First-quantization quantum-mechanical insight into the Hong-Ou-Mandel two-photon interferometer with polarizers and its role as a quantum eraser

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A straightforward derivation of a first-quantization two-photon state vector is presented for the Hong-Ou-Mandel interferometer with a possible half-wave plate and possible linear analyzers in the ports. It is shown that one is dealing with a quantum eraser in a somewhat broader physical sense: It is not strictly the interference from version I of the experiment, the interference that has been suppressed by entanglement in version II, that is revived in version III; but it is an analogous interference. Besides, the coincidence probability for arbitrary-angle analyzers in the general case of arbitrary polarization in the lower arm of the interferometer is derived. The interference phenomenon of a complete decoupling of the partial loss of interference and the coincidence events in the ports is demonstrated. It is pointed out that the phenomenon of distant polarization (a special case of distant preparation) carries all the nonlocality between the two photons in the two ports. [S1050-2947(97)05707-7]

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I. INTRODUCTION

The *first aim* of this paper is to present a simple physical understanding of the somewhat specific nature of the quantum erasure phenomenon that occurs in the two-photon Hong-Ou-Mandel interferometer [1] with polarizers.

We are discussing an experimental situation that was verified quantitatively in a real experiment [2]. To put it concisely, it was aimed at measuring *coincidence* in the two ports of a balanced Hong-Ou-Mandel two-photon interferometer with a half-wave plate as a polarizer and an analyzer in each port.

If nothing is added at X, Y, and Z (see Fig. 1 version I), then in the balanced (i.e., equal paths $ABD=ACD$) interferometer a specific *interference* phenomenon sets in, consisting of a total *lack of coincidence*. The interference is due to a spatial overlap of the two photons, both in the upper port (U) and in the lower port (L).

To show this, we denote by $|U\rangle$ the spatial state vector of a photon that has been transmitted through the beam splitter and that is in the upper port, and by $|L\rangle$ the symmetrical case. (The wave function $\langle \mathbf{r}|U\rangle$, where $\mathbf{r}$ is the radius vector, is the localization amplitude of the photon without polarization.)

The *reflected* correspondingly moving photons are then, as is well known, in the states $i|U\rangle$ and $i|L\rangle$, respectively, where i is the imaginary unit. The (unnormalized) symmetrized two-identical boson state (omitting the irrelevant equal polarizations), in which the important cancellations take place, is then

$$|\chi\rangle_{12}^{(1)} \sim (|L\rangle_1 + i|U\rangle_1)\otimes(|U\rangle_2 + i|L\rangle_2) + (|U\rangle_1 + i|L\rangle_1)\otimes(|L\rangle_2 + i|U\rangle_2) = 2i(|L\rangle_1|L\rangle_2 + |U\rangle_1|U\rangle_2). \quad (1)$$

Thus only double photons in the lower or in the upper port can be detected in version I of the experiment.

II. IS A QUANTUM ERASER IN THE STRICT SENSE POSSIBLE?

To introduce a polarization distinction of the two photons defining version II, one puts a half-wave plate in the lower

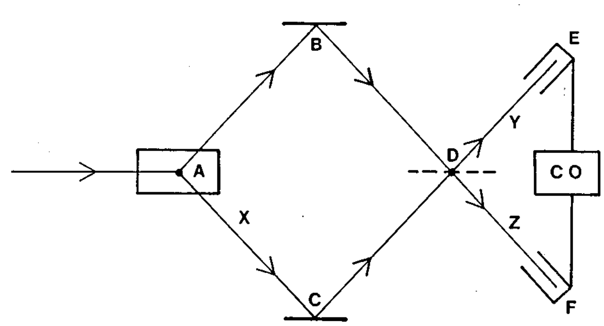

FIG. 1. A photon incoming from the left becomes down-converted (split) into two equal-energy (and equal horizontal polarization) photons in a nonlinear crystal parametric down-converter at point A. The mirrors at B and C and the equal paths ABD and ACD (upper and lower arms) make the two identical photons overlap at the beam splitter at D and in the upper (U) and the lower (L) ports DE and DF, respectively. At E and F two detectors work in coincidence. In version I, when no half-wave plate at X, and no analyzers at Y, and Z are used, one has interference consisting of a lack of coincidence. In version II, with a half-wave plate at X giving rise to vertical polarization in the lower arm, coincidence is restored. Finally, in version III, when besides the half-wave plate at X analyzers at equal angles are also put at Y and Z, the disappearance of coincidences is restored. In this way, in version III, the device acts as a kind of quantum eraser.

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arm (at point X in Fig. 1) with its optic axis at 45°. This makes the corresponding photon vertically polarized with respect to the plane of motion of the two photons. The photon in the upper arm is already polarized in the plane (horizontally polarized) due to the very downconversion.

Now one has complete distinction (analogously, as if one had different energies). As easily seen, half of all photon pairs end up in coincidence, and the interference is lost.

Scully, Englert, and Walther have pointed out [3] that in phenomena like this, interference has not been irretrievably destroyed; it is only suppressed due to entanglement. In this case the two-photon spatial state [cf. Eq. (1)], on the one hand, and the two-photon polarization state on the other become entangled.

In principle, the same interference can be revived in a so-called *quantum erasure* experiment (with some probability), if a certain two-photon polarization state is brought about in a suitable measurement.

To write down the composite-system state vector $|\chi\rangle_{12}$ $(\equiv|\chi\rangle_{12}^{(\mathrm{II})})$ of version II of the experiment in first quantization, we apply (first principle) symmetrization to the coherent sum of all four two-photon possibilities, when both photons have already passed the beam splitter and have entered the ports. We write the linear-polarization state of the photon as $|H\rangle$ in the case of horizontal, and $|V\rangle$ in the case of vertical polarization, respectively.

Enumerating the photons arbitrarily, taking into account the equal reflection and transmission probabilities on the beam splitter for each of the two photons, and the phase factor i due to reflection [cf. Eq. (1)], the entire symmetrized two-photon state vector (with polarization) is obviously

$$\begin{aligned}|\chi\rangle_{12}\equiv&(1/8)^{1/2}\{(i|H\rangle_1|U\rangle_1+|H\rangle_1|L\rangle_1)\\&\otimes(i|V\rangle_2|L\rangle_2+|V\rangle_2|U\rangle_2)+(i|V\rangle_1|L\rangle_1+|V\rangle_1|U\rangle_1)\\&\otimes(i|H\rangle_2|U\rangle_2+|H\rangle_2|L\rangle_2)\}.\end{aligned}\tag{2}$$

The normalization of the state vector $|\chi\rangle_{12}$ to 1 corresponds to preparation in which parametric down-conversion (at point A in Fig. 1) is, by definition, taken as a certain event. In the mentioned laboratory experiment [2] (to our understanding) the calculations were performed accordingly.

The entanglement (the departure from a simple tensor product) that is exhibited in Eq. (2) is due to symmetrization on account of the fact that the two photons, being of equal color, cannot be experimentally distinguished, i.e., they are identical bosons. (Note that the polarization state is included in the total state of the photon, hence it cannot be used to treat the photons as distinguishable.)

As a rule, one considers entanglement in two-particle state vectors. But, one can also take the more general view, and consider entanglement in any two-subsystem state vector, subsystems being characterized by separate tensor-factor state spaces. For our purpose of gaining insight, it will prove useful to consider polarization and the spatial degree of freedom of the two-photon system as subsystems, and to investigate the entanglement between them.

Utilizing the tensor factorization of each single-photon state vector into a polarization state vector and a spatial state vector, one can perform a suitable *rearrangement of the order* of the tensor factors in Eq. (2), grouping the polarization degrees of freedom and the spatial ones each separately together. (A similar rearrangement of tensor factors is performed in atomic and nuclear physics in transition from j-j to L-S coupling.)

In order to achieve expansion in terms of (entangled) *state vectors*, we also have to normalize the tensor factors. We write the state vectors thus obtained in square brackets. The state vectors $|H\rangle$ and $|V\rangle$ are unit vectors in the plane R_2 perpendicular to the direction of motion of the photon, and one has $\langle H|V\rangle=0$. Hence, after the rearrangement and the normalizations, one can rewrite the two-photon state vector $|\chi\rangle_{12}$ in the more suitable form

$$\begin{aligned}|\chi\rangle_{12}=&2^{-1/2}i[2^{-1/2}(|H\rangle_1|V\rangle_2+|V\rangle_1|H\rangle_2]\\&\otimes[2^{-1/2}(|U\rangle_1|U\rangle_2+|L\rangle_1|L\rangle_2]\\&+2^{-1/2}[2^{-1/2}(|H\rangle_1|V\rangle_2-|V\rangle_1|H\rangle_2)]\\&\otimes[2^{-1/2}(|L\rangle_1|U\rangle_2-|U\rangle_1|L\rangle_2].\end{aligned}\tag{3}$$

Thus, the two-photon state vector $|\chi\rangle_{12}$ in its final form (3) is seen to be a Schmidt *biorthogonal* expansion (cf. [4], where it was called the Schmidt canonical form). This means that both in the spatial and in the polarization two-photon spaces the state vectors displayed in Eq. (3) (as first and second tensor factors, respectively) are orthonormal, and the expansion coefficients are positive.

Viewing Eq. (3) as an expansion of $|\chi\rangle_{12}$ in the two state vectors

$$\begin{aligned}&[2^{-1/2}(|H\rangle_1|V\rangle_2+|V\rangle_1|H\rangle_2)],\\&[2^{-1/2}(|H\rangle_1|V\rangle_2-|V\rangle_1|H\rangle_2)]\end{aligned}\tag{4}$$

in the mentioned two-photon polarization space, we have precisely what is required for *quantum erasure* in the strict sense [3].

If one could perform in the state $|\chi\rangle_{12}$ a purely two-photon polarization-subsystem measurement defined by subbasis (4), in particular, if the former state in Eq. (4) came about, *ipso facto* one would revive the (two-photon spatial) interference state that corresponds to it in Eq. (3), i.e., the state

$$[2^{-1/2}(|U\rangle_1|U\rangle_2+|L\rangle_1|L\rangle_2)]$$

[cf. Eq. (1)] [and this would occur with the probability 1/2, cf. Eq. (3)].

Naturally, in an actual laboratory experiment one would correlate the results of the mentioned polarization measurement (in order to select out the required results) with suitable spatial measurements (that test the presence of the above spatial state). Theoretically, even delayed-choice variants might be taken into account [3].

Since a polarization-subsystem measurement of the mentioned kind would require a lack of influence on the spatial states, it is hard to see how it could be performed since the analyzers (with detectors) are always somewhere located, and hence they simultaneously also measure location. (This might be a challenge to the ingenious experimenter.)

Anyway, the Hong-Ou-Mandel two-photon intereferometer with polarizers as it stands, i.e., with the two ports meant to measure coincidence, is *not a quantum eraser in this strict sense*. Still, *it is a quantum eraser* in a physically important sense that will be explained below.

III. QUANTUM ERASER IN A BROADER SENSE

Keeping the half-wave plate at point X, we now add a linear-polarization analyzer in each of the ports: in the upper one it is at point Y and it is oriented at an arbitrary, fixed angle θ, and in the lower port it is at point Z oriented at the *same* angle (see Fig. 1).

We want to write down the symmetrized composite two-photon state vector immediately after they pass (or get extinguished in) the analyzers.

We have seen that after passing the beam splitter the photon ''remembers'' if it was reflected (transmitted) only through the presence (absence) of a phase factor i in its spatial state. Analogously, a previously polarized photon, after passing (or being extinguished in) the θ analyzer, ''remembers'' his previous polarization state through a factor (the square of which gives the probability of passing or being extinguished). These factors come from the decompositions

$$|H\rangle=\cos\theta|\theta\rangle-\sin\theta|\theta+90°\rangle,$$

$$|V\rangle=\sin\theta|\theta\rangle+\cos\theta|\theta+90°\rangle, \tag{5}$$

where $|\theta\rangle$ is the state after passing, and $|\theta+90°\rangle$ corresponds to being extinguished. (This state would actually appear if beam-splitting analyzers were used instead of ordinary ones.) Relation (5) is easily read from a R_2 diagram of unit vectors.

Thus, to write down the unnormalized composite-system state vector $|\chi\rangle_{12}^{(\mathrm{III})}$ of version III, we apply the symmetrizer $S_{12}\equiv 2^{-1}(1+E_{12})$ (where E_{12} is the exchange operator exchanging simultaneously both the polarization and the spatial single-photon state vectors) to the tensor product of the two one-photon state vectors.

Let us denote as photon 1 the one that comes from the upper arm with horizontal polarization, and as photon 2 the one coming from the lower arm with vertical polarization. Each photon has four possibilities: to be reflected from or transmitted through the beam splitter (equal probability amplitudes), and within each of these possibilities, the single photon can pass or be extinguished in the θ analyzer in the corresponding port [the probability amplitude, i.e., $\langle x|\theta°\rangle$, $x=H,V$, is clear from Eq. (5)]. Thus, we start with the unnormalized vector

$$\begin{aligned}S_{12}\{&(i\ \cos\theta|\theta\rangle_1|U\rangle_1-i\ \sin\theta|\theta+90°\rangle_1|U\rangle_1\\&+\cos\theta|\theta\rangle_1|L\rangle_1-\sin\theta|\theta+90°\rangle_1|L\rangle_1)\\&\otimes(\sin\theta|\theta\rangle_2|U\rangle_2+\cos\theta|\theta+90°\rangle_2|U\rangle_2\\&+i\ \sin\theta|\theta\rangle_2|L\rangle_2+i\ \cos\theta|\theta+90°\rangle_2|L\rangle_2)\}.\end{aligned} \tag{6}$$

Application of S_{12} in Eq. (6) makes equal those (pairs) of the 16 terms in which the two photons are in the same two-photon state (irrespectively whether photon 1 is in one of the states and 2 in the other or the other way round).

Straightforward evaluation of the action of S_{12} in Eq. (6) with subsequent rearrangement of the tensor factors and subsequent application of

$$P_{12}^{UL}\equiv I\otimes(|U\rangle_1|L\rangle_2\langle U|_1\langle L|_2+|L\rangle_1|U\rangle_2\langle L|_1\langle U|_2)$$

(''I'' being the identity operator in the two-photon polarization state) to make sure that we take into account only the coincidence photons (omit the doubles) finally gives

$$|\chi\rangle_{12}^{(\mathrm{III})}\sim S_{12}\{(|\theta+90°\rangle_1|\theta\rangle_2-|\theta\rangle_1|\theta+90°\rangle_2)\otimes|U\rangle_1|L\rangle_2\}. \tag{7}$$

It is clear from Eq. (7) that not a single pair of photons will pass the (parallel oriented) analyzers in coincidence. (Either the upper photon gets extinguished and the lower one passes or vice versa.)

This situation is *analogous* to that in version I: nothing reaches the detectors in coincidence. Two-photon-term cancellations take place owing to the prehistories of the single photons. This is a particular *interference* phenomenon similar to that in version I.

The experiment in version III, just like that in version I, brings about a nonlocal or *global* phenomenon. Namely, the single photons are at a macroscopic distance from each other, and it is, nevertheless, the whole (the two-photon state vector) that counts. Intuitively, this cannot be understood in the framework of classically trained local thinking. One wonders how do the two photons ''agree'' to eliminate the both-passing possibility without any interaction.

Namely, it might seem that what happens to the photon in the upper port and what happens to the one in the lower port should be two statistically independent events regardless of the two-photon state. But this is not so. And this is the wonder of nonseparability in quantum-mechanical distant correlations.

Finally, if we want a precise answer to the question in what sense is our device a *quantum eraser* in version III, it reads:

The point is in the term *physically analogous*, not *the same*. In version III, it is not the rearrangement of the photons into doubles (that may go into the upper or into the lower port), see version I, that takes place. It is another rearrangement. If one had beam-splitting analyzers instead of the one-way ordinary ones, one would ''see'' in the experiment what Eq. (7) tells us: that out of the four *a priori* possibilities only two take place. The pairs of photons get rearranged so that only the opposite-decision cases displayed in Eq. (7) occur.

To put it shortly, though in version III we are *not* dealing with a *quantum eraser in the strict sense* because it does not revive strictly the *same* interference phenomenon, *it is a quantum eraser* in a somewhat *broader sense* because it revives a *physically analogous interference phenomenon*.

IV. COINCIDENCE PROBABILITY IN CASE OF INDEPENDENT ARBITRARY ANALYZER ANGLES θ_U AND θ_L

The only generalization we introduce now is allowing the analyzer angles θ_U and θ_L in the two ports to be arbitrary (independently of each other). Since nothing is changed in the interferometer, the state vector $|\chi\rangle_{12}$ that describes the two-photon system entering the two ports is still given by Eq. (3).

The probability of coincidence $p_c(\theta_U,\theta_L)$ is the square norm of the projection of $|\chi\rangle_{12}$ with the (symmetrized) projector

$$P_{12}^{co}(\theta_U,\theta_L)\equiv|\theta_U\rangle_1|\theta_L\rangle_2\langle\theta_U|_1\langle\theta_L|_2\otimes|U\rangle_1|L\rangle_2\langle U|_1\langle L|_2 + |\theta_L\rangle_1|\theta_U\rangle_2\langle\theta_L|_1\langle\theta_U|_2\otimes|L\rangle_1|U\rangle_2\langle L|_1\langle U|_2 \quad (8)$$

the occurrence (result 1 in measurement) of which means passage through both analyzers.

Straightforward evaluation with the help of Eq. (3) and (8) gives

$$\begin{aligned}P_{12}^{co}(\theta_U,\theta_L)|\chi\rangle_{12} &= 8^{-1/2}\{(\langle\theta_L|_1|H\rangle_1\langle\theta_U|_2|V\rangle_2 \\ &-\langle\theta_L|_1|V\rangle_1\langle\theta_U|_2|H\rangle_2)|\theta_L\rangle_1|\theta_U\rangle_2|L\rangle_1|U\rangle_2) \\ &-(\langle\theta_U|_1|H\rangle_1\langle\theta_L|_2|V\rangle_2 \\ &-\langle\theta_U|_1|V\rangle_1\langle\theta_L|_2|H\rangle_2)|\theta_U\rangle_1|\theta_L\rangle_2|U\rangle_1|L\rangle_2\}.\end{aligned}$$

It is now suitable to decompose the linear-polarization state vectors $|H\rangle$ and $|V\rangle$ along a linear-polarization state $|\theta_X\rangle$ and its perpendicular $|\theta_X+90°\rangle$, where $X\equiv U,L$ [cf. Eq. (5)], and make substitution in the last relation

$$\begin{aligned}P_{12}^{co}(\theta_U,\theta_L)|\chi\rangle_{12} &= 8^{-1/2}\{(\cos\theta_L\sin\theta_U \\ &-\sin\theta_L\cos\theta_U)|\theta_L\rangle_1|\theta_U\rangle_2|L\rangle_1|U\rangle_2 \\ &-(\cos\theta_U\sin\theta_L-\sin\theta_U\cos\theta_L) \\ &\times|\theta_U\rangle_1|\theta_L\rangle_2|U\rangle_1|L\rangle_2\} \\ &= 8^{-1/2}\{\sin(\theta_U-\theta_L)[|\theta_L\rangle_1|\theta_U\rangle_2|L\rangle_1|U\rangle_2 \\ &+|\theta_U\rangle_1|\theta_L\rangle_2|U\rangle_1|L\rangle_2]\}.\end{aligned}$$

Hence,

$$p_c(\theta_U,\theta_L)=\sin^2(\theta_U-\theta_L)/4. \quad (9)$$

The minimal value of $p_c(\theta_U,\theta_L)$ is zero. This is the case discussed in the preceding section. The maximal value is (1/4). The probability to obtain coincidence photons (in contrast to doubles) is 1/2 [cf. Eq. (3)]. And, in the maximal-value case, 1/2 is also the probability of coincidence after passing the analyzers with the angles θ_U and $\theta_L\equiv\theta_U+90°$. We discuss the nonlocality inherent in this claim at the end of Sec. V.

V. COMPLETE DECOUPLING IN THE GENERAL CASE

If one puts a half-wave plate in the lower arm of the interferometer with its optical axis at $\phi/2$, $0<\phi\leq 90°$, i.e., if the lower-arm photon is polarized at the angle ϕ (with the horizontal plane), then we have the general case.

Now we write the two-photon state vector in the ports (before reaching the possible analyzers) as $|\chi,\phi\rangle_{12}$. We must start with generalizing Eq. (2)

$$\begin{aligned}|\chi,\phi\rangle_{12} &\equiv (1/8)^{1/2}\{(i|H\rangle_1|U\rangle_1+|H\rangle_1|L\rangle_1) \\ &\otimes(i|\phi\rangle_2|L\rangle_2+|\phi\rangle_2+|\phi\rangle_2|U\rangle_2) \\ &+(i|\phi\rangle_1|L\rangle_1+|\phi\rangle_1|U\rangle_1)\otimes(i|H\rangle_2|U\rangle_2 \\ &+|H\rangle_2|L\rangle_2)\}.\end{aligned}$$

(The normalization factor is not obvious because the eight two-photon terms are not all orthonormal. But, utilizing $\langle H|\phi\rangle=\cos\phi$, one easily evaluates that, nevertheless, $\langle\chi,\phi|_{12}|\chi,\phi\rangle_{12}=1$.) Again, the entanglement comes from symmetrization.

Rearrangement of the tensor factors as in Sec. II leads to

$$\begin{aligned}|\chi,\phi\rangle_{12} &= (1/8)^{1/2} i\{(|H\rangle_1|\phi\rangle_2+|\phi\rangle_1|H\rangle_2) \\ &\otimes(|U\rangle_1|U\rangle_2+|L\rangle_1|L\rangle_2)+(|H\rangle_1|\phi_2-|\phi\rangle_1|H\rangle_2) \\ &\otimes(|L\rangle_1|U\rangle_2)-|U\rangle_1|L\rangle_2)\}.\end{aligned}$$

It is useful to normalize the two-photon polarization and spatial tensor factors (writing the state vectors in square brackets)

$$\begin{aligned}|\chi,\phi\rangle_{12} &= i((1+\cos^2\phi)/2)^{1/2}[(2(1+\cos^2\phi))^{-1/2} \\ &\times(|H\rangle_1|\phi\rangle_2+|\phi\rangle_1|H\rangle_2)] \\ &\otimes[2^{-1/2}(|U\rangle_1|U\rangle_2+|L\rangle_1|L\rangle_2)] \\ &+2^{-1/2}\sin\phi[2^{-1/2}\sin^{-1}\phi \\ &\times(|H\rangle_1|\phi\rangle_2-|\phi\rangle_1|H\rangle_2)] \\ &\otimes[2^{-1/2}(|L\rangle_1|U\rangle_2-|U\rangle_1|L\rangle_2)]. \quad (10)\end{aligned}$$

The second (i.e., the spatial two-photon) tensor factors in Eq. (10) are orthogonal, hence the probability of *coincidence* (in contrast to detection of two photons in the same detector) without polarizers in the ports, i.e., of *pure (complete or partial) loss of interference* is obviously

$$d_{li}=\sin^2\phi/2. \quad (11)$$

We call this *degree of loss of interference*.

In the extreme case of $\phi\equiv 90°$ (when ϕ is replaced by V), this probability is one-half. This is the case of *complete loss of interference* because we have the same probability when the interferometer is unbalanced, i.e., when, e.g., the reflected spatial state $i|U\rangle$ coming from the upper arm and the transmitted state $|U\rangle$ coming from the lower arm are nonoverlapping (the time difference of arrival is larger than the coherence time), i.e., when these state vectors are orthogonal (hence distinct). Then, as an obvious consequence of the equal probability of reflection and transmission on the beam splitter, half of the photons gives coincidence and half of them goes into doubles (both photons in the same port).

The other extreme, when $\phi\equiv 0°$ (when ϕ is replaced by H), makes the second term in Eq. (10) disappear. The probability is zero and we have *interference*. If $0<\phi<90°$, we have *partial loss of interference*, i.e., the degree of loss of interference d_{1i} is larger than zero but smaller than one-half. So much for the case of *pure interference* and *loss of it*. As it was stated, we have this case as long as we do not put in the θ_U and θ_L polarizers in the upper and the lower detector ports, respectively.

Decomposing $|\phi\rangle$ along $|H\rangle$ and $|V\rangle$, i.e., substituting $|\phi\rangle=\cos\phi|H\rangle+\sin\phi|V\rangle$ in the first tensor factor state vector in the second term in Eq. (10), and utilizing Eq. (11), one obtains, perhaps surprisingly.

$$\begin{aligned}|\chi,\phi\rangle_{12}=&\,i((1+\cos^2\phi)/2)^{1/2}[(2(1+\cos^2\phi))^{-1/2}\\&\times(|H\rangle_1|\phi\rangle_2+|\phi\rangle_1|H\rangle_2)]\\&\otimes[2^{-1/2}(|U\rangle_1|U\rangle_2+|L\rangle_1|L\rangle_2)]\\&+(d_{li})^{1/2}\sin\phi[2^{-1/2}(|V\rangle_1|H\rangle_2-|H\rangle_1|V\rangle_2)]\\&\otimes[2^{-1/2}(|L\rangle_1|U\rangle_2-|U\rangle_1|L\rangle_2)].\end{aligned}\tag{12}$$

The composite two-photon state vector in the second term, which corresponds to the coincidence photons that we are interested in, is the *same* as in Eq. (3), when we had $\phi\equiv 90°$. They only differ in the expansion coefficient.

Thus, we have *complete decoupling* of the coincidence events in the ports from the loss of interference. This decoupling is another *two-particle interference phenomenon*, because it is the result of superposition of tensor products.

One may wonder if the horizontal plane plays a privileged role in the two-photon polarization state $[2^{-1/2}(|V\rangle_1|H\rangle_2-|H\rangle_1|V\rangle_2)]$ in Eq. (12) or it only appears so. The latter is the case, because, substituting here Eq. (5), this same (tensor factor) state vector becomes

$$[2^{-1/2}(|\theta+90°\rangle_1|\theta\rangle_2-|\theta\rangle_1|\theta+90°\rangle_2)]\tag{13}$$

with an arbitrary angle θ.

Two-term entanglement in a two-photon polarization state vector of the concrete form

$$[2^{-1/2}(|V\rangle_1|H\rangle_2-|H\rangle_1|V\rangle_2)]$$

has been investigated experimentally before, in the positronium-annihilation experiments of Kasday, Ullman and Wu [5,6]. (It is a variation of the Wu-Shaknov experiment [7] from 1950. See also the review article [8].)

It is a technical advantage of the Kwiat, Steinberg, and Chiao experiment [2] (for polarization analysis and detection) that the photons are not of such a high energy as in the Kasday, Ullman, and Wu experiments.

The coincidence probability $p_c(\phi,\theta_u,\theta_L)$, in the general case, is the square of the projection of $|\chi,\phi\rangle_{12}$ [given by Eq. (12)] with $P_{12}^{co}(\theta_U,\theta_L)$ [cf. Eq. (8)]. Evidently, only the second term in Eq. (12) is relevant, and it differs from $|\chi\rangle_{12}$ given by Eq. (3) only by the factor $\sin\phi$. Hence, obviously,

$$p_c(\phi,\theta_U,\theta_L)=\sin^2\phi\, p_c(\theta_U,\theta_L),$$

and, finally, utilzing Eq. (9), one obtains

$$p_c(\phi,\theta_U,\theta_L)=(1/4)\sin^2\phi\ \sin^2(\theta_U-\theta_L).\tag{14}$$

Naturally, the described decoupling of the coincidence events in the ports from the loss of interference is seen also in Eq. (14). The probability $p_c(\phi,\theta_U,\theta_L)$ generalizes $p_c(\theta_U,\theta_L)$, which is valid for $\phi=90°$. Relation (14) equals Eq. (A5) in [2] except for the factor (1/4) [9].

Conceptually, Eq. (14) should be read as follows: $(\sin^2\phi/2)$ is the probability of coincidence photons (in contrast to doubles), as seen from Eq. (11), and $(\sin^2(\theta_U-\theta_L)/2)$ is the probability that both photons pass the respective analyzers on the condition that the photons are already in one port each. [Note that in Eq. (9) (1/2) is the probability of this condition and the conditional probability is the same as in Eq. (14).]

There is a striking *nonlocality* inherent in the conditional probability $(\sin^2(\theta_U-\theta_L)/2)$. To see it, we imagine that the photon in the lower port reaches its analyzer (after some detour) somewhat later than the photon in the upper port. The latter one has a probability of 1/2 of passing the θ_U analyzer [as obvious from Eq. (13), which can replace the two-photon polarization tensor factor state vector in the second term in Eq. (12)]. Taking $\theta_L\equiv\theta_U+90°$, the conditional probability is (1/2). Hence, the photon in the lower port has a probability 1 to pass its analyzer. Namely, this photon is, by the very act of the upper port photon's passage through its polarizer, *distantly polarized* in the state $|\theta_U+90°\rangle$.

Distant polarization is a special case of distant preparation (as one calls nowadays Schrödinger's ''steering'' [10].) In our version above of the general case with the lower-port photon having a detour, after passage of the upper-port photon through its θ_U analyzer, the former photon is distantly polarized in the state $|\theta_U+90°\rangle$ [cf. Eq. (13) replaced in Eq. (12)]. The probability of passage through the θ_L analyzer is precisely $(\sin^2(\theta_U-\theta_L)/2)$ as easily seen. Thus, in general, the entire nonlocal phenomenon consists in the distant polarization. It comes about without interaction (between the two photons or between the upper-port analyzer and the lower-port photon) in an apparently magic quantum-mechanical way.

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