Excitation spectroscopy of vortex states in dilute Bose-Einstein condensed gases

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We apply linear-response analysis to the Gross-Pitaevskii equation to obtain the excitation frequencies of a Bose-Einstein condensate in a vortex state, and apply it to a system of rubidium atoms confined in a time-averaged orbiting potential trap. The excitation frequencies of a vortex differ significantly from those of the ground state, and may therefore be used to obtain a spectroscopic signature of the presence of a vortex state. [S1050-2947(97)03607-X]

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The recent attainment of quantum degeneracy conditions in magnetically trapped, dilute alkali-metal vapors [1–4] has opened an avenue for studying the many-body physics of Bose-Einstein condensates (BECs) in unprecedented detail. Recent experiments have mapped out many of the basic properties of alkali-metal BECs: the critical temperature T_c [4], the temperature dependence of the condensate fraction N_0/N [4], the contribution of particle interactions to the internal energy (i.e., departures from ideal-gas behavior) [4], and the energies of low-lying excitations [5,6].

There remain many aspects of "classical" superfluid behavior that have not yet been encountered in the atomic BECs. The one we discuss in this paper is the generation of vortices, which to our knowledge have not yet been observed in trapped gases. Recent theoretical investigations of axially symmetric harmonic traps have identified vortex states of condensates which have sharp values of the azimuthal component m of the angular momentum [7–9]. Rotation of the trap at a critical frequency ω_{crit}, which is of the order of the harmonic trap frequency ω, should force the condensate into the vortex state [7–9]. Detection of this state by current imaging techniques is complicated by present magnet geometries, which constrain viewing of a condensate to be done more or less perpendicular to the trap symmetry axis (hereafter the z axis). Most schemes acquire images of the condensate density integrated along the line of sight, and the integrated density of a vortex state perpendicular to the trap axis is not much different from that of the condensate ground state. In this paper we calculate the mechanical excitation spectrum of a vortex state, and find that it differs significantly from that of the ground state. We thus propose excitation spectroscopy as a sensitive technique for detecting the presence of vortices.

The basic framework of our method is mean-field theory as described by the Gross-Pitaevski (GP) formalism [10,11] for a condensate of a dilute Bose gas at temperature $T=0$. Current experimental BEC atomic physics appears to be practiced in a regime where this formalism is applicable: the gases are very tenuous, and a nearly pure condensate (corresponding to T very near zero) can be produced by the technique of evaporative cooling [1,3,4]. Calculations done within the GP framework yield good agreement with experiment concerning condensate shapes and sizes [7,8,12,13], and give ground-state condensate excitation frequencies within about 5% of experimental values [5,6,14].

We begin with the time-independent treatment of condensate eigenfunctions using the Gross-Pitaevskii (nonlinear Schrödinger) equation, and then calculate the excitation spectrum using the method of Bogoliubov [15], which was used by Pitaevskii to examine excitations about vortices in a homogeneous gas [19], and has been adapted to treat trapped Bose condensates [14,16–18].

The specific calculations reported here are done for a system of ^{87}Rb atoms confined in the time-averaged orbiting potential (TOP) trap currently in use at JILA [20].

In the GP formalism the interaction between atoms is approximated by the pseudopotential, $V(\mathbf{r},\mathbf{r}')=U_0\delta(\mathbf{r}-\mathbf{r}')$, where $U_0=4\pi\hbar^2 a/M$, with M being the atomic mass and a the scattering length that characterizes low-energy atomic collisions. For the triplet Born-Oppenheimer potential of $^{87}Rb_2$ that describes collisions in the JILA trap, the current best estimate [21] of a is $110a_0$, where a_0 is the Bohr radius. This value is used in the present paper. The trapping potential takes the form $V_{\text{trap}}(\mathbf{r})=M(\omega_\rho^2\rho^2+\omega_z^2z^2)/2$, in the cylindrical coordinates appropriate to the TOP trap. For this trap,

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the axial and radial frequencies are always in the same ratio, $\omega_z/\omega_\rho=\sqrt{8}$; it is convenient to characterize the trap by the single parameter $\nu_\rho=\omega_\rho/2\pi$, the value of which is often stated explicitly in experimental papers. In this paper we use ν_ρ = 74 Hz.

The time-independent GP equation that describes the condensate wave function $\psi(\mathbf{r})$ thus takes the form

$$\left(-\frac{\hbar^2}{2M}\nabla^2+V_{\rm trap}(\mathbf{r})+N_0U_0|\psi(\mathbf{r})|^2\right)\psi(\mathbf{r})=\mu\psi(\mathbf{r}), \quad (1)$$

where N_0 is the average number of condensate atoms and μ is the chemical potential, which is treated as an eigenvalue; the normalization condition

$$\int d\mathbf{r}|\psi(\mathbf{r})|^2=1 \quad (2)$$

is implied. The form of Eq. (1) is consistent with the existence of solutions

$$\psi(\mathbf{r})=\psi^{(m)}(\rho,z)\frac{e^{im\phi}}{\sqrt{2\pi}} \quad (3)$$

that are eigenfunctions (with eigenvalues $\hbar m$) of the azimuthal angular momentum operator l_z. Previous work on this system [7–9,12,13] has identified a solution with $m=0$ as the condensate ground state, and those with $|m|=1$ as the lowest vortex solutions.

As discussed elsewhere [22], we solve Eq. (1) by ''growing'' a condensate up from the noninteracting case ($U_0=0$), in which the solution is given by harmonic oscillator wave functions. The growth process may be visualized as a gradual turning-on of the interaction strength U_0 or as a gradual increase in the number of condensate atoms N_0; the two pictures are equivalent for this purpose, since N_0 and U_0 appear in the equations of motion only through the product N_0U_0. In the noninteracting case, m is a good quantum number, and it is preserved during the growth process. We start with $\psi^{(m)}(\rho,z)$ as an eigenfunction of the axially symmetric three-dimensional harmonic oscillator with quantum numbers m, the number n_z of nodes in z, and the number n_ρ of nodes in the cylindrical radial coordinate ρ. The lowest vortex state with $m=1$ has $n_z=n_\rho=0$; the number of nodes in ρ and z is also found to be conserved during the growth process. Our general representation of $\psi^{(m)}(\rho,z)$ is as a linear combination of oscillator eigenfunctions, and a set of nonlinear equations is solved iteratively to determine the coefficients at each value of N_0U_0. A cross section of the density of the vortex solution is shown in Fig. 1.

With the solution of the time-independent GP equation in hand, we calculate the response of the condensate to a weak oscillatory perturbation by standard linear-response theory [23]. The associated time-dependent driven GP equation takes the form

$$i\hbar\frac{\partial\Psi}{\partial t}=[H_0+U_0|\Psi(\mathbf{r},t)|^2+f(\mathbf{r})e^{-i\omega_p t}+f^*(\mathbf{r})e^{i\omega_p t}]\Psi(\mathbf{r},t), \quad (4)$$

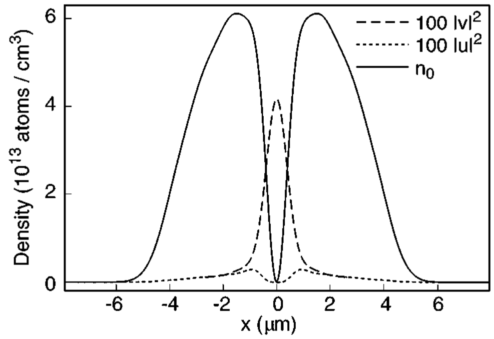

FIG. 1. The dashed line is a plot of $100|v_\lambda(\mathbf{r})|^2$ for the lowest energy excitation and the dotted line shows $100|u_\lambda(\mathbf{r})|^2$, where the factor of 100 is an arbitrary scaling factor used for plotting convenience. The solid line shows a plot of the spatial distribution of the $k=1$ vortex number density, $n_0(\mathbf{r})$. In this plot $N_0=4939$ atoms, and $\nu_\rho=74$ Hz for a TOP trap.

where $f(\mathbf{r})$ is the spatially dependent amplitude of the perturbation. We solve this equation in the linear-response limit. The details of this approach are described elsewhere [18], and we simply state the central results here. By using the form

$$\Psi(\mathbf{r},t)=e^{-i\mu t/\hbar}[N_0^{1/2}\psi(\mathbf{r})+u(\mathbf{r})e^{-i\omega_p t}+v^*(\mathbf{r})e^{i\omega_p t}] \quad (5)$$

we obtain Eq. (1) and also the linear-response equations,

$$(\mathcal{L}-\hbar\omega_p)u(\mathbf{r})+N_0U_0[\psi(\mathbf{r})]^2v(\mathbf{r})=-N_0^{1/2}f(\mathbf{r})\psi(\mathbf{r}), \quad (6)$$

$$N_0U_0[\psi^*(\mathbf{r})]^2u(\mathbf{r})+(\mathcal{L}+\hbar\omega_p)v(\mathbf{r})=-N_0^{1/2}f(\mathbf{r})\psi^*(\mathbf{r}), \quad (7)$$

where $\mathcal{L}=H_0-\mu+2N_0U_0|\psi(\mathbf{r})|^2$.

This pair of equations can be solved in a general way by expanding the condensate response in normal modes of oscillation, which are obtained by solving the Bogoliubov equations,

$$(\mathcal{L}-\hbar\omega_\lambda)u_\lambda(\mathbf{r})+N_0U_0[\psi(\mathbf{r})]^2v_\lambda(\mathbf{r})=0, \quad (8)$$

$$N_0U_0[\psi^*(\mathbf{r})]^2u_\lambda(\mathbf{r})+(\mathcal{L}+\hbar\omega_\lambda)v_\lambda(\mathbf{r})=0, \quad (9)$$

where ω_λ is an eigenvalue and $u_\lambda(\mathbf{r})$, $v_\lambda(\mathbf{r})$ are corresponding eigenfunctions. We require that $u_\lambda(\mathbf{r})$ and $v_\lambda(\mathbf{r})$ be square integrable and satisfy the conventional normalization condition

$$\int d\mathbf{r}[|u_\lambda(\mathbf{r})|^2-|v_\lambda(\mathbf{r})|^2]=1. \quad (10)$$

With this condition in force, the condensate response to an arbitrary driver $f(\mathbf{r})$ is obtained by a superposition of normal modes [18],

$$\begin{pmatrix}u(\mathbf{r})\\ v(\mathbf{r})\end{pmatrix}=-\sum_\lambda\frac{g_\lambda}{\hbar(\omega_\lambda-w_p)}\begin{pmatrix}u_\lambda(\mathbf{r})\\ v_\lambda(\mathbf{r})\end{pmatrix}, \quad (11)$$

where the amplitudes g_λ are obtained by quadrature:

$$g_\lambda = N_0^{1/2} \int d\mathbf{r} f(\mathbf{r}) [\psi(\mathbf{r}) u_\lambda^*(\mathbf{r}) + \psi^*(\mathbf{r}) v_\lambda^*(\mathbf{r})]. \quad (12)$$

Two key qualitative aspects of condensate excitation are implied by Eqs. (11) and (12). First, from Eq. (11), the response is largest when the driving frequency ω_p is equal to a normal-mode frequency ω_λ (the apparent divergence in this response, as in the case of a driven classical oscillator, is due to the neglect of damping in this treatment). Second, Eq. (12) implies selection rules for a given driver $f(\mathbf{r})$, associated with symmetries of the solutions of Eqs. (8) and (9). In particular, if $\psi(\mathbf{r})$ is given by Eq. (3), then it is straightforward to show that the normal modes will have specific angular momentum composition, in the following sense: if $u_\lambda(\mathbf{r})$ is an eigenfunction of l_z with a particular eigenvalue m_u (in units of $\hbar$), then $v_\lambda(\mathbf{r})$ will be an eigenfunction with eigenvalue $m_u - 2m$. It is appropriate to think of a normal mode as a quasiparticle moving in an effective potential created by the condensate, and for v_λ as being created by scattering of u_λ by the condensate. For $m=0$, the condensate has axial symmetry, and the normal modes thus have definite values m_u of the angular momentum; for $m \neq 0$, the condensate is not axially symmetric, and the component u_λ (with angular momentum m_u) is scattered into v_λ (with angular momentum $m_u - 2m$). In the case treated in this paper, we have $m=1$, and we will label the normal modes by the value of m_u that corresponds to the component u_λ. Thus, a "breathing-mode" driver, such as was applied to the ground state of the condensate in Ref. [5], will result here in excitation of $m_u=1$; a dipole driver, upon which we comment below, yields $m_u=0$ and 2; and a quadrupole driver, also used in Ref. [5], gives $m_u=-1$ and 3.

We have solved Eqs. (8) and (9) by an extension of the technique used in Ref. [14], in which u_λ and v_λ are expanded in trap eigenfunctions of appropriate symmetry, to obtain a system of linear eigenvalue equations. Figure 1 depicts ψ along with u_λ and v_λ as computed for the lowest normal mode, which has $m_u=2$, and would therefore be generated from the vortex condensate by a dipole excitation (e.g., oscillatory displacement of the center of the trap). The s- and d-wave characteristics of v_λ and u_λ, respectively, are apparent in the figure. An important property of this normal mode, which differs from all cases we have encountered in excitation of $m=0$, is that its frequency ω_λ is *less* than the trap frequency ω_ρ. The eigenfrequency of the lowest dipole mode for $m=0$, on the other hand, is identical to the trap frequency; that mode describes the rigid oscillation of the center of mass of the condensate ground state.

The dependence of normal mode frequencies upon N_0 is depicted in Fig. 2. It shows two candidates for excitation by dipole or quadrupole driving that have distinctive spectral signatures, in that no excitation frequencies of the $m=0$ condensate are nearby. In addition to the $m_u=2$ mode just described, there is the $m_u=-1$ mode, which could be excited by a quadrupole rotation in the sense *opposite* to that of the vortex. This frequency vanishes in the noninteracting limit, due to the m degeneracy of the cylindrical harmonic oscillator; its nonzero value elsewhere results directly from interactions of the quasiparticle with the condensate. While the figure shows only positive frequencies, the lowest $m_u=2$ normal mode would conventionally have a negative frequency, and energy; we plot the equivalent $\{-\omega, v^*, u^*\}$ mode, which has a positive frequency. This mode corresponds to placing atoms into the unoccupied ground state.

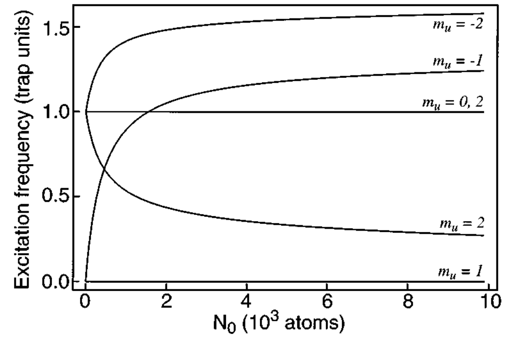

FIG. 2. Normal-mode excitation frequencies ω_λ of the vortex state in the TOP trap vs number of ^{87}Rb condensate atoms N_0, in units of the trap frequency ω_ρ. Labels indicate the angular momentum quantum number m_u of the u_λ component of the normal-mode eigenfunction, as described in the text. There are two degenerate frequencies at $\omega_\lambda = \omega_\rho$ for all values of N_0, which correspond to rigid oscillations of the vortex center of mass in the trap.

It is worth noting that the frequency spectrum in Fig. 2 is applicable to a wide range of TOP-trap geometries. Solutions of Eqs. (8) and (9), subject to the constraint of Eq. (10), apply to all TOP-trap geometries for which the parameter

$$\gamma = N_0 a (M \nu_\rho)^{1/2} \quad (13)$$

remains invariant [14]. This scaling law makes excitation spectroscopy a particularly effective tool for diagnosing the presence of vortex condensates.

In conclusion, we have calculated the excitation spectrum of vortex states of dilute Bose-Einstein condensates, in the linear-response regime. These spectra are sufficiently different from those for ground state excitation that they may provide useful diagnostics of the presence of vortices in trapped atom systems.

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