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Statistics of power-dropout events in semiconductor lasers with time-delayed optical feedback

David W. Sukow, Jeff R. Gardner, and Daniel J. Gauthier
Department of Physics and Center for Nonlinear and Complex Systems, Duke University, Box 90305, Durham, North Carolina 27708

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We measure experimentally the statistical distribution of time intervals between power-dropout events occurring in a semiconductor laser with time-delayed optical feedback operating in the low-frequency fluctuation regime. Near the laser threshold, the time-interval probability distribution displays a low-probability region, or dead zone, for short times, followed by a slow rise, and an exponential decay for long times. At higher injection currents, the distributions develop considerable structure. We compare our results to the predictions of approximate analytic models of the laser dynamics and find that no single model accurately captures the details of the observed distributions, indicating that our physical understanding of the long-term dynamics of the laser in this regime is less than complete. [S1050-2947(97)50711-6]

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Semiconductor lasers with delayed optical feedback display a complex variety of dynamical behaviors governed by the deterministic, nonlinear interaction between the electromagnetic field and the semiconducting material, and noise arising from the quantum-mechanical process of spontaneous emission of photons [1]. This situation arises when the beam generated by the laser reflects from a distant surface and is reinjected into the laser. The specific type of behavior depends sensitively on the laser-surface distance and the reflectivity of the surface. Reflectivities as low as 10^{-6}, such as that arising from spurious reflections from optical fiber junctions or compact disks, can significantly alter the laser dynamics. From a practical perspective, the performance of devices based on semiconductor laser elements is often degraded by these effects; hence it is important to uncover their origin so they can be avoided or dynamically controlled.

At present, our understanding of the long-term dynamics of a semiconductor laser with optical injection in the ''low-frequency fluctuations'' (LFF) regime is less than adequate in that some of the experimental results or their interpretation appear to be contradictory. The laser in the LFF regime occurs near threshold with moderate optical feedback and it produces an erratic train of ultrashort pulses [2–4] whose pulse width (in the range of 50–200 ps), spacing (200–1000 ps), and amplitude change from pulse to pulse. This pulsing behavior is interrupted at erratic intervals (50 ns–1 ms) by power dropout events where the average power suddenly drops, then gradually (~50 ns) builds up to its original value [5] during chaotic itinerancy [6]. While it is possible to predict the nature of the dropout event and its recovery to the high-power state in reasonable agreement with experimental observations, there is limited guidance regarding the statistics of the time intervals between power dropout events. Adding further to the complexity of the dynamics, recent observations suggest that spontaneous-emission noise [7] and multi-longitudinal-mode effects can alter significantly the time-interval statistics [5]. The primary purpose of this Rapid Communication is to present high-resolution and high-accuracy measurements of the experimentally observed time-interval distribution and to compare our observations to the predictions of approximate analytic models of the laser dynamics. We find that our results are consistent with these models for some experimental conditions, although our results indicate that new analytic treatments are necessary to capture precisely the long-term LFF dynamics.

We perform a series of experiments using a single-transverse-mode semiconductor laser (Spectra Diode Labs SDL-5401-G1, nominal wavelength $\lambda = 789$ nm, threshold injection current of the laser in the absence of external feedback $\mathcal{I}_{th} = 17.0$ mA) that is mechanically isolated and temperature-stabilized to better than 1 mK [8]. The laser operates in a single longitudinal mode with side-mode suppression >22 dB in the absence of external feedback for all injection currents $\mathcal{I}$ used in the experiment. The output (back) facet of the chip has a partial antireflection (high-reflection) coating with a power reflection coefficient of approximately 0.04 (0.95). The beam generated by the laser is collimated with a high-numerical-aperture lens and directed toward a mirror located $L \simeq 71$ cm from the laser, which redirects the beam back toward the laser. Contrary to some previously published reports, we observe power dropout events even when the external mirror is aligned to minimize the external-cavity laser threshold current $\mathcal{I}_{ext}$ (''optimum'' alignment) [9], and when $\mathcal{I} < \mathcal{I}_{th}$ [10]. The optical feedback strength is adjusted using a polarizer and a rotatable quarter-wave plate placed in the beam path, and a portion of the beam is sampled by a beam splitter and detected with a high-speed photoreceiver (New Focus 1537-LF, 6-GHz bandwidth). A long-time series of the signal generated by the detector is recorded using a high-speed digital oscilloscope (Tektronix TDS680B, 1-GHz maximum analog bandwidth, 5- Gs/s maximum sampling rate), and transferred to a computer. The time interval τ between the beginnings of successive dropout events is determined using a peak-detecting algorithm, and this procedure is repeated until a sufficient number of dropouts are recorded (typically $>$10 000 events). A histogram is generated indicating the number of instances that we observe a time interval between dropout events within τ and $\tau + \Delta\tau$, normalized by $\Delta\tau$ and the total number of recorded intervals. It approximates the dropout time-interval probability density $\eta(\tau)$, where $\eta(\tau)\Delta\tau$ is the probability of observing a time interval τ when $\Delta\tau$ is small.

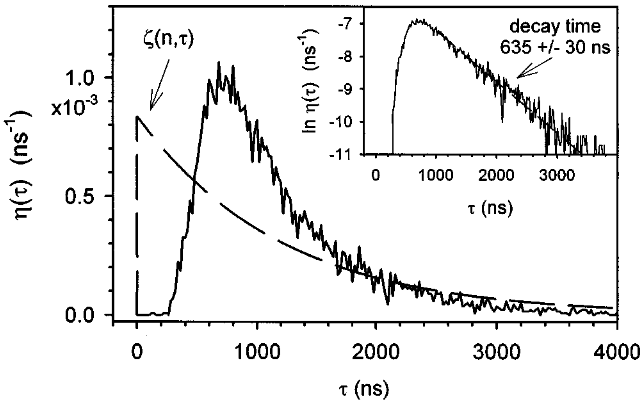

FIG. 1. Observed power dropout time-interval probability density for $\mathcal{I}=17$ mA ($P/P_1=1$) with $\Delta\tau=10$ ns (solid line) compared to the distribution predicted by the HK model (dashed line) using the measured value of $\langle\tau\rangle$. The inset demonstrates that distribution decays exponentially for long times.

Figure 1 shows $\eta(\tau)$ for the case when the injection current $\mathcal{I}=17.0$ mA and the quarter-wave plate is adjusted to give $\mathcal{I}_{ext}=14.5$ mA (corresponding to a relative threshold reduction with respect to the solitary laser threshold $\Delta C/C \equiv(\mathcal{I}_{th}-\mathcal{I}_{ext})/\mathcal{I}_{th}=0.147$). In this situation, approximately 30% of the light generated by the laser is reflected back toward the laser, although we are uncertain of its coupling efficiency. Also, the average laser power P with external feedback scales approximately linearly with $\mathcal{I}$ over the parameter range of our experiment. The laser power with external feedback when $\mathcal{I}=\mathcal{I}_{th}$ is denoted by P_1. It is seen from the figure that the probability for a dropout event is essentially zero for $\tau<260$ ns (a ''dead zone'') and that the distribution gradually increases for $\tau>260$ ns to a maximum value near $\tau=750$ ns. From the distribution, we find a mean dropout time interval $\langle\tau\rangle=1192\pm12$ ns. In addition, it can be seen in the inset that the distribution decays exponentially for long times with a decay time of 635 ± 30 ns.

For increasing injection currents, the observed dead-zone interval decreases, the distributions (solid lines) shift to shorter times, and they develop considerable structure far above threshold, as shown in Fig. 2. We note that the accuracy and resolution of our measurements, resulting from the large number of observed dropout events, makes it possible to discern the existence of the dead zone and the detailed structure of these distributions; previous measurements of $\eta(\tau)$ with fewer observed dropout events did not uncover these features [10,11]. While these distributions undergo considerable structural changes, we find that the mean time $\langle\tau\rangle$ between dropouts is a smooth function of P, as shown in Fig. 3 (filled squares).

To gain an understanding of our observations, we analyze our results using several approaches. The standard technique for investigating theoretically the dynamics of the semiconductor laser subjected to optical feedback is to integrate numerically a set of coupled nonlinear, time-delay differential equations first put forth by Lang and Kobayashi [12]. Unfortunately, this approach required prohibitively long numerical computations to obtain high enough accuracy and resolution to make a direct comparison with our observations for the range of parameters used in the experiment due to the extreme stiffness of the equations. We note that Mørk *et al.* [5] have developed an iterative numerical technique for predicting the laser dynamics on each external cavity round trip, although the computation time of this method is similar to direct numerical integration of the Lang-Kobayashi delay differential equations [13].

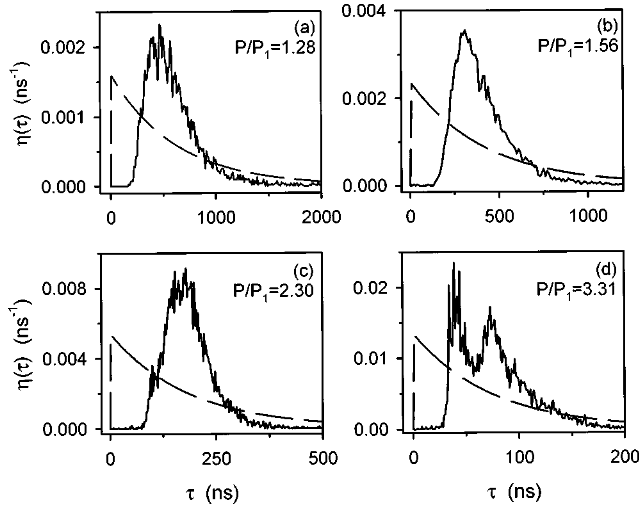

FIG. 2. Observed power dropout time-interval probability densities (solid lines) and the predictions of the HK model (dashed line) for (a) $\mathcal{I}=18$ mA ($P/P_1=1.28$), $\Delta\tau=10$ ns; (b) $\mathcal{I}=19$ mA ($P/P_1=1.56$), $\Delta\tau=10$ ns; (c) $\mathcal{I}=22$ mA ($P/P_1=2.30$), $\Delta\tau=10$ ns; and (d) $\mathcal{I}=26$ mA ($P/P_1=3.31$), $\Delta\tau=1$ ns.

In light of the computational complexity of this problem, we turn to approximate analytic techniques. One such approach was developed by Henry and Kazarinov (HK) [14]. They investigated theoretically the time-interval statistics by performing a nonlinear stability analysis of the Lang-Kobayashi equations [12] in the vicinity of the linearly stable ''maximum gain mode'' and in the presence of stochastic perturbations due to spontaneous emission of photons. They find that the laser dynamics are analogous to the motion of a stochastically driven, strongly damped particle moving in a one-dimensional potential well with a barrier, where the spatial coordinate of the potential function is the deviation of the carrier number density n away from its steady-state value. In this model, a power dropout event occurs when the particle escapes over the barrier; thus the statistics of the time intervals between dropouts can be determined by treating this as a first-passage-time problem. Hohl *et al.* [7] recently found

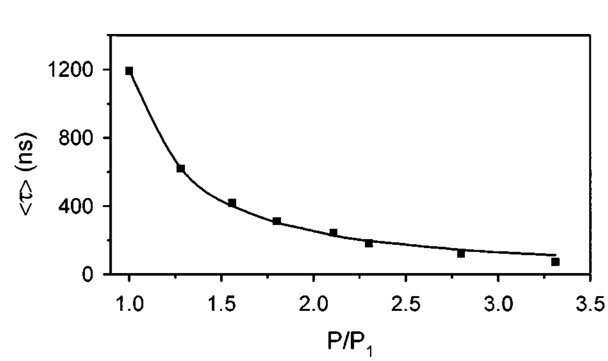

FIG. 3. Mean dropout time interval as a function of the laser power. The square symbols indicate the average time interval between the beginnings of the dropout events. The solid line illustrates the best fit of the predictions of the HK model to the observed times.

experimentally that the dependence of $\langle\tau\rangle$ on optical feedback strength for moderate feedback agrees well with the HK model, whereas its dependence on injection current $\mathcal{I}$ (and hence on P) does not agree. Cerboneschi *et al.* [11] found a dependence of $\langle\tau\rangle$ on $\mathcal{I}$ for high feedback strength that differs from the results of Hohl *et al.* [7] and that is also in disagreement with the predictions of the HK model.

Following their approach, a first-passage-time problem can be formulated to predict the first-passage-time probability density $\zeta(n,\tau_b)$ for the effective particle to go over the potential barrier in a time τ_b, given an initial location n in the potential well. The effective potential is given by [14]

$$U(n)=\gamma\left(\frac{n^2}{2}-\frac{n^3}{3n_0}\right), \tag{1}$$

where the bottom of the well (potential barrier) is located at $n=0$ ($n=n_0$), and γ is the characteristic decay rate for a particle sliding to the bottom of the well under friction.

To connect HK's theory to our experiment, we need to know the time τ_r it takes for the particle to reenter the potential well once it crosses the barrier (the reinjection time), since the experimentally measured time between dropouts $\tau=\tau_b+\tau_r$. Note that HK did not address this issue. In addition, we need to know the initial location of the particle in the well. Regarding the reinjection time, a plausible interpretation of the dropout process is that the effective particle is reinjected into the potential well as soon as the trajectory is in the neighborhood of the linearly unstable external-cavity modes, which is of the order of the external cavity round-trip time. In this case and under our experimental conditions, $\tau_r\sim 5$ ns $\ll\tau_b$, and hence $\tau_b\simeq\tau$ and $\zeta(n,\tau_b)\simeq\zeta(n,\tau)$.

Concerning the initial location of the particle, we have extended the analysis of the HK model and find that it gives us some guidance on this issue. An approximate analytic expression for $\zeta(n,\tau_b)$ is obtained by considering the moments of the integral first-passage-time distribution. We find that

$$\zeta(n,\tau_b)\simeq\frac{1}{\langle\tau_b\rangle}\exp\left(-\frac{\tau_b}{\langle\tau_b\rangle}\right), \tag{2}$$

where the mean time to cross the barrier is given approximately by

$$\langle\tau_b\rangle\simeq\frac{\pi}{\gamma}\exp\left(\frac{1}{3\mathcal{D}}\right), \tag{3}$$

$\mathcal{D}=D/3U(n_0)=2D/\gamma n_0^2$ is a dimensionless diffusion coefficient, and D is a diffusion coefficient characterizing the laser intensity fluctuations due to spontaneous emission of photons [14]. Equation (2) is valid when the diffusion coefficient is small, $\mathcal{D}\ll 1$, and the particle does not start too close to the barrier such that $n(t=0)\ll n_0(1-\sqrt{\mathcal{D}})$. Since Eq. (2) is *independent of the initial location of the particle* and the reinjection time is short, $\zeta(n,\tau_b)\simeq\eta(\tau)$.

The parameters γ and $\mathcal{D}$ depend on laser-specific quantities that are not known accurately for our system; we must determine their size to assess whether the conditions given above are satisfied. Henry and Kazarinov show that $\gamma=a(1+4P/P_1)$ and $\mathcal{D}=b/(1+P_1/4P)^3(\Delta C/C)^3$, where the laser power with external feedback at $\mathcal{I}=\mathcal{I}_{th}$ is denoted by P_1, and a and b are laser-specific parameters. We determine the parameters by fitting Eq. (3) to the observed dependence of $\langle\tau\rangle$ on P with fixed $\Delta C/C$, working under the assumption that $\langle\tau_b\rangle\simeq\langle\tau\rangle$. We find good agreement between the observed (filled squares) and predicted values (solid line) for $a=19.6$ ps^{-1} and $b=5.72\times 10^{-4}$, as shown in Fig. 3. Note that our observations are consistent with HK's model, contrary to previously published results [7,11]. Using these values, we determine that $\mathcal{D}=0.092$ and $\gamma=98$ ps^{-1} when $P/P_1=1$. Therefore, the likelihood that the conditions will be fulfilled during a typical dropout event is high since $\mathcal{D}\ll 1$, and hence Eq. (2) should well describe the first-passage- time probability density.

The dashed lines in Figs. 1 and 2 show $\zeta(n,\tau)\simeq\zeta(n,\tau_b)$ evaluated using the measured value of $\langle\tau\rangle$. It is seen that the maximum height and long-time decay of the theoretically predicted and experimentally observed distributions agree qualitatively (note that the experimentally measured and theoretically predicted distributions are characterized by the same average time interval). However, the initial dead zone and the slow rise in the distribution are not accurately captured by the simple theory. To check that this discrepancy is not due to the assumptions underlying the derivation of Eq. (2), we numerically integrate the Kolmogorov equation for the integral first-passage time using the estimated values of γ and $\mathcal{D}$ and differentiate this result to find the exact form of $\zeta(n,\tau)$. It accurately reflects the rapid rise of the approximate form of $\zeta(n,\tau)$ shown in Fig. 1 and hence we conclude that the HK model does not capture the essence of the dropout statistics, even though observed dependence of $\langle\tau\rangle$ on P is consistent with their model. We note that the potential function considered by HK does not incorporate the fine structure associated the individual external cavity modes [15]; a complete nonlinear stability analysis including this structure and the potential barrier at n_0 has yet to be performed. It is plausible that this fine structure may be responsible for the slow rise observed in the statistical distributions.

We now consider a model-independent approach suggested by Sacher *et al.* [10], who found experimentally that $\langle\tau\rangle\sim\varepsilon^{-1}$, where $\varepsilon=(\mathcal{I}-\mathcal{I}_{th})/\mathcal{I}$. One interpretation of their results is that the dropout events are a manifestation of time-inverted type-II intermittency [16], although we note that there are currently no theoretical predictions regarding the existence of type-II intermittency in this system. Figure 4 shows our observed dependence of $\langle\tau\rangle$ on ε, using the measured dependence of P on $\mathcal{I}$ together with a line of slope -1 expected for type-II intermittency. Also shown are the predictions of the HK model (solid line) with the same parameters used in Fig. 3. Note that one experimental data point for $\varepsilon=0$ cannot be shown on this plot; this data point falls close to the predictions of HK's model (see Fig. 3). It is seen that $\langle\tau\rangle\sim\varepsilon^{-1}$ for $\varepsilon\gtrsim e^{-2}$, consistent with the behavior expected for deterministic type-II intermittency [10]. Interestingly, the stochastic model of HK predicts the *same* scaling of $\langle\tau\rangle$ on ε in this region. Note that the type-II intermittency theory predicts that $\langle\tau\rangle\to\infty$ as $\varepsilon\to 0$, which is inconsistent with our highly accurate determination $\langle\tau\rangle$ for $\varepsilon=0$ (when $\mathcal{I}=\mathcal{I}_{th}$ and $P=P_1$) and inconsistent with the results of Hohl *et al.* [7]. Considering the combined results of these studies, it appears that the power dropout events are not a manifestation

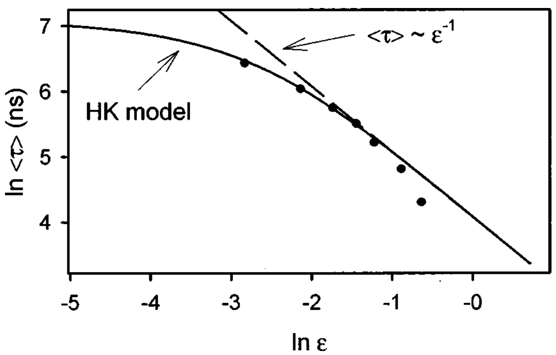

FIG. 4. Mean time interval between power dropout events as a function of $\varepsilon=(\mathcal{I}-\mathcal{I}_{th})/\mathcal{I}_{th}$. The circles are the observed values, the solid line indicates the predictions of the HK model, and the dashed line indicates the scaling expected for type-II intermittency. Note that one data point for $\varepsilon=0$ cannot be shown on this plot; it agrees well with the HK model.

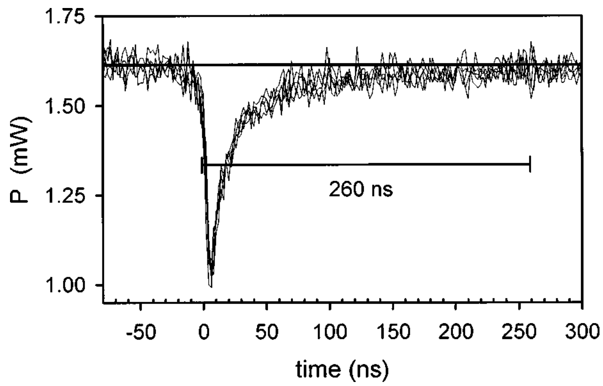

FIG. 5. Temporal evolution of the average laser power for $P/P_1=1$ during a series of power dropout events, illustrating that they are nearly identical. The length of the dropout event is approximately equal to the dead zone shown in Fig. 1. The analog bandwidth of the detection system for these measurements is 250 MHz.

of intermittency for these laser systems near the solitary laser threshold and given the choice of ε suggested by Sacher *et al.* [10].

Finally, we consider an approximate analytic treatment of the power dropouts developed by Mørk *et al.* [5] to gain an understanding of the dead zones in Figs. 1 and 2. They suggest that the dropout events arise from noise-induced switching between bistable states of the laser with external feedback (see Fig. 3 and the discussion in Sec. V of Ref. [5]), even for the case of a single-longitudinal-mode laser. During the initial buildup of the laser intensity after a dropout event, they find that bistability is absent and is only restored after the intensity nearly regains its original value. Hence, a dropout cannot occur during this time interval when P is low. Figure 5 shows an overlay of the observed temporal evolution of the laser intensity during several dropout events for $P/P_1=1$. Note that the time it takes for the laser to attain the same average laser power that it possesses before the dropout event compares well with the dead zone occurring for $\tau<260$ ns shown in Fig. 1. Thus, our observation of the dead zones shown in Figs. 1 and 2 is consistent with their predictions.

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