

Choice of consistent family, and quantum incompatibility

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In consistent-history quantum theory, a description of the time development of a quantum system requires choosing a framework or consistent family, and then calculating probabilities for the different histories that it contains. It is argued that the framework is chosen by the physicist constructing a description of a quantum system on the basis of questions he wishes to address, in a manner analogous to choosing a coarse-graining of the phase space in classical statistical mechanics. The choice of framework is not determined by some law of nature, though it is limited by quantum incompatibility, a concept that is discussed using a two-dimensional Hilbert space (spin half particle). Thus certain questions of physical interest can only be addressed using frameworks in which they make (quantum-mechanical) sense. The physicist's choice does not influence reality, nor does the presence of choices render the theory subjective. On the contrary, predictions of the theory can, in principle, be verified by experimental measurements. These considerations are used to address various criticisms and possible misunderstandings of the consistent-history approach, including its predictive power, whether it requires a new logic, whether it can be interpreted realistically, the nature of "quasiclassicality," and the possibility of "contrary" inferences. [S1050-2947(98)07903-7]

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I. INTRODUCTION

The consistent-history approach [1–7] to quantum theory provides a precise conceptual framework for describing how a closed quantum system (isolated from its environment, or with the environment itself included as part of a single closed system) develops in time. It reproduces the results of standard textbook quantum theory while avoiding the well-known paradoxes associated with the "measurement problem," Bell's inequality, and the like [7,8]. It shows promise for application to quantum optics [9], quantum computing [10] and quantum cryptography [11].

At the same time, the consistent-history approach has been the subject of various criticisms [12–20] that center upon the fact that the formalism allows the physicist to choose from a very large number of alternative frameworks or consistent families of histories, all of which are considered "equally valid" in the sense that no fundamental physical law determines which family should be used in any given case. Were this "freedom of choice" the counterpart of gauge symmetry in classical electromagnetism, it would be of no concern. However, different frameworks are often mutually incompatible in a manner which means that the use of one to describe a given physical system precludes the use of another, that certain questions can only be addressed if one uses the appropriate framework, and that results from incompatible frameworks cannot be combined to form a single quantum description. The resulting conceptual problems have given rise to the claim that consistent-history quantum theory is incomplete and lacking in predictive power [16,17], that it is logically incoherent and incompatible with physical realism [12–15], or that the physicist's choice (within this interpretation of quantum theory) must necessarily influence reality [12,15].

While all of these claims can be countered by a detailed analysis based upon consistent-history principles, as we shall see below, the fact that they have been made by scientists who have studied consistent histories in some detail means that the conceptual problems which give rise to them are likely to trouble others as well. Hence the present paper contains a detailed examination of these problems, using a systematic formulation of the consistent-history approach published under the title "Consistent histories and quantum reasoning," hereafter referred to as CHQR [7]. Indeed, the arguments presented below are a continuation of a discussion already begun in CHQR as to how the formal principles of consistent-history quantum theory, which appear to be sound, should be understood in physical terms and applied in particular situations. The basic conclusion of the present paper is that the criticisms mentioned above do not indicate any deficiency or lack of logical coherence in the consistent-history approach, once it is properly understood. Instead, the main problem is that the full import of the "single consistent family" condition, which prohibits combining results from incompatible families, has not always been appreciated, despite the fact that it was stated in the very first publication devoted to consistent histories [21], and has been repeated many times since [22]. In addition, even physicists who accept the usual Hilbert space formulation of quantum mechanics often adopt, without giving the matter enough thought, a mode of reasoning about its physical consequences that is in basic conflict with the underlying mathematics. A consequence is that one of the few interpretations of quantum theory constructed in such a way that its rules of reasoning are consistent with Hilbert space mathematics is criticized for being logically incoherent.

The structure of this paper is as follows. Section II contains a brief review of the principles of consistent-history reasoning as found in CHQR, followed by applications of these principles to a simple gedanken experiment. Various consistent families, or frameworks, which are incompatible

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with each other, are constructed, and it is shown how they can be used to derive a variety of physical conclusions. The problems of choice and of incompatibility emerging from this example are summarized at the end of the section.

The analysis of these two problems begins with a discussion of various classical analogies in Sec. III. Here it is argued that the choice of a quantum framework is, among other things, like the choice of a coarse-graining for a classical phase space, and hence can properly be considered a choice made by the physicist, not a consequence of some ‘‘law of nature.’’ However, incompatibility between frameworks is a quantum effect with no good classical analog, and must be dealt with in quantum terms. This is the topic of Sec. IV, in which properties of a spin half particle are used as a sort of quantum analogy in order to discuss general properties of quantum incompatibility. In addition, a simple example shows how the predictions of consistent histories for what goes on in a closed quantum system can, in principle, be confirmed by experimental measurements.

Section V begins with a summary of the conclusions that can be drawn from the preceding arguments, and then continues with some applications to various criticisms and (actual or potential) misunderstandings of the consistent-history approach. These include the question of whether consistent histories require a new form of logic, whether they can be interpreted in a realistic sense, a reply to the claim that the consistent-history approach lacks predictive power, comments on the nature of ‘‘quasiclassical,’’ and reasons why there cannot be a ‘‘list of true histories.’’ The Appendixes contain somewhat more technical arguments that address recent claims that consistent-history quantum mechanics allows ‘‘contrary inferences’’ [19,20], and that it cannot reproduce consequences of standard quantum mechanics even if the future is thought to be ‘‘quasiclassical’’ [18].

II. A SIMPLE EXAMPLE OF QUANTUM REASONING

The basic structure needed in order to treat quantum mechanics as a probabilistic theory is the same as for all other types of probabilistic reasoning: a sample space of mutually exclusive elementary events, one and only one of which occurs. These ‘‘events’’ in the quantum case are *elementary histories*; see Sec. II below for some examples. They are analogous to the classical histories obtained by flipping a coin several times in a row. For example, the sample space for a coin flipped three times in a row consists of eight sequences, HHH, HHT, . . . , TTT (H=heads, T=tails), one and only one of which is actually realized when the experiment is carried out.

Compound events, which are collections of elementary events, together with the elementary events themselves, constitute the Boolean *event algebra* to which probabilities are assigned. (In the example of coin flipping just mentioned, the four sequences corresponding to ‘‘H on the first toss’’ are a compound event.) In the quantum case, the algebra of events is called a *consistent family* or *framework*. It must be chosen according to appropriate quantum-mechanical rules, as discussed in CHQR. Provided these rules are satisfied, probabilities are assigned using a set of non-negative weights calculated from the unitary time evolution engendered by Schrödinger’s equation. Given these probabilities, the physical consequences of the theory, typically in the form of con-

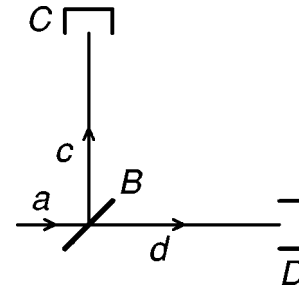


FIG. 1. Beam splitter B and detectors C and D .

ditional probabilities, are calculated following precisely the same rules as in other applications of standard probability theory.

The peculiar features of quantum theory come about from the fact that there are usually a large number of different ways to choose the sample space or consistent family, and it is necessary to pay careful attention to which of these is being used to study a particular problem. In classical physics it usually suffices to consider a single sample space, and in those instances in which more than one is used, relating the different sample spaces is relatively straightforward, unlike quantum theory, where the rules are more complicated. For further details, see CHQR.

A simple example that will serve to illustrate the principles of quantum reasoning just discussed is shown in Fig. 1. A photon that is initially in channel a passes through a beam splitter B and is later detected by one of two detectors, C and D . The unitary time evolution produced by the beam splitter takes the form

$$|a\rangle \mapsto |s\rangle = (|c\rangle + |d\rangle) / \sqrt{2}, \quad (2.1)$$

where $|a\rangle$ is a wave packet for the photon in the entrance channel a , $|c\rangle$ and $|d\rangle$ are wave packets in the exit channels c and d , and $|s\rangle$ is a coherent superposition of the latter. The interaction of the photon with each of the detectors is given by the unitary transformations

$$|c\rangle|C\rangle \mapsto |C^*\rangle, \quad |d\rangle|D\rangle \mapsto |D^*\rangle, \quad (2.2)$$

where $|C\rangle$ indicates a detector in a state ready to detect a photon, and $|C^*\rangle$ the state that results when the photon is detected. Note that $|C\rangle$ and $|C^*\rangle$ differ from each other in some macroscopic, visible property; for example, the position of a pointer. The states $|D\rangle$ and $|D^*\rangle$ of the other detector have a similar interpretation. Some other particle, such as a neutron, could replace the photon without altering the following discussion in any significant way. Or one could imagine a spin-half particle going through a Stern-Gerlach apparatus, which separates an $S_x = 1/2$ initial state into two beams with $S_z = 1/2$ and $S_z = -1/2$ directed to separate detectors.

In the consistent-history approach, the photon and the detectors are thought of as forming a single, closed quantum system, and the quantum-mechanical description applies to the system as a whole. (In principle the beam splitter should also be included, but since its quantum state plays no role in the following discussion, it is omitted so as not to clutter up the notation.) The unitary time development of the total system then takes the form

$$|\Psi_0\rangle \mapsto |sCD\rangle \mapsto |S\rangle, \quad (2.3)$$

where

$$|\Psi_0\rangle = |aCD\rangle, \quad (2.4)$$

the initial state at a time t_0 , evolves under the action of Schrödinger's equation to a state $|sCD\rangle$ at a time t_1 when the photon has passed through the beam splitter, and then to

$$|S\rangle = (|C^*D\rangle + |CD^*\rangle) / \sqrt{2} \quad (2.5)$$

at a time t_2 after the photon has been detected by one of the detectors. Since $|C^*\rangle$ and $|C\rangle$ are macroscopically distinguishable, as are also $|D\rangle$ and $|D^*\rangle$, $|S\rangle$ is a *macroscopic quantum superposition* (MQS) or ‘‘Schrödinger's cat’’ state.

One can think of (2.3) as a *quantum history* based upon the three times t_0 , t_1 , and t_2 . In the notation of CHQR it is represented as a projector

$$\Psi_0 \odot_s CD \odot S \quad (2.6)$$

on the tensor product space of histories, where we adopt the convention, here and below, that a symbol outside a ket denotes the corresponding projector, for example,

$$S = |S\rangle\langle S|. \quad (2.7)$$

For present purposes the tensor product structure is not important, and (2.6) should simply be thought of as a statement that the system started off in the initial state Ψ_0 (or $|\Psi_0\rangle$) at t_0 , was in a state sCD at t_1 , and in the state S at time t_2 . The consistent-history approach supports such a ‘‘realistic’’ interpretation for reasons indicated in CHQR and in Sec. IV C below.

The first consistent family or framework \mathcal{F}_1 we shall discuss has a sample space of elementary families consisting of (2.6) together with three additional histories:

$$\begin{aligned} \Psi_0 \odot_s CD \odot \bar{S}, \quad \Psi_0 \odot (I - sCD) \odot S, \\ \Psi_0 \odot (I - sCD) \odot \bar{S}, \end{aligned} \quad (2.8)$$

where I is the identity operator on the Hilbert space, and

$$\bar{S} = I - S \quad (2.9)$$

is the *negation* of S , and means ‘‘ S did not occur.’’ The four projectors in (2.6) and (2.8) are the elementary histories, which constitute the sample space of \mathcal{F}_1 . (They sum to Ψ_0 rather than to the history space identity because we regard \mathcal{F}_1 , along with the families we shall consider later, as having a fixed initial event; see remarks towards the end of Sec. III in CHQR.) However, quantum theory assigns a weight of zero to the histories in (2.8), which means they are dynamically impossible, and have a zero probability of occurring. When discussing other families, we will usually not display the zero-weight histories explicitly.

Because the sample space has only one history, (2.6), with finite weight, we can infer that

$$\Pr(S_2 | \Psi_0) = 1, \quad (2.10)$$

where the subscripts, in this and later formulas, always refer to the times at which the events occur: S_2 is shorthand for ‘‘ S at time t_2 .’’ Thus, given that the system was in the state $|\Psi_0\rangle$ at t_0 , it will certainly (with probability 1) be in the state $|S\rangle$ at time t_2 . As is well known, unitary time evolution of this sort is something of an embarrassment for standard quantum mechanics because physicists typically prefer to draw the conclusion that at time t_2 the system is in one of the two states $|C^*D\rangle$ or $|CD^*\rangle$, each with a certain probability, rather than in the hard-to-interpret MQS state $|S\rangle$. The endless discussions about how to interpret the state $|S\rangle$ and resolve the corresponding ‘‘measurement’’ problem have been rightly criticized by Bell [23].

The consistent history solution to this measurement problem is to introduce a distinct consistent family \mathcal{F}_2 based upon a sample space of elementary histories

$$\Psi_0 \odot_s CD \odot C^*D, \quad \Psi_0 \odot_s CD \odot CD^*, \quad (2.11)$$

together with others of zero weight, such as $\Psi_0 \odot_s CD \odot C^*\bar{D}$. The physical interpretation of the first history in (2.11) is as follows: At t_0 the photon was in state $|a\rangle$ and the detectors were ready, at t_1 the photon was in $|s\rangle$ and the detectors remained in the ready state, whereas at t_2 detector C had detected the photon, while D was still in its ready state, indicating that it had not detected the photon. The second history in (2.11) has the same interpretation with the roles of C and D interchanged.

A straightforward application of the rules for calculating probabilities given in CHQR yields the results

$$\Pr(C_2^* | \Psi_0) = 1/2 = \Pr(D_2^* | \Psi_0), \quad (2.12)$$

where, once again, the subscript 2 indicates an event at the time t_2 . In other words, Eq. (2.12) tells us that, given the initial state $|\Psi_0\rangle$ at t_0 , at time t_2 the probability is 1/2 that detector C will have detected the photon, and 1/2 that it will have been detected by D . Similarly, one can show that

$$\Pr(C_2^* \vee D_2^* | \Psi_0) = 1, \quad \Pr(C_2^* \wedge D_2^* | \Psi_0) = 0, \quad (2.13)$$

where ‘‘ \vee ’’ stands for (nonexclusive) ‘‘or,’’ and ‘‘ \wedge ’’ means ‘‘and.’’ That is, at t_2 it is certain (probability one) that one or the other of the detectors has detected the photon, and it is not true (because the probability is 0) that both detectors have detected the photon.

Thus by using family \mathcal{F}_2 , the consistent history approach provides a definite, albeit probabilistic, answer to the question ‘‘which detector detected the photon?’’ and this is the answer that everyone who uses standard quantum theory agrees is correct. Notice that this is done without the MQS state $|S\rangle$ ever entering the discussion and causing the sort of confusion pointed out by Bell [23]. Indeed, according to consistent-history rules, $|S\rangle$ at time t_2 *cannot* enter a discussion based upon \mathcal{F}_2 , because it is *incompatible* with the histories in \mathcal{F}_2 in the sense that it cannot be added to \mathcal{F}_2 without violating the rules for constructing quantum event algebras. In particular, the event algebra must be a Boolean algebra of projectors, and the projector S does commute with either C^*D or CD^* , so it cannot be part of a Boolean algebra that contains the latter. Combining any discussion of $|S\rangle$ at time t_2 with \mathcal{F}_2 violates the *single framework* rule of quantum

reasoning, CHQR Sec. III. For the very same reason, it is impossible to include either C^* or D^* at time t_2 in family \mathcal{F}_1 , and hence, if one employs \mathcal{F}_1 , one cannot, according to the rules of consistent-history quantum theory, address the question of which detector has detected the particle. Such a discussion is “meaningless” in the technical sense that this interpretation of quantum theory assigns it no meaning. In the consistent-history approach one cannot interpret $|S\rangle$ to mean “ $|C^*D\rangle$ or $|CD^*\rangle$ for all practical purposes,” or for any other purposes.

Quantum incompatibility is also the reason we *cannot* combine Eq. (2.10) with Eq. (2.12) to reach the conclusion

$$\Pr(S_2 \wedge C_2^* | \Psi_0) = 1/2, \quad (2.14)$$

a result that would follow immediately by standard probabilistic reasoning from Eqs. (2.10) and (2.12) *if these probabilities were based upon the same sample space*. But the sample spaces for \mathcal{F}_1 and \mathcal{F}_2 are distinct, and they cannot be combined to form a common sample space. This is a typical quantum-mechanical effect that serves to define when two consistent families are mutually incompatible. Consequently, the left side of Eq. (2.14) is meaningless, and assigning it the value 1/2, or any other value, is a logical error.

Now consider a third family of histories \mathcal{F}_3 based on a sample space containing two histories of finite weight,

$$\Psi_0 \odot c \odot C^*, \quad \Psi_0 \odot d \odot D^*, \quad (2.15)$$

together with additional histories of zero weight. (The projectors at t_1 and t_2 in the histories in (2.15), unlike those in (2.11), do not project onto pure states. However, one could replace c with cCD , C^* with C^*D , etc. without altering the following discussion.) The family \mathcal{F}_3 is obviously incompatible with \mathcal{F}_2 because the projectors c and d at t_1 do not commute with s , and it is obviously incompatible with \mathcal{F}_1 for the same reason, and also because of the detector states at t_2 .

The conditional probabilities (2.12) can be derived in \mathcal{F}_3 as well as in \mathcal{F}_2 ; this reflects a very general result (CHQR Sec. IV) that conditional probabilities of this type always have the same values in any framework in which they can be defined, whether or not the frameworks are incompatible. In addition, in \mathcal{F}_3 (but not \mathcal{F}_2) one has

$$\Pr(c_1 | \Psi_0) = 1/2 = \Pr(d_1 | \Psi_0), \quad (2.16)$$

a sort of microscopic analog of (2.12). In other words, given the initial state at t_0 , one can conclude that at time t_1 the photon will be either in the c channel with probability 1/2, or in the d channel with the same probability. Note that these probabilities, unlike those of standard textbook quantum theory, make no reference to measurements, which only take place some time later. That later measurements yield corresponding probabilities, Eq. (2.12), is perfectly consistent with the general principle, see Sec. IV D, that the probabilities assigned to events and histories by the consistent-history approach can always be checked, in principle, using suitable (idealized) measurements. (Measurements play no distinguished role in consistent-history quantum mechanics; if a closed quantum system contains some measuring apparatus,

the events involving the apparatus are analyzed in precisely the same way as all other events.)

Both Eq. (2.12) and Eq. (2.16) can be obtained by straightforward application of the Born rule of standard quantum theory [although its use in Eq. (2.16) might be regarded as problematical by some practitioners, since no measurement is involved]. But the next result,

$$\Pr(c_1 | \Psi_0 \wedge C_2^*) = 1, \quad \Pr(d_1 | \Psi_0 \wedge C_2^*) = 0, \quad (2.17)$$

goes beyond anything one can calculate with the Born rule. It says that given the initial state, and the fact that at t_2 it was detector C that detected the photon, one can be certain that at time t_1 the photon was in the c channel and not in the d channel. Given the arrangement shown in Fig. 1, this is a very reasonable result, and shows another way in which consistent-history methods make sense out of quantum measurements. That appropriate measurements will reveal properties a measured system had before it interacted with the apparatus is a principle used constantly in the design of experimental equipment, and the inability of standard textbook quantum mechanics to explain this is a defect just as serious as its inability to deal with Schrödinger’s cat.

Notice that the result just discussed can be obtained using \mathcal{F}_3 , but not \mathcal{F}_2 or \mathcal{F}_1 . The question “was the photon in c or d at time t_1 ?” is not meaningful in \mathcal{F}_1 and \mathcal{F}_2 because the projectors c and d needed to answer it are incompatible with these families. On the other hand, if we use \mathcal{F}_2 we can derive the result

$$\Pr(s_1 | \Psi_0 \wedge C_2^*) = 1. \quad (2.18)$$

That is, under the same conditions as in Eq. (2.17), one can conclude with certainty that at time t_1 , the particle was in the superposition state $|s\rangle$ defined in Eq. (2.1). Just as in the case of Eqs. (2.10) and (2.12) discussed earlier, because Eqs. (2.17) and (2.18) were derived in two incompatible families, they cannot be combined to reach the nonsensical conclusion

$$\Pr(c_1 \wedge s_1 | \Psi_0 \wedge C_2^*) = 1, \quad (2.19)$$

which, were it correct, would be the analog of asserting that a spin half particle has both $S_x = 1/2$ and $S_z = 1/2$ at the same time.

In summary, we have studied the gedanken experiment shown in Fig. 1 using three separate and mutually incompatible frameworks or consistent families, \mathcal{F}_1 , \mathcal{F}_2 , and \mathcal{F}_3 , each based upon its own sample space of histories involving events at the three times t_0 , t_1 , and t_2 , and have derived a number of different (probabilistic) conclusions. There are many (in fact, an uncountably infinite number of) other frameworks that could have been employed to discuss the same situation. The fact that the same physical system can be discussed using many different frameworks gives rise to the *problem of choice*: how does one decide which framework is appropriate for describing what goes on in this closed quantum system?

A partial answer is suggested by the fact that certain questions of physical interest, such as “which detector detected the photon?” and “which channel was the photon in before it was detected?” can be addressed in certain families but not in other families, because the projectors needed to provide a

quantum-mechanical representation of the question are only compatible with certain families. Thus incompatibility serves to limit choice, but at the same time it gives rise to the *problem of incompatibility*: how are we to understand the existence of, and the relationship among, incompatible histories and families?

These two problems will be discussed in the following sections, beginning with classical analogies in Sec. III, which will help us understand something of what is involved in choosing a quantum framework. Unfortunately, there is no good classical analog of quantum incompatibility, so the discussion of this concept in Sec. IV is based upon simple quantum examples. Combining the results from these two sections leads to the conclusions stated in Part A of Sec. V.

III. CLASSICAL ANALOGIES

No classical analogy is fully adequate for understanding the implications of quantum theory, which must, in the end, be considered on its own terms. Nevertheless, classical ideas can be extremely helpful in suggesting fruitful ways to think about quantum principles. This is particularly true for the problem of choice, where classical analogies suggest (though they obviously cannot prove) that the freedom a physicist has in choosing a framework does not render the resulting theory unscientific or lacking in objectivity, nor does it mean that the physicist's choice somehow "influences reality."

As a first analogy, consider a professional historian composing a history of Great Britain. Obviously, as a practical matter, he must choose to emphasize certain subjects and ignore, or at least give much less consideration, to others. The choice of topics is determined by what interests the historian (or perhaps the pressures on his professional career), and are not the consequence of some "law of history." Nor does his choice have any influence on historical events (at least those occurring prior to when his work is published). Nevertheless, the account he produces, if he does his job well, is not a purely subjective one, and other historians dealing with the same subject matter ought to concur about the historical facts, even if they disagree, as they surely will, about their significance.

A second analogy is provided by a draftsman producing a representation of a three-dimensional object on a two-dimensional piece of paper. In order to do this, he has to choose some perspective from which to view (or imagine viewing) the object. This choice is not determined by any "law of representation," though it can be influenced by pragmatic considerations: some things can be better represented using one perspective than another. Nor, obviously, does the choice influence the object itself, although it constrains the amount and type of descriptive information that can be included in the drawing.

The choice of gauge in classical electromagnetism provides a third analogy. This choice is a matter of convenience, which obviously does not influence the system being described. In addition, the same physical conclusions can be drawn using any gauge, so the choice is based entirely on pragmatic considerations. To be sure, this is different from what encounters in the case of quantum families, since, as noted repeatedly for the example in Sec. II, the topics one can discuss, and thus the conclusions one can draw, do de-

pend upon which family is employed. Nonetheless, a particular conditional probability can often be calculated in many different families, and in all families in which it can be calculated it has the same value. Thus, for example, Eq. (2.12) can be obtained using either \mathcal{F}_2 or \mathcal{F}_3 , though not \mathcal{F}_1 . Hence, relative to a particular conditional probability, the choice among the families in which it can be computed is similar to the choice of a gauge.

Classical Hamiltonian mechanics, which describes a mechanical system with N degrees of freedom by means of a $2N$ -dimensional phase space Γ , provides a fourth analogy. To be sure, once the phase space and the Hamiltonian have been specified, the only remaining choice is that of the appropriate phase space trajectory or orbit, $\gamma(t)$, and this is typically made by assuming that the state γ of the system is known at some time, say $t=0$. While this choice bears some resemblance to the quantum choice problem, the differences outnumber the similarities. Thus in the quantum case, with the exception of histories involving only one time, there is always more than one family consistent with a specification of the quantum state at an initial time. This reflects the fact that quantum mechanics is a stochastic theory, unlike classical mechanics, which is deterministic. In addition, the choice between classical trajectories is a choice between mutually exclusive alternatives: if one trajectory is a correct description of the system, all others must be incorrect. By contrast, the choice between consistent families is never a choice between mutually exclusive alternatives in this same sense, and while different quantum frameworks can be incompatible, incompatibility is a very different relationship from that of being mutually exclusive, as will be shown in Sec. IV. Nevertheless, it is probably useful to include classical mechanics among our list of analogies, if for no other reason than that the disanalogous features are sometimes employed to analyze consistent-history results, leading to the (erroneous) conclusion that the latter are unsatisfactory.

Classical *statistical* mechanics, itself a stochastic theory, provides a better analogy for the quantum choice problem than does classical mechanics. Thus our fifth analogy is a coarse-graining of the classical phase space (CHQR, Sec. VII A), a covering of Γ by a set of nonintersecting cells, which provides a coarse-grained description of the system through specifying which cell is occupied by the phase point at a succession of times. (Making these a finite set of discrete times, and allowing the cells defining the coarse graining to depend upon time, produces the closest analogy with the quantum case.)

Choosing a coarse-graining and choosing a quantum framework are analogous in that in both cases the choice is one made by the physicist in terms of the physical problems he wishes to discuss, and is not something dictated by the laws of nature. At least in the case of the classical coarse graining, it is at once obvious that this choice has no influence on the behavior the system being described, although it may very well limit the type of description that can be constructed. For example, in order to describe irreversible or other hydrodynamic behavior, one wants cells that are not too large, but also not too small. Because the coarse-graining is chosen by the physicist, the corresponding description can be said to be "subjective," but this is a harmless subjectivity, because two physicists who use the same coarse-graining

will reach identical conclusions. There is another analogy in that once a coarse-graining has been specified, one and only one coarse-grained history will actually occur, since a phase space trajectory will be in precisely one cell of the coarse-graining at any given time. In probabilistic terms, a succession of cells from the classical coarse-graining at a sequence of different times is an element of a sample space, just as an elementary history in a quantum framework is an element of a sample space, and given a sample space, one expects that one and only one of the constituent “events” will actually occur.

Along with the positive analogies just discussed, there is an important difference between descriptions based on classical coarse-grainings and those that employ quantum frameworks. Two classical coarse-grainings always possess a *common refinement*, a third coarse-graining constructed from the intersections of the cells in the two collections. Thus given any two coarse-grained histories that describe the same system (over the same time interval), there is always a finer-grained history constructed using the common refinement, which incorporates, or implies, each of the coarser histories. The ultimate refinement of all coarse grainings is, of course, a description of the system in terms of a phase space trajectory. Two *compatible* quantum frameworks possess (as a matter of definition) a common refinement, and for them the situation is quite analogous to the classical state of affairs just discussed. However, incompatible frameworks often arise in quantum theory, as illustrated in Sec. II, and their mutual relationship is necessarily different from that which exists among (always compatible) classical coarse grainings.

The solution to the problem of (quantum) choice suggested by the classical analogies discussed above (with the exception of classical mechanics, where a good analog of a quantum choice does not exist) is that the choice can be made on pragmatic grounds, that is to say, relative to the physical problems which concern the physicist who is constructing a quantum description. There is, to be sure, no guarantee that this solution is the correct one, for analogies have their limitations, and we have just noted an important one in the case of classical statistical mechanics (which also applies to the other examples in this section): There is nothing that corresponds to quantum incompatibility. On the other hand, there is also no reason to suppose that, just because they are imperfect, the analogies have led us to a false solution of the choice problem. To obtain further insight, we need to take a closer look at quantum incompatibility in quantum terms. That is the subject of the next section.

IV. QUANTUM INCOMPATIBILITY

A. Logic in two dimensions

In Sec. II we introduced three separate consistent families of histories, or frameworks, to describe the same physical system. The analogy of the draftsman in Sec. III suggests that these can be thought of as three distinct, but equally valid, perspectives from which to describe the system. However, none of the analogies of Sec. III provides insight into the fact that \mathcal{F}_1 , \mathcal{F}_2 , and \mathcal{F}_3 are incompatible alternatives. Quantum incompatibility of this sort has no good classical analog, so one needs to employ quantum examples in order to understand it. The simplest nontrivial quantum system, a

two-dimensional Hilbert space, which we shall think of as representing the angular momentum states of a spin half particle, can provide some useful insight into what is going on. The discussion given here employs a somewhat different point of view from that in CHQR, Sec. VI A, and can be thought of as supplementing the latter.

The histories of interest involve only one time, and the “event” at this time corresponds to a *ray*, or one-dimensional subspace of the Hilbert space, whose physical interpretation is that the component of spin angular momentum in a particular direction in space is $1/2$ (in units of \hbar). For example, a particular ray, which we denote by Z^+ (the same symbol denotes the ray and the projector onto the ray), corresponds to the proposition “ $S_z = 1/2$.” What corresponds to the *negation* of this proposition, to “it is not the case that $S_z = 1/2$ ”? While one might suppose it to be “the spin is in some other direction,” standard quantum mechanics identifies negation of a proposition with the orthogonal complement of the corresponding subspace, which means that the negation of “ $S_z = 1/2$ ” is the unique ray Z^- perpendicular to Z^+ , which corresponds to the proposition “ $S_z = -1/2$.” Understanding negation in this way has a certain plausibility: after all, if S_z is not equal to $1/2$, what other value can it have? Furthermore, applying negation twice in a row brings us back to the original property. Nonetheless, the translation of logical terms, such as “not,” into a quantum context is a nontrivial matter, and the reader who is not content with the traditional approach is welcome to try and improve upon it.

If we agree that the negation of a ray R in a two-dimensional Hilbert space is the orthogonal ray \tilde{R} , the next question to address is the definition of “ R and S ” ($R \wedge S$) for two rays. Guided by the rules of ordinary propositional logic, it is plausible that if $S = R$, then $R \wedge R = R$, and if S is the negation of R ,

$$\tilde{R} \wedge R = R \wedge \tilde{R} = 0, \quad (4.1)$$

where 0 is the proposition that is always false, corresponding to a projector onto the origin of the Hilbert space. But what if S is neither R nor \tilde{R} ? We might suppose that in such cases there is always a third ray T (which depends, of course, on R and S) such that

$$R \wedge S = T. \quad (4.2)$$

However, Eq. (4.2) is unsatisfactory, for consider the result

$$\tilde{R} \wedge T = \tilde{R} \wedge (R \wedge S) = (\tilde{R} \wedge R) \wedge S = 0 \wedge S = 0 \quad (4.3)$$

obtained by applying the usual logical rules governing “ \wedge .” For this to hold, T must be the same as R , since in all other cases we are assuming that $\tilde{R} \wedge T$ is another ray, not 0. However, the argument in Eq. (4.3) applied to $T \wedge \tilde{S}$ tells us that this, too, is 0, so that T must be the same as S . But T cannot be equal to both R and S , since, by hypothesis, they are unequal.

One way out of this dilemma is to follow Birkhoff and von Neumann [24], and assume that whenever R and S are unequal, “ R and S ” is false:

$$R \neq S \Rightarrow R \wedge S = 0. \quad (4.4)$$

Taking the negation of both sides of the equality in Eq. (4.4), and using the usual rules of logic, we conclude that, under the same conditions, “ R or S ” ($R \vee S$) is true:

$$R \neq S \Rightarrow R \vee S = I, \quad (4.5)$$

where I represents the proposition that is always true, corresponding to the identity operator on the Hilbert space. Alas, (4.4) and (4.5) lead to a contradiction if we employ the standard distributive law of propositional logic:

$$Q \vee (R \wedge S) = (Q \vee R) \wedge (Q \vee S). \quad (4.6)$$

The reason is that if Q , R , and S are three distinct rays, the left side of Eq. (4.6), using Eq. (4.4) and assuming that $Q \vee 0 = Q$, is Q , while the right side, see Eq. (4.5), is $I \wedge I = I$. Birkhoff and von Neumann’s solution to this problem is to adopt a “quantum logic” in which Eq. (4.6) no longer holds.

Consistent histories quantum theory has a very different way of escaping the dilemma posed by Eq. (4.3). When S is different from either R or \bar{R} , the properties corresponding to these two rays are said to be *incompatible*, and $R \wedge S$ is considered to be *meaningless*: it is not a formula composed according to the syntactical rules that govern meaningful quantum discourse. To illustrate the difference between this and the approach of Birkhoff and von Neumann, suppose that $S_z = 1/2$ is true. According to Eq. (4.4), “ $S_x = 1/2$ and $S_z = 1/2$ ” is false, as is “ $S_x = -1/2$ and $S_z = 1/2$.” The normal rules of logic would then tell us that both $S_x = 1/2$ and $S_x = -1/2$ are false, which cannot be the case, since one is the negation of the other. Consequently, the usual rules of logical reasoning have to be modified if one uses the Birkhoff and von Neumann system. By contrast, in the consistent-history approach, “ $S_x = 1/2$ and $S_z = 1/2$ ” is neither true nor false; instead, it is meaningless. Thus the truth of $S_z = 1/2$ tells us nothing at all about S_x , so we do not reach a contradiction.

Similarly, in the consistent-history approach it makes no sense to ask “is $S_x = 1/2$ or is $S_z = 1/2$?” for such a question implicitly assumes that the two can be compared: one is true and the other false, or perhaps both are true, or both are false. But none of these is possible under the rules of consistent-history reasoning [25]. Unless propositions belong to the same framework, which in the particular example we are considering (but not in general; see below and Appendix A) is equivalent to requiring that the corresponding projectors commute with each other, the consistent-history formalism allows no logical comparison between them, of any kind. In particular, it is very important to distinguish *incompatible*, which is the relationship between $S_x = 1/2$ and $S_z = 1/2$, from *mutually exclusive*, the relationship between $S_z = 1/2$ and $S_z = -1/2$. As already noted, the truth of $S_z = 1/2$ tells us nothing at all about the truth or falsity of the incompatible $S_x = 1/2$. However, the truth of $S_z = 1/2$ at once implies that $S_z = -1/2$ is false, since the two are mutually exclusive: if a spin half particle emerges in one channel from a Stern-Gerlach apparatus, it surely does not emerge in the other channel.

Standard textbook quantum mechanics escapes the dilemma of Eq. (4.3) by yet a different method, by ignoring it and treating a quantum system as a “black box,” which can be subjected to external (classical?) measurements. But ig-

oring a problem does not necessarily make it go away, and the enormous conceptual confusion that besets “the measurement problem” in standard quantum theory is an almost inevitable consequence of its inability to provide clear principles for discussing the properties of a spin half particle.

B. Generalization to histories

In CHQR, two frameworks or consistent families \mathcal{F} and \mathcal{F}' were defined to be incompatible if they lack a common refinement, that is, if there is no consistent family \mathcal{G} that contains all the histories in both \mathcal{F} and \mathcal{F}' . Consistent with that usage, we shall say that two histories Y and Y' are incompatible if there is no (single) consistent family that contains both of them, and a framework \mathcal{F} and a history Y' are incompatible if there is no consistent family \mathcal{G} that contains Y' along with all the histories in \mathcal{F} . The spin half case discussed above is best thought of as a “quantum analogy,” which illustrates some, but not all of the features of incompatibility. In particular, it shows how incompatibility can arise when the projectors representing properties at a single time do not commute with each other. This is also the source of incompatibility for the histories and families discussed in Sec. II C, where the projectors on the Hilbert space of histories [as in (2.6) and (2.11)] do not commute with each other. Incompatibility can also arise because, even though the relevant projectors commute, consistency conditions are not satisfied for the enlarged Boolean algebra. For an example, see Sec. VI D in CHQR (and Appendix A below).

Whatever the source of incompatibility, consistent-histories quantum theory deals with it in the manner suggested above in the discussion of a spin half particle, through a syntactical rule that states that all logical argumentation must take place inside a single framework or consistent history (for details, see CHQR). On the other hand, the usual rules of reasoning apply *within* a single framework, and for these purposes no modifications of standard propositional logic are required. And, just as in the case of a spin half particle, it is important to distinguish *quantum incompatibility*, which means no comparison is possible between two histories (or results calculated in two frameworks), from *mutually exclusive*, the relationship between two elementary families belonging to the same framework. Thus if A and B are mutually exclusive, the truth of one implies the falsity of the other or, as histories, the occurrence of A means that B did not occur, and vice versa. If, on the other hand, A and B are incompatible, the truth of one, which can only be defined relative to some framework which contains it, tells us nothing about the truth or falsity of the other; they are “not comparable”: no comparisons between them are possible, because such comparisons make no (quantum-mechanical) sense. In particular, if A and B are incompatible histories, the question “did A occur or did B occur?” does not make sense, for there is no framework in which they could be compared.

We have already applied these principles in discussing the examples in Sec. II, in particular when noting that no discussion of the MQS state S at time t_2 can be meaningfully combined with the histories (2.11) used in \mathcal{F}_2 to discuss which detector detected the photon. Similarly, the family \mathcal{F}_1 of unitary time evolution cannot be used to discuss, much

less answer, the question “which detector?” because \mathcal{F}_1 is incompatible with histories containing C^* or D^* at t_2 . And since, as noted above, incompatibility prevents us from asking whether the history $\Psi_0 \odot_s CD \odot S$ occurred rather than $\Psi_0 \odot_s CD \odot C^*D$ or $\Psi_0 \odot_s CD \odot CD^*$, it does not make sense to ask which of the incompatible frameworks \mathcal{F}_1 and \mathcal{F}_2 provides the correct description of this gedanken experiment.

Generalizing from this example, we conclude that whenever \mathcal{F} and \mathcal{G} are any two incompatible families, one cannot ask which provides the correct quantum-mechanical description; because the two sample spaces are mutually incompatible, they cannot be compared. But even if \mathcal{F} and \mathcal{G} are compatible, it also makes no sense to ask which is correct, for in that case their relationship is analogous to that of two classical coarse grainings, as discussed in Sec. III, and it is obvious that one cannot say that one coarse-graining provides a “correct” and another an “incorrect” physical description. Since alternative frameworks are either compatible or incompatible, we conclude that it is *never* sensible to ask “which is the correct framework?” at least in the same sense in which one can ask “which is the correct classical phase space trajectory?” The relationship between phase space trajectories is that between mutually exclusive possibilities, whereas quantum frameworks are not related to each other in this way. Of course it does make sense to ask “which framework is useful for solving this particular problem?” or “which framework provides one with the greatest physical insight?” just as in the case of classical coarse-grainings.

C. Using measurements to confirm histories

Further insight into how the descriptions provided by two or more incompatible families are related to each other can be obtained by asking how the predictions of consistent history quantum theory about the behavior of a closed quantum system might, in principle, be verified experimentally. Imagine that the system is in a closed box, the initial state Ψ_0 is known, and the consistent historian (who, needless to say, is outside the box [26]) has carried out calculations of probabilities for different histories belonging to a variety of distinct and mutually incompatible families. How might these probabilities be checked by measurements? Checking them involves opening the box, or at least letting the contents interact in some way with the external world, and this immediately raises the well-known problem that measurements might disturb what is going on in the box, so that the results they reveal might not be indicative of what would have happened in their absence. However, the very same objection can be raised in a “classical” context in which specifically quantum effects play no role. A box might contain sensitive photographic film, which would be changed if one were to look at it. Or perhaps the box contains a bird, which will escape if the box is opened.

In the end there is no ultimate proof that one’s measurement is not creating the effect observed, just as there can be no ultimate argument against solipsism. However, if he had a theory that was sufficiently powerful to predict the perturbing effects of a measurement, and if other predictions of this theory for other situations had been amply confirmed by experiment, a physicist (in contrast to a philosopher) would

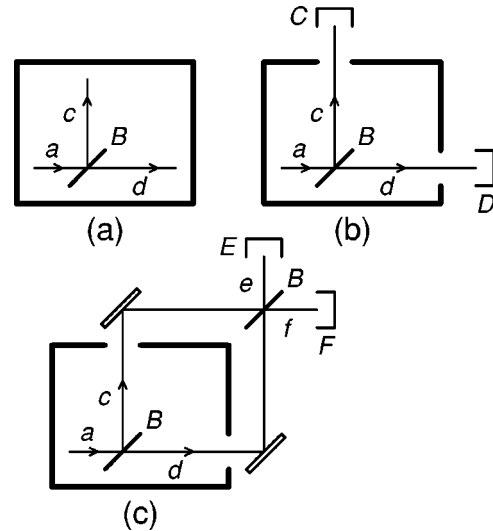


FIG. 2. Histories for the closed system in (a) can be confirmed using either the configuration shown in (b) or the one in (c).

probably be willing to use it in order to design measurements that produced minimal perturbations, or to calculate the effects of perturbations if these were unavoidable. Consistent history quantum mechanics is a theory of this sort, unlike standard textbook quantum mechanics, whose inability to relate the results of measurements to properties of the system being measured is one of its principal deficiencies. Of course, using this approach cannot demonstrate that the consistent-history predictions of what is going on in a closed box are “really true,” but it does provide the sort of internal consistency that in other areas of science (e.g., the study of the Earth’s core by means of seismic waves) is considered good evidence for the existence of phenomena that cannot be directly observed.

As an example, consider Fig. 2(a), which is a stripped down version of the gedanken experiment in Fig. 1, as the detectors C and D have been omitted from the quantum system enclosed in the box indicated by the heavy line. We consider two consistent families \mathcal{E}_1 and \mathcal{E}_2 with histories of nonzero weight:

$$\mathcal{E}_1: \quad a \odot c, a \odot d, \quad (4.7)$$

$$\mathcal{E}_2: \quad a \odot s \quad (4.8)$$

referring to events at the times t_0 and t_1 ; the notation is that of Sec. II.

To check the predictions of \mathcal{E}_1 , we open holes in the sides of the box, Fig. 2(b), at a time just before the photon would have encountered one of the walls, to allow it to pass out of the box and be detected by one of the two detectors C and D . Since the two histories in (4.7) are predicted to occur with probability 1/2, checking this requires repeating the experiment a number of times. To determine whether opening the holes and placing detectors outside the box has somehow perturbed the system, we employ a consistent-history analysis of the larger closed system that includes the original box and the detectors C and D (along with the mechanism for opening the holes, etc.). For this larger system we ask: if the photon is detected by C , was it earlier (while the holes were

still closed) in the c channel? The answer is affirmative; see Eq. (2.17) and the associated discussion. As in each realization of the experiment the particle is detected by C or D , this confirms the usual consistent-history interpretation of the two histories in (4.7) as mutually exclusive alternatives, one or the other of which must occur in any particular case.

If we employ family \mathcal{E}_2 , the single history in (4.8) is predicted to occur every time the experiment is carried out. To verify this requires the slightly more complicated arrangement shown in Fig. 2(c). Once again, the holes are opened at the last possible instant, but then a pair of mirrors and a second beam splitter are used to compare the phases of the wave packets emerging in the c and d channels. Assume the path lengths are arranged so that a photon initially in a eventually emerges in f . The fact that the photon is always detected by F , and never by E , can then be used as an experimental confirmation that it was in the state s at t_1 , since a consistent history analysis of the larger closed system yields the result

$$\Pr(s_1|\Psi_0 \wedge F_2) = 1. \quad (4.9)$$

Once again, it is the ability of consistent histories to verify that a measured system had a particular property before the measurement took place that lends plausibility to the experimental confirmation of a prediction referring to processes going on in a closed system.

Checking events at intermediate rather than final times in a family of histories can be carried out using analogous, though somewhat more complicated, arrangements. One can imagine the box to be supplied with ports through which probes of appropriate type can be inserted at appropriate times. The argument that this sort of measurement is possible in principle (that is, without violating the laws of quantum theory), and that appropriate measurements do not perturb the system in unacceptable ways, will be found in Sec. 5 of [1]. Alternatively, one can suppose that a single closed box contains both the system of interest and an appropriate measuring apparatus that interacts with the system at suitable times and records the results. At the end of the experiment the box is opened and the records are read. In either case, the consistent-history approach allows one to discuss whether the system actually did have the properties that the measurements revealed, and whether these measurements perturbed the system in unacceptable ways.

Whether the events of interest occur at intermediate or final times, checking the predictions given by different incompatible frameworks always involves alternative experimental arrangements, which are either mutually exclusive, as in Figs. 2(b) and 2(c), or of a sort in which one set of measurements makes it impossible to discuss the system using some alternative consistent family. In any case, just as there is no single ‘‘correct’’ choice of consistent family for describing the system, there is no single arrangement of apparatus that can be used to verify the predictions obtained using different families.

Note that neither the framework nor the experimental arrangement needed to check its predictions is singled out by some ‘‘law of nature.’’ Indeed, were there some such law that told us, for example, that \mathcal{E}_1 is the correct framework, we would have great difficulty interpreting the results of

measurements of the sort shown in Fig. 2(c). In this sense, the physicist’s freedom to choose a framework, as suggested by the classical analogies in Sec. III, is not only possible in the presence of quantum incompatibility, but appears necessary in order to have a theory with a consistent physical interpretation.

V. SUMMARY AND SOME APPLICATIONS

A. Overall summary

The example in Sec. II together with the classical analogies in Sec. III and the discussion of incompatibility in Sec. IV lead to the following conclusions relative to the problems of choice and incompatibility as stated at the end of Sec. II. There are many possible consistent families or frameworks that can be used to describe the same quantum system, and the choice of which of these to employ is a choice made by the physicist based upon pragmatic considerations, namely, the type of physical problem he is trying to study or the question he wants to answer. Because of quantum incompatibility, a question such as ‘‘where was the photon before it was detected?’’ can be answered in certain frameworks and not in others. Given a particular physical question there are still, in general, a large number of frameworks in which it can be discussed, but since they all yield the same values for the relevant conditional probabilities, choosing among them is, relative to this particular question, rather like the choice of gauge in classical electromagnetism: the answer does not depend upon the framework.

The choice among alternative *compatible* families is closely analogous to the choice among coarse grainings of a classical phase space, a situation in which it is transparently obvious that the physicist’s choice of a mode of description has no influence whatsoever on the physical system being described. Furthermore, such a choice is not ‘‘subjective’’ in any unacceptable way, for two physicists who use the same coarse graining will reach identical conclusions. That these same conclusions are still correct in the case of a choice between *incompatible* consistent families is not so obvious, because there is no good classical analogy. Nonetheless, their truth is supported by the discussion of quantum incompatibility in Sec. IV. To begin with, the relationship of incompatibility between frameworks (or between histories, etc.) is quite distinct from that between mutually exclusive alternatives (as in a sample space), where the correctness of one means that the others are necessarily false. Incompatible frameworks are never related in this way; and from the fact that some history has occurred, one cannot conclude that some other history incompatible with it has not occurred. Understanding the difference between mutually exclusive and incompatible helps prevent one from supposing that the physicist’s choice of a framework somehow influences the world by ‘‘preventing’’ or ‘‘interfering with’’ histories that occur in some other framework. Instead, the choice simply limits the type of description that the physicist can construct.

Additional support is provided by the discussion, in Sec. IV C, of how predictions of what goes on in a closed system based upon the consistent-histories formalism can in principle be checked experimentally. This ‘‘operational’’ point of view confirms that the choice of consistent family is up to the physicist, but that the experimental arrangements needed

to confirm the predictions made using a particular framework depend upon that framework. By contrast, assuming that the framework is determined by some “law of nature,” rather than chosen by the physicist, is rather unhelpful, since it produces new conceptual difficulties.

Incompatibility, as noted in Sec. IV A, is a specifically quantum concept, which represents one way of meeting the (necessary) process of modifying the logic of classical propositions in order to achieve descriptions that conform to the way standard quantum theory employs Hilbert space. In the consistent-history approach, incompatibility is a syntactical rule that governs which combinations of propositions about a quantum system can be said to be physically meaningful. In this respect it is quite different from the approach of Birkhoff and von Neumann [24], and has the advantage that *within* a single consistent family there is no need to replace the ordinary logic of propositions with a new “quantum logic.” (By contrast, textbook quantum mechanics in effect evades the logical issues by constructing a phenomenological theory of measurement, thus giving rise to an insoluble “measurement problem” completely absent from the consistent histories approach.)

To be sure, quantum incompatibility and the choice among incompatible frameworks for describing a particular quantum system are not matters that can be easily understood using an intuition trained largely by the “classical” world of everyday experience. This should come as no surprise: quantum mechanics is distinctly different from classical mechanics, and it is only to be expected that it contains concepts that conflict with prequantum thinking. The same is true of special relativity in relationship to Newtonian mechanics. The fact that events that occur at the same time in one coordinate system need not be simultaneous in another, and the physicist’s ability to change the time difference between them by adopting a new coordinate system, have not been considered insuperable barriers, or even serious objections to adopting special relativity as a good scientific theory of the world, and supposing that it provides a better description of physical reality than prerelativistic physics. Although the analogy is, of course, not perfect, there seems to be no reason why the physicist’s liberty to choose a consistent family renders consistent histories an unsatisfactory interpretation of quantum theory, nor why quantum incompatibility should not be included with our other ideas about what constitutes physical reality.

B. Logic, truth, and reality

The structure of quantum reasoning set forth in CHQR, which is largely compatible with earlier work by Omnès [4,6], agrees with but also differs from ordinary propositional logic, depending upon one’s point of view. As long as the discussion is confined to a single framework (or “logic,” in Omnès’ terminology) the usual rules employed for ordinary probabilistic theories apply, and no new logical concepts are required. In this respect one can agree with Omnès that consistent-histories quantum theory does not require a new “unconventional logic” [27].

On the other hand, the syntactical rules that determine what propositions can be part of a framework, whether two frameworks are compatible, and the like, are decidedly non-

classical, since their very definitions are based upon properties of the quantum Hilbert space and the corresponding unitary time transformations. As these rules are essential for correct quantum reasoning, and are thus central to the logical structure of quantum theory, it is also correct to say that, at least in this respect, consistent histories involve a new form of logic, one with no classical counterpart. Thus d’Espagnat’s assertion [28] that consistent-histories reasoning “. . . could only be valid in some as yet unspecified logic, of which it is not even known how it could be self-consistent” is not off the mark, though it is now out of date, since in CHQR the required self-consistent logic has been specified in considerable detail, filling in various items lacking in the earlier [5]. As noted in Sec. IV A, the use of some form of nonclassical reasoning is virtually inescapable once one accepts the association of propositions, and their negations, with subspaces of Hilbert space, in the way generally employed in standard quantum theory. Whereas consistent histories handles this in a very different way from the quantum logic of Birkhoff and von Neumann, it still employs nonclassical (and hence counterintuitive) ideas.

One important difference between Omnès and CHQR is in the definition of “true.” In CHQR, “true” is interpreted as “probability one.” Thus if certain data are assumed to be true, and the probability, conditioned upon these data, of a certain proposition is one, then this proposition is true. The advantage of this approach is that as long as one sticks to a single framework, “true” functions in essentially the same way as in ordinary logic and probability theory. However, because probabilities can only be discussed within some framework, comparisons of “true” between incompatible frameworks are impossible, and in this sense “true” interpreted as “probability one” must be understood as relative to a framework.

The feature just mentioned has been criticized by d’Espagnat [12,13,15]. But it is hard to see how to get around it if one wishes to maintain (as do Omnès and I) that reasoning inside a single framework should follow classical rules, and classical rules associate “true” (in a probabilistic theory) with “probability one.” Omnès’ attempt to develop an alternative definition of “true” [4,6] did not succeed [17], as he himself admits [29], and at present there seems to be no serious alternative to CHQR. In defense of the latter it is worth noting that, as a very general principle, one cannot expect to import classical concepts into quantum theory “duty free,” that is, without alterations, either in formal definitions, or in intuitive properties, or both. This is widely accepted by physicists in the case of dynamical quantities: we do not insist that quantum position and momentum commute with each other. That it also holds for logical properties and relationships deserves to be more widely appreciated; see the discussion of “not” and “and” in Sec. IV A (and of “contrary” in Appendix A). Thus it is unreasonable to expect that “quantum truth” will coincide with “classical truth” in every respect. The definition adopted in CHQR has some virtues; these include the fact that it is (almost) the same as its classical counterpart in the case of a single framework, and reduces to the usual sense of “true” in the classical (correspondence) limit in which incompatibility disappears and all frameworks possess a common refinement. Given these properties, it seems reasonable to adopt the defi-

dition given in CHQR as a suitable quantum counterpart of “true,” at least until some superior alternative appears on the scene.

Similarly, d’Espagnat [30] thinks that consistent history quantum theory cannot be interpreted in a realistic way, because it does not conform to “a basic requirement of traditional realism, that there are facts that are true quite independently of the conventions we decide to make as to which consistent family of histories we prefer to discuss.” In response, it is worth noting that some modifications of traditional realism are only to be expected if quantum theory is one of the truly revolutionary developments of twentieth century science. Nevertheless, the changes in prequantum realism required if we adopt consistent histories are perhaps less radical than d’Espagnat’s words might suggest.

Note, first of all, that the choice of framework by a physicist is not something by which he can, in any sense, render true propositions untrue, or make untrue propositions true. The reason is that as long as frameworks are compatible, choosing one or another is as “harmless” as selecting one of the alternative coarse-grainings of classical phase space, as discussed in Sec. III. If, on the other hand, some framework is chosen that is incompatible with a proposition, this choice does not make the proposition true or false; instead the proposition is “indiscussible” within this framework.

As this point is often misunderstood, it may be helpful to consider the specific example discussed in Sec. IV C. Suppose that theorist T , standing outside the box of Fig. 2, has used family \mathcal{E}_2 , (4.8), to make a prediction, and experimentalist E has set up the apparatus to test it using the arrangement in Fig. 2(c). Suppose a second theorist, T^* , carries out calculations using the incompatible family \mathcal{E}_1 instead. Will this alter what is going on inside the box? Obviously not; E ’s confirmation of T ’s prediction will be entirely unaffected by T^* ’s calculation. On the other hand, T^* will, if he limits himself to \mathcal{E}_1 , be unable to provide a coherent account of how E ’s measurement confirms T ’s prediction. In order to do that, T^* needs to employ \mathcal{E}_2 . If he does so, he will, of course, obtain the same result as T . On the other hand, if T^* can persuade E to carry out an alternative experiment using the arrangement of Fig. 2(b)—this cannot, of course, be done at the same time as the experiment in Fig. 2(c), but might be done later using a second photon—then this, or at least a sufficient number of experiments of this type, will confirm T^* ’s result, a situation that T can understand if he, too, employs \mathcal{E}_1 .

As well as showing that the choice of framework does not imply some mysterious influence of mind upon matter, this example illustrates some other aspects of quantum reality as viewed using consistent histories. The existence of a choice of framework does not render quantum theory subjective, for if T and T^* adopt the same framework, they come to the same conclusion. Predictions of what is going on in a quantum system can, in principle, be checked experimentally; the theorist’s freedom to choose a framework merely means that alternative experimental arrangements are needed to check predictions made using different frameworks.

Objective descriptions (in the sense just discussed), experimental confirmation, and the absence of peculiar influences of mind over matter are aspects of traditional realism that continue to hold true in the quantum realm if we accept

the results of consistent histories. But there is also something very different: one cannot simultaneously employ \mathcal{E}_2 and \mathcal{E}_1 for describing what is going on in the closed box. Not because using one of them makes the other false, but because there is no way to combine the results from the two incompatible families. Thus a “unicity” always present in classical physics, the ability to combine any two descriptions of the same system in a single description, is absent in quantum theory. If one wishes to maintain that this unicity is truly indispensable to realism, then, obviously, consistent-history quantum theory cannot be said to be “realistic.” The alternative, which of course I favor, is to include quantum incompatibility as part of our understanding of what quantum reality is all about, and why it differs from what reality was thought to be like before the advent of quantum theory.

C. Is the consistent-histories approach a predictive theory?

As the result of a detailed study [16,17], Dowker and Kent conclude that the consistent-history approach to quantum theory lacks predictive power, and for this reason is not satisfactory as a fundamental scientific theory. They accept the idea that once a consistent family (“consistent set” in their terminology) is specified, one and only one of the corresponding elementary histories will take place, and quantum theory can only assign a probability to the different possibilities. That the theory is probabilistic in this sense is not what concerns them. Rather, it is the fact that the consistent-history approach, as a fundamental theory, treats all frameworks or consistent families “democratically,” and provides no criterion to select out one in particular. In other words, there is no “law of nature” that specifies the framework. Thus, from their perspective, there is no way of calculating probabilities of specific histories, since probabilities cannot be computed without using some framework, and the theory does not tell one which framework to use.

Both CHQR and the present paper agree with Dowker and Kent that consistent histories, as a fundamental theory of nature, does not single out a particular framework. The difference is that Dowker and Kent regard this situation as unsatisfactory, whereas from the perspective presented here there does not seem to be any problem if one regards the physicist’s choice of framework to be like the draftsman’s choice of a perspective for representing a three-dimensional object, or like the choice of a coarse-graining of classical phase space. That is, the choice is dictated by the problems that the physicist seeks to address, the questions he is attempting to answer, and not by some law of nature. Given some question of physical interest, quantum incompatibility severely limits the choice, as noted in the case of the specific example in Sec. II, and discussed further in Sec. IV. There seems to be no reason why an element of choice of this type should render a theory unsatisfactory. It may not agree with certain aesthetic criteria for what constitutes “good science,” but such criteria tend to be somewhat subjective, as in the case of Einstein’s preference for a deterministic rather than a probabilistic quantum theory.

Insofar as Dowker and Kent regard the choice of framework as being a choice among mutually exclusive alternatives, which seems to be their perspective in Sec. 5.6 of [17], their point of view is quite different from that presented

above in Sec. IV: that if two frameworks are compatible, the relationship between them is analogous to that between classical coarse-grainings, and if they are incompatible, in the quantum sense of that term, it is still not correct to suppose that the use of one framework excludes the other in the same way that employing one trajectory in a classical phase space excludes all other trajectories. Thus one should never think of one framework as “correct” and another as “incorrect,” other than in a sense like “this is the correct framework to address the problem which I have in mind.”

A particular instance of the lack of predictive power, according to Dowker and Kent, is that consistent-history quantum theory cannot answer the question “will the world be quasiclassical tomorrow?” See CHQR, Sec. VII B, for a discussion of quasiclassicality in relationship to the ideas considered here. The term “quasiclassical” is somewhat vague, but the essence of Dowker and Kent’s concern lies in the fact that the consistent-histories formalism does not rule out the possibility of MQS states. In particular there are families in which only “normal” (non-MQS) states occur up to some particular time, and MQS states appear at later times, so that “quasiclassical” behavior up to some time is no guarantee that it will continue.

The example in Sec. II can serve to illustrate this point. Family \mathcal{F}_1 contains the “nonquasiclassical” MQS state S at t_2 , whereas \mathcal{F}_2 has “normal” detector states C^*D and CD^* ; at the earlier times t_0 and t_1 these families are identical, and the coherent superposition state s of the photon at time t_1 , because it is easily achieved and detected in the laboratory, can be considered compatible with a “quasiclassical” description. The consistent-history approach gives no reason to choose \mathcal{F}_2 rather than \mathcal{F}_1 to represent the state of affairs at t_2 . On the other hand, as pointed out in Sec. IV, a question of the form “will the system be in S , or will it be in one of the two states C^*D or CD^* at t_2 ?” is not meaningful, because it requires, at least implicitly, a comparison between mutually incompatible alternatives, and quantum incompatibility implies that no such comparison is possible, that is, it does not make sense. It is like asking, “does the spin half particle have an x or a z component of angular momentum?”

Note that “quasiclassical,” interpreted as “non-MQS,” is not a quantum-mechanical property as such, since it is not associated with a subspace of the quantum Hilbert space. Thus while both $|C^*D\rangle$ and $|CD^*\rangle$ in the example of Sec. II refer to non-MQS states, the smallest subspace that contains them also contains the MQS state $|S\rangle$. This suggests that “quasiclassical” is best thought of as a term belonging to the metalanguage of quantum descriptions, the language used to discuss these descriptions, rather than as a term that can itself enter into a quantum description. To use a classical analogy, the term “large cells” could be employed to characterize a coarse-graining of a classical phase space, and it belongs to the metalanguage, for it obviously is not correct to think of “large cells” as a property of the physical system itself. The question “will the world be quasiclassical tomorrow?” is thus comparable to “will tomorrow’s coarse-graining use large cells?” Both might make sense as part of a discussion among physicists as to how to construct a description of a physical system that best addresses the problems that interest them; neither refers directly to properties of the system being described.

To summarize, while it makes sense to compute probabilities of different histories *given* a quasiclassical framework, it does not make sense to assign a probability to such a framework, or treat it as one of a collection of mutually exclusive physical alternatives. Quasiclassicality is a property of quantum descriptions, not a property of quantum systems, and the question “is this quantum system quasiclassical?” is not meaningful, at least when understood in the same sense as “is the energy between 9 and 10 ergs?” For the same reason, quasiclassicality cannot be a condition in a quantum probability, which is why a recent argument by Kent, examined in Appendix B, is inconsistent with the rules of quantum reasoning given in CHQR.

Despite the obvious differences between the present paper and the approach of Dowker and Kent, there is a sense in which the conclusions complement each other. By studying the structure of consistent histories under the assumption that alternative consistent families are somewhat analogous to the mutually exclusive possibilities represented by a sample space, Dowker and Kent concluded that this approach does not result in a satisfactory scientific theory. That is perfectly compatible with the perspective of the present paper.

A rather different approach to consistent histories was considered by Dowker and Kent in Sec. 5.4 of [17] under the title “many histories.” See the following section for some comments.

D. List of histories

Given any consistent family of histories for a particular system, one and only one of the elementary histories will actually occur. This statement makes it tempting to suppose that it is possible to construct a list $\{\mathcal{F}_j, F_j\}$ that assigns to every consistent family \mathcal{F}_j a history F_j , understood as the history that actually occurred in a particular system or a particular realization of an experiment. This temptation should be resisted; no such list exists, at least if it is interpreted in the manner just suggested.

Note that if the \mathcal{F}_j are alternative coarse-grainings of a classical phase space, the existence of such a list is not in doubt, for one simply takes whatever classical trajectory actually occurs, and employs it to generate the history F_j by noting which cell of \mathcal{F}_j is occupied by the trajectory at each time. In the same way, if the quantum list contains only consistent families that are mutually compatible with one another, the list can be constructed by using the common refinement.

When, however, incompatible families occur in the list—as will necessarily be the case if all consistent families are included—the quantum list does not make sense. Consider the example in Sec. II. For \mathcal{F}_1 there is only one history, (2.6), with positive weight, which must therefore be F_1 . For \mathcal{F}_2 there are two histories with positive weights, (2.11), so F_2 must be one of these. But whichever it is, it makes no sense to say that a single system (and a single experimental run) can be correctly described by both F_1 and F_2 , since at t_2 the detectors cannot be said to be in the MQS state S and also in one of the states C^*D or CD^* .

To be sure, one might adopt an alternative interpretation of the list $\{\mathcal{F}_j, F_j\}$, and understand it as referring to what happens in distinct but nominally identical systems, or in

successive repetitions of an experiment, labeled by the subscript j . In such a case there is no problem, for it is not necessary to use the same framework every time an experiment is carried out, or for describing different systems. But this is obviously a very different interpretation of the list from that employed above.

Dowker and Kent's "many histories" interpretation, Sec. 5.4 of [17], involves such a list, although their interpretation of what it means is not very clear. If it is thought of as applying to a single system, or a single universe, then at least from the perspective adopted in this paper, it does not make much (quantum mechanical) sense. It is only fair to add that Dowker and Kent themselves show very little enthusiasm for their "many histories" interpretation.

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APPENDIX A: CONSISTENT HISTORIES AND CONTRARY INFERENCES

In recent work [19,20], Kent has stated that the consistent-history approach cannot be taken seriously as a fundamental theory because it allows for what he calls "contrary inferences." We shall show that the problem is not with consistent-histories quantum theory as presented in CHQR and the present paper, but rather with Kent's definition of "contrary," which fails to take proper account of quantum incompatibility when it arises from violations of consistency conditions. For a condensed version of these remarks, see [31].

The Aharonov-Vaidman [32] example discussed in CHQR Sec. VI D can be used to illustrate the central point of Kent's argument. A particle can be in one of three states $|A\rangle$, $|B\rangle$, or $|C\rangle$, and the dynamics is trivial: $|A\rangle \mapsto |A\rangle$, etc. Define

$$|\Phi\rangle = (|A\rangle + |B\rangle + |C\rangle)/\sqrt{3}, \quad |\Psi\rangle = (|A\rangle + |B\rangle - |C\rangle)/\sqrt{3}, \quad (\text{A1})$$

and (consistent with previous notation) let a letter outside a ket denote the corresponding projector, and a tilde its complement, thus:

$$A = |A\rangle\langle A|, \quad \tilde{A} = I - A = B + C, \quad (\text{A2})$$

etc. Define a consistent family \mathcal{A} of histories starting with Φ at time t_0 , followed by A or \tilde{A} at a later time t_1 , and Ψ at a still later time t_2 . It is straightforward to show, using this framework, that

$$\text{Pr}(A_1|\Phi_0 \wedge \Psi_2) = 1, \quad (\text{A3})$$

where the subscripts indicate the time associated with the corresponding event. That is, we can be sure that the particle was in state A at t_1 , given the initial state Φ at t_0 and the

final state Ψ at t_2 . An alternative framework \mathcal{B} , incompatible with \mathcal{A} , uses the same events at t_0 and t_2 , and B and \tilde{B} at t_1 . Using \mathcal{B} , one finds

$$\text{Pr}(B_1|\Phi_0 \wedge \Psi_2) = 1. \quad (\text{A4})$$

Kent defines two projectors A and B to be "contrary" provided

$$AB = BA = 0, \quad A \neq \tilde{B}, \quad (\text{A5})$$

and employs the term "contradictory" when $A = \tilde{B}$. Since A and B in the Aharonov-Vaidman example satisfy Eq. (A5), Kent would conclude that Eqs. (A3) and (A4) are two probability-one inferences based on the same data, $\Phi_0 \wedge \Psi_2$, to two contrary events, and he finds this feature of the consistent-history approach to be problematical.

To analyze this argument, we first note that the term *contrary* has a well-defined usage in classical logic [33], where it indicates the relationship between two propositions that cannot both be true, but might both be false. "The queen is in London" and "the queen is in Cambridge" are contrary propositions in this sense: they are mutually exclusive, so they cannot both be true, but they could both be false (if the queen is in some other city). Similarly, *contradictory* is reserved for the relationship of two propositions that cannot both be true, and also cannot both be false. For example, "the queen is in London" is false if and only if "the queen is not in London" is true, and vice versa.

Translating terms from classical logic into appropriate quantum counterparts is not a trivial exercise; see CHQR and Sec. IV of the present paper. As long as A and B belong to the same consistent family, (A5) is a reasonable quantum counterpart for the classical term "contrary," and agrees with the rules worked out in CHQR. The problem with Kent's argument is that he wishes to apply the same definition in a case in which A and B do not belong to the same consistent family. In the Aharonov-Vaidman example, B is *incompatible*, in the quantum-mechanical sense (Sec. IV) with the family \mathcal{A} used to obtain Eq. (A3), while A is incompatible with the family \mathcal{B} used to obtain Eq. (A4). In the case of quantum compatibility, the consistent-history approach, for reasons indicated in Sec. IV, disallows any logical comparison whatsoever. Thus as long as one is considering inferences based upon the data $\Phi_0 \wedge \Psi_2$, A and B are best thought of as "incomparable," and speaking of them as "contrary" in a sense similar to that used in classical logic is misleading.

To put the matter in another way, in order to be able to say that A and B are contrary in the sense of classical logic, consistent history rules require that they belong to the same framework. However, any framework that contains both A and B at t_1 cannot also contain both Φ at t_0 and Ψ at t_2 . Consequently, in a framework in which it would be correct to say that A and B are contrary, neither of the inferences (A3) and (A4) is possible. So, whichever way one looks at the matter, there are no "contrary inferences." The key point is that consistent-history reasoning, as clearly stated by Omnès [34] and reiterated in CHQR, must employ a single framework, and Kent's argument violates this rule through

defining [35] ‘‘contrary’’ in a manner that allows it to hold as a relationship between propositions that are not members of a single framework.

The same point can be made in a slightly different way. In Sec. II we exhibited two probability one inferences, in Eqs. (2.17) and (2.18), based upon identical conditions but carried out in two incompatible families, \mathcal{F}_3 and \mathcal{F}_2 . The nonsensical result (2.19) of combining these two inferences was blocked by the consistent-history rule that results deduced in incompatible families cannot be combined. It is precisely this same rule that applies in the case of Eqs. (A3) and (A4). The only difference is that while a quantum comparison of c and s from Eqs. (2.17) and (2.18) is obviously nonsensical, because the projectors do not commute, that of A and B as inferred in Eqs. (A3) and (A4) is not as transparently incorrect, since the projectors commute, even though one has an equally serious violation of precisely the same consistent-history rule. Note that quantum incompatibility can arise in the consistent-histories approach both because certain projectors do not commute *and* because consistency rules are violated. The incompatibility of A with B , and of B with A , is of the latter type, which is why it is a bit less evident than the incompatibility of c and s in the example of Sec. II.

In summary, to say that A and B are ‘‘contrary’’ in a logical sense when they belong to incompatible frameworks is a violation of one of the basic principles of consistent-history reasoning, at least as the subject has been developed up to now. To be sure, there may exist alternative approaches to consistent histories based upon a different set of logical rules, and one might view Kent’s argument as, in effect, proposing such an alternative. In that case there would be no reason to disagree with his conclusion, which would be that this alternative proposal constitutes a formalism that cannot be taken seriously as a fundamental theory of nature, for precisely the reasons that he points out.

Finally, a comment on the question that Kent raises in the latter part of [19]: why should the formalism of consistent histories rule out inferences to two ‘‘contradictory’’ propositions while allowing inferences to two ‘‘contrary’’ propositions? The brief response is that, according to the formalism for consistent histories developed in CHQR, neither ‘‘contrary’’ nor ‘‘contradictory’’ can be defined as logical relationships unless both of the properties (or histories) being compared are found in the same consistent family. Thus this formalism never allows an inference to either ‘‘contradictory’’ or ‘‘contrary’’ pairs of propositions, and so the question raised by Kent does not arise. Of course, alternative formulations of consistent histories that construct logical definitions in a way that allows contrary inferences to occur are subject to the conceptual problem that Kent has pointed out, and have to deal with it in some way. For one such alternative, see [20].

APPENDIX B: CONDITIONING UPON QUASICLASSICALITY

Kent has claimed that even if one assumes that the world will be quasiclassical tomorrow, the consistent-history approach does not always yield probabilistic predictions that agree with those provided by Copenhagen (i.e., standard textbook) quantum theory. The following example illustrates

his argument, and shows why it cannot be considered a serious objection to consistent histories as described in CHQR and the present paper.

Suppose the beam splitter in the example in Sec. II (Fig. 1) is replaced by one that produces beams in three exit channels c , d , and e , corresponding to a unitary time development

$$|a\rangle \mapsto (|c\rangle + |d\rangle + |e\rangle)/\sqrt{3} \quad (\text{B1})$$

in place of (2.1). A third detector E is added for the e channel, so that

$$|e\rangle|E\rangle \mapsto |E^*\rangle \quad (\text{B2})$$

in addition to (2.2). We now consider two consistent families involving two times, t_0 and t_2 :

$$\mathcal{D}_1: \Psi_0 \odot \{C^*DE, CD^*E, CDE^*\}, \quad (\text{B3})$$

$$\mathcal{D}_2: \Psi_0 \odot \{SE, CDE^*\}, \quad (\text{B4})$$

where the initial state [the counterpart of Eq. (2.4)] is

$$|\Psi_0\rangle = |aCDE\rangle, \quad (\text{B5})$$

$|S\rangle$ is the MQS combination of $|C^*D\rangle$ and $|D^*C\rangle$ defined in Eq. (2.5), and the curly brackets in (B3) and (B4) enclose alternative possibilities at t_2 . As usual, various histories of zero weight have been omitted.

A straightforward analysis using \mathcal{D}_1 yields

$$\Pr(C^*DE|\Psi_0) = \Pr(CD^*E|\Psi_0) = \Pr(CDE^*|\Psi_0) = 1/3, \quad (\text{B6})$$

that is, the probability is 1/3 that each of the detectors will have detected the photon, whereas from \mathcal{D}_2 one concludes that

$$\Pr(SE|\Psi_0) = 2/3, \quad \Pr(CDE^*|\Psi_0) = 1/3. \quad (\text{B7})$$

The fact that $\Pr(CDE^*|\Psi_0)$ is the same in both cases reflects a general property of the consistent-histories approach, as noted in Sec. II.

Kent would accept Eqs. (B6) and (B7) as correct, but would then argue that *if one conditions upon quasiclassicality*, the probability that E detects the photon is different in families \mathcal{D}_1 and \mathcal{D}_2 ; to be specific:

$$\mathcal{D}_1: \Pr(CDE^*|\Psi_0 \wedge \text{quasiclassical}) = 1/3, \quad (\text{B8})$$

$$\mathcal{D}_2: \Pr(CDE^*|\Psi_0 \wedge \text{quasiclassical}) = 1. \quad (\text{B9})$$

The argument is that all three histories in \mathcal{D}_1 are quasiclassical (no MQS states), and thus each has a probability 1/3, whether or not one conditions upon quasiclassicality. On the other hand, in \mathcal{D}_2 the only quasiclassical history is $\Psi_0 \odot CDE^*$, since the other involves the far-from-classical superposition S , and thus conditioning upon quasiclassicality yields Eq. (B9) in place of Eq. (B7).

The problem with this argument, if one adopts the point of view of CHQR, is that conditional probabilities are only defined when their arguments are projectors belonging to an appropriate Boolean algebra satisfying consistency condi-

tions. But, as noted in Sec. V C, there is no subspace of the Hilbert space, and hence no projector, corresponding to “quasiclassical,” understood in the present context as “non-MQS.” Thus the conditional probabilities (B8) and (B9) are undefined, and no argument based upon them can be valid.

Is there some way to define “quasiclassical” as a condition entering into a probability without referring to a subspace of the Hilbert space? If, as suggested in Sec. V C, the term “quasiclassical” belongs to the metalanguage rather than the language of quantum descriptions, such a definition would change the meaning of the resulting probabilities in an important way. Thus imagine that \mathcal{D}_1 and \mathcal{D}_2 were two

coarse-grainings of a classical phase space using cells of different sizes. Then by conditioning upon the initial condition and the fact that the history only involves, say, large cells, one could produce a difference analogous to that between Eqs. (B8) and (B9). But the result would obviously have no direct physical significance, although it might be useful in addressing the question of which coarse-graining is most useful for a numerical simulation.

Of course, there may be some alternative to CHQR in which “quasiclassical” refers directly to a quantum property. In that case, Kent’s argument would indicate that this alternative (in contrast to CHQR) is not likely to reproduce standard quantum physics.

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 [21] See Griffiths (Ref. [1]), Sec. 6.1.
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 [26] Even in a classical world it would be impossible for a physical system to contain a complete description of itself. Consequently, it is always simplest to imagine that a quantum description is being constructed outside the system which it purports to describe. Sometimes the question is asked as to how we can describe, in a scientific sense, the universe in which we live. This is a nontrivial philosophical issue, but it does not seem implausible to assume that some sort of analogy will work: we can imagine simple model systems inside a closed box, check that they have some features that we find in our everyday experience, and then extrapolate by imagining ourselves in a large box, etc.
 [27] See *The Interpretation of Quantum Mechanics* (Ref. [6]), p. 499.
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