Quantum radiation in a plane cavity with moving mirrors

D. F. Mundarain[1] and P. A. Maia Neto[2]
[1]Facultad de Física, Pontificia Universidad Católica, Casilla 306, Santiago 22, Chile
[2]Instituto de Física, UFRJ, Caixa Postal 68528, 21945-970 Rio de Janeiro, Brazil

(Received 10 June 1997; revised manuscript received 6 October 1997)

We consider the electromagnetic vacuum field inside a perfect plane cavity with moving mirrors, in the nonrelativistic approximation. We show that low-frequency photons are generated in pairs that satisfy simple properties associated to the plane geometry. We calculate the photon generation rates for each polarization as functions of the mechanical frequency by two independent methods: on one hand from the analysis of the boundary conditions for moving mirrors and with the aid of Green functions; and on the other hand by an effective Hamiltonian approach. The angular and frequency spectra are discrete, and emission rates for each allowed angular direction are obtained. We discuss the dependence of the generation rates on the cavity length and show that the effect is enhanced for short cavity lengths. We also compute the dissipative force on the moving mirrors and show that it is related to the total radiated energy as predicted by energy conservation. [S1050-2947(98)05202-0]

PACS number(s): 42.50.Lc, 42.50.Dv, 03.65.−w

I. INTRODUCTION

In the presence of moving boundaries, the vacuum state of the electromagnetic field may not be stable, which results in the generation of photons. This purely quantum effect, which has been known either as dynamical Casimir effect [1] or as motion- [2] or mirror- [3] induced radiation is, like the usual Casimir effect for standing mirrors, a striking illustration of the physical reality of the quantum vacuum field. Moreover, it may also be understood as a mechanical effect of the vacuum field. In fact, energy conservation entails that the radiation effect must be accompanied by a radiation reaction force that works against the motion of the mirror [3–6], and which is connected to the fluctuations of the usual (static) Casimir force by the fluctuation-dissipation theorem [7–9].

Several theoretical models have been analyzed. In the one-dimensional approximation (1D), only one direction of propagation is taken into account [10]. The quantum radiation generated inside a 1D cavity with moving mirrors was calculated in Refs. [11] and [12] in the particular case where the mechanical frequency satisfies a resonant condition for generation of photons in the lowest-order cavity field modes, whereas Ref. [2] considered a 1D cavity with partially transmitting mirrors and with no particular assumption about resonance, thereby allowing for a full analysis of the spectrum of the radiation in a more general case.

A few three-dimensional (3D) models have been recently analyzed in the literature, including moving dielectric half-spaces [13,14], and rotating [15] or collapsing dielectric spheres [16], the latter as a model for sonoluminescence. On the other hand, 3D results for the related problem of photon generation in a medium with time-dependent material coefficients ϵ and μ have been known for nearly ten years [17]. Perhaps the simplest 3D illustration of motion-induced radiation is to consider a single perfectly reflecting plane mirror moving in free space. In the perturbative regime, which is associated to the nonrelativistic limit, it is possible to derive simple results for the spectra of radiation [18], which display interesting polarization-dependent features connected to the angular distribution of the emitted photons. In this paper, we extend the method developed in Ref. [18] to analyze the radiation emitted when two parallel plane perfectly reflecting mirrors, initially a distance L apart, oscillate along the direction perpendicular to their surfaces, and according to a predefined law imposed by some external apparatus. Such geometry constitutes the simplest example, from a theoretical point of view, of a 3D cavity of length L. As compared to the previous single-mirror case, we show that the orders of magnitude for the radiation rates generated in the plane cavity may be several orders of magnitude larger, provided that L is small enough.

The paper is organized as follows. In Sec. II, we calculate the photon numbers generated inside the cavity starting from the boundary conditions of a moving perfectly reflecting mirror. The method is based on the nonrelativistic and long-wavelength approximations, which are closely connected in the context considered here [5]. In Sec. III we present an alternative derivation of the results already found in Sec. II, now employing usual time-dependent perturbation theory for an effective Hamiltonian that incorporates the motion effect in terms of a coupling via radiation pressure. This heuristic approach is considerably simpler than the previous one, since it circumvents the analysis of the moving boundaries. Furthermore, it explicitly unveils the two-photon nature of the photon emission process, and allows for the computation of the dissipative component of the radiation pressure force on the moving mirrors. In Sec. IV, we consider a specific example of motion in order to isolate the effect of a single mechanical frequency ω_0. We show that the photon numbers obtained by two independent methods in Secs. II and III grow linearly in time, allowing us to define photon production rates, whose behavior as functions of the dimensionless parameter $\omega_0 L/\pi c$ is examined in detail. Section V contains the concluding remarks.

II. BOUNDARY CONDITIONS AND INTRACAVITY QUANTUM RADIATION

For the sake of clarity we first assume that one of the mirrors is at rest. The results in the more general case where both mirrors are set to move is a simple generalization to be presented later. The moving mirror oscillates along the direction perpendicular to its surface (x direction), around the position $x=0$, its instantaneous position being given by the equation $x=\delta q(t)$.

We decompose the electromagnetic fields into their components corresponding to the electric field parallel (TM) or perpendicular (TE) to the plane of incidence. For each polarization it is possible to define a vector potential through the equations:

$$\mathbf{E}^{(\mathrm{TE})}=-\partial_t\mathbf{A}^{(\mathrm{TE})};\quad \mathbf{B}^{(\mathrm{TE})}=\nabla\times\mathbf{A}^{(\mathrm{TE})} \tag{1}$$

and

$$\mathbf{E}^{(\mathrm{TM})}=\nabla\times\boldsymbol{\mathcal{A}}^{(\mathrm{TM})};\quad \mathbf{B}^{(\mathrm{TM})}=\partial_t\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}. \tag{2}$$

The units are mks with $c=1$ and $\varepsilon_0=1$. The potentials satisfy the gauge equations

$$\nabla\cdot\mathbf{A}^{(\mathrm{TE})}=\nabla\cdot\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}=0. \tag{3}$$

As shown in Appendix A, the boundary conditions for a perfectly reflecting moving mirror are very simple when written in terms of $\mathbf{A}^{(\mathrm{TE})}$ and $\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}$, due essentially to the fact that they are both orthogonal to the direction of motion. We find

$$\mathbf{A}^{(\mathrm{TE})}(x=\delta q(t),\mathbf{r}_\parallel,t)=\mathbf{0} \tag{4}$$

and

$$(\partial_x+\delta\dot{q}(t)\partial_t)\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}(x=\delta q(t),\mathbf{r}_\parallel,t)=\mathbf{0}, \tag{5}$$

where $\mathbf{r}_\parallel=y\hat{y}+z\hat{z}$. Furthermore, the fields satisfy the usual homogeneous Dirichlet and Neumann boundary conditions on the second mirror, which is at rest at $x=L$:

$$\mathbf{A}^{(\mathrm{TE})}(x=L,\mathbf{r}_\parallel,t)=\mathbf{0};\quad \partial_x\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}(x=L,\mathbf{r}_\parallel,t)=\mathbf{0}. \tag{6}$$

We want to solve the boundary value problem as defined by Eqs. (4)–(6) for the fields in the region between the mirrors. The results for the fields outside the plane cavity are essentially the same as those for a single moving mirror in vacuum, and hence may be found in Refs. [5,18]. The essential ''ansatz'' that allows us to employ the long-wavelength approximation to solve the boundary value problem defined by Eqs. (4)–(6) is to assume that a given mechanical frequency ω_0 induces the generation of photons only in the spectral range $\omega<\omega_0$. This property is satisfied by the nonrelativistic models considered previously (see Refs. [2] and [18]). Moreover, it agrees with the intuitive notion that the radiation effect is a nonadiabatic process, so that high-frequency field modes cannot be excited since the corresponding time scales are shorter than mechanical time scales (quasistatic limit). More importantly, we show later in this section that this property is fully satisfied for the model considered here. As for the connection with the long-wavelength approximation, we note that the amplitude δq_0 of a sinusoidal nonrelativistic motion must satisfy $\omega_0\delta q_0\ll 1$. When combined with our ansatz, this condition leads to $\delta q_0\ll\lambda$, where λ is the wavelength of the emitted radiation. Actually, we may be slightly more general and consider any nonrelativistic oscillatory motion around $x=0$ such that its Fourier components satisfy the above requirements (more specifically, we shall consider a weakly damped sinusoidal nonrelativistic motion in Sec. IV).

Accordingly, we look for perturbative solutions in the form

$$\mathbf{A}^{(\mathrm{TE})}=\mathbf{A}_{\mathrm{sta}}^{(\mathrm{TE})}+\delta\mathbf{A}^{(\mathrm{TE})} \tag{7}$$

and

$$\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}=\boldsymbol{\mathcal{A}}_{\mathrm{sta}}^{(\mathrm{TM})}+\delta\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}. \tag{8}$$

$\mathbf{A}_{\mathrm{sta}}^{(\mathrm{TE})}$ and $\boldsymbol{\mathcal{A}}_{\mathrm{sta}}^{(\mathrm{TM})}$ are the fields satisfying the Dirichlet and Neumann boundary conditions for standing mirrors, whereas $\delta\mathbf{A}^{(\mathrm{TE})}$ and $\delta\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}$ represent the first-order modifications induced by the motion. As we show below, they are smaller than the fields for the static configuration by a factor of the order of $\delta q/\lambda$. We expand the fields in Eqs. (4) and (5) in Taylor series around $x=0$. Since the jth spatial derivative of a monochromatic traveling wave satisfies

$$|\partial_x^j\mathbf{A}|\leqslant(2\pi/\lambda)^j|\mathbf{A}|, \tag{9}$$

we find from Eq. (4) that the TE-polarized field $\delta\mathbf{A}^{(\mathrm{TE})}$ is given up to first order in $\delta q/\lambda$ by

$$\delta\mathbf{A}^{(\mathrm{TE})}(x=0,\mathbf{r}_\parallel,t)=-\delta q(t)\partial_x\mathbf{A}_{\mathrm{sta}}^{(\mathrm{TE})}(x=0,\mathbf{r}_\parallel,t). \tag{10}$$

Note that we have neglected the term $\delta q(t)\partial_x\delta\mathbf{A}^{(\mathrm{TE})}(x=0,\mathbf{r}_\parallel,t)$ because, as shown by the above result, $\delta\mathbf{A}^{(\mathrm{TE})}$ is already of first order in $\delta q/\lambda$. Following the same method we find the following result for TM polarization:

$$\partial_x\delta\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}(x=0,\mathbf{r}_\parallel,t)=-(\delta q(t)\partial_x^2+\delta\dot{q}(t)\partial_t)\times\boldsymbol{\mathcal{A}}_{\mathrm{sta}}^{(\mathrm{TM})}(x=0,\mathbf{r}_\parallel,t), \tag{11}$$

where now we have also neglected terms of the order of $\delta q\,\delta\dot{q}/\lambda$. According to our ansatz, when considering the generation of photons out of the vacuum field induced by a mechanical frequency ω_0, the relevant wavelengths are larger than $2\pi/\omega_0$, and thus the neglected terms are all of the order of $(\delta\dot{q})^2$. We have then transformed the homogeneous boundary conditions for the total fields at the time-dependent position $x=\delta q(t)$ given by Eqs. (4) and (5) into inhomogeneous boundary conditions for $\delta\mathbf{A}^{(\mathrm{TE})}$ and $\delta\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}$ at the position $x=0$, given by Eqs. (10) and (11), which may be solved by standard Green-function techniques.

We introduce periodic boundary conditions on the transverse plane yz over a surface of area S. In the static case, the normal mode decomposition of the fields in the interval between the mirrors, $0\leqslant x\leqslant L$, is then written as follows:

$$\mathbf{A}_{\mathrm{sta}}^{(\mathrm{TE})}(\mathbf{r},t)=i\sum_{\ell=1}^{\infty}\sum_{n}\sqrt{\frac{\hbar}{\omega_n^{\ell}SL}}\sin\left(\frac{\ell\pi}{L}x\right)\times e^{i\mathbf{k}_\parallel^n\cdot\mathbf{r}_\parallel}e^{-i\omega_n^{\ell}t}a_{n,\ell}^{(\mathrm{TE})}\hat{\boldsymbol{\epsilon}}_n+\mathrm{H.c.} \tag{12}$$

and for the TM polarization,

$$\boldsymbol{\mathcal{A}}_{\rm sta}^{\rm (TM)}(\mathbf{r},t)=i\sum_{\ell=0}^{\infty}\sum_{n}\sqrt{\frac{\hbar}{(1+\delta_{\ell 0})\omega_n^{\ell}SL}}\times\cos\left(\frac{\ell\pi}{L}x\right)e^{i\mathbf{k}_{\parallel}^{n}\cdot\mathbf{r}_{\parallel}}e^{-i\omega_n^{\ell}t}a_{n,\ell}^{\rm (TM)}\hat{\boldsymbol{\epsilon}}_n+\text{H.c.}, \tag{13}$$

where

$$\mathbf{k}_{\parallel}^{n}=2\pi(n_y\hat{\mathbf{y}}+n_z\hat{\mathbf{z}})/\sqrt{S} \tag{14}$$

represents the component of the wave vector parallel to the mirrors — the shorthand $n=(n_y,n_z)$ represents a pair of integer numbers. Note that the two potentials describing orthogonal polarizations are written in terms of the same unit vector

$$\hat{\boldsymbol{\epsilon}}_n=\hat{\mathbf{x}}\times\frac{\mathbf{k}_{\parallel}^{n}}{k_{\parallel}^{n}}. \tag{15}$$

Throughout the paper, the sum over n — as in Eqs. (12) and (13) — runs from $n_y=-\infty$ and $n_z=-\infty$ to $n_y=\infty$ and $n_z=\infty$. A given mode with indexes (n,ℓ) corresponds to a standing wave along the x direction with wave vector $k_x^{\ell}=\ell\pi/L$ traveling along a direction parallel to the mirrors with wave vector $\mathbf{k}_{\parallel}^{n}$. Its frequency is given by

$$\omega_n^{\ell}=\sqrt{\left(\frac{\ell\pi}{L}\right)^2+\frac{(2\pi)^2}{S}[(n_y)^2+(n_z)^2]}. \tag{16}$$

The bosonic field operators in Eqs. (12) and (13) satisfy the usual commutation relations

$$[a_{n,\ell}^{j},a_{n',\ell'}^{j'}]=0 \tag{17}$$

and

$$[a_{n,\ell}^{j},(a_{n',\ell'}^{j'})^{\dagger}]=\delta_{n,n'}\delta_{\ell,\ell'}\delta_{j,j'}, \tag{18}$$

where $j=\text{TE},\ \text{TM}$ represents the polarization.

It is convenient to work with a mixed Fourier representation defined as

$$\mathbf{A}_n^{\rm (TE)}[x,\omega]=\frac{1}{S}\int dt\int d^2\mathbf{r}_{\parallel}e^{-i\mathbf{k}_{\parallel}^{n}\cdot\mathbf{r}_{\parallel}}e^{i\omega t}\mathbf{A}^{\rm (TE)}(x,\mathbf{r}_{\parallel},t) \tag{19}$$

with an analogous expression for TM polarization. The Fourier-transformed fields representing the motion-induced perturbation satisfy the 1D Klein-Gordon equation

$$(\partial_x^2+\omega^2-(k_{\parallel}^{n})^2)\delta\mathbf{A}_n^{\rm (TE)}[x,\omega]=0, \tag{20}$$

$$(\partial_x^2+\omega^2-(k_{\parallel}^{n})^2)\delta\boldsymbol{\mathcal{A}}_n^{\rm (TM)}[x,\omega]=0, \tag{21}$$

and the boundary conditions at $x=0$ and $x=L$ are given by Eqs. (6), (10), and (11). The resulting boundary value problem for TE polarization is solved with the aid of the appropriate Dirichlet Green function:

$$G_{n\ \omega}^{D}(x,x')=\frac{2}{L}\sum_{\ell=1}^{\infty}\frac{\sin(\ell\pi x/L)\sin(\ell\pi x'/L)}{(\omega\pm i\epsilon)^2-\omega_n^{\ell 2}}, \tag{22}$$

where the plus (minus) sign in Eq. (22) provides the retarded (advanced) Green function. The fields with TM polarization are obtained from the Neumann Green function:

$$G_{n\ \omega}^{N}(x,x')=\frac{2}{L}\sum_{\ell=0}^{\infty}\frac{\cos(\ell\pi x/L)\cos(\ell\pi x'/L)}{(1+\delta_{\ell 0})[(\omega\pm i\epsilon)^2-\omega_n^{\ell 2}]}. \tag{23}$$

We assume that the mirror moves during a finite time interval, then returning to its initial position at $x=0$. As a consequence, we may define input and output fields, $\mathbf{A}_{{\rm in}_n}^{\rm (TE)}$ and $\mathbf{A}_{{\rm out}_n}^{\rm (TE)}$ corresponding to the limit values of very small and very large times (and likewise in the case of TM polarization), which satisfy the boundary conditions for a mirror at rest at $x=0$. They are connected by a suitable combination of retarded (superscript R) and advanced (superscript A) Green functions:

$$\mathbf{A}_{{\rm out}_n}^{\rm (TE)}[x,\omega]=\mathbf{A}_{{\rm in}_n}^{\rm (TE)}[x,\omega]+\delta\mathbf{A}_n^{\rm (TE)}[x'=0,\omega]\times[\partial_{x'}G_{n\ \omega}^{D,R}(x,x'=0)-\partial_{x'}G_{n\ \omega}^{D,A}(x,x'=0)]. \tag{24}$$

The TM output field $\boldsymbol{\mathcal{A}}_{{\rm out}_n}^{\rm (TM)}$ is related to the TM input field $\boldsymbol{\mathcal{A}}_{{\rm in}_n}^{\rm (TM)}$ by a similar expression:

$$\boldsymbol{\mathcal{A}}_{{\rm out}_n}^{\rm (TM)}[x,\omega]=\boldsymbol{\mathcal{A}}_{{\rm in}_n}^{\rm (TM)}[x,\omega]-\partial_{x'}\delta\boldsymbol{\mathcal{A}}_n^{\rm (TE)}[x'=0,\omega]\times[G_{n\ \omega}^{N,R}(x,x'=0)-G_{n\ \omega}^{N,A}(x,x'=0)]. \tag{25}$$

From Eqs. (22) and (23) we find

$$\partial_{x'}G_{n\ \omega}^{D,R}(x,x'=0)-\partial_{x'}G_{n\ \omega}^{D,A}(x,x'=0)=-\frac{2\pi^2 i}{L^2}\sum_{\ell=1}^{\infty}\frac{\ell}{\omega_n^{\ell}}\sin\left(\frac{\ell\pi x}{L}\right)(\delta(\omega-\omega_n^{\ell})-\delta(\omega+\omega_n^{\ell})), \tag{26}$$

and

$$G_{n\ \omega}^{N,R}(x,x'=0)-G_{n\ \omega}^{N,A}(x,x'=0)=-\frac{2\pi i}{L}\sum_{\ell=0}^{\infty}\frac{\cos(\ell\pi x/L)}{(1+\delta_{\ell 0})\omega_n^{\ell}}(\delta(\omega-\omega_n^{\ell})-\delta(\omega+\omega_n^{\ell})). \tag{27}$$

In general, there are no monochromatic solutions for the problem of moving boundaries, and hence it is not possible to write a normal mode decomposition for the field in this case. However, since $\mathbf{A}_{{\rm in}_n}^{\rm TE}$ and $\mathbf{A}_{{\rm out}_n}^{\rm TE}$ satisfy the boundary conditions for two standing mirrors at $x=0$ and $x=L$, we may write their normal mode decompositions as in Eqs. (12) and (13), in terms of input and output bosonic operators $a^{(\mathrm{TE})}_{\mathrm{in}_n}$ and $a^{(\mathrm{TE})}_{\mathrm{out}_n}$ (at this point our method is quite similar to the approach developed in Ref. [17] for the problem of time-dependent material coefficients). We then take the Fourier transform of Eq. (10) and replace the result, jointly with Eq. (26), into Eq. (24) in order to find the linear transformation between the input and output TE bosonic operators:

$$a^{(\mathrm{TE})}_{\mathrm{out}_{n\ell}}=a^{(\mathrm{TE})}_{\mathrm{in}_{n\ell}}+\frac{i}{L}\sum_{\ell'=1}\frac{(\ell\pi/L)\ell'\pi/L}{\sqrt{\omega_n^{\ell}\omega_n^{\ell'}}}[\delta q[\omega_n^{\ell}-\omega_n^{\ell'}]a^{(\mathrm{TE})}_{\mathrm{in}_{n\ell'}}+\delta q[\omega_n^{\ell}+\omega_n^{\ell'}](a^{(\mathrm{TE})}_{\mathrm{in}_{-n\ell'}})^{\dagger}], \tag{28}$$

where $\delta q[\omega]$ is the Fourier transform of $\delta q(t)$. The relation between TM operators is derived from Eqs. (11), (25), and (27) in a similar way:

$$a^{(\mathrm{TM})}_{\mathrm{out}_{n\ell}}=a^{(\mathrm{TM})}_{\mathrm{in}_{n\ell}}-\frac{i}{L}\sum_{\ell'=0}[(1+\delta_{\ell 0})(1+\delta_{\ell' 0})]^{-1/2}\left\{\frac{(k_{\parallel}^{n})^2-\omega_n^{\ell}\omega_n^{\ell'}}{\sqrt{\omega_n^{\ell}\omega_n^{\ell'}}}\delta q[\omega_n^{\ell}-\omega_n^{\ell'}]a^{(\mathrm{TM})}_{\mathrm{in}_{n\ell'}}+\frac{(k_{\parallel}^{n})^2+\omega_n^{\ell}\omega_n^{\ell'}}{\sqrt{\omega_n^{\ell}\omega_n^{\ell'}}}\delta q[\omega_n^{\ell}+\omega_n^{\ell'}](a^{(\mathrm{TM})}_{\mathrm{in}_{-n\ell'}})^{\dagger}\right\}. \tag{29}$$

From Eqs. (28) and (29) we may readily derive the number of photons generated inside the cavity as a quantum effect of the mirror's motion. As discussed below, the effect is associated to the creation operators appearing in the right-hand side (rhs) of Eqs. (28) and (29). We assume that the field is initially in the vacuum state. The motion of the mirror then excites a given number of photons $N^{j}_{n,\ell}$ with indexes n,ℓ and polarization j. $N^{j}_{n,\ell}$ is given by the corresponding output number operator averaged over the input vacuum state:

$$N^{j}_{n,\ell}=\langle 0,\mathrm{in}|(a^{j}_{\mathrm{out}_{n\ell}})^{\dagger}a^{j}_{\mathrm{out}_{n\ell}}|0,\mathrm{in}\rangle. \tag{30}$$

Replacing Eqs. (28) and (29) into (30) provides the photon numbers for each polarization:

$$N^{(\mathrm{TE})}_{n,\ell}=\frac{1}{L^2}\sum_{\ell'=1}\left(\frac{\ell\pi}{L}\right)^2\left(\frac{\ell'\pi}{L}\right)^2\frac{1}{\omega_n^{\ell}\omega_n^{\ell'}}|\delta q[\omega_n^{\ell}+\omega_n^{\ell'}]|^2, \tag{31}$$

and

$$N^{(\mathrm{TM})}_{n,\ell}=\frac{1}{L^2}\sum_{\ell'=0}\frac{[(k_{\parallel}^{n})^2+\omega_n^{\ell}\omega_n^{\ell'}]^2}{(1+\delta_{\ell 0})(1+\delta_{\ell' 0})\omega_n^{\ell}\omega_n^{\ell'}}\times|\delta q[\omega_n^{\ell}+\omega_n^{\ell'}]|^2. \tag{32}$$

Since the frequencies $\omega_n^{\ell'}$ are positive, we infer from Eqs. (31) and (32) that a given mechanical frequency ω_0 generates photons with frequencies $\omega_n^{\ell}\leqslant\omega_0$, thereby justifying the ansatz employed in this section.

From the above results we may directly calculate the photon production rates and then estimate the order of magnitude of the quantum radiation effect. Before addressing this question, however, we present a second derivation of Eqs. (31) and (32), which is based on usual time-dependent Hamiltonian perturbation theory. Note that the invariance of the rhs of Eqs. (31) and (32) with respect to the permutation of ℓ and ℓ' suggests that the photons are emitted in pairs. That this is indeed the case is more clearly shown by this alternative approach, to be presented in the next section.

III. CONNECTION WITH RADIATION PRESSURE

Rather than considering the boundary conditions of a moving mirror, we follow in this section the heuristic approach, first presented in Ref. [6], in which the effect of the mirror's motion is modeled by taking the perturbation Hamiltonian

$$\delta H=-\delta q(t)F, \tag{33}$$

where F is the field quantum operator representing the force on the moving mirror. Accordingly, δH corresponds to the energy supplied to the field by means of the vacuum radiation pressure effect. The total Hamiltonian of the field is

$$H=H^{(0)}+\delta H, \tag{34}$$

where the unperturbed Hamiltonian $H^{(0)}$ is written in terms of the bosonic field operators for a standing mirror [see Eqs. (12) and (13)] as

$$H^{(0)}=\sum_{n,\ell}\sum_{j=\mathrm{TE,TM}}\hbar\omega_n^{\ell}[(a^{j}_{n\ell})^{\dagger}a^{j}_{n\ell}+1/2]. \tag{35}$$

As discussed elsewhere [10], a Hamiltonian approach is not rigorously consistent with the model of perfect reflectiveness considered here. However, this model may be considered as an approximation for dielectric mirrors with large refraction index n — for which a rigorous Hamiltonian model is available [3], although such correspondence is not yet settled (according to Ref. [13], some unexpected results show up when taking the limit of large n). In any case, the formalism presented in this section is justified by comparing the results it provides with those obtained in Sec. II.

The force operator is the integral over the surface of the mirror (at its rest position at $x=0$) of the xx component of the Maxwell stress tensor:

$$F=\frac{1}{2}\int d^2\mathbf{r}_\parallel[E_x(0^+)^2-B_\parallel(0^+)^2], \tag{36}$$

where the limit $x\to 0$ is taken from positive values of x as indicated above (as in the previous section, we do not analyze the effect of the field outside the plane cavity). Since F is a quadratic operator on the field, the perturbation Hamiltonian δH excites pairs of photons as in the problem of parametric amplification by a $\chi^{(2)}$ nonlinear medium. Thus, we consider a perturbed field state of the form

$$|\Psi\rangle=\sum_{\{n\ell j,n'\ell' j'\}} c_{\{n\ell j,n'\ell' j'\}}(t)|\{n\ell j,n'\ell' j'\}\rangle+b(t)|0\rangle, \tag{37}$$

where we sum over all two-photon states $|\{n\ell j,n'\ell' j'\}\rangle$ (the symbols j and j' representing the polarizations of the photons in a given pair $\{n\ell j,n'\ell' j'\}$). Note that each pair $\{n\ell j,n'\ell' j'\}$ is included only once in Eq. (37), regardless of the ordering of the indices.

We assume that at $t\to-\infty$ the field is in the vacuum state, so that the two-photon amplitudes are initially zero: $c_{\{n\ell j,n'\ell' j'\}}(-\infty)=0, b(-\infty)=1$. We compute the buildup of the two-photon amplitude $c_{\{n\ell j,n'\ell' j'\}}(t)$ from standard first-order perturbation theory:

$$c_{\{n\ell j,n'\ell' j'\}}(t)=-\frac{i}{\hbar}\int_{-\infty}^{t}\langle n\ell j,n'\ell' j'|\delta H(t')|0\rangle \times\exp\left[\frac{i}{\hbar}(E^{(0)}_{n\ell,n'\ell'}-E^{(0)}_{\text{vac}})t'\right]dt', \tag{38}$$

with

$$E^{(0)}_{n\ell,n'\ell'}-E^{(0)}_{\text{vac}}=\hbar(\omega_n^\ell+\omega_{n'}^{\ell'}) \tag{39}$$

representing the difference between the (unperturbed) energies of the final and initial states. As discussed in the previous sections, it is meaningless to discuss two-photon amplitudes as long as the mirror is moving. Accordingly, we must take $t\to\infty$ in Eq. (38) in order to have a consistent picture of the quantum radiation effect. Then, replacing Eqs. (33) and (39) into Eq. (38) yields

$$c_{\{n\ell j,n'\ell' j'\}}(\infty)=\frac{i}{\hbar}\langle n\ell j,n'\ell' j'|F|0\rangle\,\delta q[\omega_n^\ell+\omega_{n'}^{\ell'}]. \tag{40}$$

In order to compute the matrix element appearing in the rhs of Eq. (40), we write the electric and magnetic fields in Eq. (36) in terms of the potentials $\mathbf{A}^{(\text{TE})}$ and $\boldsymbol{\mathcal{A}}^{(\text{TM})}$. It is convenient to use the Fourier series representation defined by

$$\mathbf{A}^{(\text{TE})}(x,\mathbf{r}_\parallel,t)=\sum_n \mathbf{A}_n^{(\text{TE})}(x,t)\exp(i\mathbf{k}_\parallel^n\cdot\mathbf{r}_\parallel), \tag{41}$$

and by an equivalent expression for the TM potential $\boldsymbol{\mathcal{A}}^{(\text{TM})}$. Then, the force operator is written as

$$F=\frac{S}{2}\sum_n[(k_\parallel^n)^2\boldsymbol{\mathcal{A}}_n^{(\text{TM})}(0^+,t)\boldsymbol{\mathcal{A}}_{-n}^{(\text{TM})}(0^+,t)-\partial_t\boldsymbol{\mathcal{A}}_n^{(\text{TM})}(0^+,t)\partial_t\boldsymbol{\mathcal{A}}_{-n}^{(\text{TM})}(0^+,t)-\partial_x\mathbf{A}_n^{(\text{TE})}(0^+,t)\partial_x\mathbf{A}_{-n}^{(\text{TE})}(0^+,t)]. \tag{42}$$

From Eq. (42), we obtain

$$\langle n\ell\ \text{TE},n'\ell'\ \text{TM}|F|0\rangle=0. \tag{43}$$

Therefore, the photons belonging to a given emitted pair have the same polarization. This is a general property of the plane symmetry of the problem, rather than a consequence of the specific model considered in this paper. Note, however, that it has been recently shown that TE-TM pairs may be radiated in the case of lateral motion of the mirror [14].

Using the normal mode decomposition of the field operators as given by Eqs. (12) and (13), we may calculate the TE-TE and TM-TM matrix elements. We first find

$$\langle n\ell\text{TE},n'\ell'\text{TE}|\partial_x\mathbf{A}_N^{(\text{TE})}(0^+,t)\partial_x\mathbf{A}_{-N}^{(\text{TE})}(0^+,t')|0\rangle=\frac{\hbar}{SL}\frac{(\ell\pi/L)\ell'\pi/L}{\sqrt{\omega_n^\ell\omega_n^{\ell'}}}(\delta_{n,N}\delta_{n',-N}e^{i(\omega_n^\ell t+\omega_n^{\ell'}t')}+\delta_{n,-N}\delta_{n',N}e^{i(\omega_n^{\ell'}t+\omega_n^\ell t')}), \tag{44}$$

and

$$\langle n\ell\text{TM},n'\ell'\ \text{TM}|\boldsymbol{\mathcal{A}}_N^{(\text{TM})}(0^+,t)\cdot\boldsymbol{\mathcal{A}}_{-N}^{(\text{TM})}(0^+,t')|0\rangle =\frac{\hbar}{SL}(1+\delta_{\ell 0})(1+\delta_{\ell' 0})\omega_n^\ell\omega_n^{\ell'}(\delta_{n,N}\delta_{n',-N}e^{i(\omega_n^\ell t+\omega_n^{\ell'}t')}+\delta_{n,-N}\delta_{n',N}e^{i(\omega_n^{\ell'}t+\omega_n^\ell t')}), \tag{45}$$

where use was made of the property $\omega_n^\ell=\omega_{-n}^\ell$. Combining Eqs. (44) and (45) with Eq. (42) leads to

$$\langle n\ell \mathrm{TE}, n'\ell' \ \mathrm{TE}|F|0\rangle = -\frac{\hbar}{L}\frac{(\ell\pi/L)\ell'\pi/L}{\sqrt{\omega_n^{\ell}\omega_n^{\ell'}}}\delta_{n,-n'}, \tag{46}$$

and

$$\langle n\ell \ \mathrm{TM}, n'\ell' \ \mathrm{TM}|F|0\rangle = \frac{\hbar}{L}\frac{(k_{\parallel}^{n})^2+\omega_n^{\ell}\omega_n^{\ell'}}{(1+\delta_{\ell 0})(1+\delta_{\ell' 0})\sqrt{\omega_n^{\ell}\omega_n^{\ell'}}}\delta_{n,-n'}. \tag{47}$$

From Eqs. (46) and (47), we may immediately calculate the amplitudes of creation of pairs of photons by combining them with Eq. (40). Here we write the results obtained in Appendix B for the more general case where both mirrors are moving, so that the first mirror is at $x=\delta q_1(t)$ and the second mirror at $x=L+\delta q_2(t)$. The resulting creation probabilities are

$$|c_{\{n\ell\mathrm{TE},n'\ell'\mathrm{TE}\}}|^2=\frac{1}{L^2}\left(\frac{\ell\pi}{L}\right)^2\left(\frac{\ell'\pi}{L}\right)^2\frac{1}{\omega_n^{\ell}\omega_n^{\ell'}}|\delta q_1[\omega_n^{\ell}+\omega_n^{\ell'}]-(-1)^{\ell+\ell'}\delta q_2[\omega_n^{\ell}+\omega_n^{\ell'}]|^2\delta_{n,-n'} \tag{48}$$

and

$$|c_{\{n\ell\mathrm{TM},n'\ell'\mathrm{TM}\}}|^2=\frac{1}{L^2}\frac{[(K_{\parallel}^{n})^2+\omega_n^{\ell}\omega_n^{\ell'}]^2}{(1+\delta_{\ell 0})(1+\delta_{\ell' 0})\omega_n^{\ell}\omega_n^{\ell'}}|\delta q_1[\omega_n^{\ell}+\omega_n^{\ell'}]-(-1)^{\ell+\ell'}\delta q_2[\omega_n^{\ell}+\omega_n^{\ell'}]|^2\delta_{n,-n'}. \tag{49}$$

Note that the photons in a given pair have opposite values of $\mathbf{k}_{\parallel}^{n}$, which is again a consequence of the plane symmetry [18]. As shown in Appendix C, Eqs. (44)–(49) must be slightly modified when considering the particular value $n=n'=0$ (which corresponds to the 1D limit of our 3D formalism, since such modes propagate along the x direction and do not contain any dependence on the transverse coordinates y and z).

According to Eqs. (48) and (49), the joint motion of the two mirrors selects the longitudinal modes according to the parity of the indices ℓ. When $\delta q_1=-\delta q_2$, which corresponds to the "elongation mode" of the cavity, the two photons in a pair correspond to ℓ values of the same parity, the opposite taking place when the motion is such that the cavity length is kept constant ($\delta q_1=\delta q_2$). This property is a straightforward generalization of the situation found in one-dimensional cavities [2]. It shows that the radiation effect should not be interpreted simply as a consequence of changing the optical cavity length, since it also takes place when there is no relative motion of the mirrors.

We may compute the average number of photons in a given cavity mode from

$$N_{n,\ell}^{j}=\langle\Psi|(a_{n\ell}^{j})^{\dagger}a_{n\ell}^{j}|\Psi\rangle. \tag{50}$$

Inserting Eq. (37) into Eq. (50) yields

$$N_{n,\ell}^{j}=\sum_{\ell'}|c_{\{n\ell j,-n\ell' j\}}|^2. \tag{51}$$

Equations (48) and (49), in the particular case of $\delta q_2=0$, jointly with Eq. (51) provide results for the photon numbers in full agreement with Eqs. (31) and (32) of the previous section. As for the particular case with $n=0$, Eq. (51) also needs some slight modification in order to include the contribution of the degenerate two-photon states $|0\ell,0\ell\rangle$. As shown in Appendix C, there is agreement with the results found in Sec. II in this case as well. We then conclude that the heuristic approach developed in this section yields the same final expressions for the number of photons produced in a given cavity mode. Moreover, it explicitly shows that the photons are generated in pairs, the photons in a pair having the same polarization and opposite values of $\mathbf{k}_{\parallel n}$.

With the aid of the linear response formalism [19], the perturbation Hamiltonian as given by Eq. (33) may be also applied to compute the dissipative part of the radiation pressure force δF exerted on the moving mirrors [6–9]. Such dissipative force is the mechanical effect of the quantum radiation process, and hence must be interpreted as a radiation reaction force. Since it generalizes Casimir's result for a situation where (at least) one of the mirrors is moving, it has been called motional Casimir force in Ref. [9], where a one-dimensional calculation is presented for the case of partially transmitting mirrors. We consider again the case where one of the mirrors is at rest, and then write the Fourier-transformed force $\delta F[\omega]$ as

$$\delta F[\omega]=\chi[\omega]\delta q[\omega]. \tag{52}$$

As discussed in Ref. [6], linear response theory provides a result for the imaginary part of the susceptibility function $\chi[\omega]$, which corresponds to the dissipative component of the force, in terms of the function $C_{FF}[\omega]$ representing the spectrum of fluctuations of the force operator on a standing mirror:

$$\mathrm{Im}\chi[\omega]=\frac{1}{2\hbar}(C_{FF}[\omega]-C_{FF}[-\omega]). \tag{53}$$

The spectrum of fluctuations $C_{FF}[\omega]$ is defined as the Fourier transform of the force correlation function. It may be written in terms of the two-photon matrix elements obtained above as follows [8]:

$$C_{FF}[\omega]=2\pi\sum_{\{n\ell j,n'\ell'j'\}}\delta(\omega-\omega_n^{\ell}-\omega_{n'}^{\ell'})\times|\langle n\ell j,n'\ell'j'|F|0\rangle|^2, \tag{54}$$

where, as in Eq. (37), each pair $\{n\ell j,n'\ell'j'\}$ is included only once (regardless of the ordering).

The matrix elements of the force being given by Eqs. (43), (46), and (47), we replace the rhs of Eq. (54) into Eq. (53) to find

$$\mathrm{Im}\chi[\omega]=\frac{\pi\hbar}{2L^2}\sum_{n,\ell,\ell'}\times\frac{(\ell\pi/L)^2(\ell'\pi/L)^2+[(k_\parallel^n)^2+\omega_n^{\ell}\omega_n^{\ell'}]^2}{(1+\delta_{\ell 0})(1+\delta_{\ell' 0})\omega_n^{\ell}\omega_n^{\ell'}}\times[\delta(\omega-\omega_n^{\ell}-\omega_n^{\ell'})-\delta(\omega+\omega_n^{\ell}+\omega_n^{\ell'})]. \tag{55}$$

Equation (55) provides the result for the dissipative component of the force exerted on the mirror. The term with $n=0$ in Eq. (55) is particularly interesting because it allows for a comparison with the results obtained in Ref. [9] for a 1D scalar field. As discussed in Appendix C, we find that in this case the two polarizations [represented by the two terms in the rhs of Eq. (55)] give identical contributions to the dissipative susceptibility, which are in agreement with the perfectly reflecting limit of the 1D susceptibility function derived in Ref. [9].

As mentioned above, $\mathrm{Im}\chi[\omega]$ is directly related to the number of radiated photons by energy conservation. Indeed, comparing Eqs. (31) and (32) with Eq. (55), we find

$$\sum_{n,\ell}\hbar\omega_n^{\ell}(N_{n,\ell}^{(\mathrm{TE})}+N_{n,\ell}^{(\mathrm{TM})})=\int\frac{d\omega}{2\pi}\omega(\mathrm{Im}\chi[\omega])|\delta q[\omega]|^2. \tag{56}$$

Equation (56) shows that the energy supplied to the field by the radiation pressure force $\delta F[\omega]$, given by its rhs, is equal to the total radiated energy. In the next section, we discuss in detail the properties of the radiation by taking the specific example of sinusoidal motion.

IV. PHOTON PRODUCTION RATES

In this section, we discuss in some detail the properties of the radiation emitted inside the cavity, starting from the expressions for the two-photon probabilities given by Eqs. (48) and (49), which were shown to agree with the results for the photon numbers $N_{n,\ell}$ obtained directly from the moving boundary conditions and given by Eqs. (31) and (32). We assume that the second cavity mirror is at rest at $x=L$ (hence $\delta q_2=0$), and that the first mirror oscillates around $x=0$ according to the law

$$\delta q(t)=\delta q_0 e^{-|t|/\Delta t}\cos(\omega_0 t), \tag{57}$$

where the amplitude δq_0 and frequency ω_0 satisfy the nonrelativistic condition $\omega_0\delta q_0\ll 1$. Moreover, we assume that the damping time Δt is much larger than the period of the mechanical oscillation:

$$\omega_0\Delta t\gg 1.$$

We first consider the 1D limit of the results found in Secs. II and III, by picking up the photon pairs with $n=0$. References [11] and [12] presented a 1D nonperturbative treatment for the situation where the mechanical frequency ω_0 satisfies the resonance condition

$$\omega_0=\frac{\pi(\ell+\ell')}{L} \tag{58}$$

for two longitudinal cavity modes ℓ and ℓ' (Ref. [12] considered the particular case $\ell=\ell'=1$, whereas Ref. [11] also considered the case $\ell=2,\ \ell'=1$). We may discuss the relation between such formalism and the one presented in this paper by taking the Fourier transform of Eq. (57) and computing the two-photon probabilities in the resonant case (we omit explicit reference to polarization while discussing the 1D limit). As shown in Appendix C, we find

$$|c_{\{0\ell,0\ell'\}}|^2=\frac{\pi^2\ell\ell'}{(1+\delta_{\ell\ell'})L^2}(\delta q_0)^2\Delta t^2. \tag{59}$$

According to Eqs. (51) and (59), the number of photons grows quadratically in time in this case. The same time dependence may be obtained as the short time limit of the 1D nonperturbative results found in Refs. [11] and [12]. Such behavior is related to the property that the spectrum of a 1D perfect cavity is discrete, and it was also obtained in the model of a 3D perfect closed cavity system discussed in Ref. [12]. In the case of a continuous spectrum, on the other hand, the emission probabilities grow linearly in time as long as the perturbative approximation is valid, which is well known from the derivation of Fermi's "golden rule," so that in the end the meaningful physical quantities are the photon production *rates*, as we show below. That is the case of a partially transmitting cavity, even in the 1D approximation (see Ref. [2]), as well as of a 3D open cavity, as, for instance, the two parallel infinite plates considered in this paper, even under the assumption (considered in this paper) of perfect reflectiveness.

In the 3D case, we have to sum over all possible values of $\mathbf{k}_{\parallel n}$ in order to compute the probability $\delta\mathcal{P}^{j}_{\ell_1,\ell_2}$ for emission of a pair of photons with indices ℓ_1 and ℓ_2 and polarization j. Since $\mathbf{k}_{\parallel n}$ is actually a continuous variable, we replace

$$\sum_n=\frac{S}{(2\pi)^2}\int d^2k_\parallel .$$

The probabilities do not depend on the direction of $\mathbf{k}_{\parallel n}$, hence we find, first for TE polarization,

$$\delta\mathcal{P}^{\mathrm{TE}}_{\ell_1,\ell_2}=\frac{S}{2\pi}\int_0^\infty d\omega\ \omega|c_{\{\ell_1\,\mathrm{TE},\ell_2\mathrm{TE}\}}(\omega)|^2, \tag{60}$$

where $|c_{\{\ell_1\mathrm{TE},\ell_2\mathrm{TE}\}}(\omega)|^2$ is obtained from Eq. (48):

$$|c_{\{\ell_1\mathrm{TE},\ell_2\mathrm{TE}\}}(\omega)|^2=\frac{1}{L^2}\left(\frac{\ell_1\pi}{L}\right)^2\left(\frac{\ell_2\pi}{L}\right)^2\frac{|\delta q[\omega+\tilde{\omega}_{\ell_1\ell_2}]|^2}{\omega\tilde{\omega}_{\ell_1\ell_2}},\tag{61}$$

where

$$\tilde{\omega}_{\ell_1\ell_2}=\sqrt{\omega^2-\left(\frac{\ell_1\pi}{L}\right)^2+\left(\frac{\ell_2\pi}{L}\right)^2}$$

represents the frequency of the ''twin'' photon of index ℓ_2 and which is emitted simultaneously with the photon of frequency ω and index ℓ_1. We perform the integral in Eq. (60) in the limit of very large Δt, so that $\delta q[\omega]$ is sharply peaked around ω_0. In this case, each pair ℓ_1,ℓ_2 determines completely the frequencies ω_1 and ω_2 of the two photons. Moreover, it also implies well-defined values for the angles between the direction of emission and the x direction, which we denote as θ_1 and θ_2. In fact, we have

$$\omega_1+\omega_2=\omega_0,\tag{62}$$

as in the problem of parametric amplification by a $\chi^{(2)}$ nonlinear medium;

$$\omega_1\sin\theta_1=\omega_2\sin\theta_2\tag{63}$$

expresses the plane symmetry of the cavity, and is loosely analogous to the phase matching condition in nonlinear optics. Finally we have two additional equations, which result from the boundary conditions on the two cavity mirrors:

$$\omega_i\cos\theta_i=\frac{\ell_i\pi}{L},\tag{64}$$

with $i=1,2$. Equations (62)–(64) may be solved for ω_1, ω_2, θ_1, and θ_2 as functions of ω_0, ℓ_1, and ℓ_2. We find

$$\omega_1=\frac{\omega_0}{2}\left(1+\frac{\ell_1^2-\ell_2^2}{\beta^2}\right),\tag{65}$$

where

$$\beta=\omega_0 L/\pi$$

is the ratio between the cavity round-trip time of flight and the mechanical period. Accordingly, the spectrum of photon emission, which is continuous in the case of a single moving mirror [18], becomes discrete as a consequence of the two additional conditions, given by Eq. (64), and which are associated to the presence of the second mirror that constitute the cavity. For a given value of β, the set of emitted frequencies is obtained from Eq. (65) by taking all positive integer values of ℓ_1 and ℓ_2 in the range defined by

$$\ell_1+\ell_2\leq\beta.\tag{66}$$

In the case of TM polarization, the values $\ell_1=0$ and $\ell_2=0$ are also allowed — they correspond to traveling wave modes propagating parallel to the plane of the mirror (waveguide modes). As for the spatial direction of emission, the photons are emitted along directions defined by a set of cones (whose axis of symmetry is the x direction), each pair ℓ_1, ℓ_2 defining allowed values for θ_1 and θ_2 according to Eqs. (62)–(64). As an example, consider the value $\beta=2\sqrt{2}$. Since $\beta<3$, the only allowed values for TE polarization are $\ell_1=\ell_2=1$, corresponding to a pair with $\omega_1=\omega_2=\omega_0/2$, and $\theta_1=\theta_2=45°$, which is, however, not emitted in the case of rigid motion of the cavity. For TM polarization, on the other hand, there are two additional pairs: one with $\ell_1=1$, $\ell_2=0$ (rigid motion), giving $\omega_1=9\omega_0/16$, $\omega_2=7\omega_0/16$, $\theta_1\approx 51°$, and $\theta_2=90°$; the other with $\ell_1=\ell_2=0$ (elongation motion), giving $\omega_1=\omega_2=\omega_0/2$, and $\theta_1=\theta_2=90°$.

We compute the photon production rate for emission at a given pair of allowed frequencies assuming that the integrand in Eq. (60) is the product of a slowly varying function of ω with the sharply peaked squared Fourier transform of $\delta q(t)$. This amounts to replacing the latter by a delta function, so that from Eq. (57) we derive

$$|\delta q[\omega+\tilde{\omega}_{\ell_1\ell_2}]|^2=\frac{\pi}{2}(\delta q_0)^2\Delta t\frac{\omega_0-\omega_1}{\omega_0}\delta(\omega-\omega_1),\tag{67}$$

and noting that $\omega=\omega_1$ implies $\tilde{\omega}_{\ell_1\ell_2}=\omega_2$, we find the photon production rate of TE pairs with indices ℓ_1,ℓ_2 by replacing Eq. (67) into Eq. (61) and performing the integral in Eq. (60):

$$W^{(\mathrm{TE})}_{\ell_1,\ell_2}=\frac{\delta\mathcal{P}^{(\mathrm{TE})}_{\ell_1,\ell_2}}{\Delta t}=\frac{S}{4L^2}\left(\frac{\ell_1\pi}{L}\right)^2\left(\frac{\ell_2\pi}{L}\right)^2\frac{(\delta q_0)^2}{\omega_0}.\tag{68}$$

Note that the linear time dependence found for the probability $\delta\mathcal{P}^{(\mathrm{TE})}_{\ell_1,\ell_2}$ originates from integrating over the whole width of $|\delta q[\omega]|^2$, instead of taking just the peak value as in the derivation of the 1D result given by Eq. (59). For TM photons, we find, starting from Eq. (49) and following the same method,

$$W^{(\mathrm{TM})}_{\ell_1,\ell_2}=\frac{S}{4L^2}\frac{[\omega_0\omega_1-(\ell_1\pi/L)^2]^2}{(1+\delta_{\ell_1 0})(1+\delta_{\ell_2 0})}\frac{(\delta q_0)^2}{\omega_0}.\tag{69}$$

We may also derive the total production rate for a given value of β by adding over all values of ℓ_1 and ℓ_2 in the range defined by Eq. (66):

$$W^j=\sum_{\ell_1,\ell_2}^{\ell_1+\ell_2\leq\beta}W^j_{\ell_1,\ell_2},\tag{70}$$

with $j=\mathrm{TE,TM}$. In the Fig. 1 we plot $W^{(\mathrm{TM})}$ and $W^{(\mathrm{TE})}$, both divided by the total production rate of TE photons in the case of a single moving mirror (see Ref. [18]),

$$W^{(\mathrm{TE})}_{\mathrm{single}}=\frac{1}{720\pi^2}S(\delta q_0)^2\omega_0^5,\tag{71}$$

as functions of β. The curves displayed in the figure for TE and TM polarizations are similar to those representing the decay rate of a classical dipole at the midpoint between two perfect plane mirrors along the direction parallel and perpendicular to the mirrors, respectively. Underlying both effects

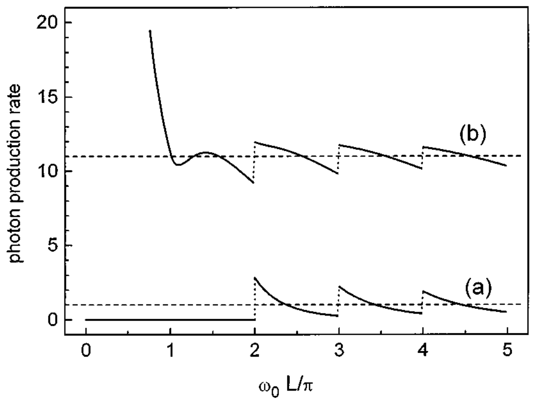

FIG. 1. Total production rates of TE (a) and TM (b) photons as functions of $\beta=\omega_0 L/\pi$, which represents the ratio between the round-trip time of flight and the mechanical period. The scale of the vertical axis is such that the value one corresponds to the generation rate of TE photons for a single moving mirror. The dashed lines provide the asymptotic limits for large β. They show that the photon production rates approach the values corresponding to the case of a single moving mirror in this limit (note that the single mirror TM photon generation rate is larger than the single mirror TE rate by a factor of 11).

are the properties of the vacuum field in the case of a plane cavity geometry and the corresponding mode spectral density function [20]. The most striking differences between the two problems are related to the two-photon nature of the quantum radiation effect considered in this paper (that explains, for instance, why, as displayed in the figure, the TE photon production rate vanishes for $\beta<2$, whereas the parallel dipole decay rate vanishes for $\beta<1$ only).

As in the problem of a decaying dipole, the photon production rates jump at integer values of β. This originates from adding the contribution of a new pair ℓ_1, ℓ_2 within the range defined by Eq. (66). The jumps for TE polarization are comparatively larger, which may be understood from the fact, discussed in detail in Ref. [18] in the case of a single mirror, that TE photons are preferably emitted near the x direction, thus being more sensitive to the discrete nature of the wave vector along that direction. For both polarizations, the jumps become smaller as β increases, and then the curves approach their asymptotic values for $\beta\to\infty$, which are indicated by the dashed lines in the figure. As expected, they correspond to the photon production rates for a single moving mirror — the rate for TM polarization being 11 times larger than the rate for TE polarization, given by Eq. (71). Alternatively, the asymptotic limits may be derived directly from the analytical results given by Eqs. (68) and (69) if we replace the sum in Eq. (70) by an integral. In fact, performing the integral in the case of TE polarization leads to the expression given by Eq. (71), whereas the result for TM polarization comes with an extra factor of 11.

Of special interest is the behavior of the TM photon production rate in the range $0<\beta<1$, where, according to the figure, $W^{(\mathrm{TM})}$ increases strongly as β decreases to zero. The precise dependence on β may be obtained by replacing $\omega_1 = \omega_2 = \omega_0/2$ in Eqs. (69) and (70) and comparing with Eq. (71):

$$W^{(\mathrm{TM})} = W^{(\mathrm{TM})}_{0,0} = \frac{45}{4}\,\frac{W^{(\mathrm{TE})}_{\mathrm{single}}}{\beta^2}. \tag{72}$$

Such dependence with β suggests that the most favorable orders of magnitude occur for $\beta<1$. In this range, the photons have frequency $\omega_0/2$ and propagate along directions parallel to the mirrors. Following Refs. [2] and [18] we rewrite the photon production rate given by Eq. (72) as

$$W^{(\mathrm{TM})} = \frac{1}{16}\,\frac{S}{\lambda_0^2}\left(\frac{v_{\mathrm{max}}}{c}\right)^2 \frac{\omega_0}{\beta^2}, \tag{73}$$

where $v_{\mathrm{max}} = \omega_0 \delta q_0$ is the maximum value of the velocity, and $\lambda_0 = 2\pi c/\omega_0$ is half the value of the wavelength of the emitted photons (we have reintroduced the speed of light c). As in Ref. [2], we take $v_{\mathrm{max}}/c = 10^{-7}$ and $\omega_0 = 2\pi \times 10^{10}\ \mathrm{sec}^{-1}$, yielding $\lambda_0 = 3$ cm. A real experiment would hardly employ moving mirrors with transverse dimensions larger than that, thus we take $S/\lambda_0^2 \approx 1$ in order to have a crude estimate of the orders of magnitude, even though diffraction effects at the borders of the mirrors, not taken into account in this paper, are of course relevant in this range. Finally, we take $L=1\ \mu$m, giving $\beta\approx 10^{-4}$. Equation (73) then yields $W^{(\mathrm{TM})} \approx 4\times 10^3$ photons/sec.

V. CONCLUSION

We have calculated the photon production rates for a plane cavity with moving mirrors by two different methods. In the first approach, we consider the boundary conditions for perfectly reflecting moving mirrors in the long-wavelength approximation and assuming the field modification due to the motion to be small. We then obtain an input-output transformation for the field bosonic operators that allows us to compute the number of emitted photons. In the second approach, we start from an effective perturbative Hamiltonian and apply usual first-order perturbation theory. This method is considerably simpler since the expressions for the fields scattered by a moving mirror are not required, and establishes a clear connection between the photon emission effect and vacuum radiation pressure. Furthermore, it explicitly unveils the fact that the photons are emitted in pairs (that satisfy simple properties expressing the symmetry of the plane geometry), essentially because the effect is contained in the time evolution of the field state vectors rather than in the evolution of the field operators. The two methods provide the same results for the photon production rates, hence justifying the somewhat heuristic Hamiltonian approach.

Radiation is generated even when the distance between the mirrors is kept constant, showing that the effect is not simply a consequence of modulating the optical cavity length. When the initial cavity length L is much smaller than $2\pi c/\omega_0$ (we have considered in detail the example of a quasisinusoidal motion at frequency ω_0), however, radiation is emitted only in the case of relative motion of the mirrors, and the generation rate is enhanced as L decreases. In this regime, the photons are generated at the subharmonic frequency $\omega_0/2$, propagate parallel to the plane of the mirror, and are TM polarized. Such an enhancement effect is closely related to the properties of the radiation emitted by a single mirror in free space [18], whose spectrum for TM polarization is sharply peaked around the frequency $\omega_0/2$.

The orders of magnitude for the photon production rate found in this paper suggest that the motion-induced quantum radiation effect may be observed under certain conditions. However, a careful analysis of the diffraction effects near the border of the mirror would be necessary if a quantitative comparison with experimental results is required, since the field wavelengths involved would probably be of the order of the transverse dimensions of the mirrors.

ACKNOWLEDGMENTS

This work was supported by the Comisión Nacional de Investigación Científica y Tecnológica, Proyecto FONDECYT No. 2960009, and by Conselho Nacional de Desenvolvimento Científico e Tecnológico.

APPENDIX A: BOUNDARY CONDITIONS FOR A PERFECT MOVING MIRROR

In this Appendix, we derive the boundary conditions in the case of a perfect plane mirror moving along its normal direction. We take a Lorentz frame $S'(t_0)$ whose trajectory in the laboratory frame S is given by $x=\delta\dot{q}(t_0)(t-t_0)+\delta q(t_0)$, so that $S'(t_0)$ represents the instantaneously comoving frame at time t_0. Quantities measured in $S'(t_0)$ are denoted by primed letters. The space-time coordinates in $S'(t_0)$ are related to those in S by

$$x=\gamma(x'+\delta\dot{q}(t_0)t')+\delta q(t_0),\quad \mathbf{r}'_\parallel=\mathbf{r}_\parallel,$$
$$t=\gamma(t'+\delta\dot{q}(t_0)x')+t_0, \tag{A1}$$

where $\gamma=[1-[\delta\dot{q}(t_0)]^2]^{-1/2}$. The electromagnetic fields $\mathbf{E}'$ and $\mathbf{B}'$ satisfy the following conditions:

$$\hat{\mathbf{x}}\times\mathbf{E}'(x'=0,\mathbf{r}'_\parallel,t'=0)=\mathbf{0};\quad \hat{\mathbf{x}}\cdot\mathbf{B}'(x'=0,\mathbf{r}'_\parallel,t'=0)=0. \tag{A2}$$

In the case of TE polarization, the condition for the electric field yields

$$\partial_{t'}\mathbf{A}^{(\mathrm{TE})\prime}(x'=0,\mathbf{r}'_\parallel,t'=0)=\mathbf{0}, \tag{A3}$$

and since $\hat{x}\cdot\mathbf{A}^{(\mathrm{TE})\prime}=0$, we have from Eqs. (A1) and (A3)

$$\gamma(\delta\dot{q}(t_0)\partial_x+\partial_t)\mathbf{A}^{(\mathrm{TE})}(x=\delta q(t_0),\mathbf{r}_\parallel,t=t_0)$$
$$=\gamma d_t\mathbf{A}^{(\mathrm{TE})}(x=\delta q(t=t_0),\mathbf{r}_\parallel,t=t_0)=\mathbf{0} \tag{A4}$$

where d_t represents the total time derivative. Since t_0 is arbitrary, Eq. (A4) implies that $\mathbf{A}^{(\mathrm{TE})}(x=\delta q(t),\mathbf{r}_\parallel,t)$ must assume a constant value, which is taken to be zero as in Eq. (4).

As for TM polarization, the condition on the electric field given by Eq. (A2) jointly with Eq. (2) yield

$$\partial_{x'}\boldsymbol{\mathcal{A}}^{(\mathrm{TM})\prime}(x'=0,\mathbf{r}'_\parallel,t'=0)=\mathbf{0}. \tag{A5}$$

On the other hand, we may write the lhs of Eq. (A5) in terms of unprimed quantities by using again Eq. (A1) and the fact that $\hat{x}\cdot\boldsymbol{\mathcal{A}}^{(\mathrm{TM})\prime}=0$:

$$\partial_{x'}\boldsymbol{\mathcal{A}}^{(\mathrm{TM})\prime}(x'=0,\mathbf{r}'_\parallel,t'=0)$$
$$=\gamma(\partial_x+\delta\dot{q}(t_0)\partial_t)\boldsymbol{\mathcal{A}}^{(\mathrm{TM})}(x=\delta q(t_0),\mathbf{r}_\parallel,t=t_0), \tag{A6}$$

and then we obtain the boundary condition as given by Eq. (5) from Eqs. (A5) and (A6).

APPENDIX B: TWO MOVING MIRRORS

In this Appendix, we consider the more general case where both mirrors move along the x direction. The first mirror is at $x=\delta q_1(t)$, whereas the second one is at $x=L+\delta q_2(t)$. As before, L represents the initial cavity length. For TE polarization, the boundary condition at the second mirror now reads

$$\mathbf{A}^{(\mathrm{TE})}(L+\delta q_2(t),\mathbf{r}_\parallel,t)=\mathbf{0}, \tag{B1}$$

which yields, in the long-wavelength and perturbative approximations, the following additional boundary condition for the motion-induced perturbation $\boldsymbol{\delta}\mathbf{A}^{(\mathrm{TE})}$:

$$\boldsymbol{\delta}\mathbf{A}^{(\mathrm{TE})}(L,\mathbf{r}_\parallel,t)=-\delta q_2(t)\partial_x\mathbf{A}^{(\mathrm{TE})}_{\mathrm{sta}}(x=L,\mathbf{r}_\parallel,t). \tag{B2}$$

Working in the mixed reciprocal space and using the normal mode decomposition of $\mathbf{A}^{(\mathrm{TE})}_{\mathrm{sta}}$ as given by Eq. (12), Eq. (B2) leads to

$$\boldsymbol{\delta}\mathbf{A}^{(\mathrm{TE})}_n(L,\omega)=-i\sum_{\ell=1}^{\infty}(-1)^{\ell}\left(\frac{\ell\pi}{L}\right)\sqrt{\frac{\hbar}{\omega_n^{\ell}SL}}$$
$$\times(\delta q_1[\omega-\omega_n^{\ell}]a^{(\mathrm{TE})}_{n\ell}+\delta q_2[\omega+\omega_n^{\ell}]$$
$$\times(a^{(\mathrm{TE})}_{-n\ell})^{\dagger})\hat{\boldsymbol{\epsilon}}_n. \tag{B3}$$

Of particular interest in Eq. (B3) is the factor $\cos(\ell\pi)=(-1)^{\ell}$ that comes from evaluating the x derivative of $\mathbf{A}^{(\mathrm{TE})}_{\mathrm{sta}}$ at $x=L$. Equation (B3) jointly with Eq. (10) defines a boundary value problem to be solved with the aid of the Green functions given by Eq. (22). We first employ the retarded Green function $G^{D,R}_{n\ \omega}(x,x')$ to solve for the total field $\mathbf{A}^{(\mathrm{TE})}_n$ in terms of the input field $\mathbf{A}^{(\mathrm{TE})}_{\mathrm{in},\ n}$:

$$\mathbf{A}^{(\mathrm{TE})}_n(x,\omega)=\mathbf{A}^{(\mathrm{TE})}_{\mathrm{in},n}(x,\omega)$$
$$+\boldsymbol{\delta}\mathbf{A}^{(\mathrm{TE})}_n(L,\omega)\partial_{x'}G^{D,R}_n(x'=L,x;\omega)$$
$$-\boldsymbol{\delta}\mathbf{A}^{(\mathrm{TE})}_n(0,\omega)\partial_{x'}G^{D,R}_n(x'=0,x;\omega). \tag{B4}$$

As in Sec. II, we also solve Eq. (B3) in terms of the output field $\mathbf{A}^{(\mathrm{TE})}_{\mathrm{out},\ n}$ with the aid of the advanced Green function $G^{D,A}_{n\ \omega}(x,x')$. The connection between output and input fields is then provided by the difference

$$\partial_{x'} G^{D,R}_{n\,\omega}(x'=L,x) - \partial_{x'} G^{D,A}_{n\,\omega}(x'=L,x) = -\frac{2\pi i}{L}\sum_{\ell=1}^{\infty}(-1)^{\ell}\left(\frac{\ell\pi}{L}\right)\sin\left(\frac{\ell\pi x}{L}\right)\frac{1}{\omega_n^{\ell}} \times[\delta(\omega-\omega_n^{\ell})-\delta(\omega+\omega_n^{\ell})]. \quad \text{(B5)}$$

As explained in Sec. II, we derive the linear transformation between output and input TE bosonic operators from Eqs. (B3)–(B5)

$$a^{(\mathrm{TE})}_{\mathrm{out},n\ell} = a^{(\mathrm{TE})}_{\mathrm{in},n\ell} - \frac{i}{L}\sum_{\ell'=1}^{\infty}\left(\frac{\ell\pi}{L}\right)\left(\frac{\ell'\pi}{L}\right)\frac{1}{(\omega_n^{\ell}\omega_n^{\ell'})^{1/2}} \times\{(\delta q_1[\omega_n^{\ell}-\omega_n^{\ell'}]-(-1)^{\ell+\ell'}\delta q_2[\omega_n^{\ell}-\omega_n^{\ell'}]) \times a^{(\mathrm{TE})}_{\mathrm{in},n\ell'} + (\delta q_1[\omega_n^{\ell}+\omega_n^{\ell'}] -(-1)^{\ell+\ell'}\delta q_2[\omega_n^{\ell}+\omega_n^{\ell'}])(a^{(\mathrm{TE})}_{\mathrm{in},-n\ell'})^{\dagger}\}. \quad \text{(B6)}$$

From Eq. (B6) we may calculate the number of photons $N^{(\mathrm{TE})}_{n,\ell}$ by taking the average of the output number operator over the input vacuum state as in Eq. (30). For TM polarization, we extend the method employed in Sec. II to take into account the motion of the second mirror exactly as discussed above for TE polarization.

Alternatively, we may compute the photon numbers from the effective perturbation Hamiltonian

$$\delta H = -\delta q_1(t)F_1 - \delta q_2(t)F_2, \quad \text{(B7)}$$

where F_i is the force exerted on mirror i by the vacuum field. Following the procedure outlined in Sec. III, we derive the two-photon creation probabilities given by Eqs. (48) and (49). As in the case of a single moving mirror, the results obtained through this method are in full agreement with those obtained directly from the boundary conditions.

APPENDIX C: PHOTONS EMITTED ALONG THE NORMAL DIRECTION

In this Appendix, we consider in detail the contribution of the degenerate two-photon states in the derivation of the photon numbers and of the susceptibility function. First note that degenerate two-photon states necessarily correspond to propagation along the direction perpendicular to the plane of the mirror, i.e., they are of the form $|n=0\ell j,n=0\ell j\rangle$. The degenerate two-photon matrix elements of the force operator are calculated from the representation of the force operator in terms of the vector potentials, given by Eq. (42), and from the normal mode decompositions given by Eqs. (12) and (13):

$$\langle n=0\ell\mathrm{TE},n=0\ell\;\mathrm{TE}|F|0\rangle = -\langle n=0\ell\mathrm{TM},n=0\ell\mathrm{TM}|F|0\rangle = -\frac{\ell\pi\hbar}{\sqrt{2}L^2}. \quad \text{(C1)}$$

These results are smaller than the values of the expressions given by Eqs. (46) and (47) at $n=0$ and $\ell=\ell'$ by a factor of $\sqrt{2}$. From them, we easily compute the degenerate two-photon probabilities by using Eq. (40), allowing us to write the correct expression for $n=0$:

$$|c_{\{0\ell\mathrm{TE},0\ell'\mathrm{TE}\}}|^2 = |c_{\{0\ell\mathrm{TM},0\ell'\mathrm{TM}\}}|^2 = \frac{1}{(1+\delta_{\ell\ell'})L^2}\left(\frac{\ell\pi}{L}\right) \times\left(\frac{\ell'\pi}{L}\right)\left|\delta q\left[\frac{\ell\pi}{L}+\frac{\ell'\pi}{L}\right]\right|^2. \quad \text{(C2)}$$

According to Eq. (C2), the degenerate two-photon probabilities are one-half the value found when replacing the values $n=0$, $\ell=\ell'$, and $\delta q_2=0$ in Eqs. (48) and (49).

The contribution of degenerate two-photon states is found from Eq. (50):

$$N^{j}_{0,\ell} = \sum_{\ell',\ell'\neq\ell}|c_{\{0\ell j,0\ell' j\}}|^2 + 2|c_{\{0\ell j,0\ell j\}}|^2. \quad \text{(C3)}$$

The factor two multiplying the degenerate two-photon amplitude in the rhs of Eq. (C3) cancels the additional factor one-half appearing in Eq. (C2) for $\ell=\ell'$, then yielding a result in full agreement with Eqs. (31) and (32). For the specific example of motion given by Eq. (57), and assuming that the mechanical frequency ω_0 satisfies the resonant condition as given by Eq. (58), we derive from Eq. (C2) the expression for the production rate of photons with $n=0$ given by Eq. (59)

Since the results for the photon numbers are not modified when taking into account the degenerate two-photon states, we expect that the formula for the dissipative component of the susceptibility function, given by Eq. (55), should also be valid for $n=0$, so as to preserve the connection between dissipation and total radiated displayed by Eq. (56). In fact, we may write separately the contribution of degenerate two-photon states to the sum over pairs $\{n\ell,n'\ell'\}$ in Eq. (54):

$$C_{FF}[\omega] = \pi\sum_j \sum_{n,\ell,\ell'}^{*}\delta(\omega-\omega_n^{\ell}-\omega_n^{\ell'})|\langle n\ell j,n\ell' j|F|0\rangle|^2 + 2\pi\sum_j\sum_{\ell}\delta(\omega-2\ell\pi/L)|\langle 0\ell j,0\ell j|F|0\rangle|^2, \quad \text{(C4)}$$

where $\Sigma^{*}_{n,\ell,\ell'}$ represents the sum over all possible values of n, ℓ, and ℓ' excluding those where simultaneously $n=0$ and $\ell=\ell'$. As before, the factor one-half found for the degenerate two-photon matrix element [given by Eq. (C2)] is canceled by the factor of two appearing in the rhs of Eq. (C4). Hence we may write the expression for the 1D dissipative susceptibility function, $\mathrm{Im}\chi_{1D}[\omega]$, by selecting directly from Eq. (55) the terms with $n=0$:

$$\mathrm{Im}\chi_{1D}[\omega]=\frac{\pi^3\hbar}{L^4}\sum_{\ell=1}^{\infty}\sum_{\ell'=1}^{\infty}\ell\ell'[\delta(\omega-(\ell+\ell')\pi/L)-\delta(\omega+(\ell+\ell')\pi/L)]. \tag{C5}$$

As for Ref. [9], the result for the (complete) susceptibility function in the perfectly reflecting limit and in the particular case where only one mirror moves reads

$$\widetilde{\chi}[\omega]=\frac{\hbar}{6\pi}\left[\frac{i\omega^3}{1-e^{2i\omega L}}+\left(\frac{\pi}{L}\right)^2(i\omega)\left(\frac{1}{2}-\frac{1}{1-e^{2i\omega L}}\right)\right]. \tag{C6}$$

In order to compare Eqs. (C5) and (C6), we must take the imaginary part of $\widetilde{\chi}[\omega]$, then yielding, after some algebra,

$$\mathrm{Im}\widetilde{\chi}[\omega]=\frac{\hbar}{6\pi}\left[\frac{\omega^3}{2}+\frac{1}{2}\left(\frac{\pi}{L}\right)^4\sum_{n=-\infty}^{\infty}n(n^2-1)\delta(\omega-n\pi/L)\right]. \tag{C7}$$

The first term in the rhs of Eq. (C7) represents the contribution of the field outside the cavity, being equal to half the value found for a two-sided single mirror in one-dimensional vacuum [4]. The second term, on the other hand, represents the contribution of the intracavity field, which is, by inspection of Eqs. (C5) and (C7), equal to half the value found from taking the 1D limit in Eq. (55), the factor of two being related to the two polarizations taken into account in the electromagnetic case.

[1] J. Schwinger, Proc. Natl. Acad. Sci. USA **90**, 958 (1993); **90**, 2105 (1993); **90**, 4505 (1993); **90**, 7285 (1993); **91**, 6473 (1994).

[2] A. Lambrecht, M.-T. Jaekel, and S. Reynaud, Phys. Rev. Lett. **77**, 615 (1996).

[3] G. Barton and C. Eberlein, Ann. Phys. (N.Y.) **227**, 222 (1993).

[4] L. H. Ford and A. Vilenkin, Phys. Rev. D **25**, 2569 (1982); P. A. Maia Neto and L. A. S. Machado, Brazilian J. Phys. **25**, 324 (1995).

[5] P. A. Maia Neto, J. Phys. A **27**, 2167 (1994).

[6] M. T. Jaekel and S. Reynaud, Quantum Opt. **4**, 39 (1992).

[7] V. B. Braginsky and F. Ya. Khalili, Phys. Lett. A **161**, 197 (1991); G. Barton, *New aspects of the Casimir effect: fluctuations and radiative reaction*, in *Cavity Quantum Electrodynamics*, Supplement: Advances in Atomic, Molecular and Optical Physics, edited by P. Berman (Academic Press, New York, 1993).

[8] P. A. Maia Neto and S. Reynaud, Phys. Rev. A **47**, 1639 (1993).

[9] M. T. Jaekel and S. Reynaud, J. Phys. I (France) **2**, 149 (1992).

[10] G. T. Moore, J. Math. Phys. **11**, 2679 (1970).

[11] C. K. Law, Phys. Rev. A **49**, 433 (1994).

[12] V. V. Dodonov and A. B. Klimov, Phys. Rev. A **53**, 2664 (1996).

[13] G. Barton and C. A. North, Ann. Phys. (N.Y.) **252**, 72 (1996).

[14] C. A. North, Sussex report.

[15] G. Barton, Ann. Phys. (N.Y.) **245**, 361 (1996).

[16] C. Eberlein, Phys. Rev. A **53**, 2772 (1996); Phys. Rev. Lett. **76**, 3842 (1996).

[17] Z. Bialynicka-Birula and I. Bialynicka-Birula, J. Opt. Soc. Am. B **4**, 1621 (1987).

[18] P. A. Maia Neto and L. A. S. Machado, Phys. Rev. A **54**, 3420 (1996).

[19] R. Kubo, Rep. Prog. Phys. **29**, 255 (1966).

[20] S. Haroche, in *Fundamental Systems in Quantum Optics*, Les Houches Summer School, Session LIII, edited by J. Dalibard, J.-M. Raymond, and J. Zinn-Justin (North-Holland, Amsterdam, 1992).