Influence of nonlinearity in one-photon processes on the relationship between field and dipole squeezing in the two-level thermal Jaynes-Cummings model

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(Received 11 November 1996; revised manuscript received 3 February 1997)

This paper describes the influence of nonlinear interaction of a two-level atom with a single-mode field via one-photon interaction on the squeezing properties of the radiation and the fluctuations of the atomic dipole variables. The thermal Jaynes-Cummings model nonlinear in occupation number is used. [S1050-2947(97)04806-3]

PACS number(s): 42.50.Dv, 32.80.−t

I. INTRODUCTION

The possibility of generating squeezed states of the radiation field in the laboratory have opened new perspectives in quantum optics and laser physics with potential applications in high-resolution spectroscopy, quantum nondemolition experiments, quantum communications, and low light level microscopy [1]. The Jaynes-Cummings model (JCM) in the rotating wave approximation has been a subject of intense investigations [2–5]. Recently, increased attention has been paid to the atomic coherent states [6]. It was found that the JCM initially in an atomic coherent state together with the vacuum field can generate field squeezing as well as squeezing of the fluctuations of the atomic dipole variables [3–5] and it has been shown that squeezed atoms can radiate squeezed light [3,4]. The relationship between the field and atomic squeezing in the thermal JCM with an initially coherent atom has been discussed by a few authors [4,7]. The problem of interaction of matter with squeezed light has been extensively studied for the past ten years [8–10]. However, the influence of nonlinearity in one-photon process on the squeezing properties of the radiation field and the fluctuations of the atomic dipole variables has not been studied earlier. Nonlinear one-photon processes are important for understanding the generation of squeezed states in off-resonant fluorescence [11]. One-photon excitation causes significant attenuation of the incident signal without being saturated. This makes it a better candidate over multiphoton process as an effective nonlinear filtering process [12]. Consequently, it will be worthwhile to examine the influence of nonlinear interaction of a two-level atom with a single-mode field via one-photon transition on the relationship between the field and atomic squeezing in the thermal JCM for an initially coherent atom.

II. MODEL

To obtain the nonlinearities in one-photon interaction processes, it is necessary to go one order higher in perturbation theory (Fig. 1), which is, in fact, a second-order correction to Rayleigh scattering. The Hamiltonian for our system of interest may be written as

$$H=H_0+H_n, \tag{1}$$

where

$$H_0=\Omega a^\dagger a+\frac{\omega}{2}S_z+g(a^\dagger S_-+aS_+) \tag{2}$$

is the usual Hamiltonian for the JCM in the rotating wave approximation and

$$H_n=\lambda a^\dagger a a^\dagger a \tag{3}$$

is the Hamiltonian nonlinear in occupation number (i.e., two one-photon process). The nonlinear term H_n is actually the second-order correction to Rayleigh scattering. An exact solution of Eq. (1) can be derived if the field is detuned from the resonance frequency of the atom by an amount equal to the nonlinear parameter λ (i.e., if $\omega-\Omega=\lambda$). The model now describes the nonlinear interaction of a two-level atom with a single-mode field via one-photon transition. S_z, S_-, and S_+ are atomic pseudospin inversion, lowering and raising operators, respectively. ω is the atomic transition frequency, $a^\dagger$ and a are boson creation and annihilation operators. Ω is the free field-mode frequency. The linear atom-field coupling constant g is retained because the transitions are still one-photon process. λ is a constant describing the

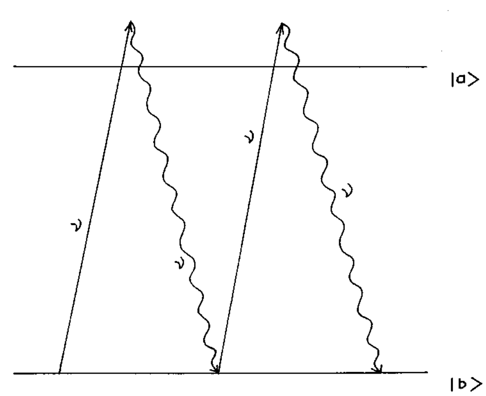

FIG. 1. Second-order correction to Rayleigh scattering.

extent of nonlinearity in occupation number. Throughout we employ $\hbar=c=1$. The density operator ρ of the thermal field is

$$\rho=\sum P_n|n\rangle\langle n|, \tag{4}$$

where the photon number distribution function P_n has the form

$$P_n=\overline{n}^n/(1+\overline{n})^{n+1}, \tag{5}$$

where $\overline{n}$ is the initial mean photon number. If the field is initially in a number state $|n\rangle$ and the atom is prepared in a coherent superposition of its ground state $|-\rangle$ and excited $|+\rangle$ states, the initial state of the atom field system reads

$$|\Psi(0)\rangle=\sin(\Theta/2)e^{-i\phi/2}|-\rangle+\cos(\Theta/2)e^{i\phi/2}|+\rangle, \tag{6}$$

here Θ denotes the distribution of the initial atom ranging from 0 to π and ϕ is the phase of the atomic coherent state. The time-dependent Schrödinger equation with the initial condition (6) gives the general time-dependent state $|\Psi(t)\rangle$ of the system with the detuning factor $\delta=\omega-\Omega=\lambda$,

$$\begin{aligned}|\Psi(t)\rangle=&\sum_{n=0}^{\infty}\cos(\Theta/2)f_n e^{-i[(n+1/2)\Omega' t+\lambda n^2 t-\phi/2]}\{\cos(1/2)Rt-i(\beta/R)\sin(1/2)Rt\}|+,n\rangle\\&-i\sum_{n=1}^{\infty}\sin(\Theta/2)f_n e^{-i[(n-1/2)\Omega' t+\lambda(n-1)^2 t+\phi/2]}(R_0'/R')\sin(1/2)R't|+,n-1\rangle\\&+\sum_{n=0}^{\infty}\sin(\Theta/2)f_n e^{-i[(n-1/2)\Omega' t+\lambda(n-1)^2 t+\phi/2]}\{\cos(1/2)R't+i(\beta'/R')\sin(1/2)R't\}|-,n\rangle\\&-i\sum_{n=0}^{\infty}\cos(\Theta/2)f_n e^{-i[(n+1/2)\Omega' t+\lambda n^2 t-\phi/2]}(R_0/R)\sin(1/2)Rt|-,n+1\rangle,\end{aligned} \tag{7}$$

where $\Omega'=\Omega+\lambda$, $|f_n|^2=P_n$, R and R' are the quantum Rabi frequencies of the oscillations of the model given as

$$R=[4\lambda^2n^2+4g^2(n+1)]^{1/2},\quad R'=[4\lambda^2(n-1)^2+4g^2n]^{1/2}, \tag{8}$$

and

$$\beta=2\lambda n,\quad R_0=2g(n+1)^{1/2},$$
$$\beta'=2\lambda(n-1),\quad R_0'=2g(n)^{1/2}.$$

The state $|-,n\rangle$ is coupled with the state $|+,n-1\rangle$ and the state $|+,n\rangle$ is coupled with the state $|-,n+1\rangle$. In order to investigate the squeezing properties of the radiation field and the atom, we follow the standard procedure of defining the slowly varying operators

$$a_1=(1/2)[ae^{i\Omega t}+a^\dagger e^{-i\Omega t}],\quad a=(1/2)i[ae^{i\Omega t}-a^\dagger e^{-i\Omega t}], \tag{9}$$

$$S_1=(1/2)[S_+e^{-i\omega t}+S_-e^{i\omega t}],$$

$$S_2=(1/2)i[S_+e^{-i\omega t}-S_-e^{i\omega t}]. \tag{10}$$

The above operators follow the commutation relations

$$[a_1,a_2]=i/2,\qquad [S_1,S_2]=iS_z. \tag{11}$$

The corresponding Heisenberg uncertainty relations are

$$(\Delta a_1)^2(\Delta a_2)^2>1/16,\quad (\Delta S_1)^2(\Delta S_2)^2>(1/4)\langle S_z\rangle^2, \tag{12}$$

III. FIELD AND ATOMIC SQUEEZING

We define the functions

$$f_i=(\Delta a_i)^2-1/4,\qquad d_i=(\Delta S_i)^2-(1/2)|\langle S_z\rangle|,\quad i=1,2. \tag{13}$$

Then field squeezing is defined if $f_i<0$ [13] and atomic squeezing if $d_i<0$ [14]. Using the general time-dependent state vector of Eq. (7), the functions of Eqs. (13) are written as

$$f_1=\overline{n}/2+(1/2)[\cos^2(\Theta/2)-q\ \sin^2(\Theta/2)]\sum_{n=0}^{\infty}(R_0^2/R^2)P_n\sin(1/2)Rt-(1/4)\sin^2\Theta$$
$$\times\left\{\sum_{n=0}^{\infty}P_n\sin[\phi-\lambda(2n-1)t][(n+1)^{1/2}(R_0/R)\sin(1/2)Rt\ \cos(1/2)R't-n^{1/2}(R_0'/R')\sin(1/2)R't\ \cos(1/2)Rt]\right\}^2, \tag{14}$$

$$f_2=\overline{n}/2+(1/2)[\cos^2(\Theta/2)-q\ \sin^2(\Theta/2)]\sum_{n=0}^{\infty}(R_0^2/R^2)P_n\sin(1/2)Rt-1/4\ \sin^2\Theta$$
$$\times\left\{\sum_{n=0}^{\infty}P_n\cos[\phi-\lambda(2n-1)t][(n+1)^{1/2}(R_0/R)\sin(1/2)Rt\ \cos(1/2)R't-n^{1/2}(R_0'/R')\sin(1/2)R't\ \cos(1/2)Rt]\right\}^2, \tag{15}$$

$$d_1=\tfrac{1}{4}-\tfrac{1}{4}\ \sin^2\Theta\sum_{n=0}^{\infty}P_n^2\ \cos^2[\phi-\lambda(2n-1)t]\{\cos(1/2)Rt\ \cos(1/2)R't-(\beta/R)(\beta'/R')\sin(1/2)Rt\ \sin(1/2)R't\}^2$$
$$-\tfrac{1}{2}\left|\tfrac{1}{2}\cos\Theta-\sum_{n=0}^{\infty}P_n(R_0^2/R^2)(\cos^2\Theta/2-q\ \sin^2\Theta/2)\sin^2(1/2)Rt\right|, \tag{16}$$

$$d_2=\tfrac{1}{4}-\tfrac{1}{4}\sin^2\Theta\sum_{n=0}^{\infty}P_n^2\sin^2[\phi-\lambda(2n-1)t]\{\cos(1/2)Rt\ \cos(1/2)R't-(\beta/R)(\beta'/R')\sin(1/2)Rt\ \sin(1/2)R't\}^2$$
$$-\tfrac{1}{2}\left|\tfrac{1}{2}\cos\Theta-\sum_{n=0}^{\infty}P_n(R_0^2/R^2)(\cos^2\Theta/2-q\ \sin^2\Theta/2)\sin^2(1/2)Rt\right|, \tag{17}$$

where $q=\overline{n}/(1+\overline{n})$.

IV. DISCUSSION

A. The case of $\overline{n}=0$

Taking the fluctuations in S_1 and a_2 as an example, we now study the relationship between dipole squeezing and field squeezing in the vacuum field. Putting $\overline{n}=0$ into Eqs. (15) and (16), we have

$$f_2=\tfrac{1}{2}\cos^2(\Theta/2)\sin^2gt$$
$$-\tfrac{1}{4}\{\sin\Theta\ \cos[\phi+\lambda t]\sin gt\ \cos\lambda t\}^2, \tag{18}$$

$$d_1=\tfrac{1}{4}-\tfrac{1}{4}\sin^2\Theta\ \cos^2[\phi+\lambda t]\cos^2gt\ \cos^2\lambda t$$
$$-\tfrac{1}{2}|\tfrac{1}{2}\cos\Theta-\cos^2\Theta/2\ \sin^2gt|. \tag{19}$$

We notice from Eqs. (18) and (19) that the maximum squeezing $A=0.0625$ can be obtained for atomic squeezing as $\phi=0$, $\lambda t=k\pi$ ($k=0$, integer), $gt=k\pi$, and $\Theta=2\pi/3$, $\pi/3$ and for field squeezing as $\phi=0$, $\lambda t=k\pi$, $gt=(k+\frac{1}{2})\pi$, and $\Theta=2\pi/3$. Taking the case of $\phi=0$, $gt=2.5\pi$ for field squeezing, and $gt=3\pi$ for atomic squeezing, Figs. 2(a) and 2(b) show how the squeezing of the radiation field (f_2) and the fluctuations of the atomic dipole variables (d_1) versus Θ changes with the nonlinear parameter λ. For $0<\Theta<\pi/2$ the fluctuations in S_1 can be squeezed but those in a_2 cannot, while for $\pi/2<\Theta<\pi$, the fluctuations in S_1 and a_2 can be squeezed almost all the time with identical squeeze duration and there exists a symmetry between the field and atomic squeezing (SFAS). With the increase of λ, we find that both field and atomic squeezing (and SFAS) start to disappear simultaneously. Furthermore, on increasing λ, the width of the the Θ interval in which squeezing appears, decreases by the same amount for both d_1 and f_2.

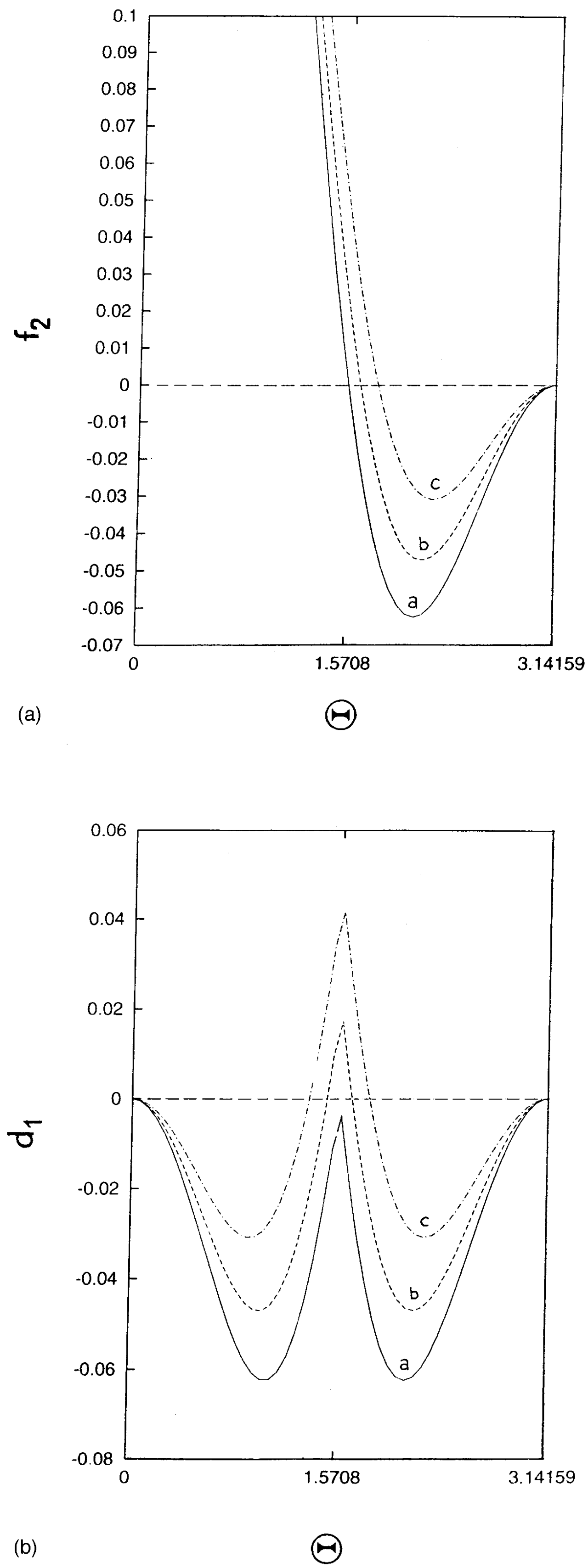

FIG. 2. (a) f_2 $(gt=2.5\pi)$ versus Θ for $\phi=0$ and (a) $\lambda t=0$, (b) $\lambda t=\pi/15$, and (c) $\lambda t=\pi/10$. (b) d_1 $(gt=3\pi)$ versus Θ for $\phi=0$ and (a) $\lambda t=0$, (b) $\lambda t=\pi/15$, and (c) $\lambda t=\pi/10$.

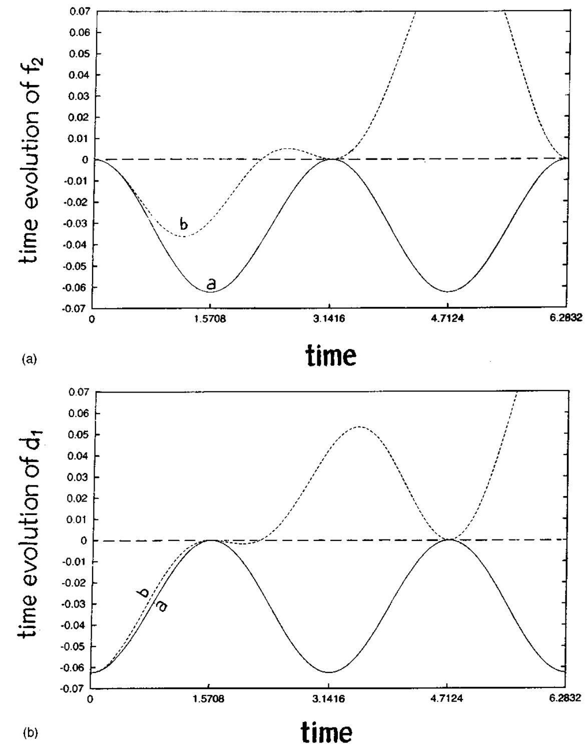

FIG. 3. (a) Time evolution of f_2 for $\Theta=2\pi/3$, $\phi=0$, $g=1.0$, and (a) $\lambda=0$ and (b) $\lambda=0.2$. (b) Time evolution of d_1 for $\Theta=2\pi/3$, $\phi=0$, $g=1.0$, and (a) $\lambda=0$ and (b) $\lambda=0.2$.

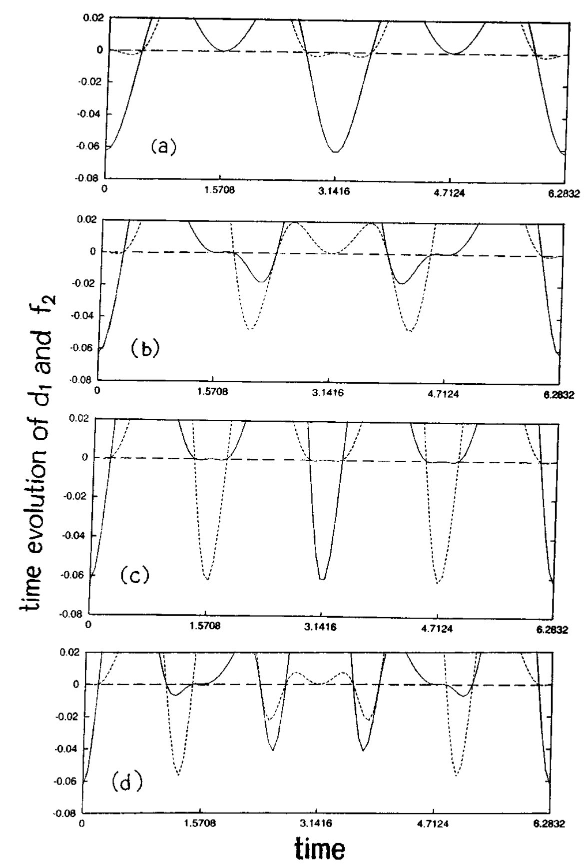

FIG. 4. Time evolution of f_2 (dotted line) and d_1 (solid line) for $\Theta=2\pi/3$, $\phi=0$, and $g=1.0$. (a) $\lambda=1.0$, (b) $\lambda=1.5$, (c) $\lambda=2.0$, and (d) $\lambda=2.5$.

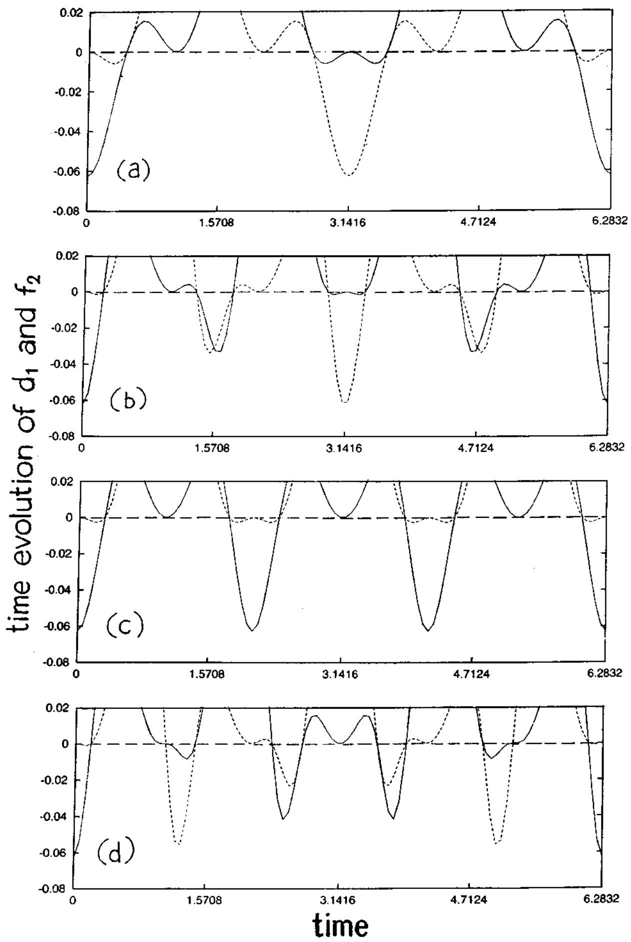

FIG. 5. Time evolution of f_2 (dotted line) and d_1 (solid line) for $\Theta=2\pi/3$, $\phi=0$, and $g=1.5$. (a) $\lambda=1.0$, (b) $\lambda=1.5$, (c) $\lambda=2.0$, and (d) $\lambda=2.5$.

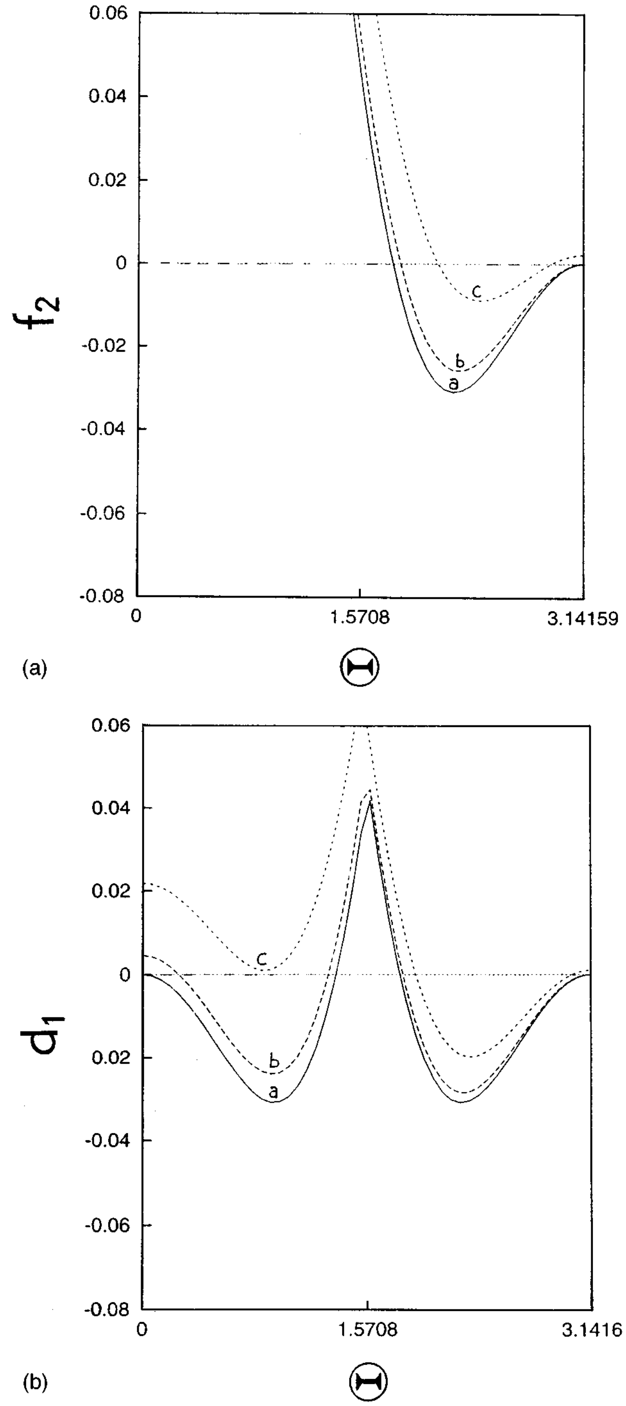

FIG. 6. (a) f_2 ($\phi=0$, $\lambda=0.1$, $g=1.5$, and $t=\pi$) versus Θ for (a) $\bar{n}=0$, (b) $\bar{n}=0.01$, and (c) $\bar{n}=0.05$. (b) d_1 ($\phi=0$, $\lambda=0.1$, $g=1.0$, and $t=\pi$) versus Θ for (a) $\bar{n}=0$, (b) $\bar{n}=0.01$, and (c) $\bar{n}=0.05$.

This demonstrates that the sensitivity of atomic and field squeezing to variations in λ is the same throughout the Θ range and at a particular instant of time, as long as squeezing occurs, variations in λ cannot destroy SFAS. In Figs. 3(a) and 3(c), we show the time evolution of d_1 and f_2 for $\Theta=2\pi/3$, $g=1$ and $\lambda<1$ (0,0.2). It is seen that for $\lambda=0$, field and atomic squeezing appear almost all the time, and the SFAS is shown clearly. For $\lambda=0.2$, we find that the initial atomic and field squeezing is revoked and is never seen to become squeezed again till $t=2\pi$, but recur in the long time scale. From the experimental point of view, the result of long time scale is not practical, so for experimentally relevant time scales, squeezing is restored periodically for large nonlinearity. Figures 4 and 5 show how the time evolution of d_1 and f_2 changes with variations in both g and large nonlinearity ($\lambda>1$). It is seen that for a fixed g and with the increase in λ, the initial squeezing is revoked but recurs periodically. The higher the value of λ, the more rapidly the squeezing is revoked and the periodical revival of squeezing as well as SFAS increases (i.e., the oscillations become more regular). However, it is noticed that with increasing g, there is no change in the periodicity of revival. Furthermore, the SFAS duration is seen to decrease with increase in λ. The field squeezing (and SFAS) present near $t=0$, gradually disappears, with increase in λ, while no such effect is seen on atomic squeezing at $t=0$. The influence of g on field squeezing at $t=0$ is seen to be exactly opposite to that of λ, while atomic squeezing at $t=0$ is insensitive to variations in g.

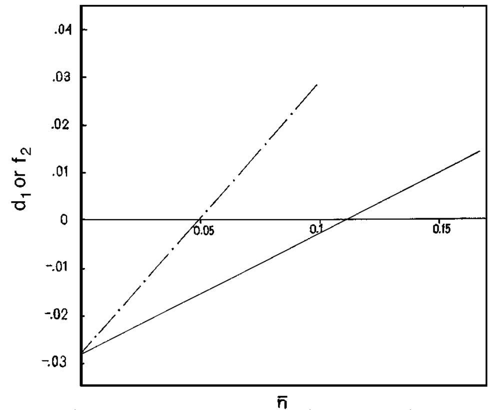

FIG. 7. f_2 ($g=1.5$, dot-dashed line) and d_1 ($g=1.0$, the line) versus $\overline{n}$ for $\Theta=2\pi/3$, $\phi=0$, $\lambda=0.1$, and $t=\pi$.

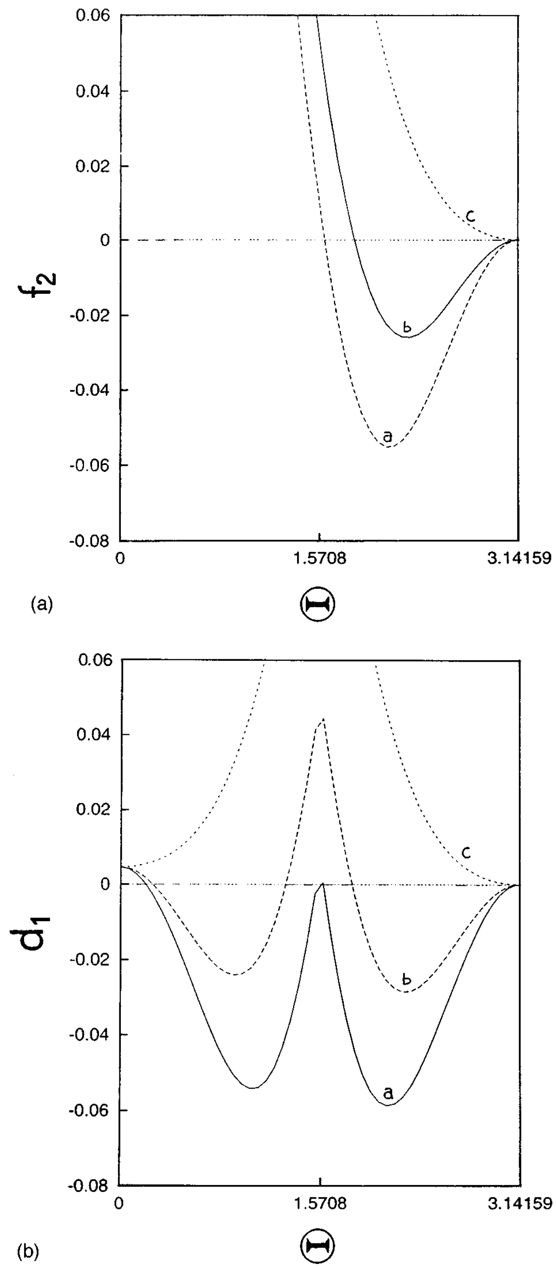

FIG. 8. (a) f_2 ($\phi=0$, $g=1.5$, $t=\pi$, and $\overline{n}=0.01$) versus Θ for (a) $\lambda=0$, (b) $\lambda=0.1$, and (c) $\lambda=0.2$. (b) d_1 ($\phi=0$, $g=1.0$, $t=\pi$, and $\overline{n}=0.01$) versus Θ for (a) $\lambda=0$, (b) $\lambda=0.1$, and (c) $\lambda=0.2$.

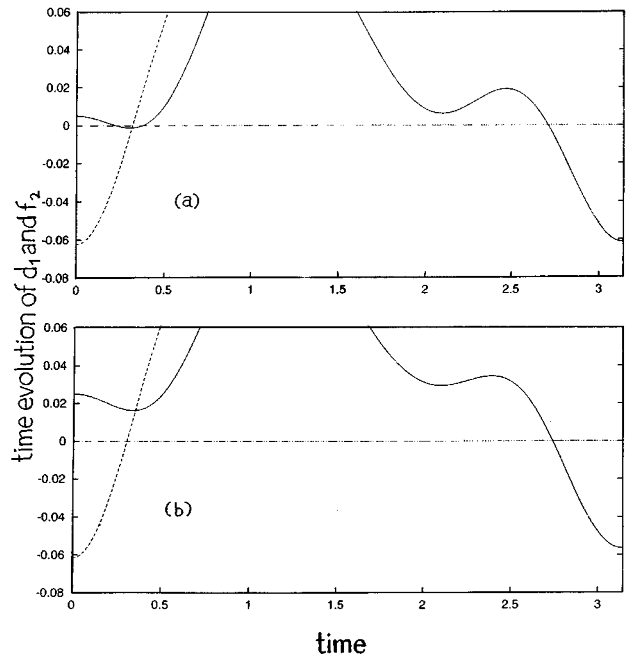

FIG. 9. Time evolution of f_2 (solid line) and d_1 (dotted line) for $\Theta=2\pi/3$, $\phi=0$, $\lambda=1.0$, $g=1.5$, and (a) $\overline{n}=0.01$ and (b) $=0.05$.

The condition for maximum squeezing to occur and/or recur for f_2 is $(\lambda/g)(k+1)\pi=k\pi$ and the times at which it will recur is $t=(\pi/g)(k+1)=k\pi/\lambda$. The maximum squeezing condition for d_1 is $\lambda=kg$ or $g=k\lambda$. In general for squeezing (not maximum) to occur is $k\pi-\cos^{-1}(\frac{2}{3})^{1/4}<\lambda t<k\pi+\cos^{-1}(\frac{2}{3})^{1/4}$.

B. The case of $\overline{n}\neq 0$

Here with the help of numerical calculations, we examine the combined effect of nonlinearity and finite number of thermal photons on the relationship between the field and atomic squeezing. Figures 6(a) and 6(b) show how the squeezing of the radiation field and the atomic dipole variable versus Θ changes in the presence of thermal photons for $\lambda=0.1$. With the increase of $\overline{n}$, we find that both field and atomic squeezing start to disappear. The width of the Θ interval in which squeezing appears is also seen to decrease with $\overline{n}$. The dipole squeezing for $0<\Theta<\pi/2$ decreases faster than for $\pi/2<\Theta<\pi$. The field squeezing is found to be much more sensitive to the presence of thermal photons than the dipole squeezing for $\pi/2<\Theta<\pi$.

From Fig. 7 we find that, when the initial photon number is 0.05, field squeezing disappears, and only the dipole squeezing for $\pi/2<\Theta<\pi$ can appear. When $\overline{n}$ is greater than 0.12 not only the field squeezing but also the atomic squeezing disappears. Figures 8(a) and 8(b) show how the squeezing versus Θ changes with the nonlinear parameter λ for $\overline{n}=0.01$. Unlike the case of $\overline{n}=0$, the dipole squeezing for $0<\Theta<\pi/2$ and field squeezing are more sensitive to the variations in λ in the presence of thermal photons than the dipole squeezing for $\pi/2<\Theta<\pi$.

Figure 9 presents the effect of $\overline{n}$ on the time evolution of the functions d_1 and f_2 for $\Theta=2\pi/3$. When $\overline{n}$ is up to 0.01, we notice from Fig. 9(a) that SFAS is almost destroyed except around $t=\pi/8$, where the squeezed atom can still radiate a squeezed field but for a very brief period. When $\overline{n}$ is increased up to 0.05 [Fig. 9(b)], SFAS is completely destroyed although the field and dipole squeezing can appear at some regions. For the case $t=0$, dipole squeezing is seen to be insensitive to the presence of thermal photons.

V. CONCLUSIONS

In conclusion, we have shown that nonlinear interaction of a two-level atom with a single-mode field via one-photon transition for vacuum field does not enhance the squeezing of the radiation field and the fluctuations of the atomic dipole variables but tends to revoke the initial squeezing. Furthermore, for experimentally relevant time scale, SFAS and initial squeezing is restored periodically for large nonlinearity. In the presence of thermal photons, the Θ interval in which squeezing appears decreases. The dipole squeezing for $0<\Theta<\pi/2$ and the field squeezing is found to be more sensitive to the presence of thermal photons than the dipole squeezing for $\pi/2<\Theta<\pi$. SFAS gradually disappears with increase in $\overline{n}$. The periodical revival of SFAS also disappears with $\overline{n}$. Hence nonlinear interaction of a two-level atom with a single-mode field via one-photon transition in the presence of thermal photons greatly enhances the destruction of squeezing and SFAS than the corresponding linear case. This study could be of interest for the micromaser experiments. In the present study dissipation have been neglected. To make the problem more realistic damping should be taken into account. This problem will be discussed elsewhere.

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