Generalized eikonal wave function of an electron in stimulated bremsstrahlung in the field of a strong electromagnetic wave

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An approximation in quantum theory of electron inelastic scattering on an arbitrary static potential in the field of a strong electromagnetic wave is developed. The obtained generalized eikonal wave function of the electron enables us to leave the framework of an ordinary eikonal approximation in stimulated bremsstrahlung, which is not applicable at large distances. [S1050-2947(97)05811-3]

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I. INTRODUCTION

Various approximations have been developed for treating the stimulated multiphoton bremsstrahlung of electrons. The main approximations are Born, low-frequency, and eikonal approximations (see, e.g., [1–5]). Though the Born and low-frequency approximations are applicable for describing free-free transitions in the field of high-intensity electromagnetic waves, they do not take into account the mutual influence of the scattering and radiation fields. The eikonal approximation describes such an influence, but the eikonal wave function can not be extended beyond the interaction region (this raises difficulties when one calculates the scattering amplitude) and seems to be inapplicable for high laser intensities.

The description of the electron eigenstates in the stimulated bremsstrahlung (SB) process particularly becomes very important for the process of above-threshold ionization of atoms in strong laser fields, which has been actively investigated during the past decades (see, e.g., [6–9]). In the theory of the above-threshold ionization process the description of the photoelectron final state still remains as one of the main problems. Such a wave function will describe multiphoton free-free transitions of the photoelectron, flying out from atom, in the fields of both atomic remainder and electromagnetic wave. So the above-threshold ionization probability essentially depends on the photoelectron SB probability. As known, the Keldysh-Faisal-Reiss [10–12] ansatz for the ionization process does not take into account the photoelectron-stimulated free-free transitions in the field of atomic remainder. In order to cover this gap attempts have been made to describe the photoelectron final state by the ''Coulomb-Volkov'' wave function [13–19], which is a product of the Coulomb wave function of elastic scattering and the wave function of a photoelectron in an electromagnetic wave. This wave function results in the factorization of the probability of multiphoton ionization and, as demonstrated in [19], such an approximation restricts both the frequency (low-frequency approximation [2]) and intensity of the electromagnetic wave. The use of another ansatz for the definition of the multiphoton ionization probabilities [20] should also be noted. However, the conditions under which such an approximation is valid are not clear; it is also not clear which approximation it is.

In this work we develop a generalized eikonal approximation (GEA) for the electron wave function in the SB process that simultaneously takes into account the influence of both the scattering and electromagnetic wave fields on the dynamics of the electron. It also allows us to describe the above-threshold ionization process of atoms with the help of such a wave function for the final state of the photoelectron, abandoning the framework of the above-mentioned approximations.

The organization of the paper is as follows. In Sec. II we present the solution of the Schrödinger equation for an electron in the field of strong electromagnetic radiation and a static potential. In Sec. III we consider the conditions of applicability of the GEA wave function and its relationship to the Born and eikonal approximations. In Sec. IV we discuss the significance of the obtained wave function and summarize our conclusions.

II. APPROXIMATE SOLUTION OF THE SCHRÖDINGER EQUATION FOR INELASTIC SCATTERING

The above-mentioned problem is reduced to the quantum-mechanical investigation of the dynamics of the SB process. The latter can be described by the Schrödinger equation for an electron in an arbitrary static potential and in the field of a plane electromagnetic wave (laser radiation)

$$\left[-\frac{\hbar^2}{2\mu}\Delta+\frac{ie\hbar}{\mu c}\mathbf{A}(t)\cdot\nabla+\frac{e^2\mathbf{A}^2(t)}{2\mu c^2}+U(\mathbf{r})\right]\times\Psi(\mathbf{r},t)=i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t), \tag{1}$$

where $U(\mathbf{r})$ is the potential energy of the electron in the static-potential field of arbitrary form, e and μ are the electron charge and mass, respectively, c is the light speed in vacuum, $\hbar$ is the Plank constant, and $\mathbf{A}(t)$ is the vector potential of the electromagnetic wave in the dipole approximation. The latter is valid if $\lambda \gg \max\{(v/c)\lambda, a\}$, where λ is the wavelength of external radiation and a is the effective dimension of the scattering potential $U(\mathbf{r})$. As long as for laser radiation and actual atomic potentials always $\lambda \gg a$, then the

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dipole approximation is applicable for nonrelativistic electrons ($v/c \ll 1$). Then the term proportional to $\mathbf{A}^2(t)$ has been removed by a unitary transformation as the only time-dependent function and consequently cannot have any physical role

$$\Psi(\mathbf{r},t)=\Psi'(\mathbf{r},t)\exp\left(-\frac{i}{\hbar}\int_{-\infty}^{t}\frac{e^2\mathbf{A}^2(t')}{2\mu c^2}dt'\right). \quad (2)$$

We shall solve Eq. (1) by assuming that the interaction with the scattering potential is not very strong. Then we seek a solution in the form

$$\Psi'(\mathbf{r},t)=\exp\left[\frac{i}{\hbar}[S_0(\mathbf{r},t)+S_1(\mathbf{r},t)]\right], \quad (3)$$

where

$$S_0(\mathbf{r},t)=\mathbf{p}\cdot\mathbf{r}-\frac{\mathbf{p}^2}{2\mu}t+\frac{e}{\mu c}\int_{-\infty}^{t}\mathbf{p}\mathbf{A}(t')dt'. \quad (4)$$

With the term proportional of $\mathbf{A}^2(t)$ [according to transformation (2)] S_0 is the classical action of an electron with the initial momentum $\mathbf{p}$ in the wave field. As a result for $S_1(\mathbf{r},t)$ we obtain the equation

$$\left[\frac{\partial}{\partial t}-\frac{i\hbar}{2\mu}\Delta+\frac{1}{\mu}\left(\mathbf{p}-\frac{e}{c}\mathbf{A}(t)\right)\nabla\right]S_1(\mathbf{r},t) =-U(\mathbf{r})-\frac{[\nabla S_1(\mathbf{r},t)]^2}{2\mu}. \quad (5)$$

Let the Oz axis be directed along the electron initial momentum $\mathbf{p}$. Then, in accordance with the solution (3), we have the initial condition

$$S_1(z=-\infty,t=-\infty)=0, \quad (6)$$

corresponding to the asymptotic behavior of the scattering potential at $z=-\infty$ [$U(z=\pm\infty)=0$] and to adiabatically switching on the electromagnetic wave at $t=-\infty$ and switching off at $t=+\infty$[$\mathbf{A}(t=\pm\infty)=0$].

The above-mentioned GEA is the following. We shall assume that the term proportional to $(\nabla \mathbf{S}_1)^2$ is small compared to $U(\mathbf{r})$,

$$(\boldsymbol{\nabla} S_1)^2 \ll 2\mu|U(\mathbf{r})|, \quad (7)$$

according to which we replace the exact equation (5) by the approximate one

$$\left[\frac{\partial}{\partial t}-\frac{i\hbar}{2\mu}\Delta+\frac{1}{\mu}\left(\mathbf{p}-\frac{e}{c}\mathbf{A}(t)\right)\boldsymbol{\nabla}\right]S_1(\mathbf{r},t)=-U(\mathbf{r}). \quad (8)$$

To solve Eq. (8) we make Fourier transformation over $\mathbf{q}$,

$$S_1(\mathbf{r},t)=\frac{1}{(2\pi)^3}\int \widetilde{S}_1(\mathbf{q},t)\exp(i\mathbf{q}\cdot\mathbf{r})d\mathbf{q}, \quad (9)$$

and obtain the equation for $\widetilde{S}_1(\mathbf{q},t)$,

$$\left[\frac{\partial}{\partial t}+\frac{i\hbar}{2\mu}\mathbf{q}^2+\frac{i}{\mu}\left(\mathbf{p}-\frac{e}{c}\mathbf{A}(t)\right)\cdot\mathbf{q}\right]\widetilde{S}_1(\mathbf{q},t)=-\widetilde{U}(\mathbf{q}), \quad (10)$$

where $\widetilde{U}(\mathbf{q})=\int U(\mathbf{r})\exp(-i\mathbf{q}\cdot\mathbf{r})d\mathbf{r}$ is the Fourier transformation of the potential energy.

We seek the solution of Eq. (10) in the form

$$\widetilde{S}_1(\mathbf{q},t)=f_1(\mathbf{q},t)+f_2(\mathbf{q}), \quad (11)$$

where

$$f_1(\mathbf{q},-\infty)=0 \quad (12)$$

and $f_2(\mathbf{q})$ is the action of the electron corresponding to the elastic scattering on the potential $U(\mathbf{r})$ in the absence of the electromagnetic wave [solution of Eq. (10) at $\mathbf{A}(t)=\mathbf{0}$]

$$f_2(\mathbf{q})=\frac{2\mu i}{\hbar}\frac{\widetilde{U}(\mathbf{q})}{\mathbf{q}^2+\dfrac{2\mathbf{p}\cdot\mathbf{q}}{\hbar}}. \quad (13)$$

Then, for $\widetilde{S}_1(\mathbf{q},t)$ we have the expression

$$\widetilde{S}_1(\mathbf{q},t)=\frac{2\mu i}{\hbar}\frac{\widetilde{U}(\mathbf{q})}{\mathbf{q}^2+\dfrac{2\mathbf{p}\cdot\mathbf{q}}{\hbar}} \times\left\{1+\frac{ie}{\mu c}\exp\left[-i\frac{\hbar\mathbf{q}^2}{2\mu}t-i\frac{\mathbf{q}}{\mu}\int\left(\mathbf{p}-\frac{e}{c}\mathbf{A}(t)\right)dt\right]\right. \times\int_{-\infty}^{t}\mathbf{q}\cdot\mathbf{A}(t') \left.\times\exp\left[i\frac{\hbar\mathbf{q}^2}{2\mu}t'+i\frac{\mathbf{q}}{\mu}\int\left(\mathbf{p}-\frac{e}{c}\mathbf{A}(t')\right)dt'\right]dt'\right\}. \quad (14)$$

We assume the electromagnetic wave to be monochromatic and of arbitrary polarization with the vector potential

$$\mathbf{A}(t)=A_0(\hat{\mathbf{e}}_1\cos\xi\cos\omega t+\hat{\mathbf{e}}_2\sin\xi\sin\omega t), \quad (15)$$

where A_0 is the amplitude of the vector potential, ω is the frequency of the plane wave in the dipole approximation because of nonrelativistic velocities of electrons, $\hat{\mathbf{e}}_1,\hat{\mathbf{e}}_2$ are unit vectors perpendicular to each other ($\hat{\mathbf{e}}_1\cdot\hat{\mathbf{e}}_2=0$, $|\hat{\mathbf{e}}_1|=|\hat{\mathbf{e}}_2|=1$), and ξ is a polarization angle.

Using the expansion by the Bessel functions

$$\exp[-i\alpha\sin(\omega t-\varphi)]=\sum_{n=-\infty}^{\infty}J_n(\alpha)\exp[-in(\omega t-\varphi)], \quad (16)$$

we carry out the integration over t' in expression (14). Then putting the resulting expression for $\widetilde{S}_1(\mathbf{q},t)$ into formula (9) we obtain

$$S_1(\mathbf{r},t)=\frac{i}{\hbar}2\mu\sum_{n=-\infty}^{\infty}e^{-in\omega t}\int\frac{\widetilde{U}(\mathbf{q})J_n(\alpha(\mathbf{q}))}{\mathbf{q}^2+2\frac{\mathbf{p}\cdot\mathbf{q}}{\hbar}-\frac{2\mu}{\hbar}n\omega-i0}\times\exp(i\{\mathbf{q}\cdot\mathbf{r}+\alpha(\mathbf{q})\sin[\omega t-\varphi(\mathbf{q})]+\varphi(\mathbf{q})n\})\times\frac{d\mathbf{q}}{(2\pi)^3},\tag{17}$$

where $\alpha(\mathbf{q})$ is the parameter of the electron interaction with both scattering and electromagnetic wave fields simultaneously

$$\alpha(\mathbf{q})=\frac{eA_0}{\mu c\omega}\eta(\mathbf{q})\tag{18}$$

and the quantities $\varphi(\mathbf{q})$ and $\eta(\mathbf{q})$ are

$$\varphi(\mathbf{q})=\arctan\left(\frac{\mathbf{q}\cdot\hat{\mathbf{e}}_2}{\mathbf{q}\cdot\hat{\mathbf{e}}_1}tg\,\xi\right),$$

$$\eta(\mathbf{q})=\sqrt{(\mathbf{q}\cdot\hat{\mathbf{e}}_1)^2\cos^2\xi+(\mathbf{q}\cdot\hat{\mathbf{e}}_2)^2\sin^2\xi}.\tag{19}$$

In the denominator of the integral in expression (17) $-i0$ is an imaginary infinitesimal, which shows how the path around the pole in the integral should be chosen to obtain a certain asymptotic behavior of the wave function, i.e., the outgoing spherical wave [to determine that we pass to the limit of the Born approximation at $\mathbf{A}(t)=\mathbf{0}$]. Expression (3) with formula (17) defines the electron wave function in the form [with the nonessential exponential (2)]

$$\Psi'(\mathbf{r},t)=\exp\left\{\frac{i}{\hbar}S_0(\mathbf{r},t)-\frac{2\mu}{\hbar^2}\sum_{n=-\infty}^{\infty}e^{-in\omega t}\times\int\frac{\widetilde{U}(\mathbf{q})J_n(\alpha(\mathbf{q}))}{\mathbf{q}^2+2\frac{\mathbf{p}\cdot\mathbf{q}}{\hbar}-\frac{2\mu}{\hbar}n\omega-i0}\times\exp(i\{\mathbf{q}\cdot\mathbf{r}+\alpha(\mathbf{q})\sin[\omega t-\varphi(\mathbf{q})]+\varphi(\mathbf{q})n\})\frac{d\mathbf{q}}{(2\pi)^3}\right\}.\tag{20}$$

III. CONDITIONS OF APPLICABILITY OF THE GEA WAVE FUNCTION: RELATION WITH THE BORN AND EIKONAL APPROXIMATIONS

Formula (20) has been obtained in the GEA (7). To estimate the latter we evaluate the expression ∇S_1 using formulas (18) and (19). Then we fix n in the denominator of expression (17) at the most probable value for the action $S_1(\mathbf{r},t)$. To determine that value of $\overline{n}$ we use the argumentation of Choudhury [21] according to which the Bessel function $J_n(z)$ takes on its largest value when its index n is roughly equal to its argument

$$\overline{n}(\mathbf{q})=\langle\alpha(\mathbf{q})\rangle,\tag{21}$$

where $\langle\alpha(\mathbf{q})\rangle$ denotes the integer value of $\alpha(\mathbf{q})$. This estimation of the Bessel function can be verified by the diagrams of Jahnke and Emde [22]. Then carrying out the summation of n in formula (17), we obtain

$$S_1\approx\frac{2i\mu}{\hbar}\int\frac{\widetilde{U}(\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2+2\frac{\mathbf{p}\cdot\mathbf{q}}{\hbar}-\frac{2\mu}{\hbar}\overline{n}(\mathbf{q})\omega-i0}\frac{d\mathbf{q}}{(2\pi)^3}.\tag{22}$$

From expressions (22) and (7) the condition of the GEA can be presented in a general form

$$\frac{2\mu}{\hbar^2}\left|\int\frac{\mathbf{q}\widetilde{U}(\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{r}}}{\mathbf{q}^2+2\frac{\mathbf{p}\cdot\mathbf{q}}{\hbar}-\frac{2\mu}{\hbar}\overline{n}(\mathbf{q})\omega-i0}\frac{d\mathbf{q}}{(2\pi)^3}\right|^2\ll|U(a)|.\tag{23}$$

Because of oscillations of the factor $e^{i\mathbf{q}\cdot\mathbf{r}}$ in the integral in Eq. (23) the main contribution is the region where $\mathbf{q}\cdot\mathbf{r}\cong 1$, i.e., $|\mathbf{q}|\simeq|\mathbf{q}_{ef}|=1/a$, where a is the dimension of the effective range of the scattering potential $U(\mathbf{r})$. Therefore, condition (23) can be written as

$$\frac{2\mu}{\hbar^2}\frac{\mathbf{q}_{ef}^2}{\left(\mathbf{q}_{ef}^2+2\frac{\mathbf{p}\cdot\mathbf{q}_{ef}}{\hbar}-\frac{2\mu}{\hbar}\overline{n}\omega\right)^2}|U(a)|\ll 1.\tag{24}$$

The $\overline{n}$ included in formula (24) is the most probable number of photons that is defined by expressions (21) and (18),

$$\overline{n}=\left\langle\frac{eA_0q_{ef}}{\mu c\omega}\overline{\eta}\right\rangle,$$

$$\overline{\eta}=\sqrt{(\hat{\mathbf{q}}_{ef}\cdot\hat{\mathbf{e}}_1)^2\cos^2\xi+(\hat{\mathbf{q}}_{ef}\cdot\hat{\mathbf{e}}_2)^2\sin^2\xi},\tag{25}$$

where $\hat{\mathbf{q}}_{ef}=\mathbf{q}_{ef}/q_{ef}$ is a unit vector along $\mathbf{q}_{ef}$.

Finally, the condition of applicability of the GEA (7) may be written in the form

$$|U|\ll\frac{1}{\mu}\left[\left(p-\frac{e}{c}A_0\right)+\frac{\hbar}{a}\right]^2.\tag{26}$$

Note that this condition leads to the condition of the generalized eikonal approximation for electron elastic scattering in potential fields [23], when $\mathbf{A}(t)\equiv\mathbf{0}$ (in the nonrelativistic limit).

The wave function (20) in the the GEA leads to the wave function of the Born approximation by the scattering potential if

$$|S_1(\mathbf{r},t)|\ll\hbar.\tag{27}$$

Then expanding the second term in the first exponent in formula (20) into the series and keeping only the terms to first order in U, we obtain

$$\Psi'_B(\mathbf{r},t)=\exp\left(\frac{i}{\hbar}S_0(\mathbf{r},t)\right)-\frac{2\mu}{\hbar^2}\sum_{n=-\infty}^{\infty}\exp\left(-\frac{i}{\hbar}\frac{p_n^2}{2\mu}t\right)$$
$$\times\int\frac{\widetilde{U}(\mathbf{q})J_n(\alpha(\mathbf{q}))}{\mathbf{q}^2+2\frac{\mathbf{p}\cdot\mathbf{q}}{\hbar}-\frac{2\mu}{\hbar}n\omega-i0}\exp(i\{\mathbf{q}\cdot\mathbf{r}+\alpha(\mathbf{q})$$
$$\times\sin[\omega t-\varphi(\mathbf{q})]+\varphi(\mathbf{q})n\})\frac{d\mathbf{q}}{(2\pi)^3}. \tag{28}$$

Taking into account Eq. (22), the condition when the wave function (28) is valid can be written from Eq. (27) as

$$|U|\ll\frac{\hbar}{\mu a}\left|\left(p-\frac{e}{c}A_0\right)+\frac{\hbar}{a}\right|. \tag{29}$$

The obtained criterion of validity of the electron wave function of the stimulated scattering in the Born approximation by the potential $U(\mathbf{r})$ includes both fast and slow particle cases. Thus, for the fast particles when $|p-(e/c)A_0|a\gg\hbar$ we have

$$|U|\ll\frac{\hbar}{\mu a}\left|p-\frac{e}{c}A_0\right|. \tag{30}$$

From the condition (29) for the slow particles when $|p-(e/c)A_0|a\lesssim\hbar$ we obtain the well-known strong criterion of the Born approximation for elastic scattering

$$|U|\ll\frac{\hbar^2}{\mu a^2}. \tag{31}$$

Comparing the condition of applicability of the GEA (26) and the conditions of the Born approximations (30) and (31), we see that for the fast particles (in strong laser fields) the wave function obtained in the GEA (20) describes the stimulated scattering in regions $|p-(e/c)A_0|a/\hbar\gg1$ times larger than Born's.

Now let us find the asymptote of the electron wave function corresponding to the Born approximation at $r\to\infty$ and justify the chosen sign at the infinitesimal $i0$ to the path around the pole in the integrals (17) and (20). To calculate the asymptote of the function (28) we temporarily direct the Oq_z coordinate axis along $\mathbf{r}$ and replace the integration variable $\mathbf{q}$ by $\mathbf{p}'=\mathbf{p}+\hbar\mathbf{q}$. Turning to spherical coordinates, we carry out the integration over the body angle by the formula

$$\exp\left(\frac{i}{\hbar}\mathbf{p}'\cdot\mathbf{r}\right)\Bigg|_{r\to\infty}\Rightarrow\frac{2\pi\hbar}{ip'r}\left[\delta(\hat{\mathbf{p}}'-\hat{\mathbf{r}})\exp\left(\frac{i}{\hbar}p'r\right)\right.$$
$$\left.-\delta(\hat{\mathbf{p}}'+\hat{\mathbf{r}})\exp\left(-\frac{i}{\hbar}p'r\right)\right], \tag{32}$$

where $\hat{\mathbf{p}}',\hat{\mathbf{r}}$ are unit vectors along $\mathbf{p}'$ and $\mathbf{r}$, respectively. Then we carry out the integration over p' in the complex plane, passing above the pole $p'=-p_n$ and below the pole $p'=p_n$, where $p_n=\sqrt{p^2+2\mu\hbar\omega n}$ (this path corresponds to the chosen sign of the infinitesimal $(-i0)$ in the denominator of the integrand). As a result, at $r\to\infty$ we obtain

$$\Psi'_B(\mathbf{r},t)=\exp\left[\frac{i}{\hbar}S_0(\mathbf{r},t)\right]-\frac{\mu}{2\pi\hbar^2 r}\sum_{n=-n_0}^{\infty}U\left(\frac{p_n\hat{\mathbf{r}}-\mathbf{p}}{\hbar}\right)$$
$$\times J_n\left[\alpha\left(\frac{p_n\hat{\mathbf{r}}-\mathbf{p}}{\hbar}\right)\right]\exp\left(\frac{i}{\hbar}\left\{p_n r-\frac{p_n^2}{2\mu}t\right.\right.$$
$$+\alpha\left(\frac{p_n\hat{\mathbf{r}}-\mathbf{p}}{\hbar}\right)\sin\left[\omega t-\varphi\left(\frac{p_n\hat{\mathbf{r}}-\mathbf{p}}{\hbar}\right)\right]$$
$$\left.\left.+\varphi\left(\frac{p_n\hat{\mathbf{r}}-\mathbf{p}}{\hbar}\right)n\right\}\right). \tag{33}$$

Here $n_0=\langle p^2/2\mu\hbar\omega\rangle$ and $\alpha[(p_n\hat{\mathbf{r}}-\mathbf{p})/\hbar]$, $\varphi[(p_n\hat{\mathbf{r}}-\mathbf{p})/\hbar]$ are defined according to formulas (18) and (19). As seen from expression (33), the asymptotic wave function at $n=0$ [if $\mathbf{A}(t)\equiv0$], corresponding to the elastic scattering of the electron in the Born approximation, describes the outgoing spherical wave at large distances, according to which the sign of the infinitesimal $i0$ in the poles of the integrals (17) and (20) had been chosen.

To obtain the wave function in the eikonal approximation of stimulated bremsstrahlung [3] from the GEA one shall neglect the term $\mathbf{q}^2$ in expression (14), which is equivalent to ignoring the second derivatives of the wave function with respect to the first ones. Then by integrating over $\mathbf{q}$ in formula (9) and taking into account expression (14), we obtain the corresponding action $S_1(\mathbf{r},t)$ in the eikonal approximation and consequently the electron wave function according to definition (3),

$$\Psi'_E(\mathbf{r},t)=\exp\left\{\frac{i}{\hbar}S_0(\mathbf{r},t)-\frac{i}{\hbar}\int_{-\infty}^{t}U\left(\mathbf{r}-\frac{\mathbf{p}}{\mu}(t-t')\right.\right.$$
$$\left.\left.+\boldsymbol{\Lambda}(t)-\boldsymbol{\Lambda}(t')\right)dt'\right\}, \tag{34}$$

where

$$\boldsymbol{\Lambda}(t)=\frac{e}{\mu c}\int_{-\infty}^{t}\mathbf{A}(\tau)d\tau \tag{35}$$

To obtain the condition of validity of the eikonal wave function (34) we use the approximate expression for S_1, which follows from formula (22) (the term $\mathbf{q}^2$ is neglected and the wave is assumed to be linear polarization)

$$S_1\approx-\frac{\mu}{\left|p-\frac{e}{c}A_0\right|}\int_{-\infty}^{z}U(\boldsymbol{\rho},z')dz', \tag{36}$$

where $|\boldsymbol{\rho}|$ is the transverse dimension of the scattering field $U(\mathbf{r})$ with respect to the electron motion. Hence the conditions of eikonal approximation for the scattering process in the field of the electromagnetic wave are (the second one follows from the condition $|\triangle S_1|\ll2|[\mathbf{p}-(e/c)\mathbf{A}_0]\nabla S_1|$; see Eq. (5))

$$|U|\ll\frac{1}{\mu}\left(p-\frac{e}{c}A_0\right)^2,\quad z\ll\left|p-\frac{e}{c}A_0\right|a^2/\hbar. \tag{37}$$

For high intensities, when $eA_0/c \gg p$ the approximation developed is valid independent of electron momentum. Here it is worthy to recall that our nonrelativistic consideration holds only if $eA_0/\mu c^2 \ll 1$. Although this condition restricts the wave intensity, it practically holds for all existing strong laser fields.

IV. CONCLUSION

The quantitative description of the scattering process of an electron on an arbitrary static potential and in the field of a strong electromagnetic wave with achievable high accuracy reduces to deriving a dynamic wave function that accounts for the effect of both fields. It is clear that the perturbation theory with only one of these fields is unable to describe the actual picture of multiphoton bremsstrahlung. Compared to other existing approximations for the description of such a process, the fairly good approximation is the eikonal one; however, as well known, this wave function has a limited range of applicability [see Eq. (37)] because in the derivation of such a solution the second derivatives of the wave function have been neglected in the equations of motion. In addition to neglecting quantum corrections (connected with the second derivatives of the wave function), this reduces to well-known difficulties in the derivation of the cross sections even in the case of elastic scattering (see [23]). From this point of view the so-called generalized eikonal approximation has been developed in the present work, which takes into account the quantum corrections in the eikonal limit and allows us to apply the obtained wave function at arbitrary distances, particularly at asymptotic large distances $r \to \infty$. For this reason the second derivatives of the wave function have been taken into account in the current ansatz. The only approximation made in the derivation of this wave function is the neglect of the nonlinear term $[(\nabla S_1)^2]$ in the equation of motion [see Eq. (7)]. Concerning a more accurate quantitative description of stimulated multiphoton bremsstrahlung by the GEA wave function, it is evident, that the obtained wave function provides higher accuracy because in different limits this wave function turns into the Born approximation wave function, when conditions (29)–(31) are fulfilled, and into the wave function of the eikonal approximation (34), when conditions (37) are satisfied. In particular, as seen from the comparison of formulas (26), (29) and (30) for the scattering of fast particles, the wave function in approximation (20) is applicable for $|p-(e/c)A_0|a/\hbar \gg 1$ times stronger scattering potentials than is allowed by the wave function in the Born approximation. So, even in the quantum limit this wave function has an essential contribution and consequently it should not be accepted as a reformed semiclassical wave function.

As follows from the above consideration, the cross sections calculated by the GEA wave function will contribute different corrections to the scattering cross sections of known approximations, which has been shown in Ref. [23] for the elastic scattering (in the absence of an electromagnetic wave). The multiphoton cross sections of stimulated bremsstrahlung, as well as above-threshold ionization of atoms in the GEA, will be presented elsewhere.

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