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Laser-noise-induced heating in far-off resonance optical traps

T. A. Savard, K. M. O'Hara, and J. E. Thomas
Physics Department, Duke University, Durham, North Carolina 27708-0305
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Using a simple model, we calculate the heating rates arising from laser intensity noise and beam-pointing fluctuations in far-off resonance optical traps. Intensity noise causes exponential heating, while beam-pointing noise causes heating at a constant rate. The achievement of heating time constants well beyond 10 sec imposes stringent requirements on the laser noise power spectra. Noise spectra are measured for a commercial argon-ion laser to illustrate the expected time scales.
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Far-off resonance optical traps offer an attractive means of confining atoms with a large restoring force and potentially long trapping times. Large detunings from resonance are used to achieve low spontaneous scattering rates. This increases the maximum trapped atom density by reducing heating [1,2], by reducing repulsive scattering forces [3], and by reducing excited-state trap loss collisions [4]. The large detuning also permits trapping of multiple atomic spin states in nearly identical potentials [1]. This is useful for fundamental studies of collisions and for atomic clocks. For these reasons, far-off resonance optical traps and lattices have been extensively explored by a number of groups [1,2,5–9].

Several groups have attempted to obtain long storage times by utilizing extreme detunings from resonance. Heinzen and co-workers employed red detunings of up to 65 nm and achieved storage times of 200 msec, limited by background gas collisions [1]. A sodium trap using a krypton-ion laser operating at 647 and 676 nm also has been explored [5]. Storage times of a few seconds have been demonstrated with a red detuned trap based on a YAG (yttrium aluminum garnet) laser at 1.06 μm [5,6]. Most recently, trapping of cesium atoms has been achieved using a CO_2 laser operating at 10.6 μm. In this case, spontaneous emission was limited to the Rayleigh scattering rate, of order 10^{-3} $\sec^{-1}$. However, the background pressure limited storage times to less than 1 s, with extrapolated storage times of 3 sec at zero pressure [7]. In principle, by using very-far-off resonance traps in a high vacuum, the ideal limit of a conservative potential and very long trapping times can be nearly achieved.

Unfortunately, in all traps employing red detunings, storage times have been limited below 10 sec [5–7]. Further, it has been noted that red detuned traps appear to suffer from unexplained heating rates. For the YAG laser trap, the trapping time is reported to be inversely proportional to the laser intensity and independent of density [5,6]. Hence, attempts have been made to circumvent the heating problem by using blue detuned traps that confine atoms via a repulsive potential [5,10,11]. In this way, atoms spend a minimum time in the optical field and hopefully experience less heating.

In this paper, we show that laser intensity fluctuations and beam-pointing fluctuations may play an important role in determining the minimum heating rates that can be obtained in both red and blue detuned optical traps. Although it has been obvious for some time that trap fluctuations can cause heating, to our knowledge, no explicit expressions for the expected heating rates have been given previously. We show in the framework of a simple one-dimensional harmonic-oscillator model that the attainment of long storage times imposes stringent requirements on the trap stability and hence on the intensity noise and pointing stability of laser beams used in optical traps. We measure the intensity and pointing noise power spectra for an argon-ion laser and estimate heating times as a function of trap frequency. The mechanisms described induce heating in any harmonic trap. For brevity, we consider here only heating rates, and defer detailed discussion of trap loss and trap state dynamics to future work.

In a far-off resonance red detuned trap, the effective potential can be written as

$$V(x) = -\frac{1}{4}\alpha|\mathcal{E}(x)|^2, \tag{1}$$

where α is the atomic polarizability and $\mathcal{E}(x)$ is the slowly varying field amplitude [1]. For detunings less than 10% off resonance, a two-level model in the rotating-wave approximation yields $\alpha = -\mu^2/(\hbar\Delta)$ where μ is the transition dipole moment and $\Delta = \omega - \omega_0$ is the detuning. Hence, the potential is attractive for a red detuning. Very far below resonance, the rotating-wave approximation is not valid and the polarizability approaches the static limit [12]. For a focused Gaussian laser beam of $1/e$ intensity radius a, the potential can be approximated as $-V_0 + \frac{1}{2}kx^2$, where $V_0 = \alpha|\mathcal{E}_0|^2/4$ is the maximum light shift. The spring constant $k = 2V_0/a^2$. Since both V_0 and k are proportional to the fluctuating laser intensity $I(t)$, the spring constant fluctuates in time. This leads to exponential heating as we now show.

To determine the heating rate due to laser intensity fluctuations, we take the model Hamiltonian for a trapped atom of mass M to be

$$H = \frac{p^2}{2M} + \frac{1}{2}M\omega_{tr}^2[1+\epsilon(t)]x^2. \tag{2}$$

Here, $\omega_{tr}^2 = k_0/M$ is the mean-square trap oscillation frequency and k_0 is proportional to the time averaged laser intensity I_0. The fluctuation in the spatially constant poten-

tial V_0 exerts no force and hence does not cause heating. The spring constant exhibits a fractional fluctuation $\epsilon(t)$, where

$$\epsilon(t)=\frac{I(t)-I_0}{I_0} \tag{3}$$

is the fractional fluctuation in the laser intensity.

The heating rate is easily determined using first-order time-dependent perturbation theory to calculate the average transition rates between quantum states of the trap. The perturbation of interest is given by

$$H'(t)=\frac{1}{2}\epsilon(t)M\omega_{tr}^2x^2. \tag{4}$$

For an atom in the state $|n\rangle$ at time $t=0$, the average rate to make a transition to state $|m\neq n\rangle$ in a time interval T is

$$R_{m\leftarrow n}\equiv\frac{1}{T}\left|\frac{-i}{\hbar}\int_0^T dt' H'_{mn}(t')e^{i\omega_{mn}t'}\right|^2$$

$$=\left(\frac{M\omega_{tr}^2}{2\hbar}\right)^2\int_{-\infty}^{\infty}d\tau e^{i\omega_{mn}\tau}\langle\epsilon(t)\epsilon(t+\tau)\rangle|\langle m|x^2|n\rangle|^2. \tag{5}$$

Here, we have assumed that the averaging time T is short compared to the time scale over which the level populations vary, but large compared to the correlation time of the fluctuations so that the range of τ extends formally to $\pm\infty$. The correlation function for the fractional intensity fluctuations is defined as

$$\langle\epsilon(t)\epsilon(t+\tau)\rangle\equiv\frac{1}{T}\int_0^T dt\ \epsilon(t)\epsilon(t+\tau). \tag{6}$$

Using the transition matrix elements $(m\neq n)$ of x^2 and $\omega_{n\pm2,n}=\pm2\omega_{tr}$ in Eq. (5), the transition rates are given by

$$R_{n\pm2\leftarrow n}=\frac{\pi\omega_{tr}^2}{16}S_\epsilon(2\omega_{tr})(n+1\pm1)(n\pm1). \tag{7}$$

In Eq. (7), $S_\epsilon(\omega)$ is the one-sided power spectrum of the fractional intensity noise,

$$S_\epsilon(\omega)\equiv\frac{2}{\pi}\int_0^\infty d\tau\cos\omega\tau\langle\epsilon(t)\epsilon(t+\tau)\rangle. \tag{8}$$

The one-sided power spectrum is defined so that

$$\int_0^\infty d\omega\ S_\epsilon(\omega)=\int_0^\infty d\nu\ S_\epsilon(\nu)=\langle\epsilon^2(t)\rangle\equiv\epsilon_0^2, \tag{9}$$

where ϵ_0 is the root-mean-square fractional intensity fluctuation, and $\omega=2\pi\nu$, with ν the frequency in Hertz.

Assuming that the trapped atoms occupy state $|n\rangle$ with probability $P(n,t)$ at time t, the average heating rate is just

$$\langle\dot{E}\rangle=\sum_n P(n)2\hbar\omega_{tr}(R_{n+2\leftarrow n}-R_{n-2\leftarrow n})$$

$$=\frac{\pi}{2}\omega_{tr}^2S_\epsilon(2\omega_{tr})\langle E\rangle, \tag{10}$$

where the average energy $\langle E\rangle$ is $\langle E(t)\rangle=\Sigma_nP(n,t)(n+1/2)\hbar\omega_{tr}$.

Equation (10) shows that the average energy increases exponentially,

$$\langle\dot{E}\rangle=\Gamma_\epsilon\langle E\rangle, \tag{11}$$

where the rate constant Γ_ϵ is given by

$$\Gamma_\epsilon\equiv\frac{1}{T_I(\text{sec})}=\pi^2\nu_{tr}^2S_\epsilon(2\nu_{tr}). \tag{12}$$

Here ν_{tr} is the trap oscillation frequency in hertz and T_I is the energy e-folding time in seconds (time to increase $\langle E\rangle$ by a factor e).

An interesting feature of the heating rate is that $\hbar$ does not appear explicitly. Hence, the heating rate can be calculated classically. In this case, the time averaged rate of change of the total energy is just

$$\langle\dot{E}\rangle=\left\langle\frac{dH}{dt}\right\rangle=\left\langle\frac{\partial H}{\partial t}\right\rangle=\frac{M\omega_{tr}^2}{2T}\int_0^T dt\ \dot{\epsilon}(t)x^2(t), \tag{13}$$

where, according to Hamilton's equations, only the explicit time dependence in the Hamiltonian determines the rate of change of the energy. Integrating by parts and keeping terms in $x^2(t)$ to first order in $\epsilon(t)$ yields the same result as Eqs. (11) and (12).

The heating rate is proportional to the energy because the mean-square force fluctuations increase as the square of the distance from the trap center. The dependence of the heating rate on the second harmonic of the trap frequency shows that it is a parametric heating process. Laser intensity dependence arises through the trap oscillation frequency. The heating is not eliminated by using a blue detuned trap with the same oscillation frequency and fractional fluctuation in the spring constant.

We note that for a three-dimensional trap, assuming that the energy is distributed equally in all three dimensions, the effective energy e-folding rate will be the mean of the rates for the x, y, and z directions. The energy e-folding time directly limits the time that atoms can remain in the ground state of a dipole force trap. In addition, atoms will leave the trap when their mean energy $\langle E(t)\rangle$ is approximately equal to the well depth, V_0.

Equation (12) yields an energy e-folding time that is strongly dependent on the trap oscillation frequency. For the parameters of the red detuned traps that have been explored, we find that trap oscillation frequencies ν_{tr} span a wide range. For the CO_2 laser trap [7], we estimate an axial frequency of 6 Hz and a radial oscillation frequency of 300 Hz. For the YAG laser trap [5], we estimate an oscillation frequency of 8.5 kHz ($8.5\sqrt{2}$ kHz) in (perpendicular to) the plane of the crossed beams. For the krypton-ion laser trap [6], the axial (radial) oscillation frequency is reported as 10 Hz (2.3 kHz) for a waist of 36 μm, and we estimate 3.5 kHz

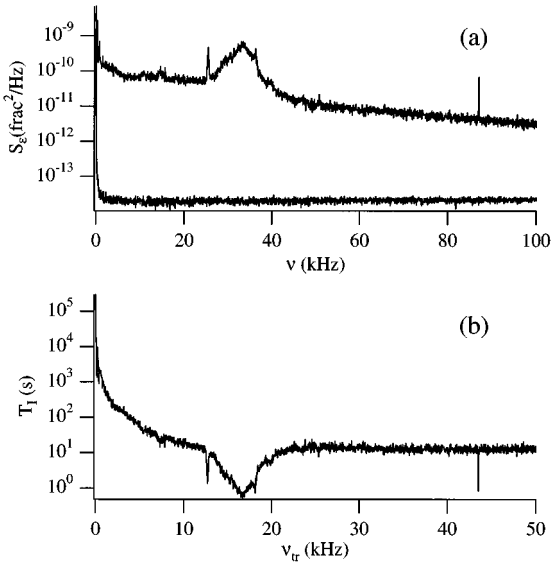

FIG. 1. (a) Intensity noise power spectrum $S_\epsilon(\nu)$ for an argon-ion laser with current regulation only. $S_\epsilon(\nu)$ is in units of fraction squared per hertz where the fractional intensity fluctuation is defined as $\epsilon(t) \equiv [I(t) - I_0]/I_0$. The lower curve shows the electronic noise spectrum, using a flashlight to obtain the same detector current; (b) the calculated energy e-folding time T_I is plotted versus trap frequency ν_{tr}.

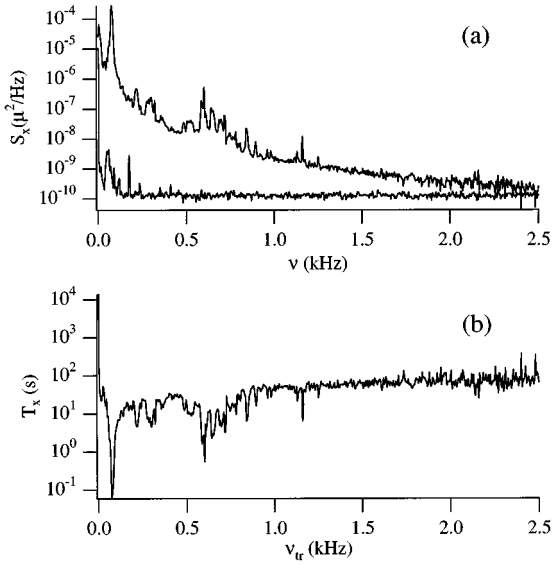

FIG. 2. (a) Position noise power spectrum $S_x(\nu)$ in units of micrometers squared per hertz (μ^2/Hz) for an argon-ion laser with current regulation only. The lower curve shows the electronic noise contribution to the spectrum, using a flashlight to obtain the same single detector current (one detector blocked); (b) the calculated mean energy doubling time T_x is plotted versus trap frequency ν_{tr}, for a sample confined initially to a 1 μm region, x_{rms}. Note that $T_x \propto x_{rms}^2$.

(119 kHz) for the reported waist of 5 μm. According to Eq. (12), to achieve an energy e-folding time of 100 sec in a trap with an oscillation frequency of 10 kHz requires $\sqrt{S_\epsilon(2\nu_{tr})} = 3\times10^{-6}$ Hz$^{-1/2}$. Hence, if most of the intensity noise were evenly distributed over a 40-kHz bandwidth, the rms fractional intensity noise must be better than $\epsilon_0 = 6\times10^{-4}$.

Since argon-ion lasers are often used to pump Ti:sapphire and dye lasers that are utilized in red detuned traps, and have noise spectra comparable to krypton-ion lasers, we have measured the fractional intensity noise power spectrum $S_\epsilon(\nu)$ for a Coherent model Innova 310-argon-ion laser at a power of 4 W, Fig. 1(a). This is accomplished using a low noise diode detector [13] illuminated with 0.7 mW and a Tektronix TDS644B digital oscilloscope with a low pass filter at 100 kHz (3-dB point). Spectra taken at higher bandwidths show that aliasing is not a problem. The integrated power spectrum yields a rms noise $\epsilon_0 = 0.26\times10^{-2}$, consistent with the manufacturer's specification and with the directly measured rms intensity fluctuations. Shot-noise contributes $S_{SN}(\nu) = 2h\nu_L/P$, where ν_L is the laser frequency in hertz, and P is the power in watts. This is of order 10^{-19} Hz^{-1} at 4 W and is negligible. Figure 1 (b) shows how the energy e-folding time T_I calculated from Eq. (12) varies with the choice of trap frequency for a laser with this noise power spectrum. A bad choice of trap frequency is near 17 kHz; however, low trap frequencies yield long heating times. Unfortunately, low trap frequencies usually correspond to smaller well depths, and less confinement. At high frequencies, the spectrum scales as ν^{-2}, and the energy e-folding time is constant at 10 sec.

From Eq. (12), we see that the heating rate scales with trap laser power P, since the trap frequency $\nu_{tr} \propto \sqrt{P}$. In addition, the laser noise power spectrum also may vary with laser power. For the Innova 310, we find that the intensity noise power decreases inversely as the laser power, but the spectrum remains constant. For other lasers, the amplitude of the noise spectrum may be independent of power. If the spectrum is also flat near the trap frequency, the energy e-folding time will scale as ν^2 and hence linearly with laser power.

In addition to intensity noise, laser-beam-pointing noise also must be stringently controlled [14]. In this case, the effective Hamiltonian can be taken to be of the form

$$H = \frac{p^2}{2M} + \frac{1}{2}M\omega_{tr}^2[x - \epsilon(t)]^2, \tag{14}$$

where $\epsilon(t)$ is now the fluctuation in the location of the trap center [15]. In this case, analogous to the methods used to obtain Eq. (11), quantum and classical calculations based on Eq. (14) yield

$$\langle \dot{E} \rangle = \frac{\pi}{2}M\omega_{tr}^4 S_x(\omega_{tr}). \tag{15}$$

Shaking the trap causes heating that is independent of the trap energy. Here, $S_x(\omega)$ is the one-sided power spectrum of the position fluctuations in the trap center, i.e., $\int_0^\infty d\omega S_x(\omega) = \epsilon_x^2$ is the mean-square variation in the trap center position, analogous to Eq. (9).

An energy-doubling time T_x can be defined as the time needed to increase the energy by the average energy at $t = 0$: $\langle \dot{E} \rangle / \langle E(0) \rangle \equiv 1/T_x$. Then, using $\langle E(0) \rangle = M\omega_{tr}^2 \langle x^2 \rangle$, where $\langle x^2 \rangle$ is the mean-square position of an atom in the trap at $t = 0$, one obtains

$$\frac{\langle \dot{E} \rangle}{\langle E(0) \rangle} \equiv \frac{1}{T_x(\mathrm{sec})} = \pi^2 \nu_{tr}^2 \frac{S_x(\nu_{tr})}{\langle x^2 \rangle}. \quad (16)$$

According to Eq. (16), if an atom is confined in a trap to a dimension of a 1 μm with an oscillation frequency of 10 kHz, achievement of an energy-doubling time of 100 sec requires a position stability of $\sqrt{S_x(\nu_{tr})} = 3 \times 10^{-6}$ μm/Hz$^{1/2}$.

We have measured the pointing noise for the Innova 310 argon-ion laser using a balanced detection system [13]. Quantum beam-pointing noise produces negligible heating [16], while classical noise can be important. In the experiments, two laser beams of power $P_0 = 0.7$ mW are generated with a beam splitter and focused onto two detectors with lenses ($f = 15$ cm). The intensity $1/e$ radii are $w = 14$ μm, comparable to dimensions used in some traps. One detector monitors a 50% attenuated beam, while the other detector measures the power transmitted past a razor blade that blocks half of one beam at the lens focus. The difference in the detector currents is proportional to the power change Δp arising from the beam displacement $\epsilon(t)$: $\Delta p = P_0 \epsilon(t)/(w\sqrt{\pi})$. Figure 2(a) shows the position noise power spectrum $S_x(\nu)$. This includes pointing noise from both the laser and the optical mounts. Figure 2(b) shows the energy-doubling time T_x calculated from this spectrum for a sample of atoms initially confined to a rms distance $x_{rms} = \sqrt{\langle x^2 \rangle} = 1$ μm. Micrometer dimensions are typical for the quantum scale of a low-frequency trap (100 Hz, Rb atoms) and for temperatures of a few hundred μK in a tight trap. Note that T_x scales as x_{rms}^2. We have not yet fully explored how the classical pointing noise scales with the beam spot size, w, but we expect the relative fluctuation ϵ/w to be independent of the lens focal length.

In conclusion, we have demonstrated that the attainment of heating times well beyond 10 sec imposes stringent requirements on the laser intensity and beam-pointing noise for both red and blue detuned optical traps. Observed heating rates in far-off resonance traps may arise from these noise sources, but it is not yet clear if additional mechanisms are at work. Cooling beams that excite atoms may cause additional fluctuations in the effective spring constant [17]. Subsequent parametric heating may reduce the cooling efficiency. All harmonic traps are affected by the same mechanisms in principle. For magnetic traps, $S_\epsilon(\nu)$ is determined by the power supply stability. In practice, however, heating rates in magnetic traps can be minimized, provided that the mechanical stability and power supplies are carefully controlled. Currently, we are measuring additional laser noise spectra and investigating the trap state dynamics and trap loss rates induced by laser fluctuations in a red detuned dipole force trap.

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