PHYSICAL REVIEW A VOLUME 57, NUMBER 2 FEBRUARY 1998

Photon correlations in the spectrum of a superradiant system in a strong cavity field

H. M. Castro-Beltran,[1] S. M. Chumakov,[2] and J. J. Sanchez-Mondragon[1]
[1]Instituto Nacional de Astrofísica, Optica y Electrónica, Apartados Postales 51 y 216, 72000 Puebla, Puebla, Mexico
[2] Instituto de Investigaciones en Matemáticas Aplicadas y en Sistemas, Universidad Nacional Autónoma de México, Apartado Postal 48-3, 62251 Cuernavaca, Morelos, Mexico

(Received 17 July 1997; revised manuscript received 7 October 1997)

We investigate correlations among photons from the components of the resonance fluorescence spectrum of a cooperative system of two-level atoms interacting with a strong field of a single-mode lossless cavity. The properties of the central band are more closely related to the superradiant effect in free space than the sidebands. The central band field has reduced fluctuations and the collective dipole has its largest uncertainty when half the atoms are initially excited. Partial atomic excitation destroys the sub-Poissonian character of the sideband field and leads to a bunching-antibunching transition in the correlation of a sideband photon followed by a central band photon. [S1050-2947(98)05402-X]

PACS number(s): 42.50.Fx, 42.50.Dv

Superradiance or coherent spontaneous emission is one of the most important problems in physics [1]. It is the enhancement of the spontaneous emission rate of a symmetrically excited atomic system, particularly a half-excited system, with emission proportional to the square of the number of atoms. It has been studied mainly for a vacuum field in free space, its most notable features being [1,2] the enhanced decay rate in the form of a hyperbolic secant pulse, narrowing with increasing cooperativity; the reduction of the fluctuations of the field, approaching that of a coherent state; and a large energy loss per atom. For moderate and high-Q cavities it has been shown that a large number of atoms increase the Rabi frequency leading to oscillatory behavior [3,4]. We recently showed that when there is a strong field in a lossless cavity an initially half-excited atomic system experiences an *average* loss of energy, which remains in the cavity, in contrast to the free-space case [5].

Strong fields (average photon number $\overline{n}$ much larger than number of atoms A) strongly modify the atomic response, and the resonance fluorescence (RF) spectrum consists of a triplet similar to the one-atom Mollow spectrum [6], plus a series of small cooperative or extra sidebands [7–9]. In free space the components of the triplet have an intensity proportional to A^2 [7,8], while in a lossless cavity the central band intensity changes from A dependent for all atoms in the same state, to A^2 dependent for half-atomic excitation [10]. This suggests considering superradiance from the spectral point of view, but most of the studies on cooperative emission in the presence of a strong field do not consider the effect of partial excitation, and, certainly, not every collective radiation process is superradiant. One must analyze the fluctuations of both the scattered field and atoms.

In this Brief Report we investigate the photon correlations among the components of the RF spectrum of a partially excited system of two-level atoms in a *lossless* cavity with a *strong* field. Our aim is to determine the collective and superradiant properties of this spectrum. We also analyze the cooperative modifications to the correlations of photons from different components of the spectrum, extending previous results limited to atoms initially in the ground state [11].

We consider the interaction of A two-level atoms with a strong field of a single-mode loss-free cavity. The atoms and field have the same frequency, and the spread of the atomic sample is much less than the wavelength of the mode but the atoms are coupled only by the same field. We use our quantum electrodynamic perturbation method [9,10], whose lowest-order Hamiltonian is $\mathcal{V}_0=2\Omega_N L^X$, just like the semiclassical one, L^X is the angular momentum operator in the x-axis, but the Rabi frequency $\Omega_N \equiv g\sqrt{N-A/2+1/2}$ is still of QED form, where $N=n+m$ (number of photons n plus number of excited atoms m) is the excitation number, and g is the coupling constant, assumed equal for all atoms. Then the eigenvalues are

$$\Lambda_p^N=\Omega_N(A-2p), \tag{1}$$

and $|\underline{N,p}\rangle$ are the lowest-order eigenvectors. The indices p, which span an $(A+1)$-dimensional subspace, $0\leqslant p\leqslant A$, label the semiclassical dressed states in the manifold with N quanta.

The relationship between the bare and the dressed bases is given by the unitary rotation matrix α whose elements are $\alpha_{mp}=\langle n,m|\underline{N,p}\rangle$,

$$\alpha_{mp}=\sqrt{\frac{m!p!}{2^A(A-m)!(A-p)!}}\sum_{j=0}^{\min\{m,p\}}\frac{(-2)^j(A-j)!}{j!(m-j)!(p-j)!}. \tag{2}$$

The calculation of the spectrum and photon correlations is simpler in this basis, where the collective atomic transition operators have the matrix elements

$$\langle\underline{q}|L^{\pm}|\underline{p}\rangle=\left(\frac{A}{2}-p\right)\delta_{qp}\pm\tfrac{1}{2}\sqrt{p(A-p+1)}\,\delta_{qp-1}\mp\tfrac{1}{2}\sqrt{(p+1)(A-p)}\,\delta_{qp+1}. \tag{3}$$

Using this basis, we found in Refs. [9,10] that the spectrum consists of three bands, each a multiplet with peaks located at the frequencies $\omega_{qp}^N\equiv\Lambda_p^N-\Lambda_q^{N-1}$,

$$\omega_{p+kp}^{N}=2k\Omega_N+\frac{g^2}{2\Omega_N}(A-2p-2k), \qquad (4)$$

where $k=-1,0,1$ denote the left, central, and right bands. The absence of the elastic peak gives a doublet envelope to the central band [10]. The separation of two neighboring peaks is g^2/Ω_N, but the fine structure and extra sidebands disappear in the strong-field limit. The integrated intensities of the bands are

$$I_{\pm1}(m)=\frac{A^2+A}{16}-\frac{m(A-m)}{8},$$
$$I_0(m)=\frac{A}{4}+\frac{m(A-m)}{2}. \qquad (5)$$

The sidebands have an A^2 behavior for any number of excited atoms. Moreover, for $m=0,A$ and $A\geqslant 4$ the intensity of a sideband is larger than the central band, but for $m=A/2$ it drops to almost half the intensity it has at $m=0,A$. The central band is more interesting in this respect. For $m=0,A$, the intensity of the central peak is proportional to A [11], in contrast to the free-space case [7] where the entire triplet is A^2 dependent. However, when $m=A/2$, the central band also has an A^2 dependence, and its intensity is four times that of a sideband [10].

The dependence of the central band on the initial atomic state is reminiscent of superradiance in the absence of a driving field. But, as the sidebands are also A^2 dependent, we must study the fluctuations of both the radiated field at the frequencies of the bands and of the initial atomic state to characterize the cooperative emission further.

In the rotating-wave approximation only transitions between two neighbor manifolds are allowed, and the only pairs of correlated photons are those of the transitions in consecutive manifolds $N\rightarrow N-1\rightarrow N-2$ with frequencies ω_{qp}^{N} and ω_{rq}^{N-1}. It is clear that the multiplicity of a given two-photon sequence can modify the statistical properties of the photon emission.

The probability that a photon of frequency ω_a is followed by a photon of frequency ω_b is given by

$$G^{(2)}=4|\alpha_{mp}|^2|\langle \underline{N-2,r}|L^-L^-|\underline{N,p}\rangle|^2. \qquad (6)$$

The above expression contains all the allowed two-photon transition amplitudes, which we calculate using the rotated basis, Eq. (3),

$$\begin{aligned}4\langle \underline{r}|L^-L^-|\underline{p}\rangle &=(A-2p)^2\delta_{rp}-(A-2p-2)\sqrt{(p+1)(A-p)}\,\delta_{rp+1}\\ &\quad+(A-2p+2)\sqrt{p(A-p+1)}\,\delta_{rp-1}\\ &\quad-(A-2p)\sqrt{(p+1)(A-p)}\,\delta_{rp+1}\\ &\quad+\sqrt{(p+1)(p+2)(A-p)(A-p-1)}\,\delta_{rp+2}\\ &\quad-p(A-p+1)\,\delta_{rp}+(A-2p)\sqrt{p(A-p+1)}\,\delta_{rp-1}\\ &\quad-(p+1)(A-p)\,\delta_{rp}\\ &\quad+\sqrt{p(p-1)(A-p+1)(A-p+2)}\,\delta_{rp-2}. \qquad (7)\end{aligned}$$

These terms correspond to the following photon emission orders (first-second): (1) center-center, (2) right-center, (3) left-center, (4) center-right, (5) right-right, (6) left-right, (7) center-left, (8) right-left, and (9) left-left, respectively, for a total of $9A-1$ pairs (from the restriction $0\leqslant p,r\leqslant A$). In the complete spectrum there are $2A+1$ bands with $(A+1)^3$ two-photon transitions grouped into $(2A+1)^2$ sets.

However, knowing the difficulty of resolving the cooperative fine structure of the bands, we consider the sum of all the two-photon sequences of a given pair of bands, that is, the probability that a photon of frequency $\overline{\omega_a}$ is followed by a photon of frequency $\overline{\omega_b}$ $(\overline{\omega_k}=2k\Omega)$ is

$$G_{a,b}^{(2)}=\sum_{p=0}^{A}|\alpha_{mp}|^2\langle L_a^+L_b^+L_b^-L_a^-\rangle, \qquad (8)$$

where L_k^- means the emission of a photon of frequency $\overline{\omega_k}$, or, in the normalized form,

$$g_{a,b}^{(2)}=\frac{G_{a,b}^{(2)}}{I_aI_b}, \qquad (9)$$

to clearly reveal the statistical meaning of the correlations. The intensities I_k are given by Eq. (5). A very interesting feature is that the spectrally selected photon correlations are a consequence of the eigenvalue spectrum, and the influence of the initial photon statistics is small, as long as it is narrow enough so that the spectral bands are well separated.

To obtain the total probabilities of the nine sets of amplitudes Eq. (7), we need the moments of the distribution of dressed states $\langle p^l\rangle_p\equiv\Sigma_{p=0}{}^{A}p^l|\alpha_{mp}|^2$, $l=1$, 2, 3, and 4, obtained generalizing the treatment in Ref. [10]:

$$\langle p\rangle_p=\frac{A}{2},\quad \langle p^2\rangle_p=\frac{A(A+1)}{4}+\frac{m(A-m)}{2},$$

$$\langle p^3\rangle_p=\frac{A^3+3A^2+6Am(A-m)}{8},$$

$$\begin{aligned}\langle p^4\rangle_p=\tfrac{1}{16}[&6m^4-12Am^3-(6A^2+6A-10)m^2\\ &+(12A^3+6A^2-10)m+(A^4+6A^3+3A^2-2A)].\end{aligned}$$

The exact expresions for the correlations of photons of given bands as a function of the initial atomic excitation are

$$\begin{aligned}G_{0,0}^{(2)}=\tfrac{1}{16}[&6m^4-12Am^3+(6A^2-6A+10)m^2\\ &+(6A^2-10A)m+3A^2-2A], \qquad (10)\end{aligned}$$

$$\begin{aligned}G_{\pm1,\pm1}^{(2)}=\tfrac{1}{256}[&6m^4-12Am^3+(10A^2+2A-30)m^2\\ &-(4A^3+2A^2-30)m+A^4+2A^3-5A^2+2A], \qquad (11)\end{aligned}$$

$$\begin{aligned}G_{\pm1,\mp1}^{(2)}=\tfrac{1}{256}[&6m^4-12Am^3+(10A^2+2A+2)m^2\\ &-(4A^3+2A^2+2A)m+A^4+2A^3+3A^2+2A], \qquad (12)\end{aligned}$$

$$\begin{aligned}G_{0,\pm1}^{(2)}=\tfrac{1}{64}[&-6m^4+12Am^3-(8A^2-2A+10)m^2\\ &+(2A^3-2A^2+10A)m+A^3-A^2+2A], \qquad (13)\end{aligned}$$

$$G^{(2)}_{\pm 1,0}=\tfrac{1}{64}[-6m^4+12Am^3-(8A^2-2A-14)m^2+(2A^3-2A^2-14A)m+A^3+3A^2-2A]. \tag{14}$$

It is useful to write $g^{(2)}_{a,b}$ for the particularly interesting cases of atomic excitation $m=0,1,A/2$, which are actually the same as for $m=A,A-1,A/2$, due to the symmetry the photon correlations exhibit when the interaction is on resonance (finite atom-field frequency detunings modify to an important degree the results for the on-resonance case, adding other symmetries [12,13]). Our results agree with more restricted cases studied by Buzek and Quang for a single atom [14], and by Shumovsky and Quang for all atoms in the ground state [11].

The correlation of two photons from the same band gives the fluctuations of the light at the frequency $\overline{\omega_a}$. For $g^{(2)}_{a,a}=1$ emission is Poissonian, as for a coherent state, while for $g^{(2)}_{a,a}<1$ $(g^{(2)}_{a,a}>1)$ emission is sub-Poissonian (super-Poissonian), i.e., the photon distribution is narrower (broader) than the Poisson distribution. That $g^{(2)}_{a,a}<1$ also indicates that the atomic system cannot emit two photons simultaneously (antibunching) [15,16].

For the central band the normalized correlation function gives

$$g^{(2)}_{0,0}=\begin{cases} 1+\dfrac{2(A-1)}{A}, & m=0,A \\ 1+\dfrac{3(A-1)(A-2)}{(3A-2)^2}, & m=1,A-1 \\ 1+\dfrac{A(A+2)-8}{2A(A+2)}, & m=A/2, \end{cases}$$

which indicates that emission at central frequencies is Poissonian only for $A=1$ [14], and $A=2$ with $m=1$. In general, the emitted photons from the central band are super-Poissonian (bunched).

For two photons from the same sideband we have

$$g^{(2)}_{\pm 1,\pm 1}=\begin{cases} 1-\dfrac{2(3A-1)}{A(A+1)^2}, & m=0,A \\ 1-\dfrac{2(A^2-13A+14)}{(A^2-A+2)^2}, & m=1,A-1 \\ 1+\dfrac{A(A+2)+8}{2A(A+2)}, & m=A/2. \end{cases}$$

When all the atoms are in the same state the sidebands tend to be less sub-Poissonian for increasing A [11]. A small number of atoms can be used to generate highly nonclassical radiation at the frequencies $\pm 2\overline{\Omega}$. Partial excitation, on the other hand, spoils the above situation to the level of converting all this emission to super-Poissonian for half-atomic excitation. For $m=1$, 12 atoms are needed to generate a sub-Poissonian field.

This incoherence of the bands $(g^{(2)}_{a,a}>1)$ seems to present a contradiction to the definition of superradiance as "coherent" spontaneous emission, given the A^2 dependence in their intensities. To our knowledge, this important point has not been addressed before. In fact, due to the absence of an elastic peak, our spectrum is fully incoherent to first order in the limit where there are more photons than atoms and, as the results above show, driven superradiance is not second order coherent.

Another question arises: What about the relationship between the fluctuations of the atomic system and of the scattered radiation? We will not discuss definitions of atomic coherence [17], a very interesting problem in itself. Here we only consider the width of the distribution of dressed states $|\alpha_{mp}|^2$, which is a measure of the uncertainty of the atomic component in the x axis, and reads

$$\Delta p=\sqrt{\langle p^2\rangle_p-\langle p\rangle_p^2}=\sqrt{\frac{A}{4}+\frac{m(A-m)}{2}}. \tag{15}$$

For the state $|m=0,A\rangle$, the width $\Delta p=\sqrt{A}/2$ is at a minimum, and increasing A the central band becomes more incoherent $(1\leqslant g^{(2)}_{0,0}<3)$, and the sidebands lose their sub-Poissonian character $(0\leqslant g^{(2)}_{\pm 1,\pm 1}\leqslant 1)$. On the other hand, for $|m=A/2\rangle$, $\Delta p=\sqrt{A^2/8+A/4}$ is at a maximum, but the statistics of the central band are closer to the coherent behavior, $1\leqslant g^{(2)}_{0,0}<1.5$ than in the $m=0,A$ case, while the sidebands become less super-Poissonian, $2\geqslant g^{(2)}_{\pm 1,\pm 1}>1.5$, for increasing cooperativity. So, in general, for $N\gg A$ and $m=A/2$, the collective emission is not coherent, it is just less incoherent.

Superradiance in a cavity is, therefore, a consequence of enhanced atomic fluctuations, while showing *reduced* field fluctuations, even though this is super-Poissonian. Then only the central band is superradiant. Let us note that, while superradiance in free space involves significant energy loss per atom [2], in a lossless cavity there is an average atomic energy loss which is taken by the field to keep the total energy constant [5].

The cross correlations indicate whether there is a time order of emission of the two photons [18]. Two photons from the opposite sidebands are always bunched:

$$g^{(2)}_{\pm 1,\mp 1}=\begin{cases} 1+\dfrac{2}{A(A+1)}, & m=0,A \\ 1+\dfrac{2(3A^2-3A+2)}{(A^2-A+2)^2}, & m=1,A-1 \\ 1+\dfrac{A(A+2)+8}{2A(A+2)}, & m=A/2. \end{cases}$$

After emission of one photon of frequency $\overline{\omega_{\pm 1}}$, with increasing A, there is an enhanced probability of emission of a $\overline{\omega_{\mp 1}}$ photon, but certainly to a lesser degree than for two $\overline{\omega_0}$ photons. The degree of bunching is reduced with increasing cooperativity for any m.

The correlation of a photon from the central band followed by one of a sideband is

$$g^{(2)}_{0,\pm1}=\begin{cases}1-\dfrac{2(A-1)}{A(A+1)}, & m=0,A\\[2ex] 1-\dfrac{6(A^2-3A+2)}{(3A-2)(A^2-A+2)}, & m=1,A-1\\[2ex] 1-\dfrac{A(A+2)-8}{2A(A+2)}, & m=A/2.\end{cases}$$

In the case $m=0,A$ there is a small degree of antibunching, but for $A\gg 1$ these photons tend to be emitted independently [11]. For $m=A/2$, on the other hand, the degree of antibunching can be as large as $\frac{1}{2}$.

Finally, the correlation of a photon from any sideband followed by a photon of the central band is

$$g^{(2)}_{\mp1,0}=\begin{cases}1+\dfrac{2(A-1)}{A(A+1)}, & m=0,A\\[2ex] 1-\dfrac{2(A^2+5A-6)}{(3A-2)(A^2-A+2)}, & m=1,A-1\\[2ex] 1-\dfrac{A(A+2)+8}{2A(A+2)}, & m=A/2,\end{cases}$$

but now we have a bunching to antibunching transition when only one atom or $A-1$ atoms are excited. Actually, there is complete antibunching for a two-atom system when only one of them is excited. The large effect of the cooperativity limit of the previous case also occurs here. We note that the cross correlations reflect the statistics of the bands, but the properties of the central band (reduction of fluctuations for partial excitation) dominate over those of the sideband.

In conclusion, we have investigated spectrally selected photon correlations for a system of many two-level atoms interacting with a strong cavity field. While emission at the frequency of the central band presents signatures of superradiance as in free space, the sidebands have cooperative behavior but not of the superradiant type. Also, partial atomic excitation destroys the sub-Poissonian character of the sidebands. In the case of an ideal cavity, the role of the large atomic energy loss of the free-space superradiant process, is taken by the largest fluctuations of the atomic system when this is initially half-excited. The correlation of a sideband photon followed by a central band photon becomes antibunched when there is partial excitation.

[1] R. H. Dicke, Phys. Rev. **93**, 99 (1954).
[2] L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms* (Wiley, New York, 1975).
[3] J. M. Raimond, P. Goy, M. Gross, C. Fabre, and S. Haroche, Phys. Rev. Lett. **49**, 117 (1982); **49**, 1924 (1982); Y. Kaluzny, P. Goy, M. Gross, J. M. Raimond, and S. Haroche, *ibid.* **51**, 1175 (1983).
[4] M. Kozierowski, S. M. Chumakov, and A. A. Mamedov, Physica A **180**, 435 (1992).
[5] H. M. Castro-Beltran, J. J. Sanchez-Mondragon, and S. M. Chumakov, Opt. Commun. (to be published).
[6] B. R. Mollow, Phys. Rev. **188**, 1969 (1969).
[7] I. R. Senitzky, Phys. Rev. Lett. **40**, 1334 (1978); H. J. Carmichael, *ibid.* **43**, 1106 (1979); G. S. Agarwal, R. Saxena, L. M. Narducci, D. H. Feng, and R. Gilmore, Phys. Rev. A **21**, 257 (1980); P. D. Drummond and S. S. Hassan, *ibid.* **22**, 662 (1980).
[8] G. S. Agarwal, L. M. Narducci, D. H. Feng, and R. Gilmore, Phys. Rev. Lett. **42**, 1260 (1979).
[9] H. M. Castro-Beltran, J. J. Sanchez-Mondragon, and S. M. Chumakov, Phys. Rev. A **53**, 4420 (1996).
[10] H. M. Castro-Beltran, S. M. Chumakov, and J. J. Sanchez-Mondragon, Opt. Commun. **129**, 184 (1996).
[11] A. S. Shumovsky and Tran Quang, Phys. Lett. A **132**, 164 (1988).
[12] N. N. Bogolubov, Jr., E. I. Aliskenderov, A. S. Shumovsky, and Tran Quang, J. Phys. B **20**, 1885 (1987).
[13] P. L. Knight and Tran Quang, Phys. Rev. A **41**, 6255 (1990).
[14] V. Buzek and Tran Quang, Opt. Commun. **78**, 274 (1990).
[15] H. J. Carmichael and D. F. Walls, J. Phys. B **9**, 1199 (1976); H. J. Kimble and L. Mandel, Phys. Rev. A **13**, 2123 (1976); H. J. Kimble, M. Dagenais, and L. Mandel, Phys. Rev. Lett. **39**, 691 (1977).
[16] G. S. Agarwal, A. C. Brown, L. M. Narducci, and G. Vetri, Phys. Rev. A **15**, 1613 (1977).
[17] J. M. Radcliffe, J. Phys. A **4**, 313 (1971); F. T. Arecchi, E. Courtens, R. Gilmore, and H. Thomas, Phys. Rev. A **6**, 2211 (1972).
[18] C. Cohen-Tannoudji and S. Reynaud, Philos. Trans. R. Soc. London, Ser. A **293**, 223 (1979); A. Aspect, G. Roger, S. Reynaud, J. Dalibard and C. Cohen-Tannoudji, Phys. Rev. Lett. **45**, 617 (1980).