produced by electron impact excitation of atomic hydrogen was reported by Fite and Brackmann [11]. Values for the polarization were determined from the angular distribution of the Lyman-α radiation. However, the data were essentially of a preliminary nature, and have very large error bars.

The only subsequent measurement reported in the literature is that of Ref. [12]. These authors used a tungsten oven to dissociate molecular hydrogen and an oxygen filter and iodine vapor photon counter to isolate and detect the Lyman-α radiation reflected from a LiF crystal mounted at the Brewster angle in their polarization analyzer. The polarization data of Ref. [12] have been widely used in the literature to correct the H$(2P)$ cross section data of Ref. [13] for polarization effects in order to obtain values for the integral cross section.

Accurate polarization data play a pivotal role in the measurement of integral cross sections, not only for comparison with various theoretical approximations but, as importantly, for establishing the secondary standards for spectroscopic

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modeling of stellar and planetary atmospheres. In this paper, measurements are presented of the polarization of Lyman-α radiation produced by electron-impact excitation of atomic hydrogen in the extended energy range from near threshold to 1800 eV. In addition, convergent-close-coupling (CCC) calculations of the Lyman-α polarization are performed. The present polarization data and our previous measurement of the optical excitation function of H$(2P)$ (Ref. [14]) together describe the Lyman-α emission integrated over all electron-scattering angles.

The present experimental approach takes advantage of recent developments in VUV polarization analyzers (Ref. [15]) and in H-atom sources (Slevin and Stirling [16]) which are capable of producing atomic densities three orders of magnitude greater than previously available. Furthermore, the use of a 0.2-m VUV monochromator in the present experimental apparatus permits the unambiguous isolation of the Lyman-α radiation. This leads to a more accurate determination of the molecular contribution to the signal than in previous work, and opens the possibility of extending the present measurements in the future to higher members of the Lyman series.

II. POLARIZATION OF LINE RADIATION

Dipole radiation emitted in the relaxation of an atom excited by electron impact will, in general, be polarized due to the anisotropy of the collision process. The present experiment has a crossed-beam geometry, with the incident electron beam defining an axis of symmetry. It can be shown (see, for example, Ref. [17]) that for such cylindrical symmetry the radiation can be completely characterized by a single integrated Stokes parameter S_1 which is defined by

$$S_1=\frac{I(0°)-I(90°)}{I(0°)+I(90°)}, \tag{1}$$

where $I(0°)$ (also referred to as $I_{\parallel}$) and $I(90°)$ (or $I_{\perp}$) are the photon intensities observed at 90° to the electron-beam axis with electric vector parallel or perpendicular to the beam, respectively. This parameter is often given the symbol P $(=S_1)$, and is referred to as the polarization of the radiation.

A comprehensive theoretical treatment of the polarization of radiation was given by Percival and Seaton [18]. This corrected some limitations in earlier work by Oppenheimer [19–21] and Penney [22]. More recently, Blum [23] and Andersen, Gallagher, and Hertel [17] set polarization measurements in the wider context of a description of collisionally excited atoms in terms of state multipoles. Excited atomic states populated by electron impact on ground-state atoms evolve under the influence of spin-orbit and hyperfine interactions, and decay with the emission of radiation. The relationship between the cross sections for populating the various degenerate magnetic sublevels of the excited state and the resulting polarization is characterized by a set of constants that depend on the relative magnitudes of these interactions. For the Lyman series the polarization takes the following form:

$$P(nP)=\frac{3(Q_0-Q_1)}{7Q_0+11Q_1}, \tag{2}$$

where Q_M is the cross section for excitation of the magnetic sublevel M related to the orbital angular momentum, and it is assumed that hyperfine interactions and radiation damping can be neglected.

Experimental observations of emitted radiation made at an angle of 90° to the electron-beam axis must be corrected for the polarization of the radiation in order to determine the integral cross section for the excitation process. Such raw experimental data yield values of the apparent cross section (Q_{90}), which are related to the true integral cross section (Q_T) by

$$Q_T=Q_{90}\left(1-\frac{P}{3}\right). \tag{3}$$

Measurements of the polarization are thus not only of interest in determining magnetic sublevel cross sections, but also to provide a means for correcting polarization sensitive data.

At high energies, where the Bethe approximation is expected to be valid, the polarization P of electron-impact-induced radiation from an atomic state j can be calculated from an expression derived by McFarlane [24]. This high-energy limit of the polarization was discussed in some detail by Heddle [25], who showed that the parameter P can be represented in this approximation by the expression

$$P=P_0\left[3-\ln\left(4c_j\frac{E}{R}\right)\right]\left[(2-P_0)\ln\left(4c_j\frac{E}{R}\right)+P_0\right]^{-1}, \tag{4}$$

where P_0 is the polarization at threshold produced by electron impact of monoenergetic electrons of energy E_j, c_j is a parameter which describes the angular distribution of the scattered electrons, and R is the Rydberg constant. The parameter P_0 can be calculated exactly from angular momentum conservation considerations and has a value of 0.42 for nP excitations in atomic hydrogen (Percival and Seaton [18]). Using a Bethe approach to the excitation, Inokuti [26] obtained a value of 0.408 for c_j. With these values for the constants P_0 and c_j, Eq. (4) determines the high-energy Bethe limit for the polarization.

One consequence of the above formulation is that the polarization has a value of zero at an energy given by $E=e^3R/4\,c_j$. Using the above value for c_j, the polarization of Lyman-α radiation is zero at an impact energy of 167 eV. An experimental determination of this quantity is therefore of considerable interest.

For an optically allowed excitation process, Heddle [27] shows that if the polarization is plotted against ln E, then the gradient (G) of this curve at the energy (E_p), where the polarization passes through zero is given by

$$G(1+\beta)=-\frac{P_0}{(6-2P_0)}, \tag{5}$$

where β is the fractional cascade component of the observed radiation at energy E_p. This allows an experimental estimate of P_0 to be made well away from the threshold energy region.

III. EXPERIMENTAL APPROACH

A. Experimental apparatus

The experimental apparatus consists of an electron-impact collision chamber equipped with an atomic hydrogen source, in tandem with a 0.2-m VUV monochromator (resolving power 250) and silica reflection linear polarization analyzer (Ref. [15]) positioned after the exit slit of the monochromator. The electrostatic electron gun and monochromator systems have been described in detail in an earlier publication (Ref. [14]).

The use of an electrostatic electron gun over the entire energy range from near threshold to 1800 eV is an important feature of the present experimental configuration. As pointed out in Ref. [12], polarization measurements using magnetically confined electron beams may be subject to systematic errors due to spiraling and other effects associated with magnetic-field confinement. This is especially true at low energies close to threshold, and results in a reduction in the observed polarization. While electrostatically focused electron beams present the experimenter with the difficult task of minimizing energy-dependent beam overlap variations, a crucial problem for a measurement of the optical excitation function, these effects are not important in polarization measurements since the experimental data relate to a *ratio* of $I_{\parallel}$ and $I_{\perp}$ signals measured at each energy and overlap variations cancel.

A Faraday cup designed to eliminate backscattered secondary electrons is used to monitor the electron-beam current (typically 5 μA). The energy spread of the electron beam is approximately 0.4 eV, with an uncertainty in the beam energy of ± 0.1 eV, as measured from the appearance potential for excitation of the Lyman-α transition.

The atomic hydrogen source was described in detail by Slevin and Stirling [16]. Hydrogen molecules are dissociated in a discharge, excited within a (rf) cavity, resonant at 36 MHz. Hydrogen atoms effuse from a water-cooled Pyrex discharge tube, past a quartz photon trap and through a 1-mm capillary into a field-free interaction region where they are crossfired by the electron beam. Photons emitted from the interaction region are dispersed by the VUV monochromator, with slit widths chosen to ensure adequate separation of atomic line emissions. The VUV monochromator provides precise wavelength selection, a factor that is critical in quantifying the molecular contribution to the observed Lyman-α signal. The use of an oxygen filter in the previous work of Ref. [12] introduced an uncertainty in precisely what spectrum was transmitted to the detector.

The polarization analyzer is shown in Fig. 1, and was described in detail by Chwirot *et al.* [15], who also compared its performance to other analyzer designs. The optical constants of the silica mirror require an angle of incidence of 70° to reflect a single plane of polarization only. A value of 0.85 (± 0.03) for the polarizance ε (or extinction ratio for the two orthogonal polarizations) of the analyzer used in the present measurements was measured by Chwirot *et al.* [15] for Lyman-α radiation using the geometry shown in Fig. 2. A channeltron positioned at the reflector angle is used as the photon detector with a CsI-coated entrance cone to enhance the quantum efficiency for Lyman-α radiation. The use of a fully characterized (and stable) silica reflection polarization analyzer in the present experiment represents a considerable advantage over previous measurements that employed a LiF reflector. LiF crystals are hygroscopic, and degrade over a period of time when exposed to the atmosphere. It is thus difficult to maintain their long-term stability and their use as polarizers adds a measure of uncertainty to the experimental data.

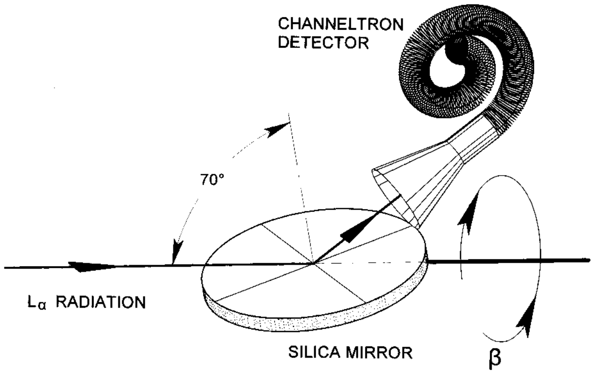

FIG. 1. Schematic diagram of the silica reflection linear polarization analyzer. The optical constants of the silica mirror require an angle of incidence of 70° to reflect a single plane of polarization only. A channeltron positioned at the reflector angle is used as the photon detector with a CsI-coated entrance cone to enhance the quantum efficiency for Lyman-α radiation.

In order to eliminate any polarization effects that may be induced by the monochromator and detector systems, the grating is rotated such that the plane defined by the monochromator entrance slit and optic axis is at 45° to the electron-beam axis (James *et al.* [14]). Clout and Heddle [28] and Donaldson, Hender, and McConkey [29] described the theoretical basis for this orientation in detail.

Polarization measurements are made in the conventional manner by aligning the analyzer axis such that signals proportional to $I_{\parallel}$ and $I_{\perp}$ reach the detector. These correspond to orthogonal values for the angle β shown in Fig. 1. This is achieved by rotating the analyzer mirror and detector assembly using a stepper motor. Using an identical polarizer to that in the present experiment, Chwirot *et al.* [15] reported measurements of the full angular distribution of Lyman-α radiation, fitted to the well-known functional form, confirming the

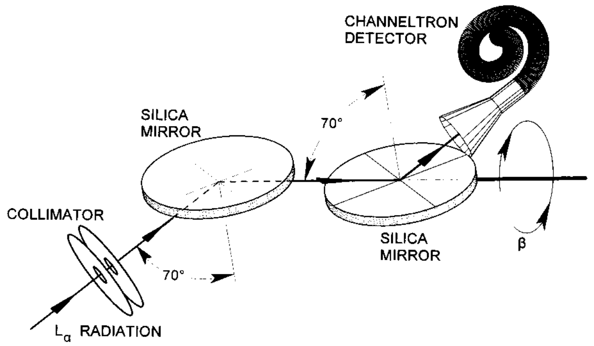

FIG. 2. Schematic diagram of the geometry of the double-reflection polarizance measurement. A value of 0.85 (± 0.03) for the polarizance ε of the analyzer used in the present measurements was measured by Chwirot *et al.* [15] for Lyman-α radiation using this geometry.

validity and accuracy of this experimental procedure.

The entire experimental system is interfaced to a personal computer that controls the electron-beam energy and the stepper motor used to change the polarization analyzer orientation. Measured signals are normalized to the electron-beam current and hydrogen source pressure, eliminating these potential sources of systematic error. Data are accumulated in a multiple scanning mode to reduce the effects of any drifting in other experimental parameters.

B. Correction procedure for molecular contribution

Since the hydrogen beam is not fully dissociated, the observed Lyman-α photon signal at 121.57 nm contains a contribution from molecular emission, which must be quantified. The molecular component results from Lyman-α radiation produced by dissociative excitation of H_2, as well as radiation from molecular bands transmitted by the bandpass of the monochromator (full width at half maximum 2.4 nm at typical slit widths of 600 μm). In order to correct the measured polarization data for this molecular contribution, the dissociation fraction must be measured, together with the polarization of a pure molecular hydrogen target produced with the rf discharge off.

The dissociation fraction is established in the manner described in Ref. [14] by tuning the monochromator to a H_2 molecular band at 110 nm (with the bandpass adjusted to exclude any atomic component from the Lyman-α radiation) and measuring the molecular emission with the discharge on and off at the same hydrogen source driving pressure and electron beam current. The dissociation fraction D is then related to these two signals S_1(on) and S_2(off) by the relationship

$$1-D=\left(\frac{T_2}{T_1}\right)^{1/2}\frac{S_1}{S_2}, \tag{6}$$

where T_1 and T_2 are the effective kinetic temperatures in the gas beam with the discharge on and off, respectively. Woolsey, Forand, and McConkey [30] and Forand *et al.* [31] measured these kinetic temperatures in a similar source and found that the two temperatures were equal, confirming the reasonable assumption that the source produces a thermal beam of hydrogen. A typical value for the measured dissociation fraction is 0.65 ± 0.02.

If the signals measured at the two orthogonal orientations of the polarization analyzer axis are defined as I^+ and I^-, and the subscripts 1 or 2 correspond to signals produced by atomic or molecular hydrogen targets, respectively, then for the pure molecular beam produced with the rf discharge off, the measured molecular polarization ($P_{(\mathrm{H2})}$) is given by

$$P_{(\mathrm{H2})}=\frac{I_2^+-I_2^-}{I_2^++I_2^-}. \tag{7}$$

With the rf discharge on, the beam contains both hydrogen atoms and molecules, and the ratio of atoms to the total number of particles in the beam is given by the dissociation fraction D. Under these conditions,

$$I^+=I_1^++(1-D)I_2^+ \quad \text{and} \quad I^-=I_1^-+(1-D)I_2^-. \tag{8}$$

Thus the apparent polarization (P') measured with the rf discharge on is given by

$$P'=\frac{I^+-I^-}{I^++I^-}. \tag{9}$$

The polarization of the radiation resulting from excitation of hydrogen atoms alone, P_{H}, is then given by

$$P_{\mathrm{H}}=\frac{I_1^+-I_1^-}{I_1^++I_1^-}. \tag{10}$$

Finally, we obtain

$$P_{\mathrm{H}}=P'+(1-D)\kappa(P'-P_{(\mathrm{H2})}), \tag{11}$$

where

$$\kappa=\frac{I_2^++I_2^-}{I_1^++I_1^-}=\left[\frac{I^++I^-}{I_2^++I_2^-}-(1-D)\right]^{-1}.$$

Thus the polarization for atomic hydrogen (P_{H}) can be obtained from separate measurements of the polarization with the rf discharge on and off (giving P' and $P_{(\mathrm{H2})}$, respectively), together with a measurement of the dissociation fraction D.

The above analysis assumes that the polarizance of the analyzer is unity. The true polarization of the atomic radiation is obtained by dividing the result derived using Eq. (11) by the polarizance ε. The present data are corrected by the polarizance value of 0.85 for Lyman-α radiation measured by Chwirot *et al.* [15].

C. Resonance trapping

Since trapping of the resonance Lyman-α radiation by ambient atomic hydrogen generally leads to a reduction in the polarization, it is essential to ensure that the column density of atomic hydrogen is such that the probability of absorption of a Lyman-α photon en route to the detector is negligibly small. To ensure the absence of resonance trapping effects in the present experiment, measurements are made under conditions where the detected photon signal is proportional to the hydrogen source pressure. Operating under Knudsen conditions at the beam source preserves a linear relationship between the source pressure and the number density in the interaction region. Previous measurements with this source described by James *et al.* [14] verify the absence of resonance trapping and associated depolarization effects for source pressures less than 46 m torr. The present experiment was carried out at a source pressure of ~40 m torr.

IV. THEORETICAL APPROACH

The CCC method for e-H scattering was given by Bray and Stelbovics [32]. The scattering amplitudes for the $2P$ excitation are calculated after partial wave T-matrix elements are evaluated. The spin-averaged magnetic-sublevel-dependent integrated cross sections Q_m are then obtained and used to define the polarization fraction P via

$$P=\frac{Q_0-Q_1}{2.375Q_0+3.749Q_1}. \tag{12}$$

This formulation takes into account the effects of hyperfine structure [18].

V. RESULTS AND DISCUSSION

The experimental Lyman-α polarization data measured in the present work over the electron-impact energy range from near threshold to 1800 eV are listed in Table I, together with the previous experimental data of Ref. [12]. Included in the table are the results of our theoretical CCC calculations, the $(1s\text{-}2s\text{-}2p)$ close-coupling (CC) calculations of Ref. [33], and the multipseudostate CC calculations of van Wyngaarden and Walters [34] and Callaway [35]. In addition, the distorted-wave second Born approximation (DWSBA) calculations of Kingston and Walters [36] and Bubelev *et al.* [37] are listed, together with selected Bethe values of McFarlane [24]. All of these data are illustrated in Figs. 3 and 4, showing the general agreement between the present experiment and the various theoretical calculations. The stated errors in the present experimental data correspond to one standard deviation in the signal statistics, combined with an additional contribution from identifiable sources of systematic error, as detailed in Sec. V. For clarity, the stated error bars in the Ref. [12] data are omitted in Fig. 3 but are listed in Table I.

Clearly, the agreement between the present experimental polarization data and the CCC calculations is excellent over the entire energy range of these calculations. Overall excellent agreement with CCC theory was also found in our previous measurement of the optical excitation function of H$(2P)$ (James *et al.* [14]). CCC calculations of the H$(2P)$ cross section were in excellent agreement with the experimental cross-section data over the entire electron-impact energy range from near threshold to 1800 eV, providing further confirmation of the validity of the CCC methodology.

Comparison of the present data with the $(1s\text{-}2s\text{-}2p)$ close-coupling calculations of Ref. [33] shows good agreement in the energy range from 13 to 20 eV. However, the Ref. [33] value for the polarization at 54.4 eV is significantly lower than both the experimental data and all the other theoretical calculations presented in Table I.

The multipseudostate calculations of van Wyngaarden and Walters [34] and Callaway [35] are also in good agreement with the present experiment at energies in the range 16–350 eV. However, a meaningful comparison of the present experimental data with the calculations of Callaway [35] in the near-threshold region from 12.00 to 12.66 eV is not possible since the ~0.4 eV energy width of the electron beam used in the present experiment prevents measurement of the narrow resonance structure shown in the calculations.

Agreement between the present data and the distorted-wave second Born approximation calculations of Kingston and Walters [36] is good for energies above 54.4 eV where this approximation is likely to be most accurate. At energies greater than 100 eV, the convergence of the present experimental data to Bethe values for the polarization given by McFarlane [24] can be seen in Fig. 3. This convergence of experiment and Bethe theory at high energies provides further evidence that the present experimental method is free from any significant unknown systematic effects (for example, in the polarizance of the analyzer, or the presence of low-energy secondary electrons).

The previous experimental polarization data of Ref. [12] fall below both the present experimental data and available theoretical calculations in the energy region from 13 to 25 eV, and exhibit an anomalous rise above both the present experiment and available theories in the region 100–400 eV, suggesting the possible presence of systematic errors. The truncation of the Ref. [12] data at 700 eV also makes it difficult to assess whether Bethe convergence has been achieved. The data of Ref. [12] are, however, in general agreement with the present polarization data in the limited electron impact energy range 25–100 eV. At the important energy of 54.4 eV, for example, the present measurement of Lyman-α polarization yields 0.118 ± 0.007, compared to an interpolated value of 0.125 ± 0.004 in Ref. [12], and a theoretical value of 0.109 given by the present CCC calculations.

The use of a monochromator for wavelength selection in the present experiment (permitting accurate subtraction of the molecular component of the Lyman-α signal), as well as a more stable polarization analyzer, and better agreement with theoretical calculations over the entire range of electron-impact energies from near threshold to 1800 eV suggest fewer sources of systematic error in the present experiment than in that of Ref. [12].

It is noteworthy that neither the present experimental Lyman-α polarization data nor the data of Ref. [12] tend to the Percival and Seaton [18] limit of 0.42 at threshold. However, the threshold polarization is likely to be masked by the electron beam energy resolution of ~0.4 eV obtaining in the present experiment which prevents the measurement of very narrow resonance structure. Our CCC calculations (Fig. 4) show considerable structure in the polarization function is evident in the near-threshold region, with the theoretical polarization falling from a value not inconsistent with the Percival–Seaton [18] limit at threshold to values consistent with experiment at just 0.1 eV above threshold. It should also be noted that the lack of convergence cannot be attributed to the effect of cascade in either experiment since the $n=3$ cascade threshold is at ~12.1 eV.

The existence of resonances is well known to have a profound effect on polarization functions (see, for example, Refs. [38,9]). Thus the energy resolution of the electron gun is crucial. If this is larger than the resonance widths, or if multiple overlapping resonances are contributing to the observed signal, significant distortion of the measured polarization function results. In $2\,^1P$ excitation in helium, where no resonance contribution occurs until ~1 eV above threshold, the predicted threshold value of P is clearly observed (Noren *et al.* [8]). It is reasonable to argue that the low near-threshold value of P measured in the present experiment is an indication of the perturbing effects of the resonances in this energy region.

Based on Eq. (5), the gradient of the P vs $\ln E$ curve at the energy ($E_p=184\pm20$ eV) at which the measured polarization function passes through zero yields an experimental value for P_0 of 0.38 ± 0.06, using a value of $\beta=0.05$ (James *et al.* [14]). This is consistent with the theoretical value of 0.42 within the (rather large) error bars.

TABLE I. Experimental and theoretical values for the polarization of Lyman-α radiation from atomic hydrogen excited by electron impact from threshold to 1800 eV.

	Experiment				Theory						
					CCC	$1s$-$2s$-$2p$ CC	Multi-pseudostate CC		DWSBA		Bethe approx.
Energy (eV)	Present work	Error in present work	Ott *et al.* [12]	Error in Ott *et al.* [12]	Present work	Burke *et al.* [33]	van Wyngaarden and Walters [34]	Callaway [35]	Kingston and Walters [36]	Bubelev *et al.* [37]	McFarlane [24]
10.15			0.130	0.006							
10.20			0.159	0.007							
10.30			0.140	0.007							
10.40			0.158	0.006							
10.50	0.129	0.012	0.139	0.006							
10.60			0.142	0.005							
10.70			0.137	0.006							
10.80			0.141	0.004							
10.90			0.165	0.005							
11.00	0.137	0.010	0.171	0.005		0.2090					
11.10			0.175	0.006							
11.20			0.190	0.005							
11.30			0.186	0.005							
11.40			0.193	0.005							
11.50	0.178	0.010	0.209	0.006							
11.60			0.221	0.006							
11.70			0.222	0.007							
11.80			0.228	0.005							
11.90			0.241	0.007							
12.00	0.236	0.011	0.225	0.007				0.2620			
12.10			0.242	0.006				0.2616			
12.13								0.2650			
12.17								0.2674			
12.20			0.223	0.007				0.2698			
12.24								0.2721			
12.30			0.229	0.006							
12.31								0.2754			
12.38								0.2771			
12.40			0.240	0.008							
12.44								0.2742			
12.49								0.2648			
12.50	0.240	0.011	0.246	0.006				0.2582			
12.51								0.2433			
12.53								0.2483			
12.54								0.2662			
12.55								0.2927			
12.57								0.2987			
12.58								0.3007			
12.59								0.2983			
12.60			0.241	0.007							
12.61								0.2889			
12.62								0.2859			
12.63								0.2818			
12.65								0.2785			
12.66								0.2694			
12.70			0.234	0.008							

TABLE I. *(Continued).*

	Experiment				Theory						
					CCC	$1s$-$2s$-$2p$ CC	Multi-pseudostate CC		DWSBA		Bethe approx.
Energy (eV)	Present work	Error in present work	Ott *et al.*[12]	Error in Ott *et al.*[12]	Present work	Burke *et al.* [33]	van Wyngaarden and Walters [34]	Callaway [35]	Kingston and Walters [36]	Bubelev *et al.* [37]	McFarlane [24]
12.80			0.221	0.005							
13.00	0.254	0.011	0.246	0.005							
13.10			0.250	0.008							
13.20			0.226	0.007							
13.30			0.236	0.008							
13.40			0.220	0.006							
13.50			0.246	0.008							
13.60			0.219	0.008		0.2835					
13.70			0.243	0.006							
13.80			0.249	0.006							
14.00	0.268	0.011	0.243	0.006	0.3063						
14.20			0.246	0.007							
14.30			0.229	0.007							
14.50					0.2954						
15.00	0.270	0.011	0.247	0.010	0.2946					0.3159	
15.60					0.2905						
16.00	0.267	0.011	0.234	0.010	0.2858						
16.46						0.2799		0.2860			
17.00	0.271	0.011	0.234	0.010	0.2811						
17.60					0.2794						
18.00	0.266	0.011	0.253	0.010	0.2768						
19.00	0.262	0.011			0.2731						
19.58						0.2667		0.2700			
20.00	0.254	0.011	0.231	0.009	0.2668					0.2894	
22.00	0.254	0.011			0.2506						
25.00	0.234	0.010	0.216	0.008	0.2332					0.2476	
30.00	0.208	0.009	0.209	0.005	0.1997						
30.61						0.1800					
31.50										0.2019	
35.00	0.184	0.008	0.171	0.007	0.1743						
35.40								0.1760			
40.00	0.159	0.008	0.166	0.004	0.1534					0.1576	0.2075
45.00	0.144	0.007			0.1380						0.1790
50.00	0.130	0.007	0.137	0.004	0.1202						0.1562
54.40	0.118	0.007			0.1090	0.0855	0.110	0.1060	0.135	0.1078	0.1396
60.00	0.107	0.006	0.109	0.004	0.0959					0.0964	0.1218
70.00	0.082	0.006	0.090	0.005	0.0792					0.0759	0.0969
80.00			0.074	0.005						0.0620	0.0777
85.00	0.053	0.006			0.0559						0.0697
90.00			0.055	0.006						0.0486	0.0624
100.0	0.049	0.006	0.052	0.004	0.0418		0.039	0.0380	0.050	0.0371	0.0498
110.0			0.053	0.005						0.0279	0.0393
120.0	0.025	0.006	0.051	0.005	0.0228					0.0216	0.0302
130.0										0.0156	0.0223
140.0			0.040	0.005						0.0096	0.0154
150.0	0.014	0.006			0.0075					0.0033	0.0093
160.0			0.028	0.005							0.0038
175.0										−0.0096	−0.0035

TABLE I. *(Continued).*

	Experiment				Theory						
					CCC	$1s$-$2s$-$2p$ CC	Multi-pseudostate CC		DWSBA		Bethe approx.
Energy (eV)	Present work	Error in present work	Ott *et al.* [12]	Error in Ott *et al.* [12]	Present work	Burke *et al.* [33]	van Wyngaarden and Walters [34]	Callaway [35]	Kingston and Walters [36]	Bubelev *et al.* [37]	McFarlane [24]
180.0			0.027	0.005							−0.0058
200.0	0.006	0.006	0.020	0.007	−0.0167		−0.015		−0.013	−0.0185	−0.0137
225.0										−0.0241	−0.0220
250.0	−0.017	0.007	−0.001	0.009	−0.0311					−0.0292	−0.0291
275.0										−0.0345	−0.0351
290.0											−0.0383
300.0	−0.045	0.007	−0.030	0.020	−0.0418		−0.042		−0.040		−0.0403
302.0										−0.0395	−0.0407
350.0							−0.054				−0.0490
400.0	−0.054	0.008	−0.028	0.020	−0.0568				−0.058	−0.0560	−0.0560
500.0	−0.054	0.008	−0.048	0.020	−0.0667				−0.070	−0.0630	−0.0667
600.0	−0.038	0.009	−0.040	0.020	−0.0742						−0.0747
680.0									−0.084	−0.0604	−0.0798
700.0	−0.070	0.009	−0.062	0.020	−0.0800						−0.0810
800.0	−0.078	0.010			−0.0848						−0.0861
900.0	−0.094	0.010			−0.0879						−0.0904
999.9										−0.0557	−0.0940
1000	−0.070	0.011			−0.0909						−0.0940
1200	−0.115	0.012			−0.0957						−0.1000
1400	−0.092	0.012			−0.1000						−0.1047
1600	−0.116	0.013			−0.1048						−0.1086
1800	−0.093	0.013			−0.1079						−0.1119

VI. ERROR ANALYSIS

A comprehensive analysis of statistical and systematic errors was performed in order to determine the limiting accuracy of the present measurements. The total error in our values of the polarization relates to the errors in the following parameters: (1) the observed count rates $I_i^{\pm}$, (2) the dissociation fraction D, and (3) the polarizance ε (since the value of P_{H} must be divided by ε in order to obtain the true polarization).

The error in the measured polarization is calculated in the usual way by carrying out a Taylor series expansion of the quantity $[P_{\mathrm{H}}/\varepsilon]$ with respect to all of the above quantities, and then combining the individual error contributions in quadrature. For example, typical values for the individual relative errors at an impact energy of 54.4 eV are 2.8% (for discharge on count rates), 0.25% (for discharge off count rates), $<0.1\%$ (for D) and 3.5% (for ε), combined in quadrature to yield a total error of ± 0.006 in a measured polarization of 0.118. As expected, larger errors occur at electron-impact energies where the smaller cross section results in lower signal rates.

The silica reflector in the polarization analyzer will, in practice, accept a cone of angles of incidence of the detected radiation on the mirror surface about the nominal angle of incidence of 70°. The VUV monochromator has an f 4.5 optical system with the diffraction grating used that would result in an acceptance cone half-angle at the reflector surface of 6.3°. However, aperture stops are included both at the exit of the interaction region and at the entrance of the analyzer that effectively restrict the acceptance cone half-angle of the analyzer to approximately 3°. The resulting depolarization effect can be shown to be negligible (Chwirot *et al.* [15]).

It might be argued that the present polarization data could be affected by H($2S$) metastables being quenched within the interaction volume by stray electric fields. This process would cause a reduction in the measured polarization. The interaction region was, however, rigorously shielded to ensure the absence of any stray fields. The earlier measurement in this laboratory by James *et al.* [14] of the optical excitation function of the H($2P$) state found no evidence of any significant contribution to the signal arising from quenching of the H($2S$) metastable population by field effects.

Finally, it should be noted that in the present experiment the observed Lyman-α signal will include a contribution from H($2P$) states populated by cascade processes from higher-lying states rather than by direct excitation. Cascade can occur from directly excited S and D states $[1S\rightarrow nS,\ nD\rightarrow 2P\ (n\geqslant 3)]$ or from directly excited P states via S and D states $[1S\rightarrow nP\rightarrow n'S,\ n'D\rightarrow 2P\ (3\leqslant n'<n\geqslant 4)]$. The cascade contribution to the measured H($2P$) cross section is most significant at low electron impact energies (James *et al.*

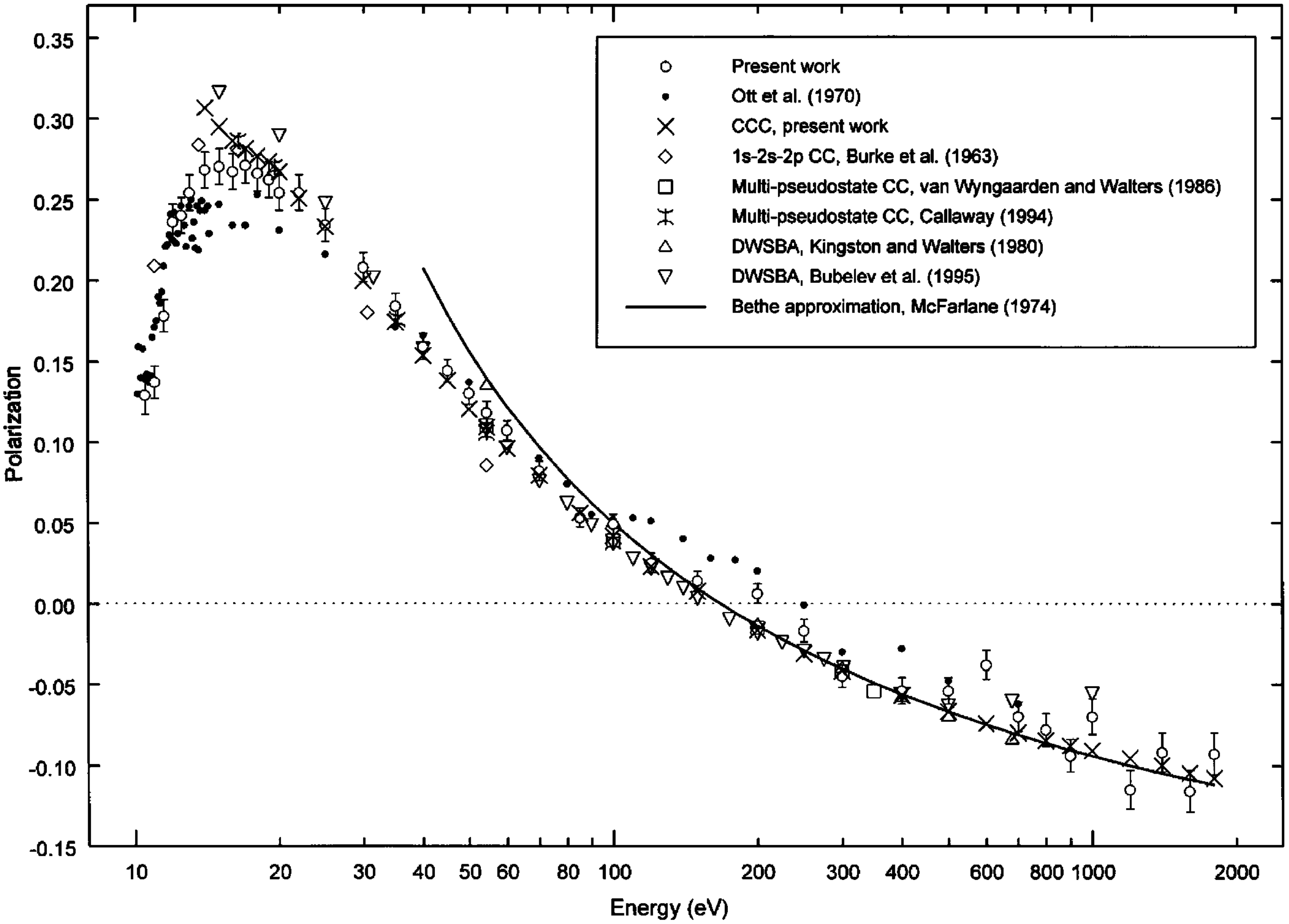

FIG. 3. Experimental and theoretical values for the polarization of Lyman-α radiation from atomic hydrogen excited by electron impact over the energy range from threshold to 1800 eV. For clarity, only a few representative values of the present CCC calculations are shown for energies $\leqslant 14$ eV; the detailed near-threshold behavior of our CCC calculations (10.2–14 eV) is shown in Fig. 4. Similarly, the calculations of Callaway [35] are shown only for energies $\geqslant 16$ eV. The dotted line representing zero polarization is added for clarity. Error bars on the Ref. [12] data are omitted but listed in Table I.

[14]) since $1S \rightarrow nS$, nD excitations are dipole-forbidden. Calculations indicate that the cascade component represents $\sim 27\%$ of the observed Lyman-α emission at 14-eV electron-impact energy, falling to $\sim 3\%$ at 1800 eV (James *et al.* [14]). If it is assumed that cascade processes from higher-lying S states populate the $2P$ magnetic sublevels equally, and that cascade from nD states produces only weak polarization, then the measured Lyman-α polarization values at energies above the threshold for cascading transitions (~ 12.1 eV) will be reduced by similar factors. However, due to the considerable uncertainties involved in a correction procedure, it was decided not to attempt a cascade correction. In these circumstances the present data represent a lower limit to the true polarization in so far as possible cascade effects are concerned.

VII. CONCLUSIONS

The polarization of Lyman-α radiation, produced by electron-impact excitation of atomic hydrogen in the extended energy range from near threshold to 1800 eV, has been measured in a crossed-beam experiment using a silica reflection linear polarization analyzer. The present experimental results have been compared to the data of Ref. [12] and to the latest theoretical calculations. They are in excellent agreement with the present CCC results over the entire energy range of the calculations, are also in good agreement with other close-coupling calculations, and converge to Bethe values at high energies. The present data show significant differences from the previous experimental data of Ref. [12], and it is argued that the present data are likely to be more accurate.

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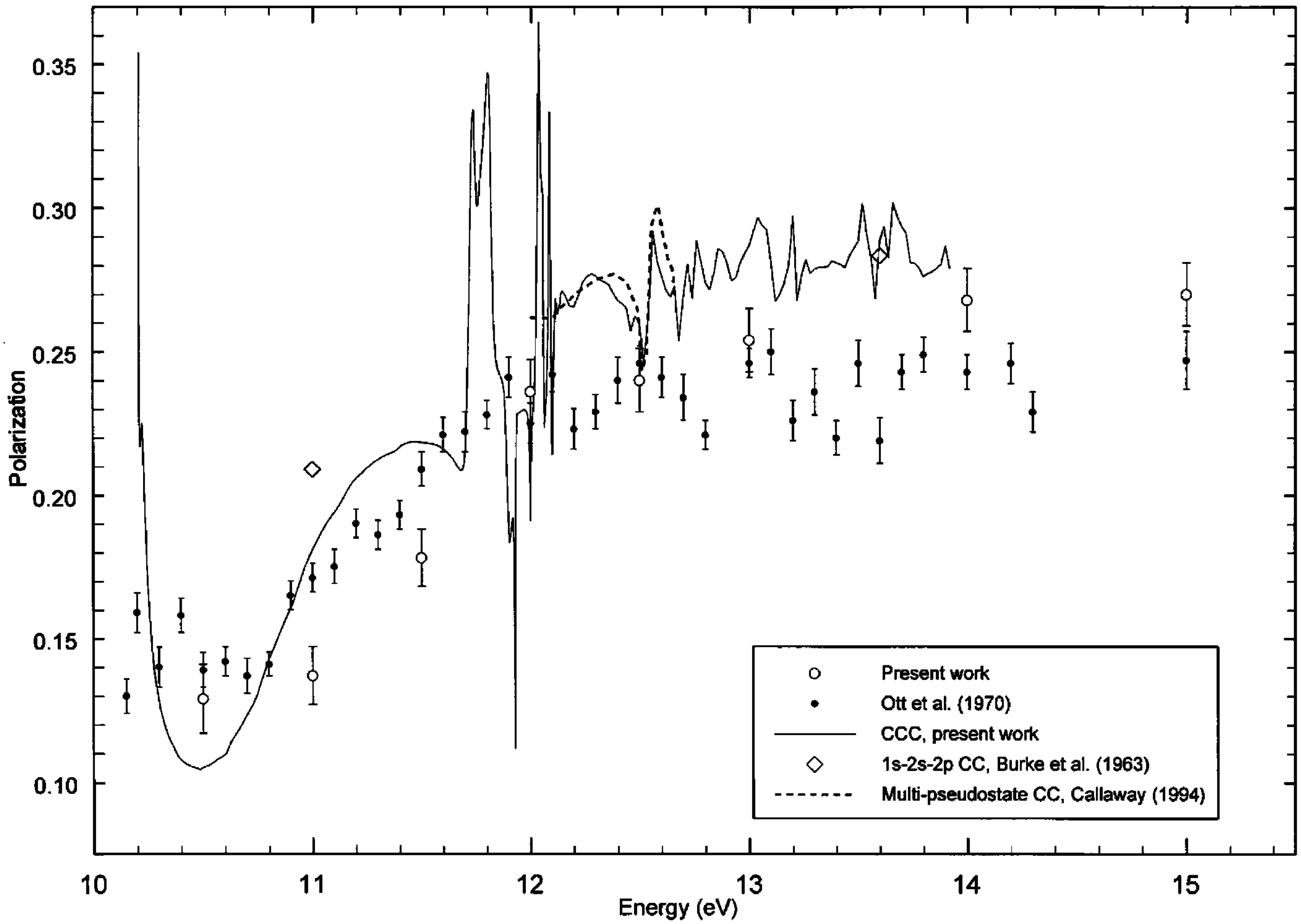

FIG. 4. Experimental and theoretical values for the polarization of Lyman-α radiation from atomic hydrogen excited by electron impact over the energy range from threshold to 14.1 eV. The theoretical calculations of Callaway [35] are shown up to an energy of 12.66 eV only.

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