Limits of the measurability of the local quantum electromagnetic-field amplitude

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The precision with which the amplitude of the free electromagnetic field can be measured locally in QED is evaluated by analyzing a well-known gedanken experiment originally proposed by Bohr and Rosenfeld (BR). The analysis is performed by applying standard theoretical techniques familiar in quantum optics. The main result obtained for the precision is significantly different from the generally accepted Bohr-Rosenfeld result. This leads to questioning the widely accepted notion of the compensating field, fostered by these authors. A misconception at the origin of this notion is pointed out by a careful investigation of the self-force acting on the apparatus designed to measure the field. The correct expression for this self-force is found to be at variance with that proposed by Bohr and Rosenfeld and generally accepted. It is argued that, as a consequence of this new expression and in contrast with the generally accepted view, no compensating force of nonelectromagnetic nature is required in order to perform measurements of the quantum field amplitude with any desired accuracy. It is shown that the only limitations to the precision of the measurement, in the BR gedanken experiment, arise from the time-energy uncertainty principle, as well as from the finite dimensions of the measuring apparatus. [S1050-2947(98)06903-0]

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I. INTRODUCTION

The present paper deals with the precision with which the amplitude of the electric component $\mathbf{E}^{\text{ext}}$ of an electromagnetic field can be measured in the neighborhood of a space-time point, in the context of nonrelativistic QED. This problem has a long history and dates back to a paper published by Landau and Peierls in the early days of quantum field theory [1]. These authors, paving the way for others who took up the problem up to recent times, analyzed a gedanken experiment in which the momentum increase of a massive charged test body under the influence of the field is measured and is related to the amplitude of the field in a simple way. They realized the impossibility of discerning the force exerted on the test body by the external field $\mathbf{E}^{\text{ext}}$ from the force exerted by the field created by the same test body during the measurement. The latter field is intrinsically uncertain due to the quantum nature of measurement the and consequently causes an uncertainty in the total field. This uncertainty must be compared with the the minimum precision needed to reveal the quantum features of the electromagnetic field. Landau and Peierls used a pointlike test body and they found that the former uncertainty exceeds the latter minimum precision in all cases. Thus they concluded that no meaningful field amplitude measurement is possible in QED.

In a subsequent paper Bohr and Rosenfeld proposed to spread the test body of large mass M over a volume V of finite linear dimensions a, thereby obtaining a perfectly rigid, finite, and uniform charge density ρ [2]. They also emphasized the need for a finite time interval $\tau = t_1'' - t_1'$ between the initial and final momentum measurements at t_1' and t_1'' and they constrained the test body (which in this paper we shall often call the *pointer* according to modern usage) to move rigidly along the **1** direction in order to measure the **1** component of the electric field. Their protocol for the measurement of the pointer momentum is such that at t_1', and within a small time interval $\Delta t \ll \tau$, the pointer gets rigidly displaced over a distance Q along **1**. Due to the large value of M, Q does not change much for the rest of the measurement time τ until t_1'', when the second and final momentum measurement brings the body back to the original configuration ($Q=0$) in a time interval Δt. The finite extent of the test body ensures that the electromagnetic forces, caused by the acceleration within Δt during either momentum measurement, have a negligible effect on the motion of the pointer, thereby disposing of the difficulty of the Landau-Peierls treatment for a pointlike test body, at least during the initial and final acts of momentum measurement.

Another important feature of the gedanken apparatus contrived by Bohr and Rosenfeld (BR), in contrast with the Landau-Peierls setup, is that in the initial undisplaced configuration the test body is perfectly neutralized by a fixed body of identical shape and charge density $-\rho$. BR solve Maxwell equations by prescribing a gatelike form for $Q(t)$ [i.e., $Q(t)=0$ for $t<t_1'$ and for $t>t_1''$; $Q(t)=Q$ for $t_1'<t<t_1''$] and they obtain an expression for the electric field E_1 created by the system (neutralizing body plus pointer) during τ, under the reasonable assumption of negligible magnetic effects. From the BR expression for E_1 it follows immediately that the force F exerted on the pointer and stemming from E_1, which in this paper we shall call the *self-force*, is proportional to Q according to the expression

$$F(t_2) = \rho^2 Q \int_V d^3\mathbf{x}_1 \int_V d^3\mathbf{x}_2 \int_\tau dt_1 A_{xx}^{(1,2)},$$
$$A_{xx}^{(1,2)} = -\left(\frac{\partial^2}{\partial x_2 \partial x_1} - \frac{1}{c^2}\frac{\partial^2}{\partial t_2 \partial t_1} \right) \frac{1}{r}\, \delta\!\left(t_2 - t_1 - \frac{r}{c} \right). \tag{1}$$

In this expression F, Q, x_1, and x_2 are along **1**, the space integrations are over the volume V of the pointer in its undisplaced configuration of the beginning of the experiment, the time integration extends over τ from t_1' to t_1'', $\mathbf{x}_i \equiv (x_i, y_i, z_i)$ for $i=1$ or 2, $r=|\mathbf{x}_2-\mathbf{x}_1|$, $t_1'<t_1$, $t_2<t_1''$, and c is the velocity of light.

The self-force (1) contributes an impulse to the pointer proportional to Q, which adds to the impulse provided by the external field E_1^{ext}. In view of the uncertainty principle, this contribution linear in Q cannot be measured with arbitrary precision simultaneously with the pointer momentum increase between t_1' and t_1''. On the other hand, both quantities should be accurately measured in order to determine precisely E_1^{ext}, or rather its space-time average $\bar{\bar{E}}_1^{\mathrm{ext}}$ defined in Eq. (23), from the momentum balance equation. Indeed, using expression (1), BR find that the precision with which $\bar{\bar{E}}_1^{\mathrm{ext}}$ is measured cannot exceed the limit

$$(\Delta \bar{\bar{E}}_1^{\mathrm{ext}})_{\min} \sim \sqrt{\hbar |\bar{\bar{A}}_{xx}^{(I,I)}|},$$

$$\bar{\bar{A}}_{xx}^{(I,I)} = \frac{1}{V^2\tau^2} \int_V d^3\mathbf{x}_1 \int_V d^3\mathbf{x}_2 \int_\tau dt_1 \int_\tau dt_2 A_{xx}^{(1,2)}. \tag{2}$$

This is less stringent than the limit estimated by Landau and Peierls for a pointlike test body. Nevertheless, it seems enough to preclude the measurement of field amplitudes in the range where quantum effects become evident since, in view of the form of the field commutation relations, only fields weaker than $[\hbar|\bar{\bar{A}}_{xx}^{(I,I)}|]^{1/2}$ display quantum features in view of the form of the field commutation relations. It is essential at this point to note that the limit (2) estimated by BR is truly fundamental, since it depends only on the properties of the free field and not on the structure of the pointer. Consequently, it would seem impossible in principle to measure field amplitudes in the quantum range. In order to circumvent this difficulty, BR proceed to modify their gedanken apparatus by the addition of an elastic force $-F$ of nonelectromagnetic nature, which acts on the pointer in such a way as to cancel the self-force F evaluated in Eq. (1). This trick leads to the disappearance of the contribution to the impulse which is linear in Q, thereby yielding a simple momentum balance equation which relates directly the increase in pointer momentum with $\bar{\bar{E}}_1^{\mathrm{ext}}$. In this way $\bar{\bar{E}}_1^{\mathrm{ext}}$ can be measured with any desired precision, by measuring only the momentum gained by the pointer, also in the case of weak fields in the quantum domain.

This argument seems to have been accepted in all the subsequent work on the subject [3–5] and the additional nonelectromagnetic force has been incorporated in the Lagrangian of the field-pointer system in the context of the algebraic treatment of the theory of measurement [6]. The BR compensation mechanism has also been used in more recent work on the theory of measurement of more general quantum fields [7,8], although it must be noted that Pauli fails to mention it in the latest edition of his book on quantum mechanics [9].

The BR argument, however, is unsatisfactory for at least two reasons. First, the introduction of a nonelectromagnetic force, which is apparently necessary in order to eliminate a limitation of quantum origin in the measurability of the electromagnetic field, seems to indicate a fundamental lack of self-consistency of QED. Second, the physical nature of the self-force F in Eq. (1) is rather unclear. In fact for $t_1'<t_2<t_1''$ expression (1) can be partitioned as

$$F = -KQ + \rho^2 Q \int_V d^3\mathbf{x}_1 \int_V d^3\mathbf{x}_2 \int_\tau dt_1 \frac{1}{c^2}\frac{\partial^2}{\partial t_2 \partial t_1}\frac{1}{r} \times \delta\left(t_2 - t_1 - \frac{r}{c}\right),$$
$$K = \rho^2 \int_V d^3\mathbf{x}_1 \int_V d^3\mathbf{x}_2 \frac{\partial^2}{\partial x_2 \partial x_1}\frac{1}{r}\,\theta\left(t_2 - t_1' - \frac{r}{c}\right). \tag{3}$$

The first contribution in Eq. (3) is the linear approximation to the Coulomb attraction between two identical initially overlapping oppositely charged bodies of arbitrary shape. Thus one would expect the second contribution to coincide with the radiation-reaction force arising because of the motion of the pointer. The textbook expression of the latter, however, is [10]

$$F_{RR}(t) = \sum_{n=0}^{\infty} \frac{(-1)^n}{c^{n+2}}\frac{1}{n!} Q^{(n+2)} \int_V d^3\mathbf{x}_1 \int_V d^3\mathbf{x}_2 \rho(\mathbf{x}_1)\rho(\mathbf{x}_2) \times\left[\frac{n+1}{n+2} - \frac{n-1}{n+2}\left(\frac{x_2-x_1}{r}\right)^2\right] r^{n-1},$$
$$Q^{(n)} = \frac{\partial^n Q}{\partial t^n} \tag{4}$$

for a body of spherically symmetric charge density, for $t > a/c$, for small velocity of the pointer and for negligible nonlinear terms in Q and in its time derivatives. Evidently expression (4) looks rather different from the second contribution in Eq. (3) since, for example, it does not contain any term linear in Q. It must be emphasized, however, that in contrast with Eq. (1) the validity of expression (4) is restricted to a body of spherical symmetry and in the absence of the neutralizing charge distribution. Nevertheless the form of Eq. (4) renders unclear both the physical interpretation of the self-force (1) and its connection with the radiation-reaction force.

Thus the above remarks indicate the opportunity for a closer consideration of the measurement of the amplitude of the quantum electromagnetic field, particularly in view of its basic conceptual importance. This summarizes the aim and the scope of the present paper.

II. QED OF THE POINTER-FIELD SYSTEM

In this section we present an *ab initio* quantum-mechanical treatment of the dynamics of the pointer interacting with the local quantized field. We develop the treatment in the Coulomb gauge and we use an approximation familiar to quantum opticians, namely, the electric dipole approximation.

In the presence of the neutralizing body, the BR gedanken apparatus can be schematically represented, for small displacements, by a harmonic oscillator coupled to the quantum electromagnetic field. Thus the pointer-field Hamiltonian in the minimal coupling scheme and in the electric dipole approximation is [11]

$$H=\frac{1}{2M}\mathbf{P}^2+\frac{1}{2}K\mathbf{Q}^2+H_F-q\frac{1}{Mc}\mathbf{P}\cdot\mathbf{A}_\perp(\mathbf{R})+q^2\frac{1}{2Mc^2}\mathbf{A}_\perp^2(\mathbf{R}),$$

$$H_F=\frac{1}{8\pi}\int\left(\frac{1}{c^2}\dot{\mathbf{A}}_\perp^2(\mathbf{x})+[\boldsymbol{\nabla}\wedge\mathbf{A}_\perp(\mathbf{x})]^2\right)d^3\mathbf{x}. \tag{5}$$

In Eq. (5) K is given by Eq. (3) with $t_1'=-\infty$, $\mathbf{P}=M\dot{\mathbf{Q}}+q\mathbf{A}_\perp(\mathbf{R})/c$ is the canonical momentum, $q=\rho V$ is the total charge of the pointer, and

$$\mathbf{A}_\perp(\mathbf{R})=\frac{1}{V}\int_V d^3\mathbf{x}\mathbf{A}_\perp(\mathbf{x}) \tag{6}$$

is the (transverse) vector potential in the electric dipole approximation, coinciding with the space average of $\mathbf{A}_\perp(\mathbf{x})$ within the volume V occupied by the pointer at equilibrium $(\mathbf{Q}=\mathbf{0})$. The difference between the displacement $\mathbf{Q}$ of the pointer from its equilibrium position and $\mathbf{R}$, the position of the center of mass of the pointer at equilibrium, should be noted since $\mathbf{Q}$ is a dynamic variable whereas $\mathbf{R}$ is not. For the moment we shall treat $\mathbf{Q}$ as a three-dimensional vector and we shall constrain it later to the **1** direction.

It is convenient to second quantize the field as

$$\mathbf{A}_\perp(\mathbf{x})=\sum_{kj}\left(\frac{2\pi\hbar c^2}{L^3\omega_k}\right)^{1/2}\mathbf{e}_{kj}(a_{kj}e^{i\mathbf{k}\cdot\mathbf{x}}+a_{kj}^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}}), \tag{7}$$

where L^3 is the field quantization volume, a_{kj} are the usual field annihilation operators, and $\mathbf{e}_{kj}$ are real polarization vectors for photons in the mode $\mathbf{k}_j$ of frequency ω_k. Using Eq. (7) it is easy to show that

$$[A_{\perp l}(\mathbf{x}),A_{\perp m}(\mathbf{x}')]=[A_{\perp l}(\mathbf{x}),\mathbf{A}_\perp^2(\mathbf{x}')]=0,$$

$$[A_{\perp l}(\mathbf{x}),H_F]=-i\hbar cE_{\perp l}(\mathbf{x}), \tag{8}$$

where

$$\mathbf{E}_\perp(\mathbf{x})=i\sum_{kj}\left(\frac{2\pi\hbar\omega_k}{L^3}\right)^{1/2}\mathbf{e}_{kj}(a_{kj}e^{i\mathbf{k}\cdot\mathbf{x}}-a_{kj}^\dagger e^{-i\mathbf{k}\cdot\mathbf{x}}). \tag{9}$$

It should be noted that, consistently with the electric dipole approximation, all sums over $\mathbf{k}$ are restricted to $k<k_M\sim\pi/2a$ and this is equivalent to assigning finite dimensions to the pointer. Use of Eqs. (6) and (8) leads to

$$\ddot{Q}_l=-\frac{i}{\hbar}\left[\frac{1}{M}\left(\mathbf{P}-q\frac{1}{c}\mathbf{A}_\perp(\mathbf{R})\right)_l,H\right]=-\frac{K}{M}Q_l+\frac{1}{M}qE_{\perp l}(\mathbf{R}), \tag{10}$$

where $\mathbf{E}_\perp(\mathbf{R})$ is the transverse electric field at the pointer location in the electric dipole approximation, as obtained by the same kind of averaging leading to Eq. (6). Moreover we have

$$[a_{kj},A_{\perp l}(\mathbf{R})]=\left(\frac{2\pi\hbar c^2}{L^3\omega_k}\right)^{1/2}(\mathbf{e}_{kj})_l e^{-i\mathbf{k}\cdot\mathbf{R}},$$

$$[a_{kj},\mathbf{A}_\perp^2(\mathbf{R})]=2\mathbf{A}_\perp(\mathbf{R})\cdot[a_{kj},\mathbf{A}_\perp(\mathbf{R})]. \tag{11}$$

Thus, after some algebra,

$$\dot{a}_{kj}=-\frac{i}{\hbar}[a_{kj},H]=-i\omega_k a_{kj}+\frac{i}{\hbar}q\left(\frac{2\pi\hbar}{L^3\omega_k}\right)^{1/2}(\mathbf{e}_{kj})_l\dot{Q}_l e^{-i\mathbf{k}\cdot\mathbf{R}}. \tag{12}$$

We now constrain the pointer to move along **1** using

$$Q_l^{(n)}\equiv Q^{(n)}\delta_{l1}. \tag{13}$$

This gives

$$\dot{a}_{kj}=-i\omega_k a_{kj}+\frac{i}{\hbar}q\left(\frac{2\pi\hbar}{L^3\omega_k}\right)^{1/2}(\mathbf{e}_{kj})_1\dot{Q}e^{-i\mathbf{k}\cdot\mathbf{R}}, \tag{14}$$

which can be straightforwardly integrated as

$$a_{kj}(t)=a_{kj}(0)e^{-i\omega_k t}+\frac{i}{\hbar}q\left(\frac{2\pi\hbar}{L^3\omega_k}\right)^{1/2}(\mathbf{e}_{kj})_1 e^{-i\mathbf{k}\cdot\mathbf{R}}e^{-i\omega_k t}\int_0^t e^{i\omega_k t'}\dot{Q}(t')dt'. \tag{15}$$

Substitution of Eq. (15) in Eq. (9) yields

$$E_{\perp 1}(\mathbf{R},t)=E_{\perp 1}^{\mathrm{ext}}(\mathbf{R},t)-4\pi q\int_0^t\dot{Q}(t')\frac{1}{L^3}\sum_{kj}(\mathbf{e}_{kj})_1^2\cos\,\omega_k(t-t')dt', \tag{16}$$

where

$$E_{\perp 1}^{\mathrm{ext}}(\mathbf{R},t)=i\sum_{kj}\left(\frac{2\pi\hbar\omega_k}{L^3}\right)^{1/2}(\mathbf{e}_{kj})_1[a_{kj}(0)e^{i(\mathbf{k}\cdot\mathbf{R}-\omega_k t')}-hc] \tag{17}$$

is the transverse field amplitude operator in the absence of the pointer, which is the object of the measurement. Further, it is not difficult to show that

$$\frac{1}{L^3}\sum_{kj}(\mathbf{e}_{kj})^2\cos\,\omega_k(t-t')=-\frac{1}{3\pi^2c^3}\frac{d^2}{dt'^2}\frac{\sin\,k_Mc(t-t')}{t-t'}=-\frac{1}{3\pi c^3}\delta''(t-t'). \tag{18}$$

The last equality in Eq. (18) is symbolic and valid only in the limit $k_M\to\infty$, which strictly speaking we cannot take in view of the electric dipole approximation. It is, however, useful to adopt it as an approximate equality, in which case it yields

$$\int_0^t \dot{Q}(t')\,\frac{1}{L^3}\sum_{kj}\,(\mathbf{e}_{kj})_1^2 \cos\,\omega_k(t-t')dt' = \frac{1}{3\pi c^3}\left(\frac{c}{\pi}\,\ddot{Q}(t)\int_0^{k_M}dk - \frac{1}{2}\,\dddot{Q}(t)\right). \tag{19}$$

Use of Eq. (19) in Eq. (16) gives

$$E_{\perp 1}(\mathbf{R},t) = E_{\perp 1}^{\text{ext}}(\mathbf{R},t) - \frac{4}{3\pi c^2}\,q\ddot{Q}(t)\int_0^{k_M}dk + \frac{2}{3c^3}\,q\dddot{Q}(t). \tag{20}$$

The last two terms in Eq. (20) represent the radiation-reaction contribution to the electric field acting on the pointer [12] in the electric dipole approximation. Substitution of this expression in Eq. (10) yields the equation of motion of the pointer in the form

$$M\ddot{Q}(t) = -KQ(t) + qE_{\perp 1}^{\text{ext}}(\mathbf{R},t) - \frac{4k_M}{3\pi c^2}\,q^2\ddot{Q}(t) + \frac{2}{3c^3}\,q^2\dddot{Q}(t). \tag{21}$$

Inspection of Eq. (21) leads to identification of the first term on the right-hand side as the linearized Coulomb attraction on the pointer by the neutralizing body and the last two terms as the radiation-reaction force operator acting on the pointer. In the latter contribution higher-order derivatives in Q are absent due to the electric dipole approximation. Clearly the radiation-reaction force in Eq. (21) seems compatible with the textbook expression (4) and in disagreement with the BR expression linear in Q which can be derived from Eq. (3).

III. MEASURABILITY OF THE LOCAL ELECTRIC FIELD

We cast the equation of motion of the pointer between the two momentum measurements and in the presence of the neutralizing body in the form

$$M\ddot{Q} = -KQ + \rho V\overline{E_1^{\text{ext}}} + F_{RR},$$

$$F_{RR} = -\frac{4k_M}{3\pi c^2}\,q^2\ddot{Q} + \frac{2}{3c^3}\,q^2\dddot{Q}. \tag{22}$$

In this expression $\overline{E_1^{\text{ext}}}$ is the space average of the external field within the volume of the pointer and coincides with $E_{\perp 1}^{\text{ext}}(\mathbf{R})$. Moreover the assumption of large M entails that all time derivatives of Q are small and entitles us to neglect F_{RR} in Eq. (22). Subsequent time integration yields the following operator equation for the momentum balance:

$$p(t_1'') - p(t_1') = -\int_\tau KQ\;dt_2 + \rho V\tau\bar{\bar{E}}_1^{\text{ext}},$$

$$\bar{\bar{E}}_1^{\text{ext}} = \frac{1}{V\tau}\int_V d^3\mathbf{x}\int_\tau dt\;E_1^{\text{ext}}(\mathbf{x},t), \quad p = M\dot{Q}. \tag{23}$$

It must be emphasized that in the absence of the neutralizing body it is $K=0$, and Eq. (23) expresses perfect correlation of the pointer momentum operator with the space- and time-averaged external field operator. Consequently a measurement of the latter field can be performed by measuring the increase in pointer momentum, without additional uncertainties of quantum nature, provided the neutralizing body is eliminated from the experimental setup. It is thus clear already at this point that, in contrast with the BR conclusions, there is no need to invoke an additional nonelectromagnetic force in order to compensate the effects of the self-force contribution linear in Q, since it is sufficient to eliminate the neutralizing body from the gedanken apparatus.

Equation (23) is exact within the electric dipole approximation and, in some sense, constitutes the final result of our treatment. The rest of this section is devoted to discussing the impact of this result on the BR analysis. To this aim we shall implement the same procedure used by BR to derive expression (2). More precisely, following BR, we assume that the momentum measurement at t_1' is contrived in such a way that the operator Q does not depend on time between t_1' and t_1'', and we also assume that Q can be taken out of the t_2 integration in Eq. (23). This leads to

$$p(t_1'') - p(t_1') = -\overline{K}\tau Q + \rho V\tau\bar{\bar{E}}_1^{\text{ext}}, \quad \overline{K} = \frac{1}{\tau}\int_\tau K(t_2)dt_2. \tag{24}$$

It is appropriate to remark at this point that the expression for $\bar{\bar{E}}_1^{\text{ext}}$ is given by

$$\bar{\bar{E}}_1^{\text{ext}} = \frac{1}{\tau}\int_\tau dt\;E_{\perp 1}^{\text{ext}}(\mathbf{R},t)dt = i\sum_{kj}\left(\frac{2\pi\hbar\omega_k}{V}\right)^{1/2}(\mathbf{e}_{kj})_1[a_{kj}(0)e^{i(\mathbf{k}\cdot\mathbf{R}-\omega_k t_1')} \times F(\omega_k,t) - hc],$$

$$F(\omega_k,t) = \frac{1-e^{-i\omega_k\tau}}{i\omega_k\tau}. \tag{25}$$

The quantity $F(\omega_k,\tau)$ appearing in this expression is 1 for $\omega_k \ll \tau^{-1}$ and vanishes for $\omega \gg \tau^{-1}$. Thus the components of the field of frequency $\omega_k > \tau^{-1}$ are more or less severely distorted by the measurement process. Such a distortion, however, is of a rather trivial nature because it is implicit in the model of measurement and it is not related to any quantum effect.

We now note that Eq. (24) is the counterpart of the BR momentum balance equation. In fact the latter is exactly of the same form as Eq. (24), the only difference being that the BR coefficient of the term linear in Q is $\rho^2V^2\tau^2\bar{\bar{A}}_{xx}^{(I,I)}$ rather than $-\overline{K}\tau$. Since BR obtain expression (2) from their momentum balance equation, proceeding in the same way we derive from Eq. (24) that the minimum uncertainty with which $\bar{\bar{E}}_1^{\text{ext}}$ can be measured is

$$(\Delta\bar{\bar{E}}_1^{\text{ext}})_{\min} \sim \left(\frac{\hbar\overline{K}}{\rho^2V^2\tau}\right)^{1/2}. \tag{26}$$

This expression is the counterpart of the BR expression (2).

The contrast between Eqs. (2) and (26) is manifest, since the former is independent of the structure of the detector (by which we mean the system constituted by the pointer and by the neutralizing body) and depends only on the properties of the free electromagnetic field [13], whereas in the latter the influence of the physical structure of the detector is present through the quantity $\overline{K}$ which, for example, vanishes in the absence of the neutralizing body. Consequently the uncertainty (26), contrary to Eq. (2), does not constitute a fundamental limit for the precision of the measurement even in the presence of the neutralizing body, since this uncertainty decreases with increasing τ.

On the other hand, expression (26) has an interesting physical meaning that is worth pointing out. We note that for long measurements $\tau > 2a/c$ we can roughly evaluate $\overline{K}$ from Eqs. (3) and (24) as

$$\overline{K} \sim \rho^2 \int_V d^3\mathbf{x}_1 \int_V d^3\mathbf{x}_2 \, \frac{1}{r^3} = \rho^2 V^2 \overline{\left(\frac{1}{r^3} \right)} \sim \rho^2 V. \tag{27}$$

Thus, squaring both sides of Eq. (26), we get

$$V(\Delta \overline{\overline{E}}_1^{\mathrm{ext}})^2_{\mathrm{min}} \sim \hbar/\tau. \tag{28}$$

Since in a field measurement of duration τ within a volume V an energy density $\hbar/\tau V$ is inevitably conferred to the field because of the time-energy uncertainty principle, expression (28) indicates that this additional energy is at the origin of the minimum field uncertainty (26).

Thus we reach the conclusion that no impassable limit exists to prevent measuring the field amplitude in the quantum range and that no necessity arises for the introduction of a compensating force of nonelectromagnetic nature, in contrast with the BR conclusion.

It must be emphasized, however, that the approach to the measurement of the field amplitude presented in Secs. II and III is in some sense complementary to the BR approach. In fact BR choose to discuss the problem in a domain where the zero-point quantum fluctuations of the field are smaller than the field uncertainty related to the commutation relations of the field in disjoint space-time regions, in such a way that classical electrodynamics can be used in a first approximation. This requirement leads BR to concentrate on the case $a > c\tau$ [2], although this BR procedure was subsequently criticized by Corinaldesi [3]. In contrast, it is possible to show that our approach, which relies on the electric dipole approximation, involves $a < c\tau$. In fact in our framework the finite value of a involves a finite value of $k_M \sim \pi/2a$ such that all components of $\mathbf{E}^{\mathrm{ext}}$ with wave vectors larger than k_M do not contribute to the impulse imparted by $\mathbf{E}^{\mathrm{ext}}$ to the pointer, and consequently cannot give rise to measurable effects. This has been taken into account simply by truncating at k_M the sums over k in expressions like Eq. (9). Clearly such a truncation introduces a limitation in the time domain, since it prevents considering phenomena of duration shorter than $\omega_M^{-1} = 2a/\pi c$. This condition must be satisfied also by the measurement duration, which leads to $\tau > a/c$ in our treatment. Thus the doubt might arise that the difference between results (2) and (26) is simply a reflection of the different domains of validity of the BR and of the present theory. This forces us to consider in more detail the original BR argument. We shall devote the next section to this task.

IV. THE BR SELF-FORCE

It is easy to convince oneself that expression (2) follows if the BR expression (1) for the self-force is assumed. Thus we concentrate on rederiving expression (1), following closely the BR method, in order to understand the origin of the discrepancy between Eqs. (2) and (26).

It is convenient to consider first the field created by the pointer disregarding the contribution of the neutralizing body during the measurement. The classical electromagnetic potentials in the Lorentz gauge, generated at point $\mathbf{x}_2$ and time t_2 by a classical charge density $\rho(\mathbf{x}_1, t_1)$ and by a classical current density $\mathbf{j}(\mathbf{x}_1, t_1)$ are [10]

$$\begin{aligned} \phi(\mathbf{x}_2, t_2) &= \int d^3\mathbf{x}_1 \int dt_1 \, \frac{\rho(\mathbf{x}_1, t_1)}{r} \, \delta\!\left(t_1 - t_2 + \frac{r}{c} \right), \\ \mathbf{A}(\mathbf{x}_2, t_2) &= \frac{1}{c} \int d^3\mathbf{x}_1 \int dt_1 \, \frac{\mathbf{j}(\mathbf{x}_1, t_1)}{r} \, \delta\!\left(t_1 - t_2 + \frac{r}{c} \right). \end{aligned} \tag{29}$$

Except where otherwise indicated, space integrations extend over all space whereas time integrations are between t_1' and t_1''. The latter condition is equivalent to assuming that the charge density is rigidly fastened to the reference frame and neutralized up to time t_1' and after time t_1''.

If $\rho(\mathbf{x}_1)$ represents the undisplaced charge density of the pointer for $t_1 < t_1'$, its time development $\rho(\mathbf{x}_1, t_1)$ can be described as due to a rigid displacement over a small distance $\mathbf{Q}(t_1)$. Then clearly $\rho(\mathbf{x}_1, t_1) = \rho(\mathbf{x}_1 - \mathbf{Q}(t_1))$, and it is convenient to introduce the displacement operator

$$T^{(i)}(\mathbf{Q}) = \sum_{n=0}^{\infty} \frac{1}{n!} \, (\mathbf{Q} \cdot \boldsymbol{\nabla}^{(i)})^n, \tag{30}$$

where the superscript (i) indicates differentiation with respect to $\mathbf{x}_i$ $(i = 1,2)$. The action of $T^{(i)}$ on any function of $\mathbf{x}_i$ is to change its argument to $\mathbf{x}_i + \mathbf{Q}$. Thus in Eq. (29) we have

$$\begin{aligned} \phi(\mathbf{x}_2, t_2) &= \int d^3\mathbf{x}_1 \int dt_1 \rho(\mathbf{x}_1 - \mathbf{Q}) \, \frac{1}{r} \, \delta\!\left(t_1 - t_2 + \frac{r}{c} \right) \\ &= \int d^3\mathbf{x}_1 \int dt_1 \rho(\mathbf{x}_1) T^{(1)}(\mathbf{Q}) \, \frac{1}{r} \, \delta\!\left(t_1 - t_2 + \frac{r}{c} \right), \end{aligned} \tag{31}$$

$$\begin{aligned} \mathbf{A}(\mathbf{x}_2, t_2) &= \frac{1}{c} \int d^3\mathbf{x}_1 \int dt_1 \rho(\mathbf{x}_1 - \mathbf{Q}) \dot{\mathbf{Q}}(t_1) \, \frac{1}{r} \, \delta\!\left(t_1 - t_2 + \frac{r}{c} \right) \\ &= \frac{1}{c} \int d^3\mathbf{x}_1 \int dt_1 \rho(\mathbf{x}_1) \dot{\mathbf{Q}}(t_1) T^{(1)}(\mathbf{Q}) \, \frac{1}{r} \\ &\quad \times \delta\!\left(t_1 - t_2 + \frac{r}{c} \right), \end{aligned}$$

where we have performed a change of the space integration variable from $\mathbf{x}_1$ to $\mathbf{x}_1 - \mathbf{Q}(t_1)$ and where we have used $\mathbf{j} = \rho \dot{\mathbf{Q}}$. Neglecting terms of order Q^2 and $Q\dot{Q}$ for $Q/a, \dot{Q}/c \ll 1$, we get from Eq. (31), for $t_1' < t_2 < t_1''$,

$$\phi(\mathbf{x}_2,t_2)=\int d^3\mathbf{x}_1\rho(\mathbf{x}_1)\,\frac{1}{r}\,\theta\left(t_2-t_1'-\frac{r}{c}\right)$$
$$+\int d^3\mathbf{x}_1\int dt_1\rho(\mathbf{x}_1)\mathbf{Q}(t_1)\cdot\boldsymbol{\nabla}^{(1)}\,\frac{1}{r}$$
$$\times\delta\left(t_1-t_2+\frac{r}{c}\right),$$
$$\mathbf{A}(\mathbf{x}_2,t_2)=\frac{1}{c}\int d^3\mathbf{x}_1\int dt_1\rho(\mathbf{x}_1)\dot{\mathbf{Q}}(t_1)\,\frac{1}{r}\,\delta\left(t_1-t_2+\frac{r}{c}\right). \tag{32}$$

Thus the electric field created by the pointer at $\mathbf{x}_2,t_2$ for $t_1'<t_2<t_1''$ is

$$\mathbf{E}(\mathbf{x}_2,t_2)=-\nabla^{(2)}\phi(\mathbf{x}_2,t_2)-\frac{1}{c}\,\frac{\partial}{\partial t_2}\,\mathbf{A}(\mathbf{x},t_2)$$
$$=-\int d^3\mathbf{x}_1\rho(\mathbf{x}_1)\nabla^{(2)}\,\frac{1}{r}\,\theta\left(t_2-t_1'-\frac{r}{c}\right)$$
$$-\int d^3\mathbf{x}_1\int dt_1\rho(\mathbf{x}_1)\nabla^{(2)}\left[\mathbf{Q}(t_1)\cdot\boldsymbol{\nabla}^{(1)}\,\frac{1}{r}\right.$$
$$\left.\times\delta\left(t_1-t_2+\frac{r}{c}\right)\right]$$
$$-\frac{1}{c^2}\int d^3\mathbf{x}_1\int dt_1\rho(\mathbf{x}_1)\dot{\mathbf{Q}}(t_1)\,\frac{\partial}{\partial t_2}\,\frac{1}{r}$$
$$\times\delta\left(t_1-t_2+\frac{r}{c}\right). \tag{33}$$

We shall neglect the magnetic field, since we have already assumed $\dot{Q}(t_1)\ll c$. Specializing to a displacement $\mathbf{Q}$ along **1** and transforming the time integration in the last term of Eq. (33) by the use of the well-known properties of the δ function, we obtain for the component of $\mathbf{E}$ along **1**

$$E_1(\mathbf{x}_2,t_2)=E_0(\mathbf{x}_2,t_2)+E_D(\mathbf{x}_2,t_2),$$
$$E_0(\mathbf{x}_2,t_2)=-\int d^3\mathbf{x}_1\rho(\mathbf{x}_1)\,\frac{\partial}{\partial\mathbf{x}_2}\,\frac{1}{r}\,\theta\left(t_2-t_1'-\frac{r}{c}\right), \tag{34}$$
$$E_D(\mathbf{x}_2,t_2)=-\int d^3\mathbf{x}_1\int dt_1\rho(\mathbf{x}_1)Q(t_1)$$
$$\times\left(\frac{\partial^2}{\partial\mathbf{x}_2\partial\mathbf{x}_1}-\frac{1}{c^2}\,\frac{\partial^2}{\partial t_2\partial t_1}\right)\frac{1}{r}$$
$$\times\delta\left(t_2-t_1-\frac{r}{c}\right).$$

E_0 can be visualized as the electric field created by a replica of the pointer appearing at time t_1' in the original undisplaced configuration and E_D as the electric field of a distribution of dipoles generated by the displacement of the pointer. Consequently the **1** component of the self-force acting on the pointer at time t_2 is

$$F(t_2)=\int d^3\mathbf{x}_2\rho(\mathbf{x}_2,t_2)E_1(\mathbf{x}_2,t_2)=\int d^3\mathbf{x}_2\rho(\mathbf{x}_2$$
$$-\mathbf{Q})E_1(\mathbf{x}_2,t_2)=\int d^3\mathbf{x}_2\rho(\mathbf{x}_2)T^{(2)}(\mathbf{Q})E_1(\mathbf{x}_2,t_2), \tag{35}$$

where we have performed a change of the integration variable from $\mathbf{x}_2$ to $\mathbf{x}_2-\mathbf{Q}$. Remembering that $\mathbf{Q}$ in Eq. (34) is along **1**, substituting Eq. (34) in Eq. (35), and neglecting terms of $O(Q^2)$ we get

$$F(t_2)=F_{00}(t_2)+F_{0D}(t_2)+F_D(t_2),$$
$$F_{00}(t_2)=-\int d^3\mathbf{x}_1\int d^3\mathbf{x}_2\rho(\mathbf{x}_1)\rho(\mathbf{x}_2)\,\frac{\partial}{\partial x_2}\,\frac{1}{r}$$
$$\times\theta\left(t_2-t_1'-\frac{r}{c}\right), \tag{36}$$
$$F_{0D}(t_2)=-\int d^3\mathbf{x}_1\int d^3\mathbf{x}_2\rho(\mathbf{x}_1)\rho(\mathbf{x}_2)Q(t_2)\,\frac{\partial^2}{\partial x_2^2}\,\frac{1}{r}$$
$$\times\theta\left(t_2-t_1'-\frac{r}{c}\right),$$
$$F_D(t_2)=-\int d^3\mathbf{x}_1\int d^3\mathbf{x}_2\int dt_1\rho(\mathbf{x}_1)\rho(\mathbf{x}_2)Q(t_1)$$
$$\times\left(\frac{\partial^2}{\partial x_2\partial x_1}-\frac{1}{c^2}\,\frac{\partial^2}{\partial t_2\partial t_1}\right)\frac{1}{r}\,\delta\left(t_2-t_1-\frac{r}{c}\right).$$

In this expression F_{00} arises from E_0 and is the force exerted by the replica upon the pointer taken in the initial configuration. F_{0D} also arises from E_0 and is the force exerted by the replica on the distribution of dipoles generated by the displacement of the pointer. Finally, F_D arises from E_D and it can be visualized as the force on the pointer due to the field created by the distribution of dipoles and evaluated up to terms linear in Q. Clearly F_{00} vanishes, since for any $f(r)$ it is

$$\frac{\partial f(r)}{\partial x_2}=(x_2-x_1)\,\frac{1}{r}\,\frac{\partial f(r)}{\partial r}.$$

Thus the contribution from an infinitesimal volume with a given x_2-x_1 in the integration in Eq. (36) is canceled by the contribution of the infinitesimal volume obtained by exchanging x_2 and x_1. Thus the only contributions to F are F_{0D} and F_D which, specializing to the uniformly charged pointer, can be written as

$$F(t_2)=F_{0D}(t_2)+F_D(t_2),$$
$$F_{0D}(t_2)=-\rho^2Q(t_2)\int_V d^3\mathbf{x}_1\int_V d^3\mathbf{x}_2\,\frac{\partial^2}{\partial x_2^2}\,\frac{1}{r}\,\theta\left(t_2-t_1'-\frac{r}{c}\right),$$

$$F_D(t_2)=-\rho^2\int_V d^3\mathbf{x}_1\int_V d^3\mathbf{x}_2\int dt_1 Q(t_1)\times\left(\frac{\partial^2}{\partial x_2\partial x_1}-\frac{1}{c^2}\frac{\partial^2}{\partial t_2\partial t_1}\right)\frac{1}{r}\,\delta\left(t_2-t_1-\frac{r}{c}\right), \tag{37}$$

where the space integrations are over the arbitrarily shaped volume V occupied by the pointer in the initial configuration.

Setting F_{0D} aside for the moment, we concentrate on F_D, which has the interesting property of coinciding with the total self-force when the neutralizing body is present. In fact the effect of the latter is simply to annihilate the replica and its field, yielding immediately $F(t_2)=F_D(t_2)$. We remark that if we could take $Q(t_1)$ out of all integrations in F_D, we would obtain the BR expression (1) for the self-force in the presence of the neutralizing body. This, however, is not a trivial step as we shall now show. Indeed, performing the time integration in F_D for $t_1'<t_2<t_1''$ and remembering that $Q(t_1')=Q(t_1'')=0$, we find

$$F_D(t_2)=-\rho^2\int_V d^3\mathbf{x}_1\int_V d^3\mathbf{x}_2\left[\frac{\partial^2}{\partial x_2\partial x_1}\frac{1}{r}\,Q\left(t_2-\frac{r}{c}\right)\times\theta\left(t_2-t_1'-\frac{r}{c}\right)+\frac{1}{c^2 r}\,\ddot{Q}\left(t_2-\frac{r}{c}\right)\theta\left(t_2-t_1'-\frac{r}{c}\right)+\dot{Q}\left(t_2-\frac{r}{c}\right)\delta\left(t_2-t_1'-\frac{r}{c}\right)\right]. \tag{38}$$

This shows that taking $Q(t_1)$ out of the integral (37) in F_D, in order to obtain the BR result in the presence of the neutralizing body, is incorrect. Moreover the singular nature of the distribution $A_{xx}^{(1,2)}$ defined in Eq. (2) and appearing in the expression (37) for F_D introduces a dependence of Q on r which must be properly dealt with. This can be conveniently done by the series expansion

$$Q\left(t_2-\frac{r}{c}\right)=\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\left(\frac{r}{c}\right)^n Q^{(n)}(t_2),$$

$$Q^{(n)}(t_2)=\frac{\partial^n Q(t_2)}{\partial t_2^n}. \tag{39}$$

This leads to

$$F_D(t_2)=F_Q(t_2)+F_{RR}(t_2),$$

$$F_Q(t_2)=-\rho^2 Q(t_2)\int_V d^3\mathbf{x}_1\int_V d^3\mathbf{x}_2\,\frac{\partial^2}{\partial x_2\partial x_1}\frac{1}{r}\times\theta\left(t_2-t_1'-\frac{r}{c}\right), \tag{40}$$

$$F_{RR}(t_2)=-\rho^2\sum_{n=1}^{\infty}\frac{(-1)^n}{n!}\frac{1}{c^n}Q^{(n)}(t_2)\times\int_V d^3\mathbf{x}_1\int_V d^3\mathbf{x}_2\,\frac{\partial^2}{\partial\mathbf{x}_2\partial\mathbf{x}_1}r^{n-1}\theta\left(t_2-t_1'-\frac{r}{c}\right)-\rho^2\sum_{n=0}^{\infty}\frac{(-1)^n}{n!}\frac{1}{c^{n+2}}\left[Q^{(n+2)}(t_2)\int_V d^3\mathbf{x}_1\int_V d^3\mathbf{x}_2\times r^{n-1}\theta\left(t_2-t_1'-\frac{r}{c}\right)+Q^{(n+1)}(t_2)\times\int_V d^3\mathbf{x}_1\int_V d^3\mathbf{x}_2 r^{n-1}\delta\left(t_2-t_1'-\frac{r}{c}\right)\right].$$

Clearly the only contribution to F_D linear in Q is F_Q. Since for any $f(r)$ it is $\partial f(r)/\partial x_1=-\partial f(r)/\partial x_2$ in the absence of the neutralizing body we have $F_Q=-F_{0D}$. As we have seen, F_{0D} is the electrostatic force exerted by the replica on the distribution of dipoles generated by the displacement of the pointer; hence F_Q can be interpreted as the force exerted by the dipolar field on the pointer taken in the original undisplaced configuration. In the absence of the neutralizing body this force F_Q is canceled by F_{0D}, so that the total self-force is simply $F=F_{RR}$; in the presence of the neutralizing body F_Q contributes to the self-force according to $F=F_Q+F_{RR}$ and corresponds to the attractive force the neutralizing body exerts on the pointer displaced over a distance Q, when $\dot{\mathbf{A}}$ can be neglected. In fact expression (40) for F_Q is currently used to evaluate the plasma frequency of a macroscopic body in solid state physics [14].

Moreover we shall show that F_{RR} in Eq. (40), which is the only contribution to the self-force F surviving in the absence of the neutralizing body, reduces to the usual radiation-reaction force under appropriate circumstances. To this aim we use

$$\frac{\partial^2}{\partial x_2\partial x_1}r^{n-1}\theta\left(t_2-t_1'-\frac{r}{c}\right)=r^{n-1}\left\{\frac{1-n}{r^2}\left[1+(n-3)\left(\frac{x_2-x_1}{r}\right)^2\right]\theta\left(t_2-t_1'-\frac{r}{c}\right)+\frac{1}{cr}\left[1+(2n-3)\left(\frac{x_2-x_1}{r}\right)^2\right]\delta\left(t_2-t_1'-\frac{r}{c}\right)+\frac{1}{c}\left(\frac{x_2-x_1}{r}\right)^2\frac{\partial}{\partial r}\delta\left(t_2-t_1'-\frac{r}{c}\right)\right\} \tag{41}$$

in Eq. (40), which we specialize to the case of measurements of long duration $\tau\gg 2a/c$. For such measurements we are entitled to consider times $t_2>t_1'+r/c$ for any pair of points $\mathbf{x}_1$ and $\mathbf{x}_2$ within the pointer. Restriction to these times t_2 amounts to assuming that any part of the pointer is causally related to all others. With this restriction expression (40) yields

$$F_{RR}(t_2) = -\frac{\rho^2}{c^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \frac{1}{c^n} Q^{(n+2)}(t_2) \times \int_V d^3\mathbf{x}_1 \int_V d^3\mathbf{x}_2 r^{n-1} \times \left[\frac{n+1}{n+2} - \frac{n-1}{n+2} \left(\frac{x_2 - x_1}{r} \right)^2 \right]. \quad (42)$$

We note that this result is valid for an arbitrarily shaped pointer of uniform density. Thus its domain of validity is different from that of Eq. (4). The two domains, however, coincide if we take V spherical in the former and if in addition we take ρ uniform within the pointer in the latter. With this choice expressions (4) and (42) coincide. We conclude that F_{RR} in Eq. (40) is the genuine radiation-reaction contribution to the self-force, which exists independently of the presence of the neutralizing body.

Thus we reach the following conclusions:

(1) In the presence of the neutralizing body, the only term in the self-force proportional to the displacement Q arises from the attraction by this body on the pointer. This term is electrostatic in nature and is expressed by F_Q as in Eq. (40) rather than by the BR expression in Eq. (1). In this case the self-force is given by $F = F_Q + F_{RR}$.

(2) In the absence of the neutralizing body no contribution proportional to Q exists in the expression for the self-force, which is simply given by $F = F_{RR}$.

These conclusions seem in agreement with the results of our theory presented in Secs. II and III. Thus the origin of the discrepancy between Eqs. (2) and (26) is related to the approximation implicit in expression (1). As we have shown, such an approximation has far-reaching consequences. Indeed the introduction of a compensating force of nonelectromagnetic nature, proposed by BR, seems superfluous and misleading in the circumstances considered, since from Eq. (26) it is clear that the same result can be obtained simply by eliminating the neutralizing body from the gedanken apparatus, provided the mass of the test body is large enough to make the radiation-reaction force negligible.

Finally we remark that the only condition for the validity of our results in this section is that terms nonlinear in Q and in its time derivatives, as well as magnetic effects, should be negligible. These constraints are respectively equivalent to assuming that Q should be negligible with respect to a and that $\dot{Q}$ should be negligible with respect to c. A sufficiently large value of M should ensure the validity of both assumptions.

V. CONCLUSIONS

We have reconsidered the old problem of the precision with which the electromagnetic-field amplitude can be measured in the neighborhood of a space-time point in a nonrelativistic context. We have applied theoretical techniques, familiar in quantum optics, to the gedanken experiment devised long ago by BR to discuss this problem. In analogy with the results of the BR analysis, our theory yields a minimum uncertainty with which the field amplitude can be measured locally. Our expression for this minimum, however, is significantly different from the BR expression. Moreover it has a clear physical meaning in terms of the energy $\hbar/\tau$ which is inevitably conveyed to any object in the course of a quantum measurement of finite duration τ [15] because of the time-energy uncertainty principle, and it vanishes after a straightforward modification of the BR gedanken apparatus consisting of the elimination of the neutralizing body. This leads us to conclude that there is no ground for the introduction of nonelectromagnetic forces, fostered by BR in order to perform an exact measurement of the field amplitude. We note that this conclusion establishes QED as a self-consistent theory also in the context of the quantum theory of measurement. In addition, we have investigated the discrepancy between our results and those obtained by BR, and we have shown that it stems from an approximation in the BR treatment which we have carefully avoided in our theory.

Finally, we make three further comments. First, in the theory presented in this paper we have been concerned only with measurements of the local field amplitude in the region occupied by the pointer (''single field averages'' in BR's language). Thus the question of the measurability of field correlations (''twofold averages'' in BR's language), which is discussed at length in the BR paper [2], remains an open question. Second, we emphasize that problems of self-acceleration and runaway solutions [16] are out of the scope of this paper. It should be mentioned, however, that expression (40) for the radiation-reaction force indicates the presence of a contribution proportional to $\dot{Q}$ for times shorter than that taken by light to traverse the pointer. It might be interesting to speculate if the Abraham-Lorentz paradox survives in the presence of this damping term, which of course vanishes for sufficiently long times. Third, one might raise doubts about the relevance of the present work for QED as a branch of modern science, since the modern point of view emphasizes the concept of photon rather than that of field amplitude. The notion of photon, however, seems inextricably related to that of normal modes of the field and, as it has been pointed out recently, there are cases where the electromagnetic field cannot be represented in terms of normal modes [17], particularly in the physically important case of time-dependent boundaries [18]. In these cases the description of the quantized field in terms of photons, defined as quanta of the excitation of the normal modes, apparently fails. One is then led to adopt other descriptions of the field. In this paper we have shown that a description in terms of field amplitudes is feasible and self-consistent, since the field amplitude can be measured in the context of QED without making recourse to compensating forces of nonelectromagnetic nature.

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[1] L. Landau and R. Peierls, Z. Phys. **69**, 56 (1931), translated and reprinted in *Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, NJ, 1983), p. 465.

[2] N. Bohr and L. Rosenfeld, Mat. Fys. Medd. K. Dan. Vidensk. Selsk. **12**, No. 8 (1933), translated and reprinted in *Quantum Theory and Measurement*, edited by J. A. Wheeler and W. H. Zurek (Princeton University Press, Princeton, NJ, 1983), p. 479.

[3] E. Corinaldesi, Nuovo Cimento Suppl. **10**, 83 (1953).

[4] W. Heitler, *The Quantum Theory of Radiation* (Oxford University Press, New York, 1954); J. Kalckar, in *Foundations of Quantum Mechanics*, Varenna International School of Physics ''Enrico Fermi,'' Course No. IL, 1970, edited by B. d' Espagnat (Academic, New York, 1971), p. 127; O. Darrigol, Rev. Hist. Sci. **44**, 137 (1991).

[5] P. G. Bergmann and G. J. Smith, Gen. Relativ. Gravit. **14**, 1131 (1982).

[6] B. S. De Witt, in *Relativity*, *Groups and Topology*, edited by C. De Witt and B. De Witt (Blackie, London, 1964), p. 585.

[7] H. H. Von Borzeskowski and H. J. Treder, Found. Phys. **12**, 1113 (1982).

[8] H. H. Von Borzeskowski and M. B. Mensky, Phys. Lett. A **188**, 249 (1994).

[9] W. Pauli, *General Principles of Quantum Mechanics* (Springer-Verlag, New York, 1980).

[10] See, e.g., J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1962).

[11] D. P. Craig and T. Thirunamachandran, *Molecular Quantum Electrodynamics* (Academic, New York, 1984).

[12] P. W. Milonni, *The Quantum Vacuum* (Academic, New York, 1994).

[13] L. Rosenfeld, in *Niels Bohr and the Development of Physics*, edited by W. Pauli (Pergamon, New York, 1955), p. 70.

[14] See, e.g., N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt, Rinehart and Winston, New York, 1976).

[15] A. Peres and W. K. Wootters, Phys. Rev. D **32**, 1968 (1985); A. Peres, *ibid.* **39**, 2943 (1989).

[16] E. J. Moniz and D. H. Sharp, Phys. Rev. D **15**, 2850 (1977); D. H. Sharp, in *Radiation Reaction in Nonrelativistic Quantum Theory*, edited by A. O. Barut (Plenum, New York, 1980), p. 127.

[17] A. E. Siegman, in *Coherence and Quantum Optics VII*, edited by J. Eberly, L. Mandel, and E. Wolf (Plenum, New York, 1996), p. 229.

[18] G. Barton and C. Eberlein, Ann. Phys. (N.Y.) **227**, 222 (1993).