Selective creation of quasiparticles in trapped Bose condensates

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We investigate theoretically the use of time-varying magnetic fields to selectively create and manipulate quasiparticles in magnetically trapped Bose condensates. To maximize the transition matrix element connecting two desired quasiparticle states, the spatial symmetry of the applied magnetic field must be tailored to exploit the different spatial distributions of magnetization in the two quasiparticle states. This ''spatial magnetic resonance'' effect is analogous to the Franck-Condon factor in electric dipole transitions in diatomic molecules. Experimentally, the spatial magnetic resonance technique may allow the creation of coherences between quasiparticle states, the inversion of quasiparticle state populations, the measurement of quasiparticle lifetimes (T_1) and decoherence times (T_2), the creation of quasiparticle echoes, etc., in analogy with conventional spin magnetic resonance. [S1050-2947(97)04306-0]

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The experimental realization of Bose-Einstein condensation (BEC) in cold trapped alkali-metal atoms [1–3] has generated great interest in the properties of these weakly interacting quantum gases. Among the important properties to be investigated are excitations of the trapped condensates, with comparison to the behavior of superfluid ^{4}He. The mean-field Bogoliubov-Hartree (BH) theory suggests that below the critical temperature for BEC, the coherent mean field of an interacting Bose condensate mixes the collective and quasiparticle excitations [4,5]. Calculations show that the excitation spectrum of a trapped, interacting condensate includes a very large number of quasiparticle modes that are each unique superpositions of ''bare states'' of the external trapping potential [6]. Recently, several quasiparticle modes, including three center-of-mass ''sloshing'' modes, have been created and studied in the trapped rubidium and sodium condensates (by Jin *et al.* [7] and Mewes *et al.* [8], respectively) (''sloshing'' denotes the oscillatory center-of-mass motion of an extended fluidlike object). To create these condensate excitations, both groups used parametric drive (or modulation) of the electric currents that produce the magnetic trapping field for their condensates [7,8]. Results based on the mean-field BH theory [6] agree well with these experimental observations. Nevertheless, it should be possible to create many more quasiparticle modes in trapped condensates [6].

In this paper, we investigate theoretically a technique for the selective creation and manipulation of all quasiparticles in trapped condensates. In this excitation scheme, the spatial symmetry of resonant variations in the longitudinal magnetic field is crafted to exploit the different spatial distributions of magnetization in different quasiparticle states and thus, to maximize the transition matrix element between these states. We refer to this condensate excitation technique as spatial magnetic resonance (SMR). Experimental application of SMR will facilitate measurement of the condensate excitation spectrum to provide a comparison with the low-energy excitation spectrum in superfluid ^{4}He. Furthermore, SMR should allow the selective creation of coherent coupling among different quasiparticle states, thus effectively mapping transitions between trapped quasiparticle states into transitions in a multilevel atom. This condensate-atom correspondence will allow techniques developed in quantum optics and magnetic resonance for creating and manipulating coherence within multilevel atoms to be extended to condensate excitations.

The physics of SMR is straightforward, and may be thought of as an optimized variation of the trap potential: optimized to excite the condensate into higher quasiparticle states with the desired spatial symmetry, and with a minimum of variation in the trap potential and, hence, a minimum of perturbation of the condensed system. In contrast, simple resonant variations of the trap potential will be largely spatially symmetric in the radial and azimuthal directions, and, hence, will not be generally efficient at coupling the ground and excited states; in addition, such nonoptimal trap potential variations will significantly perturb the condensed system during application [9].

Figure 1 illustrates the effectiveness of SMR for two quasiparticle states in a one-dimensional trap [10]. The two trap state wave functions $\psi_i(\vec{r})$ and $\psi_f(\vec{r})$ describe the spatial

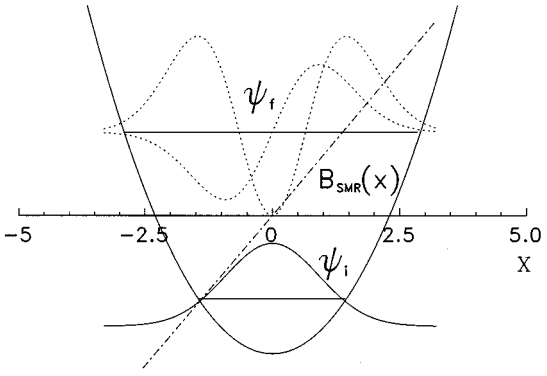

FIG. 1. SMR utilizes a spatially dependent resonant magnetic field to selectively couple two quasiparticle states in a trap (denoted by solid horizontal lines). For example, to excite a spatially symmetric ground state $\psi_i(x)$ to spatially symmetric or antisymmetric excited states, denoted by $\psi_f(x)$, the applied magnetic field $B_{\mathrm{SMR}}(x)$ could be chosen to be spatially uniform or to have a linear position dependence, respectively. The figure shows $B_{\mathrm{SMR}}(x)$ for this latter case.

extent of the respective quasiparticles, while the internal spin state $|\text{in}\rangle$ of the magnetic dipole moment of the atoms remains the same for both trapped states [10]. We introduce a time- and space-dependent longitudinal magnetic field $\vec{B}_{\text{SMR}}(\vec{r},t)=\hat{e}_B(\vec{r})B_{\text{SMR}}(\vec{r})F(t)$, where $F(t)=\mathcal{F}(t)(e^{-i\omega_B t}+e^{i\omega_B t})$, $\mathcal{F}(t)$ is a slowly varying envelope function, and ω_B is close to resonance with the transition of the two states $\omega_f-\omega_i$. The unit vector $\hat{e}_B(\vec{r})$ denotes the direction of the magnetic field (assumed here to be everywhere parallel to the z axis). Then the magnetic dipole transition matrix element between two quasiparticle states is

$$-{}_i\langle\psi_i(\vec{r})|_i\langle\text{in}|\vec{\mu}_m\cdot\vec{B}_{\text{SMR}}(\vec{r},t)|\text{in}\rangle_f|\psi_f(\vec{r})\rangle_f \approx-\mu_m\langle\psi_i(\vec{r})|B_{\text{SMR}}(\vec{r})|\psi_f(\vec{r})\rangle F(t), \tag{1}$$

where the magnetic dipole moment of the atom is $\vec{\mu}_m=\mu_m\hat{e}_m(\vec{r})$, and we have assumed that the strong trapping field completely spin polarizes the atoms and is pointing along the z axis [10]. We see that the transition matrix element is a product of two parts: an internal part (a constant), which comes from $\vec{\mu}_m$, and a spatial part, which is the magnetic dipole transition analog of the Franck-Condon factor in electric dipole transitions in diatomic molecules [11]. Now if $B_{\text{SMR}}(\vec{r})$ is spatially symmetric, as in a uniform trap potential variation, then the transition matrix element is zero when the initial and final quasiparticle states have opposite spatial parity. As long as $B_{\text{SMR}}(\vec{r})$ has some spatial asymmetry, however, the matrix element is nonzero. For optimal SMR in this example with $\psi_i(\vec{r})$ and $\psi_f(\vec{r})$, $B_{\text{SMR}}(\vec{r})$ should be antisymmetric.

In the following we develop the idea of spatial magnetic resonance using the mean-field Bogoliubov theory [12,13]. The second quantized Hamiltonian for a system of N bosonic atoms in a trapping potential $V_t(\vec{r})$ [10] is $\mathcal{H}=\mathcal{H}_0+\mathcal{H}_{\text{SMR}}$, with

$$\mathcal{H}_0=\int d\vec{r}\ \hat{\Psi}^\dagger(\vec{r})\left[-\frac{\hbar^2}{2M}\nabla^2+V_t(\vec{r})-\mu\right]\hat{\Psi}(\vec{r}) +\frac{1}{2}\hbar u_0\int d\vec{r}\ \hat{\Psi}^\dagger(\vec{r})\hat{\Psi}^\dagger(\vec{r})\hat{\Psi}(\vec{r})\hat{\Psi}(\vec{r}),$$

$$\mathcal{H}_{\text{SMR}}=-\int d\vec{r}\ \hat{\Psi}^\dagger(\vec{r})\vec{\mu}_m\cdot\vec{B}_{\text{SMR}}(\vec{r},t)\hat{\Psi}(\vec{r}), \tag{2}$$

where $\mathcal{H}_{\text{SMR}}$ describes the SMR interaction, $\hat{\Psi}(\vec{r})$ [$\hat{\Psi}^\dagger(\vec{r})$] is the atomic (bosonic) annihilation (creation) field, and M is the atomic mass. The two-body interaction takes the familiar contact (pseudopotential) form, with $\hbar u_0=4\pi\hbar^2 a_{\text{sc}}/M$ and a_{sc} being the scattering length. μ inside the square brackets in $\mathcal{H}_0$ denotes the chemical potential of the trapped atoms. With the Bogoliubov approximation [12,13], the interacting gas can be described by a set of noninteracting quasiparticles:

$$\mathcal{H}_0\rightarrow\frac{1}{2}\hbar\alpha p_0^2+\sum_{n\neq0}\hbar\tilde{\omega}_n\tilde{g}_n^\dagger\tilde{g}_n. \tag{3}$$

Here, $\tilde{g}_n^\dagger$ ($\tilde{g}_n$) are the bosonic quasiparticle creation (annihilation) operators. (Here, we only discuss the zero temperature limit; however, our result can easily be generalized to finite temperatures.) The index $n=0,1,2,\ldots$ labels quasiparticles in ascending order of $\tilde{\omega}_n$. As discussed in detail in [13], the α-dependent term describes the dephasing of the condensate.

The quasiparticles are defined as [6,12,13]

$$\tilde{g}_n=\int d\vec{r}[U_n(\vec{r})\hat{\Psi}(\vec{r})+V_n(\vec{r})\hat{\Psi}^\dagger(\vec{r})], \tag{4}$$

with the inverse transformations [12,13]

$$\hat{\Psi}(\vec{r})=\sum_{n=0}^{\infty}[U_n^*(\vec{r})\tilde{g}_n-V_n(\vec{r})\tilde{g}_n^\dagger] \tag{5}$$

(and their Hermitian conjugates). Both $U_n(\vec{r})$ and $V_n(\vec{r})$ are wave functions for quasiparticles [6]. The interaction $\mathcal{H}_{\text{SMR}}$ can be rewritten as

$$\mathcal{H}_{\text{SMR}}=\mathcal{F}(t)\hbar\sum_{k,k'=0}^{\infty}[(u_{kk'}+v_{k'k})\tilde{g}_k^\dagger\tilde{g}_{k'}-\gamma_{k'k}^*\tilde{g}_k\tilde{g}_{k'} -\gamma_{kk'}\tilde{g}_k^\dagger\tilde{g}_{k'}^\dagger], \tag{6}$$

where we have neglected a constant term and introduced the following SMR coupling matrix elements:

$$\hbar u_{kk'}=-\mu_m\int d\vec{r}\ U_k(\vec{r})U_{k'}^*(\vec{r})B_{\text{SMR}}(\vec{r}),$$

$$\hbar v_{kk'}=-\mu_m\int d\vec{r}\ V_k^*(\vec{r})V_{k'}(\vec{r})B_{\text{SMR}}(\vec{r}),$$

$$\hbar\gamma_{kk'}=-\mu_m\int d\vec{r}\ U_k(\vec{r})V_{k'}(\vec{r})B_{\text{SMR}}(\vec{r}). \tag{7}$$

By arranging the $B_{\text{SMR}}(\vec{r})$ to have certain spatial symmetries, the magnitudes of these SMR coupling matrix elements can be manipulated. For a spherically symmetric trap, the (ground) condensate state is described by spherically symmetric functions $U_0(r)$ and $V_0(r)$, and the excited state wave functions factor into radial parts [$U_{nLL_z}(r)$ and $V_{nLL_z}(r)$] and angular parts [the spherical harmonics $Y_{LL_z}(\theta,\phi)$]. (L is the angular momentum and L_z is its projection on the z axis; n is the quantum number for the radial direction.) Following Eq. (7), if $B_{\text{SMR}}(\vec{r})$ is chosen to be proportional to $Y_{L'L_z'}(\theta,\phi)$, then only quasiparticle states with the same angular symmetry will be created. [A similar argument applies for the radial dependence of $B_{\text{SMR}}(\vec{r})$.]

The system described by Eq. (3) and (6) can be solved using a general Bogoliubov transformation, since only terms quadratic in creation or annihilation operators are involved. However, the analysis of the resulting time-dependent quasiparticles is quite complicated. To better express the essential physics of SMR, we consider a simpler model based on the rotating wave approximation. We assume that the applied magnetic field is near resonance between two quasiparticle

states ($\omega_B \sim \omega_f - \omega_i$, taking $\omega_f = \tilde{\omega}_n$ and $\omega_i = \tilde{\omega}_0 = 0$). Neglecting all other quasiparticle states, we then obtain

$$\mathcal{H}_0 = \frac{1}{2}\hbar\,\overline{\alpha}(\tilde{g}_0 + \tilde{g}_0^\dagger)^2 + \hbar\,\delta_n \tilde{g}_n^\dagger \tilde{g}_n,$$

$$\mathcal{H}_{\mathrm{SMR}} = \hbar\,\Omega(\tilde{g}_0^\dagger \tilde{g}_n + \tilde{g}_n^\dagger \tilde{g}_0) - \hbar\,\gamma^* \tilde{g}_0 \tilde{g}_n - \hbar\,\gamma \tilde{g}_0^\dagger \tilde{g}_n^\dagger, \quad (8)$$

where we have used the rotating wave approximation. We have also absorbed $u_{nn} + v_{nn}$ into a renormalized $\tilde{\omega}_n$ and simplified the notation: $\alpha = 2\overline{\alpha}$, $\delta_n = \tilde{\omega}_n - \omega_B$, $\Omega = \mathcal{F}(t)(u_{n0} + v_{0n})$, and $\gamma = \mathcal{F}(t)(\gamma_{0n} + \gamma_{n0})$. Without loss of generality we also take Ω to be real.

From the Hamiltonian (8), we determine the Heisenberg operator equations of motion to be

$$\dot{\tilde{g}}_0 = -i[\,\overline{\alpha}(\tilde{g}_0 + \tilde{g}_0^\dagger) + \Omega \tilde{g}_n - \gamma \tilde{g}_n^\dagger],$$

$$\dot{\tilde{g}}_n = -i[\,\delta_n \tilde{g}_n + \Omega \tilde{g}_0 - \gamma \tilde{g}_0^\dagger]. \quad (9)$$

For a resonant drive (i.e., $\delta_n = 0$), the solutions to Eq. (9) are [14]

$$\begin{aligned} \tilde{g}_0(t) &= \tilde{g}_0(0)\cos[A(t)] - i e_n(0)\sin[A(t)] \\ &\quad - i\,\overline{\alpha}[\tilde{g}_0(0) + \tilde{g}_0^\dagger(0)] f_{cc} - \overline{\alpha}[e_n(0) - e_n^\dagger(0)] f_{sc}, \\ e_n(t) &= e_n(0)\cos[A(t)] - i\,\tilde{g}_0(0)\sin[A(t)] \\ &\quad - \overline{\alpha}[\tilde{g}_0(0) + \tilde{g}_0^\dagger(0)] f_{cs} + i\,\overline{\alpha}[e_n(0) - e_n^\dagger(0)] f_{ss}, \end{aligned} \quad (10)$$

where we have defined

$$e_n = (\Omega \tilde{g}_n - \gamma \tilde{g}_n^\dagger)/\Lambda,$$

$$f_{\mathrm{ss}'}(t) = \int_0^t dt'\, \mathrm{s}[A(t')]\,\mathrm{s}'[A(t) - A(t')], \quad (11)$$

with s and s′ denoting the cos (c) and sin (s) functions. $A(t) = \int_0^t dt' \Lambda(t')$, with $\Lambda = \sqrt{\Omega^2 - \gamma^* \gamma}$, is analogous to the area of the the pulse or Rabi angle in conventional magnetic resonance.

Equation (10) describes the dynamics of two arbitrary quasiparticle states (0 and n), coupled by a resonant SMR field. If neither of the two coupled quasiparticle states is the condensate, one simply puts $\overline{\alpha} = 0$. Then, Eq. (10) represents a coherent Rabi oscillation. When one of the two quasiparticle states coupled by SMR is the condensate state, then the $\overline{\alpha}$ terms in Eq. (10) must be retained, creating a dephasing of the induced coherence between the two coupled quasiparticle states. In this case, complete (i.e., 180°) SMR-induced Rabi oscillations between the ground state and an excited quasiparticle state cannot occur because the constancy of the quasiparticle energy level structure ($\tilde{\omega}_n$, etc.) requires that most atoms stay in the condensate ground state [6]. As ground state atoms are pumped by SMR into an excited state, the $\tilde{\omega}_n$'s begin to shift and $U_n(\vec{r})$ and $V_n(\vec{r})$ begin to change, thereby altering the coupling constants Ω and γ and destroying the SMR resonance condition: i.e., by the time a significant fraction of condensed atoms are excited, the target-excited quasiparticle state will not be in resonance with the SMR driving field. As a consequence, the SMR matrix element coupling the ground and excited quasiparticle states goes to zero with increasing excited state occupation, and any induced coherence between the ground and excited quasiparticle states is destroyed due to the constant ground state dephasing. Therefore, when one of the quasiparticle states coupled by SMR is the condensate ground state, the physically relevant limit of the solution for the quasiparticle dynamics [Eq. (10)] is the early time limit when only a negligible fraction of atoms are excited. (Note that novel techniques such as chirped SMR may allow the condensate to be completely driven into an excited quasiparticle state. This interesting possibility will be explored elsewhere.)

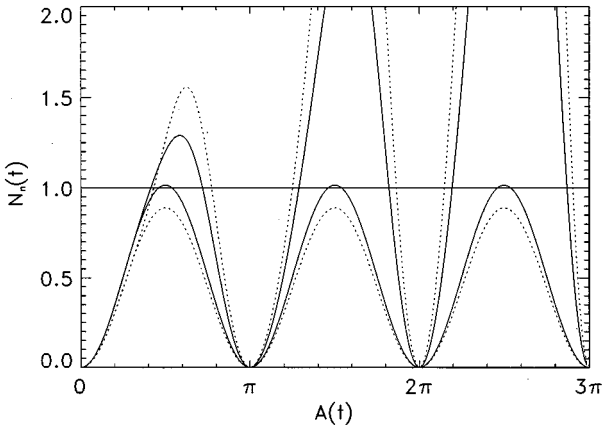

FIG. 2. The normalized population of the excited quasiparticle state $N_n(t) = \langle \tilde{g}_n^\dagger(t)\tilde{g}_n(t)\rangle / N_0(0)$, assuming the lower quasiparticle state is initially a coherent state (dashed line) or a Fock state (solid line) as a function of the SMR pulse area $A(t)$. We have used $\Omega = 4.0$, $\gamma = (0.25, 0.25)$, and $N_0(0) = \langle \tilde{g}_i^\dagger(0)\tilde{g}_i(0)\rangle = 1000$. The two lower curves are for the case when the initial state is not the condensate, while the two upper curves (going off scale) are for the case when the initial state is the condensate. Note: for the latter case, the plotted results are only valid for small $A(t)$.

We now calculate the time-dependent occupation of the excited quasiparticle state n, coupled by SMR from a lower-lying state i, assuming state n to be unoccupied initially (a vacuum state). If the initial state i is not the condensate ground state, then we obtain

$$\begin{aligned} \langle \tilde{g}_n^\dagger(t)\tilde{g}_n(t)\rangle_F &= [\Omega^2 N_0 + \gamma\gamma^*(N_0 + 1)]\sin^2[A(t)]/\Lambda^2, \\ \langle \tilde{g}_n^\dagger(t)\tilde{g}_n(t)\rangle_C &= [(\Omega^2 - \Omega\gamma - \Omega\gamma^*)N_0 \\ &\quad + \gamma\gamma^*(N_0 + 1)]\sin^2[A(t)]/\Lambda^2 \end{aligned} \quad (12)$$

for an initial Fock (F) or coherent (C) state i, with the average number of atoms $\langle \tilde{g}_i^\dagger(0)\tilde{g}_i(0)\rangle = N_0$. Expressions for the case where the lower energy state of the two coupled quasiparticle states is the condensate ground state can also be obtained, but are too complicated to be presented here [14]. Typical results for these SMR Rabi oscillations are shown in Fig. 2 for constant SMR coupling matrix elements with $\mathcal{F}(t) = \theta(t)$. Note that, because the chemical potentials for

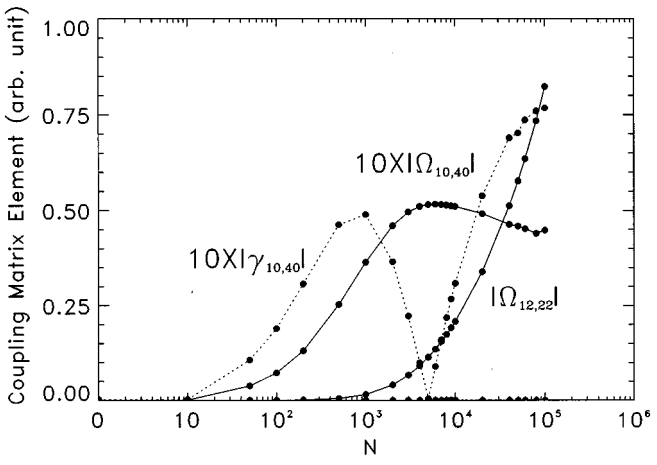

FIG. 3. Various SMR coupling constants calculated for the JILA TOP trap for ^{87}Rb atoms, with $a_{sc}=5.2$ nm, trap frequencies $(\omega_x:\omega_y:\omega_z)=(1:1:8^{1/2})\times$ 10 Hz. A spatially uniform SMR magnetic field was assumed. The dots denote the numerically computed points.

the quasiparticles are zero, their total populations are not conserved, i.e., $\langle \tilde{g}_n^\dagger \tilde{g}_n\rangle + \langle \tilde{g}_i^\dagger \tilde{g}_i\rangle \neq$ const. This explains why the peak of the Rabi oscillation of the excited state population can be more than one in Fig. 2. Also note that the early period for excitation out of the condensate shows growth behavior similar to that of the case where the lower quasiparticle state is not the condensate. Clearly, in each case the difference between an initial coherent or Fock state is quite noticeable.

To illustrate the potential efficacy of the SMR technique, we next present sample calculations of the SMR coupling matrix elements for the JILA TOP (time-averaged, orbiting potential) trap [1,7]. To simplify the presentation, we consider here only the case of a spatially uniform SMR field. The coupling matrix elements are therefore simply related to the overlap integrals of the quasiparticle wave functions (with $\mathcal{F}=1$). We plot in Fig. 3 the dependence of the modulus of various SMR coupling constants on the number of condensed atoms N. The solid lines denote the normal coupling Ω, while the dashed lines are for the anomalous coupling γ. Because of the cylindrical symmetry of the JILA TOP trap, the quasiparticle states are labeled as nL_z [i.e., according to their angular momentum projection L_z along the symmetric z axis, as well as an integer quantum number $(n=1,2,3,\ldots)$ for enumerating in ascending order states of the same L_z]. Therefore, $\Omega_{10,40}$ and $\gamma_{10,40}$ are the SMR coupling constants from the ground condensate state (lowest $L_z=0$ state) to the fourth-lowest $L_z=0$ state; and $\Omega_{12,22}$ and $\gamma_{12,22}$ are the SMR coupling constants from the lowest $L_z=2$ state (already observed in [7]) to the next higher $L_z=2$ state. (Note that $\gamma_{12,22}$ is practically zero, and falls on the x axis of the plot.) The interesting dependence of the SMR coupling constants on the number of atoms (and thus the density of the condensate) may be helpful in probing the condensate properties.

In summary, we have investigated theoretically a mechanism for selectively creating and manipulating quasiparticles in trapped Bose condensates. We refer to this condensate excitation technique as spatial magnetic resonance, since it involves a magnetic dipole moment matrix element (coupling quasiparticle states) that depends on changes of the spatial part of the quasiparticle wave function, not the internal (spin) part. Condensate SMR is an analog of the Franck-Condon factor in electric dipole transitions in diatomic molecules [11]. Engineering of the applied resonant magnetic field to have a particular spatial symmetry, so as to exploit the different spatial distributions of magnetization in different quasiparticle states, should allow the selective creation of quasiparticles and coherent superpositions of quasiparticles with minimal trap potential perturbation.

If the proposed scheme can be realized experimentally with sufficient control, then, adopting magnetic resonance terminology, one could apply 90° pulses, 180° pulses, etc., using a resonant magnetic field, to create coherences between quasiparticle states, to invert the population of such states, to measure quasiparticle lifetimes (T_1) and decoherence times (T_2), to create quasiparticle echoes, etc. Of course, these ideas require further investigation.

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[9] Note that the spirit of the SMR excitation scheme is similar to a parametric drive technique recently employed in the JILA TOP trap [7]. However, the SMR field we consider here is assumed to come from sources in addition to the coils generating the magnetic trapping field. In principle, this will allow greater freedom in tailoring the spatial symmetry of the resonant magnetic field and hence greater control of the SMR effect, during which quasiparticles are created, etc.

[10] Magnetic trapping relies on the interaction between the magnetic dipole moment of an atom $\vec{\mu}_m$ and a (static) trapping field $\vec{B}_t(\vec{r})$. In general, both the direction and magnitude of $\vec{B}_t(\vec{r})$ varies in space, while $\vec{\mu}_m$ Larmor precesses locally. For time scales longer than the Larmor period, $\vec{\mu}_m$ may be considered to be aligned with $\vec{B}_t(\vec{r})$ on average. Consequently, we approximate the trapping potential by $V_t(\vec{r}) \approx -\mu_m |\vec{B}_t(\vec{r})|$.

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