Perturbative expansions for the fidelities and spatially correlated dissipation of quantum bits

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We construct generally applicable short-time perturbative expansions for some fidelities, such as the input-output fidelity, the entanglement fidelity, and the average fidelity. Successive terms of these expansions yield characteristic times for the damping of the fidelities involving successive powers of the Hamiltonian. The second-order results, which represent the damping rates of the fidelities, are extensively discussed. As an interesting application of these expansions, we use them to study spatially correlated dissipation of quantum bits. Spatial correlations in the dissipation are described by a correlation function. Explicit conditions are derived for independent decoherence and for collective decoherence. [S1050-2947(97)02712-1]

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In various fields of physics, the study of open quantum systems plays an important role. For example, decoherence was recognized as a major problem in realizing quantum computation [1,2]. Decoherence in quantum computers mainly results from the coupling of quantum bit (qubits) to environment. In general, it is not practical to look for exact solutions of these complicated systems, which consist of many spatially correlated qubits interacting with a reservoir.

In this paper, we propose a perturbative approach to the study of open quantum systems. We construct generally applicable short-time perturbative expansions for some fidelities, such as the input-output fidelity, the entanglement fidelity, and the average fidelity. A similar perturbative expansion for coherence loss has been proposed in a recent paper [3], where coherence loss is measured by the quantity $\delta(t) = \mathrm{tr}[\rho(t) - \rho^2(t)]$, in which $\rho(t)$ indicates the reduced density operator of the system. Since $\delta(t)$ has no direct physical meanings, in this paper, we choose the fidelities rather than $\delta(t)$ as measures of decoherence. The fidelities are important quantities and have been widely used in quantum coding theory [4–7]. Through the perturbative expansions, we can get some general relations between the fidelities, such as the demonstration that the entanglement fidelity decays more rapidly than the average fidelity.

As an interesting application of this perturbative approach, we use it to study spatially correlated dissipation of qubits. The question is of great practical importance in designing quantum computers. Spatial correlations in pure dephasing of the qubits have been analyzed in Refs. [8] and [9]. However, the calculations of spatial correlations in general dissipation of the qubits, including the dephasing and the relaxation, are far more involved. This question is hard to solve either by any existing exact approaches or by solving the master equation [10]. Fortunately, here we show, through the perturbative expansions, spatial correlations in general dissipation of the qubits can be easily analyzed. Spatial correlations are described by a correlation function. From this result, we derive explicit conditions for independent deco-

herence and for collective decoherence, which are two important ideal circumstances in dissipation of the qubits.

I. PERTURBATIVE EXPANSIONS FOR THE FIDELITIES

We consider an open quantum system, the total Hamiltonian of which is expressed as

$$H_T = H_0 + H_I + H_{\mathrm{env}}, \tag{1}$$

where H_0 and H_{env} indicate the free Hamiltonians of the system and of the environment, respectively. H_I is the interaction Hamiltonian between the system and the environment. First, we suppose that the system is initially in a pure state $|\Psi_0\rangle$. If there is no coupling between the system and the environment, at time t the system is in the state

$$|\Psi(t)\rangle = e^{-iH_0 t/\hbar}|\Psi_0\rangle = U_0(t)|\Psi_0\rangle. \tag{2}$$

But in reality, coupling of the system to environment is inevitable. So at time t the system is in fact described by the reduced density operator

$$\rho(t) = \mathrm{tr}_{\mathrm{env}}[\exp(-iH_T t/\hbar)\rho_{\mathrm{env}}(0)\otimes|\Psi_0\rangle$$
$$\times \langle\Psi_0|\exp(iH_T t/\hbar)], \tag{3}$$

where $\rho_{\mathrm{env}}(0)$ is the initial density operator of the environment. Decoherence of the system due to this inevitable coupling can therefore be measured by the fidelity between the state (2) and (3), which has the form

$$F(t) = \langle\Psi_0|U_0^+(t)\rho(t)U_0(t)|\Psi_0\rangle. \tag{4}$$

The input-output fidelity is first defined in Ref. [4]. Here the definition is slightly modified to Eq. (4) to make it more suitable for measuring decoherence of the system.

In general, the reduced density operator $\rho(t)$ in Eq. (3) is hard to calculate. So there is difficulty in obtaining the fidelity (4). Fortunately, in practice, short-time behaviors of the system under dissipation are of most interest. Hence we do not need to exactly calculate the fidelity (4), but expand it into a short-time power series

$$F(t) = 1 - \frac{t}{\tau_1} - \frac{t^2}{\tau_2^2} - \cdots . \qquad (5)$$

Following Eqs. (2), (3), and (4), it is not difficult to obtain the expansion coefficients. Up to order t^2 they read explicitly as

$$\frac{1}{\tau_1} = 0, \qquad (6)$$

$$\frac{\hbar^2}{2\tau_2^2} = \langle \Psi_0 | \mathrm{tr}_{\mathrm{env}} \{ [H_I, [H_I, \rho_{\mathrm{env}}(0) \otimes |\Psi_0\rangle\langle\Psi_0|]]$$

$$+ [[H_I, H_0], \rho_{\mathrm{env}}(0) \otimes |\Psi_0\rangle\langle\Psi_0|] \} | \Psi_0 \rangle$$

$$= \langle H_I^2 \rangle_{s,\mathrm{env}} - \langle \langle H_I \rangle_s^2 \rangle_{\mathrm{env}}, \qquad (7)$$

where the symbol $\langle \cdots \rangle_{s,\mathrm{env}}$ stands for the average value over the system and the environment, i.e., $\langle H_I \rangle_s = \langle \Psi_0 | H_I | \Psi_0 \rangle$, and $\langle H_I \rangle_{s,\mathrm{env}} = \mathrm{tr}_{\mathrm{env}} [\rho_{\mathrm{env}}(0)\langle H_I \rangle_s]$. It is obvious that the expansion coefficients in Eq. (5) furnish characteristic times τ_n associated with decoherence processes involving H_I to order n. The second-order coefficient $1/\tau_2^2$ is of special interest. It is the coefficient of the first nontrivial term in the expansion. Under the short-time approximation, $1/\tau_2$ measures the damping rate of the fidelity. In most cases, it is sufficient to consider the expansion up to order t^2.

In the above, we have assumed that the system is initially in a pure state. The input-output fidelity (4) does not apply for mixed states. For a mixed input state of the system, there are some fidelities defined, such as the entanglement fidelity and the average fidelity [5]. These two fidelities are widely used in quantum coding theory. Suppose $|\Psi_{rs}\rangle$ is a purification of the density operator ρ_s, i.e., the state $|\Psi_{rs}\rangle$ satisfies $\mathrm{tr}_r(|\Psi_{rs}\rangle\langle\Psi_{rs}|) = \rho_s$, where the symbol r denotes an ancillary system. Similar to Eq. (4), the entanglement fidelity F_e is defined as

$$F_e(t) = \langle \Psi_{rs} | U_0^+(t) \mathrm{tr}_{\mathrm{env}} [\exp(-iH_T t/\hbar) \rho_{\mathrm{env}}(0) \otimes |\Psi_{rs}\rangle$$

$$\times \langle \Psi_{rs} | \exp(iH_T t/\hbar)] U_0(t) | \Psi_{rs} \rangle$$

$$= 1 - \frac{t}{\tau_{1e}} - \frac{t^2}{\tau_{2e}^2} - \cdots, \qquad (8)$$

where $F_e(t)$ is furthermore subjected to a short-time power series expansion. In Ref. [5], it has been proven that the entanglement fidelity defined above is an intrinsic quantity of the system s, i.e., it does not depend on the specific purification $|\Psi_{rs}\rangle$. From Eq. (5), we easily obtain

$$\frac{1}{\tau_{1e}} = 0, \qquad (9)$$

$$\frac{\hbar^2}{2\tau_{2e}^2} = \langle H_I^2 \rangle_{s,\mathrm{env}} - \langle \langle H_I \rangle_s^2 \rangle_{\mathrm{env}}. \qquad (10)$$

Comparing Eqs. (6) and (7) with Eqs. (9) and (10), we see that the characteristic times for the entanglement fidelity have the same forms as those for the input-output fidelity. The only change is that the symbol $\langle H_I \rangle_s$ now means $\langle H_I \rangle_s = \mathrm{tr}_s(\rho_s H_I)$. So the entanglement fidelity is a natural exten-

sion of the input-output fidelity to include the mixed input states. Equations (9) and (10) also show the characteristic times for the entanglement fidelity are all intrinsic properties of the system s.

The initial density operator of the system can be expressed as a mixture of pure states, i.e.,

$$\rho_s = \sum_i p_i |\Psi_i\rangle\langle\Psi_i|, \qquad (11)$$

where p_i satisfy $\Sigma_i p_i = 1$. The average fidelity F_a is defined as

$$F_a(t) = \sum_i p_i F(|\Psi_i\rangle) = 1 - \frac{t}{\tau_{1a}} - \frac{t^2}{\tau_{2a}^2} - \cdots, \qquad (12)$$

where $F(|\Psi_i\rangle)$ indicates the input-output fidelity for the pure state $|\Psi_i\rangle$. Since the expression (11) for the density operator ρ_s is not unique, unlike the entanglement fidelity, the average fidelity is not solely defined for a definite ρ_s. From Eqs. (12) and (4), we get the characteristic times for the average fidelity

$$\frac{1}{\tau_{1a}} = 0, \qquad (13)$$

$$\frac{\hbar^2}{2\tau_{2a}^2} = \langle H_I^2 \rangle_{s,\mathrm{env}} - \left\langle \sum_i p_i \langle H_I \rangle_i^2 \right\rangle_{\mathrm{env}}, \qquad (14)$$

where the symbol $\langle H_I \rangle_i$ means $\langle \Psi_i | H_I | \Psi_i \rangle$. Comparing Eq. (10) with Eq. (14), we get an interesting inequality. For any interaction Hamiltonian H_I, there is the operator inequality

$$\sum_i p_i \langle H_I \rangle_i^2 \geqslant \left(\sum_i p_i \langle H_I \rangle_i \right)^2 = \langle H_I \rangle_s^2. \qquad (15)$$

Hence Eqs. (10) and (14) yield

$$\frac{1}{\tau_{2e}} \geqslant \frac{1}{\tau_{2a}}, \qquad (16)$$

which suggests that the entanglement fidelity decays more rapidly than any average fidelities. In Ref. [5] it has been proven that the entanglement fidelity is always less than the average fidelity. Here we show this fact from another aspect.

II. SPATIALLY CORRELATED DISSIPATION OF THE QUBITS

As an interesting application of the perturbative approach to open quantum systems, we consider a practical question: decoherence in quantum computers. This decoherence is due to the inevitable coupling of the qubits to the external environment. A few papers [1,2,8,9,11–14] have been published on this subject with some simplifications, such as omitting spatial correlations in the decoherence [11–14], or omitting relaxation of the qubits [8,9], or omitting both of them [1,2]. However, in real circumstances, such as in the ion-trapped quantum computers [15], relaxation of the qubits has notable contributions to decoherence [11–13]. On the other hand, spatial correlation properties of decoherence play an impor-

tant role in the choice of the decoherence-reducing strategies. For independent decoherence and for collective decoherence, the decoherence-reducing strategies are quite different. So here we consider a more practical model of decoherence. This decoherence is described by a spatially correlated amplitude damping, which includes both the dephasing and the relaxation of the qubits. The environment is modeled by a bath of oscillators with infinite degrees of freedom and the mode functions of the bath field are chosen as plane waves. The Hamiltonian describing this decoherence process has the form

$$H = H_0 + \sum_{l=1}^{L} \sum_k [\hbar g_k (e^{-ikr_l} a_k + e^{ikr_l} a_k^\dagger)(\lambda_1 \sigma_l^x + \lambda_2 \sigma_l^y)]$$

$$+ \sum_k (\hbar \omega_k a_k^\dagger a_k), \tag{17}$$

where the Pauli operator σ_l represents the l qubit and a_k is the annihilation operator of the bath mode k. The symbol r_l denotes the site of the l qubit, and λ_1, λ_2, and g_k are coupling constants. H_0 in Eq. (17) describes the free evolution and the internal interaction of the qubits. The qubits, whether in memory or in quantum gate operations, can all be described by the Hamiltonian (17).

The Hamiltonian (17) is very complicated and it is hard to find its exact solutions. Fortunately, with the perturbative approach developed in the previous section, this complex system can be easily treated. To analyze the decoherence, we use the perturbative expansion for the entanglement fidelity, which returns to the input-output fidelity if the initial state of the qubits is pure. The environment is supposed in thermal equilibrium, i.e., the initial density operator $\rho_{\text{env}}(0)$ of the bath has the following form in the coherent representation:

$$\rho_{\text{env}}(0) = \prod_k \int d^2\alpha_k \frac{1}{\pi \langle N_{\omega_k} \rangle} \exp\left(-\frac{|\alpha_k|^2}{\pi \langle N_{\omega_k} \rangle}\right) |\alpha_k\rangle\langle\alpha_k|, \tag{18}$$

where the mean photon (or phonon) number

$$\langle N_{\omega_k} \rangle = 1 \bigg/ \left[\exp\left(\frac{\hbar \omega_k}{k_B T}\right) - 1\right]. \tag{19}$$

With this density operator, substituting the Hamiltonian (17) into Eq. (10), we obtain the decoherence rate $1/\tau_{2e}$ for this system,

$$\frac{1}{\tau_{2e}^2} = \sum_{l_1, l_2 = 1}^{L} \Omega^2(r_{l_1} - r_{l_2}) \langle \Delta A_{l_1} \Delta A_{l_2} \rangle_s, \tag{20}$$

where the operator is

$$A_{l_i} = \lambda_1 \sigma_{l_i}^x + \lambda_2 \sigma_{l_i}^y \quad (i = 1, 2) \tag{21}$$

and the spatial correlation function is

$$\Omega^2(r_{l_1} - r_{l_2}) = 2 \sum_k \left\{ |g_k|^2 \cos[k(r_{l_1} - r_{l_2})] \coth\left(\frac{\hbar \omega_k}{2k_B T}\right) \right\}. \tag{22}$$

Spatial correlation properties of the decoherence are completely determined by the correlation function $\Omega^2(r_{l_1} - r_{l_2})$.

In the following, we discuss two important ideal circumstances for the correlation function. Equation (22) can be rewritten as

$$\Omega^2(r_{l_1} - r_{l_2}) = x \sum_k f(k) \cos[k(r_{l_1} - r_{l_2})], \tag{23}$$

where $f(k)$ is a normalized distribution satisfying $\Sigma_k f(k) = 1$ and x is the normalization constant

$$x = 2 \sum_k |g_k|^2 \coth\left(\frac{\hbar \omega_k}{2k_B T}\right). \tag{24}$$

The expression of $f(k)$ is given by comparing Eq. (23) with Eq. (22). Its explicit form depends on the coupling coefficient $|g_k|^2$, whereas the latter is determined by the specific physical model for quantum computers. As a simplification, here we assume that the distribution $f(k)$ can be approximated by a Gaussian function with an expectation value $\bar{k}$ and a variance Δk, respectively. With this simplification, Eq. (23) reduces to

$$\Omega^2(r_{l_1} - r_{l_2}) \approx x \, \cos[\bar{k}(r_{l_1} - r_{l_2})] \exp[-\tfrac{1}{2}(\Delta k)^2 (r_{l_1} - r_{l_2})^2]. \tag{25}$$

Suppose d is distance between the adjacent qubits. If d satisfies

$$(\Delta k)d \gg 1, \tag{26}$$

Eq. (25) yields

$$\Omega^2(r_{l_1} - r_{l_2}) \approx x \delta_{l_1 l_2}, \tag{27}$$

and then from Eq. (20) the decoherence rate is simplified to

$$\frac{1}{\tau_{2e}^2} = \sum_{l=1}^{L} x \langle (\Delta A_l)^2 \rangle_s, \tag{28}$$

which suggests that the total decoherence rate of L qubits equals the sum of the decoherence rates of individual qubits. So in this circumstance, the qubits are decohered independently. Equation (26) is the condition for independent decoherence. Most of the existing quantum error correction schemes are designed to correct for the errors induced by independent decoherence [16–24].

Apart from the independent decoherence, there is another ideal circumstance for the spatial correlation function. Suppose there are $2L$ qubits. Two adjacent qubits make up a qubit pair. So we have L qubit pairs. The two qubits in the l qubit pair are denoted by l and l', respectively. If distance d between the adjacent qubits satisfies the conditions

$$\bar{k}d \ll 1, \quad (\Delta k)d \ll 1, \tag{29}$$

from Eq. (25) the spatial correlation function remains a constant for the two qubits in each qubit pair. The decoherence rate $1/\tau_{2e}$ for L qubit pairs thus becomes

$$\frac{1}{\tau_{2e}^2} = \sum_{l_1,l_2=1}^{L} \Omega^2(r_{l_1}-r_{l_2})\langle\Delta(A_{l_1}+A_{l_1'})\Delta(A_{l_2}+A_{l_2'})\rangle_s. \tag{30}$$

So under the condition (29) the two qubits in each qubit pair are decohered collectively. In the collective decoherence, the decoherence rate is sensitive to the type of the initial states. If the initial state of the qubit pairs is a coeigenstate of all the operators A_l+A_l', from Eq. (30), the second-order decoherence rate $1/\tau_{2e}$ reduces to zero. With these states decoherence of the qubits can therefore be much reduced. The coeigenstates of the operators A_l+A_l' are called the subdecoherent states. In fact, an arbitrary input state of L qubits can be transformed into the corresponding subdecoherent state of L qubit pairs by the following encoding:

$$|-1\rangle\rightarrow|-1,+1\rangle,$$
$$|+1\rangle\rightarrow|+1,-1\rangle, \tag{31}$$

where $|-1\rangle$ and $|+1\rangle$ are two eigenstates of the operator A_l. Obviously, the encoded state is a coeigenstate of the operators A_l+A_l' with the eigenvalue 0. This encoding has been mentioned in [8] and [25] and extended in [26] and [27] to reduce decoherence in general circumstances. Here we derive the working condition (29) for this encoding.

In the above, $\bar{k}$ and Δk are introduced phenomenologically. It is assumed that we have no knowledge about the coupling coefficient $|g_k|^2$. There is an interesting case in which the spatial correlation function can be calculated exactly. Consider one-dimensional quantum computers. In the continuum limit, it can be assumed that

$$\sum_k |g_k|^2 \cdots \propto \int_0^\infty d\omega_k\omega_k e^{-\omega_k/\omega_c}\cdots, \tag{32}$$

where ω_c is the cutoff frequency whose specific value depends on the particular nature of the physical qubit under investigation. The form (32) of $|g_k|^2$ was also used in Refs. [1] and [8]. We consider two circumstances. In the high-temperature limit, i.e., $T\gg\hbar\omega_c/k_B$, the correlation function (22) is simplified to

$$\Omega^2(r_{l_1}-r_{l_2})\propto\frac{4k_BT}{\hbar}\int_0^\infty d\omega_k e^{-\omega_k/\omega_c}\cos\left[\frac{\omega_k}{v}(r_{l_1}-r_{l_2})\right]$$
$$=\frac{4k_BT}{\hbar\omega_c}\frac{\omega_c^2}{1+[\omega_c(r_{l_1}-r_{l_2})/v]^2}, \tag{33}$$

where v indicates velocity of the noise field. If distance d between the adjacent qubits satisfies $d\gg v/\omega_c$, Eq. (33) tends to a delta function, and the qubits are therefore decohered independently. On the other hand, if $d\ll v/\omega_c$, we have $\Omega^2(d)\approx\Omega^2(0)$ and the adjacent qubits are then decohered collectively. In the low-temperature limit, things are much similar. If $T\ll\hbar\omega_c/k_B$, $\coth(\hbar\omega_k/2k_BT)\approx1$ and Eq. (22) yields

$$\Omega^2(r_{l_1}-r_{l_2})\propto2\int_0^\infty d\omega_k\omega_k e^{-\omega_k/\omega_c}\cos\left[\frac{\omega_k}{v}(r_{l_1}-r_{l_2})\right]$$
$$=2\omega_c^2\frac{1-\omega_c^2(r_{l_1}-r_{l_2})^2/v^2}{[1+\omega_c^2(r_{l_1}-r_{l_2})^2/v^2]}. \tag{34}$$

Equation (34) suggests that the above conditions for independent decoherence and for collective decoherence still hold at low temperature. The type of decoherence is mainly determined by the distance between adjacent qubits and by the cutoff frequency. The temperature of the environment hardly influences the decoherence type, though it determines the decoherence rate.

As an illustration of the above conditions, we consider the ion trap quantum computers [15]. There are two important sources of decoherence [11–13]. One is spontaneous emission and the other is thermal vibration of the qubits. For these two sources of decoherence, v/ω_c can be estimated respectively by the optical wavelength λ_{opt} or by the acoustic wavelength λ_{aco}. Since different ions should be separately addressable by a laser, we have $d\gg\lambda_{opt}$. On the other hand, it is relatively easy to make $d\ll\lambda_{aco}$. So in spontaneous emission, the qubits are decohered independently; and in thermal vibration, they may be decohered collectively.

III. CONCLUSION

In this paper, we develop short-time perturbative expansions for some widely used fidelities. From the expansions, we demonstrate some interesting relations between the fidelities. Perturbative expansions for the fidelities can be used to study open quantum systems. As an example, we consider spatially correlated dissipation of the qubits in the quantum computer. Spatial correlations in the dissipation are described by a correlation function and we successfully derive the explicit conditions for independent decoherence and for collective decoherence. This example suggests that the perturbative expansions for the fidelities may be proven as a useful tool for studying open quantum systems.

The study of spatially correlated dissipation of the qubits is a subject that must be considered in depth if quantum error correction is ever to be made to work in practice. There are still many interesting open problems in this direction. For example, until now many sources of decoherence have been found. What are the spatial correlation properties of these sources of decoherence? Quantum error correction is designed to correct a single or a certain number of errors. Are they applicable for reducing general spatially correlated decoherence? If so, to what extent can the decoherence be reduced? And if not, are there more effective decoherence-reducing strategies? With the techniques applied in this paper, some of these open problems may become accessible.

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