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Macroscopic coherence for a trapped electron

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We investigate the possibility of generating quantum macroscopic coherence phenomena by means of relativistic effects on a trapped electron. [S1050-2947(97)50409-4]

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One of the fundamental aspects of quantum mechanics is the exsistence of interference among quantum states, which signifies the difference between a superposition of states and a mixture of states. However, as soon as the superposition principle is extended to the macroscopic world the Schrödinger-cat paradox [1] arises. Generally the visibility of such curious superposition states at the macroscopic level is precluded by the decoherence phenomena [2]. In fact, the coherence vanishes rapidly with the ''macroscopic separation'' between the superposed states [3]. This subject has always attracted the attention of physicists, but recently, due to improved technology, there has been growing interest in the possibility of observing such superposition states, called Schrödinger-cat states.

Several proposals for the generation of linear superpositions of coherent states in various nonlinear processes [4] and in quantum nondemolition (QND) measurements [5] have been made. Actually, only ''mesoscopic'' cats have been observed in trapped ions and in cavity QED [6].

In order to generate the cat state in nonlinear systems, the crucial point is the ratio between the strength of nonlinearity and the decoherence rate. The interference effects could be preserved by slowing down the decoherence, but the proposed methods [7,8] encounter some difficulties in practical realization. Hence the search for systems where such a ratio is sufficiently high.

A high nonlinearity with respect to the damping, and consequently the dilatation, of the decoherence time, could be obtained in charged trapped systems, even at the microwave level. In this Rapid Communication we shall present, as a system for the generation of cat states, an electron trapped in a Penning trap [9] whose relativistic motion induces nonlinear effects. Although the relativistic correction is very small, its effect is, however, observable [10]. The macroscopic character relies in this case on the possibility of high coherent excitations of one mode of the electron motion. We shall also suggest appropriate measurement techniques useful for revealing the quantum macroscopic coherence.

In a Penning trap one considers the motion of an electron in a uniform magnetic field B along the positive z axis, driven by an electric field circularly polarized on the xy plane and a static quadrupolar potential. As is well known [11], the motions of that electron in the trap are well separated in energy scale and, in a typical experimental situation [12], the interesting frequencies are 160 GHz for the cyclotron motion, 64 MHz for the axial motion, and 12 kHz for the magnetron motion. In what follows we shall consider only the cyclotron and the axial degrees of freedom, neglecting the slow magnetron motion. To simplify our presentation, we assume the *a priori* knowledge of the electron's spin; then we neglect all the spin-related terms in the Hamiltonian that, for an electron of rest mass m and charge $-|e|$, can be approximated by [11,13]

$$\hat{H}=\frac{1}{2m}\left[\hat{\mathbf{p}}-\frac{e}{c}\hat{\mathbf{A}}\right]^2-\frac{1}{8m^3c^2}\left[\hat{\mathbf{p}}-\frac{e}{c}\hat{\mathbf{A}}\right]^4+eV_0\frac{\hat{x}^2+\hat{y}^2-2\hat{z}^2}{4d^2}, \tag{1}$$

with

$$\hat{\mathbf{A}}=\left(i\frac{c}{\omega_p}\langle\epsilon^* e^{i\omega_p t}-\epsilon e^{-i\omega_p t}\rangle-\frac{B}{2}\hat{y},\frac{c}{\omega_p}\langle\epsilon^* e^{i\omega_p t}+\epsilon e^{-i\omega_p t}\rangle+\frac{B}{2}\hat{x},0\right), \tag{2}$$

where c is the speed of light and ϵ the amplitude of the driving field at angular frequency ω_p; d characterizes the dimensions of the trap and V_0 is the potential applied to its electrodes. The second term on the right-hand side of Eq. (1) represents the correction due to the relativistic shift of the electron mass (we have neglected all contributions of higher order). It is now convenient to introduce the raising and lowering operators for the cyclotron motion,

$$\hat{a}=\frac{1}{2}\left[\beta(\hat{x}-i\hat{y})+\frac{1}{\beta\hbar}(\hat{p}_y+i\hat{p}_x)\right], \tag{3}$$

$$\hat{a}^\dagger=\frac{1}{2}\left[\beta(\hat{x}+i\hat{y})+\frac{1}{\beta\hbar}(\hat{p}_y-i\hat{p}_x)\right], \tag{4}$$

with $\beta=(m\omega_c/2\hbar)^{1/2}$ and $\omega_c=|e|B/mc$ being the cyclotron angular frequency. Analogously, for the axial motion we define

$$\hat{a}_z=\left[\frac{m\omega_z}{2\hbar}\right]^{1/2}\hat{z}+i\left[\frac{1}{2m\hbar\omega_z}\right]^{1/2}\hat{p}_z, \tag{5}$$

$$\hat{a}_z^\dagger=\left[\frac{m\omega_z}{2\hbar}\right]^{1/2}\hat{z}-i\left[\frac{1}{2m\hbar\omega_z}\right]^{1/2}\hat{p}_z, \tag{6}$$

with $\omega_z^2=|e|V_0/md^2$.

Thus, by using these new operators, in the dipole and rotating-wave approximation, the Hamiltonian (1) becomes

$$\hat{H}=\hbar\hat{\omega}_M(\hat{a}^\dagger\hat{a}+\tfrac{1}{2})-\hbar\mu(\hat{a}^\dagger\hat{a})^2+i\hbar k(\epsilon\hat{a}^\dagger e^{-i\omega_p t}-\epsilon^*\hat{a}e^{i\omega_p t})+\hbar\omega_z(\hat{a}_z^\dagger\hat{a}_z+\tfrac{1}{2}), \tag{7}$$

where

$$\mu=\frac{\hbar\omega_c^2}{2mc^2},\quad k=\frac{|e|}{\omega_p}\left(\frac{\omega_c}{2\hbar m}\right)^{1/2}, \tag{8}$$

and $\hat{\omega}_M$ is an operator defined by

$$\hat{\omega}_M=\omega_c\left[1-\frac{\hat{p}_z^2}{2m^2c^2}-\frac{\hbar\omega_c}{2mc^2}\right]. \tag{9}$$

All terms not containing the raising or lowering operators have been omitted. We have also neglected the anharmonicity of the axial motion, which is smaller by a factor $(\omega_z/\omega_c)^2$ with respect to that of the cyclotron motion. In the expression for $\hat{\omega}_M$ we have neglected a correction to the bare cyclotron angular frequency due to the presence of the quadrupolar potential, which is of the order $(\omega_z/\omega_c)^2$. Thus, we can see in Eqs. (7) and (9) that the relativistic correction results in a coupling between axial and cyclotron motions allowing QND measurements of cyclotron excitations in the absence of the pumping field [14], and in a further anharmonicity term.

The initial state for the cyclotron motion should be the ground state, considering for simplicity that no excitations are present in the cyclotron motion due to the thermal bath (however, such excitations could easily be introduced). Then we suppose the driving field to be acting initially as a kick, and strong enough so that within its duration, say τ, the remaining evolution can be neglected. This situation could be realized if τ were shorter than the characteristic periods, i.e., $\tau\ll 2\pi/\omega_z\ll 2\pi/\mu$ once one has chosen ω_p close enough to ω_c. Hence, we may write the effective initial cyclotron state as

$$\hat{\rho}(0)=\hat{D}(\alpha_0)|0\rangle\langle 0|\hat{D}^\dagger(\alpha_0), \tag{10}$$

with the displacement operator, in a frame rotating at the frequency ω_p, given by

$$\hat{D}(\alpha_0)=\exp[\alpha_0\hat{a}^\dagger-\alpha_0^*\hat{a}],\quad \alpha_0=k\epsilon\tau. \tag{11}$$

After that the Hamiltonian governing the electron's motion (again in the rotating frame) is

$$\hat{H}=\hbar(\hat{\omega}_M-\omega_p)(\hat{a}^\dagger\hat{a}+\tfrac{1}{2})-\hbar\mu(\hat{a}^\dagger\hat{a})^2+\hbar\omega_z(\hat{a}_z^\dagger\hat{a}_z+\tfrac{1}{2}). \tag{12}$$

Furthermore, the axial motion relaxes much faster than the cyclotron one [11]; then we can first average over the axial degrees of freedom and the Hamiltonian (12) simply reduces to

$$\hat{H}=\hbar(\omega_M-\omega_p)(\hat{a}^\dagger\hat{a}+\tfrac{1}{2})-\hbar\mu(\hat{a}^\dagger\hat{a})^2, \tag{13}$$

where ω_M is no longer an operator, and is determined by the equilibrium temperature T

$$\omega_M=\omega_c\left[1-\frac{k_BT}{2mc^2}-\frac{\hbar\omega_c}{2mc^2}\right], \tag{14}$$

while the initial state of the cyclotron motion is the coherent state (10). The Hamiltonian (13) represents the same model studied in Ref. [15]. By choosing $\omega_p=\omega_M$ and regarding the anharmonicity as the interaction part, the discussed initial coherent state $|\alpha_0\rangle$ will evolve, after a time $t=\pi/2\mu$, in a superposition of coherent states

$$\frac{1}{\sqrt{2}}[e^{-i\frac{\pi}{4}}|\alpha_0\rangle-e^{i\frac{\pi}{4}}|-\alpha_0\rangle], \tag{15}$$

which could be macroscopically distinguishable due to the opposite phase whenever a sufficiently strong driven field ϵ is used.

However, a complete treatment of the problem has to include the interaction of the cyclotron motion with the environment. The master equation for the (reduced) density matrix $\hat{\rho}$ of the cyclotron motion can be derived by standard procedures [16] to get, in the interaction picture,

$$\frac{\partial\hat{\rho}}{\partial t}=i\mu[(\hat{a}^{\dagger}\hat{a})^{2},\hat{\rho}]+\frac{\gamma}{2}[2\hat{a}\hat{\rho}\hat{a}^{\dagger}-\hat{a}^{\dagger}\hat{a}\hat{\rho}-\hat{\rho}\hat{a}^{\dagger}\hat{a}], \quad (16)$$

with γ representing the energy relaxation rate of the cyclotron motion and for simplicity the temperature of the bath considered to be zero because the number of thermal excitations at angular frequency ω_c is negligible at the usual temperature of performed experiments; i.e., $T=4\,K$. Equation (16) may be converted into a partial differential equation for the Husimi [17] function $Q(\alpha,t)=\langle\alpha|\hat{\rho}(t)|\alpha\rangle$, which in turn should be solved subject to the initial condition $Q(\alpha,0)=\exp(-|\alpha-\alpha_0|^2)$. The solution can be found as in Ref. [18] and reads

$$Q(\alpha,t)=e^{-|\alpha|^2-|\alpha_0|^2}\sum_{p,q=0}^{\infty}\frac{(\alpha\alpha_0^*)^p}{p!}\frac{(\alpha^*\alpha_0)^q}{q!}Z_{p,q}(t), \quad (17)$$

$$Z_{p,q}(t)=\exp\left\{-\frac{p+q}{2}[\gamma+2i\mu(p-q)]t+\gamma|\alpha_0|^2\frac{1-e^{-[\gamma+2i\mu(p-q)]t}}{\gamma+2i\mu(p-q)}\right\}. \quad (18)$$

It is worth noting that the best way of observing the microscopic system from the outside world is through the measurement of the current due to the induced charge on the cap electrodes of the trap, as a consequence of the axial motion of the electron along the symmetry axis [11]. Even though the relativistic effect assures a coupling between the cyclotron and the axial motions, the detection models we have in mind are based on specific couplings induced by Hamiltonians immediately before the measurement process. These Hamiltonians could be obtained by suitable modifications of the external fields and are extensively discussed in Ref. [19] where the reconstruction of the whole Wigner function of the cyclotron state is also considered.

Usually cat states are very fragile with respect to the introduction of dissipation effects; however, the present system has the great advantage of obtaining a high ratio between the nonlinearity μ and the damping coefficient γ. In fact the energy loss of the cyclotron motion can be reduced by cavity effects [20] or by an off-resonant situation [21] to obtain $\gamma\approx 1\ \mathrm{s}^{-1}$, so that [11] $\mu/\gamma\approx 10^2\gg 1$. Hence, the cat state may survive many cycles due to the long decoherence time, which is of the order of $(\gamma|\alpha_0|^2)^{-1}$, as can be extracted from Eq. (18).

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