Classical underpinnings of gravitationally induced quantum interference

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We show that the gravitational modification of the phase of a neutron beam [the Colella-Overhauser-Werner (COW) experiment] has a classical origin, being due to the time delay that classical particles experience in traversing a background gravitational field. Similarly, we show that classical light waves also undergo a phase shift in traversing a gravitational field. We show that the COW experiment respects the equivalence principle even in the presence of quantum mechanics. [S1050-2947(98)03802-5]

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In a landmark series of experiments [1,2] Colella, Overhauser, and Werner and subsequent workers (see, e.g., [3–5] for overviews) detected the modification of the phase of a neutron beam as it traverses the Earth's gravitational field, to thus realize an experiment that involved both quantum mechanics and gravity. A typical generic experimental setup is shown in the schematic Fig. 1, in which a neutron beam from a reactor is Bragg split at point A into a horizontal beam AB and a vertical beam AC (we take the Bragg angle to be 45° for illustrative simplicity in the following), with the subsequent scatterings at B and C then producing beams that interfere at D, after which they are then detected. If the neutrons arrive at A with velocity v_0 ($v_0 \sim 2.8\times10^5$ cm sec^{-1} is typical [4]) and $ABCD$ is a square of side H ($\sim$4.8 cm), then the phase difference $\phi_{COW}=\phi_{ACD}-\phi_{ABD}$ is given by $-mgH^2/\hbar v_0$ to lowest order in the acceleration g due to gravity [1] and is actually observable despite the weakness of gravity, since even though $\int \overline{p}\ d\overline{r}$ differs only by the very small amount $m(v_{CD}-v_{AB})H=-mgH^2/v_0$ between the CD and AB paths, nonetheless this quantity is not small compared to Planck's constant, to thus give an observable fringe shift ($\sim$56.5 rad [4]) even for H as small as a few centimeters.

The detected Colella-Overhauser-Werner (COW) phase is extremely intriguing for two reasons. First, it shows that it is possible to distinguish between different paths that have common end points, with the explicit global ordering in which the horizontal and vertical sections are traversed leading to observable consequences. Second, it yields an answer that explicitly depends on the mass of the neutron even while the classical neutron trajectories (viz., the ones explicitly followed by the centers of the wave packets of the quantum-mechanical neutron beam) of course do not. The COW result thus invites consideration of whether the detected ordering is possibly a topological effect typical of quantum mechanics and of whether quantum mechanics actually respects the equivalence principle. As we shall see, the ordering effect is in fact already present in the motion of classical particles in gravitational fields and even in the propagation of classical waves in the same background, with this latter feature enabling us to establish below that the mass dependence of the neutron beam COW phase is purely kinematic with the equivalence principle then not being affected.

To address these issues specifically we have found it convenient to carefully follow the neutron as it traverses the interferometer, to find that the two beams do not in fact arrive at the same point D or even at the same time, with this spatial offset and time delay not only producing the interference effect, but also being present in the underlying classical theory. Quantum mechanics thus does not cause the time delay; rather it only serves to make it observable. Since gravity is a relativistic theory we shall need to introduce curvature (which we do below), but we have found it more instructive to consider the nonrelativistic limit first. Since we can treat the neutron beams as rays, their motions round the $ABCD$ loop can be treated purely classically between the various scatterings. Moreover, the various scatterings themselves at A, B, C, and D introduce no additional phases, are energy conserving, and give angles of reflection equal to the angles of incidence [6]. Thus the entire motion of the neutron is the same as that of a spinless macroscopic particle that undergoes classical mirror reflections.

A nonrelativistic classical neutron that goes up vertically from A arrives at C with a velocity $(0,v_0-gH/v_0)$. The neutron AC travel time is $t(AC)=(H+\delta)/v_0$ (where $\delta=gH^2/2v_0^2$) and the standard nonrelativistic classical action $S_{CL}=\int(\overline{p}\ d\overline{r}-E_0dt)$ ($E_0=mv_0^2/2$) undergoes a change $S(AC)=mv_0(H-\delta)-E_0t(AC)$. On scattering at C the neutron is then reflected so that it starts off toward D with a velocity $(v_0-gH/v_0,0)$. On its flight it dips slightly to arrive at the next scattering surface at the point D_1 with coordinates $(H-\delta,H-\delta)$, so that there is a change in the end point of the motion that is first order in g and thus relevant to our

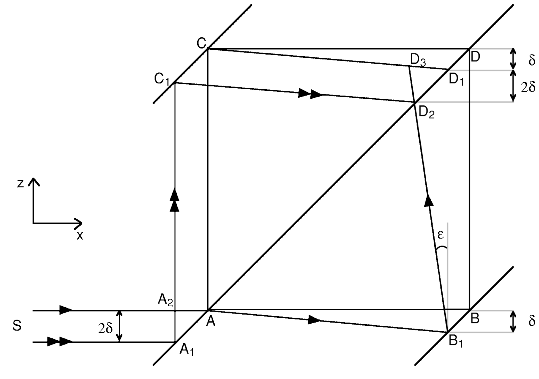

FIG. 1. COW wave paths.

*Electronic address: mannheim@uconnvm.uconn.edu discussion. At D_1 the neutron has a velocity $(v_0 - gH/v_0, -gH/v_0)$, with the CD_1 segment taking a time $t(CD_1) = (H+\delta)/v_0$ and contributing an amount $S(CD_1) = mv_0(H-3\delta) - E_0 t(CD_1)$ to S_{CL}. A classical neutron that starts horizontally from A arrives not at B but at the point B_1 with coordinates $(H-\delta, -\delta)$ and with a velocity $(v_0, -gH/v_0)$. The AB_1 segment takes a time $t(AB_1) = (H-\delta)/v_0$ and the action changes by $S(AB_1) = mv_0(H-\delta) - E_0 t(AB_1)$. After scattering at B_1 the neutron sets off toward D with velocity $(-gH/v_0, v_0)$ and arrives not at D or D_1 but rather at the point D_2 with coordinates $(H-3\delta, H-3\delta)$ and reaches there with velocity $(-gH/v_0, v_0 - gH/v_0)$. The B_1D_2 segment takes a time $t(B_1D_2) = (H-\delta)/v_0$ and the action changes by $S(B_1D_2) = mv_0(H-3\delta) - E_0 t(B_1D_2)$.

As regards the neutron's path around the loop, we see from Fig. 1 that the small vertical dip δ during each of the two horizontal legs causes both of these neutron paths to be an amount δ shorter in the horizontal than they would have been in the absence of gravity, to thus provide a first order in g modification to $\int \overline{p} \, d\overline{r}$ in each of these legs, even while these same vertical dips themselves only contribute to the action in second order. However, for the two horizontal sections, each leg is shortened by the same amount in the horizontal, so that the difference in $\int \overline{p} \, d\overline{r}$ between the CD_1 and AB_1 legs still takes the value $-mgH^2/v_0$ quoted earlier. As regards the two vertical legs, we note that even though the AC leg is completely in the vertical, since the neutron beam starts the B_1D_2 leg with a small horizontal velocity, during this leg the neutron beam changes its horizontal coordinate by an amount 2δ, thereby causing it to reach D_2 after having also traveled a distance 2δ less in the vertical than it would travel in the AC leg. Consequently, there is both a spatial offset $(2\delta, 2\delta)$ between D_1 and D_2, and a time delay $t(ACD_1) - t(AB_1D_2) = 4\delta/v_0$ between the arrival of the two beams, with $\int \overline{p} \, d\overline{r}$ thus not taking the same value in each of the two vertical legs. However, our calculation shows that all these modifications actually compensate in the overall loop with there being no difference in $\int \overline{p} \, d\overline{r}$ between the ACD_1 and AB_1D_2 paths. However, even though there is still a net change in the action $S(ACD_1) - S(AB_1D_2) = -mgH^2/v_0$ because of the net time delay, we cannot identify this quantity with the COW phase $\hbar \Delta\phi_{COW}$ since the beams have not interfered due to the spatial offset between D_1 and D_2.

Before discussing the issue of this spatial offset, it is instructive to ask where the classical neutron paths would have met had there been no third crystal at D to get in the way. Explicit calculation shows that the paths would in fact have met at the asymmetric point D_3 with coordinates $(H-3\delta, H-\delta)$ with the CD_3 and B_1D_3 segments taking times $t(CD_3) = (H-\delta)/v_0$ and $t(B_1D_3) = (H+\delta)/v_0$, respectively, while yielding action changes $S(CD_3) = mv_0(H-5\delta) - E_0 t(CD_3)$ and $S(B_1D_3) = mv_0(H-\delta) - E_0 t(B_1D_3)$. The neutron paths would thus meet at D_3 without any time delay and with $S(ACD_3) - S(AB_1D_3) = -2mgH^2/v_0$. We thus see that for purely classical particles reflecting off mirrors at B_1 and C the quantity $\int \overline{p} \, d\overline{r}$ evaluates differently for the two paths ACD_3 and AB_1D_3.

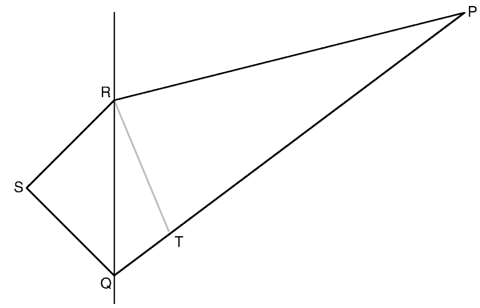

FIG. 2. Double-slit wave paths.

This is thus a global, path-dependent effect in purely classical mechanics in a background classical gravitational field that is completely independent of quantum mechanics [7]. However, since the classical action is not observable in classical mechanics, it is only in the presence of quantum mechanics that phase differences become observable. (In classical mechanics what is observable is that the neutron paths meet at D_3 rather than on the AD axis.)

Returning now to the COW experiment itself, in order to understand the implications of the time and spatial offsets between D_1 and D_2, it is instructive to consider the Young double-slit experiment with purely classical light. As shown in Fig. 2, light from a source S goes through slits Q and R to form an interference pattern at points such as P, with the distance $\Delta x = QT$ representing the difference in path length between the two beams. Given this path difference, the phase difference between the two beams is usually identified as $k\Delta x$, from which an interference pattern is then readily calculated. However, because of this path difference, the SQP ray takes the extra time $\Delta t = \Delta x/c$ to get to P, to thus give a net change in the phase of the SQP beam of $k\Delta x - \omega\Delta t$, which actually vanishes for light rays. The relative phase of the two light rays in the double-slit experiment thus does not change at all as the two beams traverse the interferometer. However, because of the time delay, the SRP beam actually interferes with an SQP beam that had left the source a time Δt earlier. Thus, if the source is coherent over these time scales, the SQP beam carries an additional $+\omega\Delta t$ phase from the very outset. This phase then cancels the $-\omega\Delta t$ phase it acquires during the propagation to P (a cancellation that clearly also occurs for quantum-mechanical matter waves moving with velocities less than the velocity of light), leaving just $k\Delta x$ as the final observable phase difference, a quantity that is nonzero only if there is in fact a time delay. We thus see that the double-slit device itself actually produces no phase change for light. Rather, the choice of point P on the screen is a choice that selects which time delays at the source are relevant at each P, with the interference pattern thus not only involving a time delay at the source, but in fact even requiring one.

With this in mind, we now see that we also need to monitor the time delay of the neutron in the COW experiment. However, since the total energy of the neutron does not change as it goes through the interferometer, the time-delay contribution will still drop out of the final phase-shift expression (explicitly but not implicitly). However, for the COW experiment we noted above that as well as a time delay between the ACD_1 and AB_1D_2 paths, there was also a spatial offset. Consequently, the AB_1D_2 path interferes not with the ACD_1 path, but rather with the indicated offset $A_1C_1D_2$ path, a very close path that in fact is found to lie a distance 2δ vertically below AB, an offset distance that is within the resolution of the beam [8]. The evaluation of the phase shift is then exactly as before with $S(A_1C_1D_2)$ taking the exact same value as $S(ACD_1)$ to lowest order in g. Now we noted above that all the $\int \overline{p}\, d\overline{r}$ contributions actually cancel for this particular set of paths. However, because of the spatial offset between D_1 and D_2, the AB_1D_2 path beam has to travel an extra horizontal distance $A_2A=2\delta$ to first get to the interferometer (to therefore provide an analog to the distance $\Delta x = QT$ in the double-slit experiment, with A_1 and A_2 acting just like the pair of slits Q and R). Now in traveling this extra A_2A distance this beam actually acquires yet another time delay $t(A_2A)$, to therefore impose yet another relative phase condition at the source, which then identically cancels the associated $-E_0t(A_2A)$ change in the action. Moreover, in traveling this extra A_2A the integral $\int \overline{p}\, d\overline{r}$ acquires yet one more contribution mgH^2/v_0, and this term then emerges as the only contribution in the entire circuit that is not canceled, to thus yield $\Delta\phi_{COW} = -mgH^2/\hbar v_0$ as the final observable COW phase shift.

Turning now to a fully covariant analysis [9], we need to look at solutions to the Klein-Gordon equation $\phi^{\mu}{}_{;\mu} - (mc/\hbar)^2\phi = 0$ (ϕ^{μ} denotes $\partial\phi/\partial x_{\mu}$) in the background field of the Earth, viz., $d\tau^2 = B(r)c^2dt^2 - dr^2/B(r) - r^2d\Omega$, where $B(r) = 1 - 2MG/c^2r$. First we note that the nonrelativistic reduction of this Klein-Gordon equation is straightforward, with the substitution $\phi = \exp(-imc^2t/\hbar)\psi$ then yielding $i\hbar\partial\psi/\partial t + (\hbar^2/2m)\nabla^2\psi = mc^2[B(r)-1]/2 = -mMG/r$ for slowly moving particles. We thus see that the inertial mass m that is defined via the starting Klein-Gordon equation thus also serves as the passive gravitational mass that serves to couple massive particles to gravity, so that the particle modes associated with the quantization of the Klein-Gordon field thus automatically obey the equivalence principle, precisely because of quantum mechanics in fact [10].

As regards the covariant Klein-Gordon equation, it is convenient to make the substitution $\phi(x) = \exp[iS(x)/\hbar]$, so that the phase then obeys $S^{\mu}S_{\mu} + m^2c^2 = i\hbar S^{\mu}{}_{;\mu}$. In the eikonal or ray approximation the $i\hbar S^{\mu}{}_{;\mu}$ term can be dropped, so that the phase $S(x)$ is then seen to obey the classical Hamilton-Jacobi equation $S^{\mu}S_{\mu} + m^2c^2 = 0$, an equation whose solution is the stationary classical action between relevant end points. We thus establish that in the eikonal approximation the phase of the wave function of a material particle is in fact the classical action just as required for the discussion of the COW experiment we gave earlier. In the eikonal approximation we can also identify S^{μ} as the momentum $p^{\mu} = mc\,dx^{\mu}/d\tau$, so that we can set $S(x) = \int p_{\mu}dx^{\mu}$, with the covariant differentiation of the Hamilton-Jacobi equation then yielding [11] $p^{\mu}p^{\nu}{}_{;\mu} = 0$, which we recognize as the massive particle geodesic equation.

In order to actually calculate the geodesics in the gravitational field of the Earth it is convenient to rewrite the Schwarzschild metric in terms of a Cartesian coordinate system $x = r\,\sin\theta\cos\phi$, $y = r\,\sin\theta\sin\phi$, $z = r\,\cos\theta - R$ erected at a point on the surface of the Earth. With z being normal to the Earth's surface, to lowest order in x/R, y/R, z/R, $MG/c^2R(=gR/c^2)$ the Schwarzschild line element is then found [12] to take the form $d\tau^2 = [1-a(z)]c^2dt^2 - dx^2 - dy^2 - [1+a(z)]dz^2 - (4g/c^2)(xdx+ydy)dz$ where $a(z) = 2g(R-z)/c^2$. For this metric [13] the nonrelativistic geodesics for material particles are given by $\ddot{x}=0$, $\ddot{y}=0$, $\ddot{z}=-g$, to thus enable us to completely justify our earlier nonrelativistic calculation [14].

As regards the purely classical, massless case, on defining $\phi(x) = \exp[iT(x)]$, we can this time identify the eikonal phase derivative T^{μ} with the wave number $k^{\mu} = dx^{\mu}/dq$, where q is a convenient affine parameter that can be used to measure distances along trajectories. In the massless case the Hamilton-Jacobi equation takes the light cone form $g_{\mu\nu}k^{\mu}k^{\nu} = 0$ and yields the requisite massless particle geodesic equation $k^{\mu}k^{\nu}{}_{;\mu} = 0$ just as in the massive case. Given these geodesics, the motion of a classical light wave around the $ABCD$ interferometer loop is readily calculated, with explicit calculation [15] then showing that the ensuing light rays precisely follow Fig. 1 around the interferometer. However, even while there is still a spatial offset $A_1A_2 = gH^2/c^2$ just as before (explicitly because of the gravitational bending that the light rays experience), for light neither a time delay nor any net phase shift is found between the $A_1C_1D_2$ and AB_1D_2 paths. However, just as with the neutron case, the spatial offset itself leads to a time delay A_2A/c, so that there is still observable interference. Then, with $2\pi/\lambda$ (where λ is the wavelength of the incident beam) replacing $mc/\hbar$ in the normalization of the phase shift, we thus find that in traversing the interferometer the two light beams acquire a final observable net relative phase shift $\Delta\phi_{cl} = -2\pi gH^2/\lambda c^2$, where cl denotes classical light. Now while H would have to be of the order of 10^5 cm for $\Delta\phi_{cl}$ to actually be detectable in a Bragg scattering interferometer of the same sensitivity as the COW experiment (actually a quite conservative requirement since $\Delta\phi_{COW} \sim 56.5$ rad) [16], nonetheless, we can still identify this phase shift as a, in principle, completely classical effect that reveals the intrinsically global nature of classical gravity.

Now that we have obtained $\Delta\phi_{cl}$ it is instructive to compare it with $\Delta\phi_{COW}$. If we introduce the neutron de Broglie wavelength $\lambda_n = h/mv_0$, we may rewrite $\Delta\phi_{COW}$ in the form $-2\pi gH^2/\lambda_n v_0^2$. A comparison with $\Delta\phi_{cl}$ thus reveals a beautiful example of wave particle duality, with the quantum-mechanical matter wave inheriting its interference aspects from the behavior of the underlying classical wave. Thus, even while $\Delta\phi_{COW}$ does depend on the mass of the neutron [17], its dependence is strictly kinematic (the time delay needed for COW interference is independent of m) with gravity only coupling via λ_n (a quantity whose measurement is thus a measurement of the neutron's passive gravitational mass), with the $\Delta\phi_{COW}$ formula thus apparently being completely compatible with the equivalence principle [18].

Note added. For some recent related studies of atoms in gravitational fields see [19].

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[2] R. Colella, A. W. Overhauser, and S. A. Werner, Phys. Rev. Lett. **34**, 1472 (1975).

[3] D. M. Greenberger and A. W. Overhauser, Rev. Mod. Phys. **51**, 43 (1979).

[4] S. A. Werner, Class. Quantum Grav. **11**, A207 (1994).

[5] J. Anandan and H. R. Brown, Found. Phys. **25**, 349 (1995).

[6] If the neutron slows down or dips a little (numerically the deviation angle $\epsilon = gH/v_0^2$ shown in Fig. 1 is of order 0.01 arcsec), then at each subsequent scattering the neutron beam would not quite be incident with a wavelength and angle that satisfy a Bragg peak condition. However, elastic scattering events for which the momentum transfer remains perpendicular to the scattering surface will still be within the (~ 1.0 arcsec) Darwin width of the peak.

[7] Even while the ACD_3 path, for instance, is composed of the two subpaths AC and CD_3, each one of which is stationary between its own end points, nonetheless the sum of the two subpaths is not stationary between the overall A and D_3 end points and thus for the overall path $\overline{p}$ cannot be written as the derivative of a classical action, so that $\int \overline{p}\, d\overline{r}$ is then path dependent. In passing, we note that the fact that the sum of two stationary paths is not necessarily stationary between the overall end points is also the origin [see, e.g., P. D. Mannheim, Am. J. Phys. **51**, 328 (1983)] of the existence of the many paths of the Feynman path-integral description of quantum mechanics.

[8] Numerically the offset distance 2δ is found to be of order 30 Å, which is within the typical neutron beam longitudinal (100 Å), transverse (50 000 Å), and vertical (50 Å) spatial coherences that have been reported in the literature [H. Kaiser, S. A. Werner, and E. A. George, Phys. Rev. Lett. **50**, 560 (1983); H. Rauch, H. Wölwitsch, H. Kaiser, R. Clothier, and S. A. Werner, Phys. Rev. A **53**, 902 (1996)].

[9] For previous discussion of the role of curvature see J. Anandan, Phys. Rev. D **15**, 1448 (1977); **30**, 1615 (1984).

[10] We note that since the starting covariant Klein-Gordon equation only possesses one intrinsic mass scale, it would appear that the equivalence principle has to emerge. However, there is actually a hidden assumption in the use of the covariant Klein-Gordon equation (see [11]) since it is not the most general equation in curved space that can reduce to the flat Klein-Gordon equation in flat space. Rather, the curved space equation could also possess additional explicitly curvature-dependent terms, terms that would then vanish in the flat space limit, but would modify the particle's coupling to gravity in curved space in a potentially equivalence principle violating manner. Now such possible terms (which phenomenologically would have to be very small) are simply ignored in standard gravity without any apparent justification as far as we can tell, with the standard gravitational phenomenology only in fact following in their assumed absence. Thus, in passing, it is of interest to note that any such possible additional curvature dependent terms are not in fact allowed to appear [11] in conformal gravity, a currently viable candidate alternative to standard gravity [see, e.g., P. D. Mannheim, Found. Phys. **24**, 487 (1994); Astrophys. J. **479**, 659 (1997)].

[11] P. D. Mannheim, Gen. Relativ. Gravit. **25**, 697 (1993).

[12] W. Moreau, R. Neutze, and D. K. Ross, Am. J. Phys. **62**, 1037 (1994).

[13] Since this metric yields a uniform gravitational acceleration, it must therefore be flat, and indeed it can readily be transformed into $d\tau^2 = c^2 dt'^2 - dx^2 - dy^2 - dz'^2$ via the coordinate transformation $t' = t[1 - g(R-z)/c^2]$, $z' = z[1 + g(R-z/2)/c^2] + gt^2/2 + g(x^2+y^2)/c^2$. However, under such a transformation the sign of any time delay cannot change, with the fact of a time delay between the two beams at the source (even one as small as the $\delta/v_0 \simeq 10^{-12}$ sec one typically associated with the COW kinematics) thus being a covariant indicator for the COW effect.

[14] In passing we note that the energy associated with the nonrelativistic reduction of the Klein-Gordon equation actually also includes the rest energy mc^2 in addition to the kinetic energy of motion $E_0 = mv_0^2/2$ usually considered in the standard nonrelativistic classical action. While this rest energy would actually make a nonzero contribution to quantities such as $S(ACD_1) - S(AB_1D_2)$ because of the net time delay, precisely because of this same time delay the relevant relative phase selected at the source (where now $\hbar\omega = mc^2 + mv_0^2/2$) would then serve to cancel any specific mc^2 dependence in the overall action, with the rest energy term then not affecting the observable COW phase shift at all.

[15] P. D. Mannheim, Found. Phys. **26**, 1683 (1996); and unpublished.

[16] While 10^5 cm is certainly sizable for an interferometer, its interest lies in the fact that it allows us to detect, in principle at least, the gravitational bending of light using laboratory-sized distance scales rather than solar-system-sized ones. Thus it would also be of interest to see what dimension interferometer might serve as a gravitational wave detector or be sensitive to any possible neutrino masses in neutrino beam interferometry.

[17] The reason why $\Delta\phi_{COW}$ actually depends on m at all is that even though the position of the minimum of $S_{cl} = -mc\int d\tau$ is independent of m, nonetheless, the actual value of S_{cl} in this minimum does depend on m (though only as a kinematic overall multiplying factor); and even though the value of S_{cl} is not observable classically, nonetheless, it is observable quantum mechanically as the phase of the wave function whose normalization then explicitly depends (kinematically) on m.

[18] Now that we have established what it specifically is that the equivalence principle actually requires in the presence of quantum mechanics, it is somewhat disquieting to note that experi-

mentally a persistent 1% discrepancy between experiment and theory continues to be found [see, e.g., the very precise recent COW study of K. C. Littrell, B. E. Allman, and S. A. Werner, Phys. Rev. A **56**, 1767 (1997)], even while no analogous discrepancy is apparent in the (far less extensive) studies of neutron interferometry in noninertial accelerating frames [U. Bonse and T. Wroblewski, Phys. Rev. Lett. **51**, 1401 (1983); Phys. Rev. D **30**, 1214 (1984)]. Thus unlike the extraordinarily well established classical-mechanical equivalence principle, at the present time the quantum-mechanical equivalence principle has yet to be confirmed to better than the 1% level.

[19] P. Szriftgiser *et al.*, Phys. Rev. Lett. **77**, 4 (1996); M. K. Oberthaler *et al.*, Phys. Rev. A **54**, 3165 (1996).