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Satellite test of special relativity using the global positioning system

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A test of special relativity has been carried out using data of clock comparisons between hydrogen maser clocks on the ground and cesium and rubidium clocks on board 25 global positioning system (GPS) satellites. The clocks were compared via carrier phase measurements of the GPS signal using geodetic receivers at a number of stations of the International GPS Service for Geodynamics (IGS) spread worldwide. In special relativity, synchronization of distant clocks by slow clock transport and by Einstein synchrony (using the transmission of light signals) is equivalent in any inertial frame. A violation of this equivalence can be modeled using the parameter $\delta c/c$, where c is the round-trip speed of light ($c=299\,792\,458$ m/s in vacuum) and δc is the deviation from c of the observed velocity of a light signal traveling one way along a particular spatial direction with the measuring clocks synchronized using slow clock transport. In special relativity $\delta c/c=0$. Experiments can set a limit on the value of $\delta c/c$ along a particular spatial direction (henceforth referred to as ''direction of δc''). Within this model our experiment is sensitive to a possible violation of special relativity in any direction of δc, and on a nonlaboratory scale (baselines $\geqslant$20 000 km). The results presented here set an upper limit on the value of $\delta c/c<5\times10^{-9}$ when considering all spatial directions of δc and $\delta c/c<2\times10^{-9}$ for the component in the equatorial plane. [S1050-2947(97)06712-7]

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I. INTRODUCTION

The equivalence of distant clock synchronization in inertial frames by slow clock transport and by Einstein synchrony [1] is fundamental to the theories of special and general relativity. It can be tested directly by comparing the one-way propagation times (using slow clock transport synchronization) of light signals along known paths, but in different spatial directions (in the past, often referred to as a test of the isotropy of the one-way speed of light). The only such test was carried out by Krisher *et al.* [2], who compared the phases of two hydrogen masers separated by a distance of 21 km and linked via an ultrastable fiber-optics link of the NASA deep-space network. The sensitivity of this experiment, expressed as a limit on the parameter $\delta c/c$, was $\delta c/c<3.5\times10^{-7}$. Riis *et al.* [3] tested the variation with spatial direction of the first-order Doppler shift of light emitted by an atomic beam (and indirectly thereby the synchrony equivalence) using fast-beam laser spectroscopy, obtaining what is currently the smallest limit, $\delta c/c<3\times10^{-9}$ [4]. Both of these experiments relied on the rotation of the Earth to change the direction of signal transmission and were therefore only sensitive to a component of the direction of δc that lies in the equatorial plane. The Gravity Probe A rocket experiment [5] can be used to measure the first-order Doppler shift of the link between the ground and on-board masers, giving a limit of $\delta c/c<3.2\times10^{-9}$ in one particular spatial direction. The only experiment sensitive in any spatial direction of δc was carried out by Turner and Hill [6], who measured the first-order Doppler shift in a Mössbauer rotor, obtaining a limit of $\delta c/c<3\times10^{-8}$. We present here the results of a test of special relativity sensitive in any spatial direction of δc. Using the clocks on board the global positioning system (GPS) satellites (providing baselines $\geqslant$20 000 km), we obtain a limit of $\delta c/c<5\times10^{-9}$ when considering all spatial directions and $\delta c/c<2\times10^{-9}$ for the component that lies in the equatorial plane. These results, together with those obtained by previous experiments are summarized in Table I. In Sec. II the principle of the experiment is explained, Sec. III provides details about the experimental procedure and data treatment, and Sec. IV shows the results. We consider possible systematic effects in Sec. V with a final discussion and conclusion in Sec. VI.

II. PRINCIPLE OF THE EXPERIMENT

Satellites of the GPS constellation are distributed in six orbital planes, at an inclination of 55° in near circular orbits with a period corresponding to 0.5 sidereal days (718 min) [7]. Each satellite is equipped with an on-board atomic clock and a dual-frequency signal transmission system.

The emission time of a signal as measured by the on-board clock τ_e and its reception time as measured by the ground-clock τ_r are recorded. The difference $T=\tau_r-\tau_e$ represents the transmission time of the signal plus some initial, unknown, phase difference between the clocks when these are synchronized using slow clock transport. Defining D as the distance along a straight line from the satellite (at the moment of emission) to the ground station (at the moment of

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TABLE I. Tests of special relativity showing the limits they set on $\delta c/c$ and their respective sensitivities to the possible spatial directions of δc.

	Limits on $\delta c/c$	
Direct measurements		
T. P. Krisher *et al.* (1990)	$\delta c/c < 3.5\times 10^{-7}$	Component in equatorial plane
GPS test (this experiment)	$\delta c/c < 5\times 10^{-9}$	All spatial directions
GPS test (this experiment)	$\delta c/c < 2\times 10^{-9}$	Component in equatorial plane
Indirect measurements		
E. Riis *et al.* (1987)	$\delta c/c < 3\times 10^{-9}$	Component in equatorial plane
R. F. C. Vessot *et al.* (1979)	$\delta c/c < 3\times 10^{-9}$	Component in one particular direction
K. C. Turner and H. A. Hill (1964)	$\delta c/c < 3\times 10^{-8}$	All spatial directions

reception) in a geocentric, inertial (nonrotating) coordinate system, one can write

$$T - \frac{D}{c} = \Delta_0, \tag{1}$$

where Δ_0 is a constant characterizing the initial phase difference of the two clocks. Special relativity requires that, for a series of measurements in different directions (e.g., during a complete passage of the satellite), $T - D/c$ should remain constant, after correction for the relative rate of the two clocks due to the gravitational redshift, second-order Doppler shift and the intrinsic (proper) frequency difference of the clocks.

In the theoretical framework generally used for the interpretation of similar tests of special relativity, the speed of light as measured by distant clocks that are synchronized using slow clock transport is $c + \delta c$ in one direction and $c - \delta c$ in the opposite direction along a particular spatial axis (in an inertial frame). The experiments then determine whether the special relativistic postulate $\delta c = 0$ is confirmed within the uncertainty of the experiment and set an upper limit on the parameter $\delta c/c$. A more sophisticated theoretical approach to all tests of special relativity was developed by Mansouri and Sexl [8]. A detailed interpretation of our experiment in this framework is presented elsewhere [9]; only the results are given here (see Sec. VI).

The effect of a nonzero value of δc on the transmission time of an individual link would be $(\delta c/c)(D/c)\cos\alpha$, where α is the angle between the direction of δc and of the transmitted signal, resulting in a measurable variation of T as a function of direction of signal transmission. However, a violation of special relativity might also affect the determination of the satellite ephemerides, and therefore the value of D, leaving the difference $T - D/c$ unchanged. A meaningful test of special relativity using the above principle therefore requires a method of satellite orbit determination that is insensitive to a nonzero value of δc.

This is the case for the GPS ephemerides obtained by the Center for Orbit Determination in Europe (CODE) of the IGS [10]. The method used adjusts a post-Keplerian, nonrelativistic orbit model to doubly differenced GPS timing data (see Fig. 1). The effects of a nonzero value of δc on the individual links cancel (to first order) when the double differences are formed (see Fig. 1). The IGS-CODE method is used to simultaneously adjust a number of parameters, including the satellite ephemerides and the ground station coordinates, thereby providing values of D that are unaffected by δc.

Additionally one has to ensure that corrections applied to the raw timing data used for orbit determination and the measurement of T do not presuppose the validity of special relativity. In fact, two corrections are routinely applied to GPS timing data, which are of relativistic origin and therefore do imply $\delta c = 0$ [7]: the correction for the gravitational redshift and the second-order Doppler shift of the rate of the satellite clock with respect to coordinate time, and the correction for the so-called Sagnac effect, which is due to the rotation of the Earth during signal transmission. Both of these are small corrections of order c^{-2}; hence the effect of an error in these corrections due to a nonzero value of δc would be negligible with respect to the first-order effect on T (for a more detailed theoretical analysis see, e.g., [14]).

Observation of GPS satellites in varying spatial directions thus provides a meaningful test of special relativity via relation (1), with D obtained using IGS-CODE ephemerides and station coordinates.

III. EXPERIMENTAL PROCEDURE

The IGS is a global network of ground stations that continuously observe the GPS satellites for civil, geodetic pur-

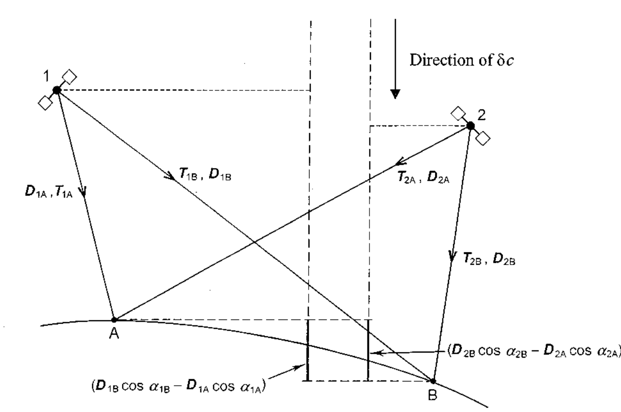

FIG. 1. Double difference (Y) for pairs of stations (A and B) and satellites (1 and 2), $Y = (T_{1B} - T_{1A}) - (T_{2B} - T_{2A})$. The effect of a nonzero value of δc on Y, given by $[(D_{1B}\cos a_{1B} - D_{1A}\cos a_{1A}) - (D_{2B}\cos a_{2B} - D_{2A}\cos a_{2A})]\delta c/c^2$, where α is the angle between the directions of δc and of the transmitted signal, vanishes.

poses [10]. From the raw data the IGS processing centers calculate (among other parameters) precise satellite ephemerides and ground station coordinates. These, together with the raw observations, are freely available through the internet via anonymous ftp [10]. We use data from eight ground stations for our experiment: Brussels (Belgium), Algonquin (Canada), Yellowknife (Canada), Fairbanks (Alaska, USA), Kokee Park (Hawaii, USA), Fortaleza (Brazil), Santiago (Chile), and Hobart (Australia). The motivation for this choice of ground stations is to ensure global coverage while providing maximum ground clock stability [for averaging times ≈ 6 h (one passage)] by using only stations that are equipped with hydrogen-maser clocks. The geodetic GPS receivers used provide raw phase measurements of the two GPS carrier frequencies at a sample interval of 30 s. The data sets cover six days (1994, September 18 to 23) and contain observations of all 25 GPS satellites available at the time. During this period (coinciding with the military intervention in Haiti) all GPS signals were free of the intentional degradation [selective availability (SA)] which is imposed by the US military. In general, this affects all but two satellites, making them unusable for the experiment described here. Of the 25 satellites used, 19 are equipped with cesium clocks and six with rubidium clocks.

From the raw data the differences $T-D/c$ are formed, taking into account corrections for the variable part of the gravitational redshift and second-order Doppler shift, the Sagnac effect, the ionospheric delay (using an ionosphere-free combination of the two frequencies), and the tropospheric delay [using the NATO Standardization Agreement (STANAG) tropospheric model [7]]. The amount of data is reduced by averaging over nine 30-s points, in order to save computer space and to make the data more manageable.

For a test of special relativity, one is interested in the variation of the difference $T-D/c$ during individual passages of the satellite over the ground station, i.e., variations over time scales of roughly six hours. Therefore we first filter the data, excluding all long-term variations ($\geqslant 5$ d), effectively subtracting the relative rate of the two clocks. Then an arbitrary offset per passage is adjusted as only the *variation* of $T-D/c$ during the passage is of interest.

As is well known, measurements of the GPS carrier phase are subject to an unknown phase ambiguity error of an integer number of cycles. This does not present a problem for our purposes as long as the induced error remains constant during each passage, which is the case if the receiver stays locked onto the satellite over the complete passage. Therefore all passages that were incomplete (data gaps indicating a possible loss of the satellite) were excluded.

The 0.5 sidereal day period of the GPS satellites implies that a station ‘‘sees’’ each passage of a particular GPS satellite at the same time of day (in sidereal days) and in the same directions (in a geocentric nonrotating frame). So, to see the data more clearly, all passages can be projected onto the same day, by shifting them individually by an integer number of sidereal days. Figure 2 shows the residuals of a typical data set after filtering and adjustment of an offset per passage, and with all passages shifted onto the same day. The standard deviation of these residuals for the complete data set (all stations and satellites) is 2.2 ns.

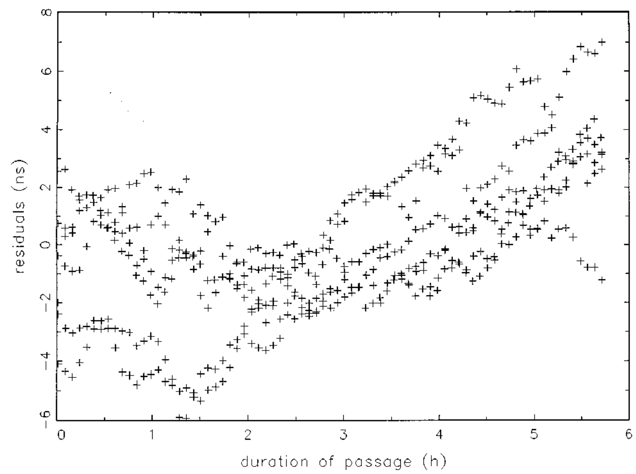

FIG. 2. Residuals of $(T-D/c)$ after filtering and adjustment of an offset per passage. The graph shows six passages of GPS satellite vehicle 22 over Brussels shifted onto the same day.

IV. RESULTS

Figure 3 shows the spatial directions of signal transmission for the individual links in a nonrotating geocentric frame. There are no links at colatitudes below $\approx 20°$ and above $\approx 163°$, which is due to the 55° inclination of the satellite orbits and implies that the experiment was least sensitive in the N-S direction.

The effect of a nonzero value of δc on the transmission time T for a particular link is given by (D/c) $(\delta c/c)\cos\alpha$, where α is the angle between the direction of signal transmission and the direction of δc. This model was fitted to the data using the least-squares method, adjusting the value of $\delta c/c$ and an offset per satellite-station pair. The adjustment was performed for a range of directions of δc, spanning colatitudes and longitudes from 0 to π in a grid of 0.1 rad$\times$0.1 rad. It is sufficient to cover half of all possible spatial directions, as opposing directions correspond to the same magnitude of $\delta c/c$ with the opposite sign. Directions are given in the nonrotating geocentric frame that is coinci-

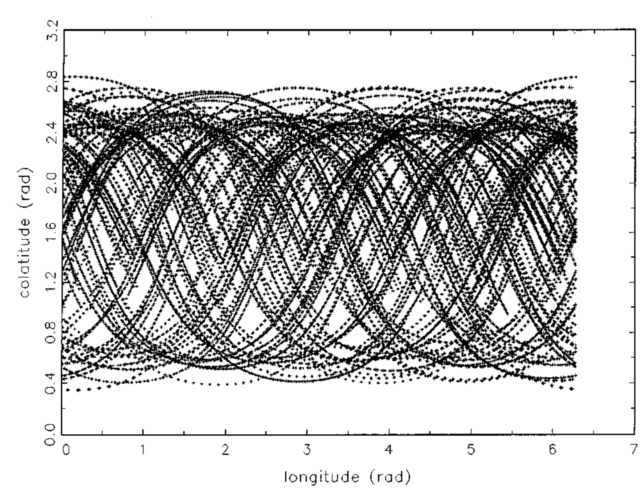

FIG. 3. Directions of signal transmission, in a geocentric nonrotating frame, for all satellite station links.

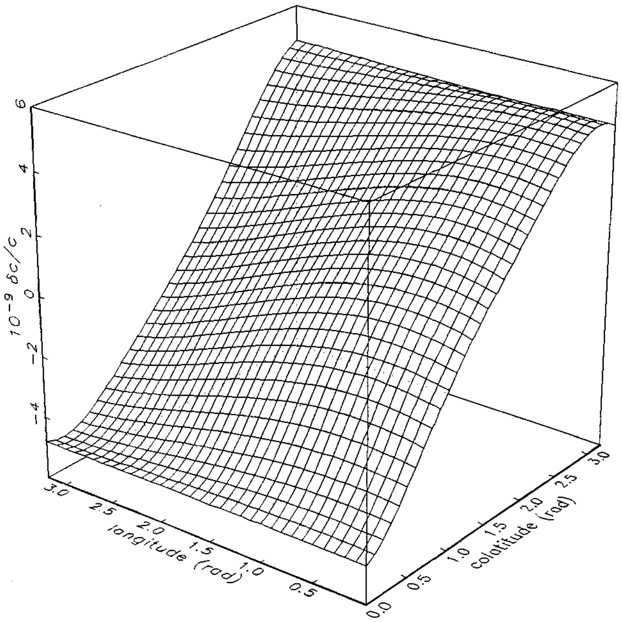

FIG. 4. Adjusted values of $\delta c/c$ as a function of direction of δc in a geocentric nonrotating frame.

dent with the Earth fixed frame of the International Earth Rotation Service (the ITRF) on March 7, 1995, 0 h 00 min universal time (UTC).

Figure 4 shows the adjusted values of $\delta c/c$ as a function of their direction. The extremum value of $\delta c/c$ is 4.9×10^{-9} at a colatitude of 2.9 rad and a longitude of 0.5 rad. In the equatorial plane (colatitude of 1.6 rad) the extremum value of $\delta c/c$ is 1.6×10^{-9} at a longitude of 0.6 rad. The formal statistical standard uncertainties of these values are below 3×10^{-10}, but the overall uncertainty of the experiment is largely dominated by systematic effects (Sec. V) that are responsible for the final uncertainty (Sec. VI).

V. SYSTEMATIC EFFECTS

The three main systematic effects that could affect the data are the satellite clock instability, ephemerides errors, and uncertainties in the modeled tropospheric delays. Prelaunch measurements of the relative frequency stability of the GPS cesium clock showed an instability of $\sigma_y(\tau)\approx1.5 \times10^{-13}$ (standard Allan deviation) for integration times of ≈ 6 h [11]. On-orbit measurements showed satellite clock instabilities of order 10^{-13} for integration times of one day [12], which corresponds to $\sigma_y(6\,\mathrm{h})\approx2\times10^{-13}$ when extrapolated assuming a $\tau^{-1/2}$ dependence. This translates into an accumulated time error over one passage of the satellite of $\delta\tau\approx\tau\sigma_y(\tau)/\sqrt{6}\approx2$ ns. Beutler *et al.* [13] estimate the uncertainty of the IGS-CODE satellite ephemerides to be 15 to 20 cm, corresponding to a timing error of $\delta\tau\approx0.7$ ns.

The tropospheric delays were estimated using the standard STANAG model [7]. More accurate values for these delays can be obtained through estimations in a regional or global network (as done, for example, by the IGS). We have studied such estimates (four values per day and station) for a period of five days in September 1995 and for six of the eight stations used (no IGS estimates were available for the remaining two stations). The differences between the zenithal tropospheric delay obtained from the STANAG model and the five-day averages of the IGS estimates do not exceed 200 ps for any station with the standard deviation of the averages <110 ps. For low elevations these differences can increase to maximum 1.1 ns (at elevations of 10°). We conclude that, for our purposes, the STANAG model is sufficient, as the dominant limitation of the sensitivity of our experiment is more likely to arise from satellite clock instabilities. All other, unmodeled, systematic effects, such as the residual ionospheric delay (in f^3) or the variation of the gravitational redshift due to the quadrupole moment of the Earth, should not induce uncertainties exceeding 100 ps.

VI. DISCUSSION AND CONCLUSION

Timing errors due to the systematic effects discussed in the previous section could give rise to values of $\delta c/c$ of order 10^{-8} $[\delta c/c\approx\delta\tau/(D/c)]$, assuming that no cancellation takes place between the systematic effects of different satellite-station pairs. This is an order of magnitude larger than the maximum value observed. The experiment therefore cannot suggest a violation of special relativity.

It is unlikely that, in a global treatment, the systematic errors are correlated with the signature of a nonzero value of δc, as they are expected, in general, to correlate differently with the effect of $\delta c/c$ for each satellite-station pair and, to some extent, for each passage. So the results from a number of randomly chosen subsets of the data can provide an estimate of the reliability of the results presented in Sec. IV. The least-squares adjustment was repeated for five different subsets of the complete data, removing observations from three to six satellites (i.e., 12% to 24% of the total data). The mean of the five adjusted extremum values of $\delta c/c$ is 4.6×10^{-9} when considering all spatial directions and 2.2×10^{-9} for the component in the equatorial plane, with a standard deviation about the mean of 1.3×10^{-9} in both cases. These values should be compared to $\delta c/c$ of 4.9×10^{-9} and 1.6×10^{-9} obtained from the complete data set (cf Sec. IV). So, assuming no correlation (and resulting cancellation) between the systematic effects in a global treatment and the effect of a possible nonzero value of δc (for the reasons mentioned above), we can set a limit of $\delta c/c<5\times10^{-9}$ when considering all spatial directions and $\delta c/c<2\times10^{-9}$ for the component of the direction of δc that lies in the equatorial plane.

Interpretation of the experiment, using the test theory developed by Mansouri and Sexl [8], shows that the value of $\delta c/c$ obtained here is related to the parameter α of the test theory (in special relativity $\alpha=-1/2$) by the well-known relationship (see [9] for details)

$$\frac{\delta c}{c}=(1+2\alpha)\,\frac{v}{c}, \tag{2}$$

where v is the velocity of the Earth with respect to the ''universal frame'' defined in the theory (the frame in which slow clock transport and Einstein synchronism are equivalent), and the parameter α is dependent on the choice of Einstein synchronism in this universal frame [15]. Additionally one obtains an explicit expression for the double difference Y (cf Fig. 1) that is independent of α to first order in v/c [9], which confirms the validity of the experimental principle in this test theory. Taking v as the velocity of the Earth with respect to the ''mean rest frame of the universe'' ($v \approx 300$ km/s) in the direction of the dipole anisotropy of the cosmic microwave background (declination$\approx -1°$, right ascension$\approx$11 h) [16,17], we obtain a limit of $|\alpha+1/2|<1\times10^{-6}$ ($\delta c/c<2\times10^{-9}$ in this direction). This, to our knowledge, is the smallest limit for the parameter α published to date [4].

In principle, any space mission flying a highly stable atomic clock that is equipped with a time transfer system could be used for a similar test of special relativity. An example is the experiment on timing, ranging and atmospheric sounding (ExTRAS) mission [18] that is currently ''on hold.'' This mission is of interest, as the space clock will be a hydrogen maser expected to be significantly more stable in the short term than the GPS clocks, and because the optical two-way time transfer system planned should diminish the effects of the time transfer (in particular, tropospheric) and ephemerides uncertainties [18].

Finally we would like to emphasize that an experiment like the one described in this paper can be carried out at minimal cost by virtually anyone, as all the IGS data are freely available on the internet via anonymous ftp. This is of particular interest in view of a recent U.S. presidential decision [19] to switch off selective availability completely on all GPS satellites within the next ten years.

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