Decoherence and Schrödinger-cat states in a Stern-Gerlach-type experiment

Anu Venugopalan*
Theoretical Physics Group, Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400 005, India
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A Stern-Gerlach setup for the measurement on a quantum system (spin-$\frac{1}{2}$) with a macroscopic quantum apparatus (the particle trajectory) produces a Schrödinger-cat-like superposition in which the spin states correlate with wave packets centered around macroscopically distinguishable positions and momenta. When an interaction with an environment is included, the pure density matrix of the system-apparatus combination reduces to a statistical mixture in the spin space, and *exact solutions* show that the decoherence time for this reduction goes inversly as the *macroscopic separation* between the two parts of the superposition correlating with up- and down-spin states. This is consistent with Zurek's approximate result for the decoherence time, and the persistence of system-meter correlations at large times makes it an interesting candidate to look at experimentally. [S1050-2947(97)08211-5]

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In spite of the tremendous success of quantum mechanics, some of its concepts seem absurd when related to the world of our experience, the familiar (classical) physical world. For example, when the linear superposition principle of quantum mechanics is extrapolated to macroscopic systems which are conventionally described by classical mechanics, we are faced with the counterintuitive prospect of ''Schrödinger's cat'' [1], a classic illustration of the clash between the predictions of quantum theory and familiar classical perceptions. A closely related problem is that of quantum measurement [2], where the coupling between a microscopic system and a macroscopic measuring apparatus results in an entangled state where quantum mechanics seems to allow the apparatus (''meter'') to exist in a coherent superposition of macroscopically distinct states, a situation which is hard to imagine in terms of classical intuition in the real world. Such concepts raise several questions about quantum theory's connection with the emergence of classicality and the elusive boundary between quantum and classical worlds [3]. It was postulated by von Neumann [2] that an irreversible reduction process takes the quantum superposition into a statistical mixture which is classically meaningful and interpretable. However, the nonunitary nature of this reduction seems to imply that the mechanism lies outside the realm of quantum mechanics, thus questioning its validity.

Recently, there has been much progress in the theoretical and experimental understanding of quantum decoherence, which is now widely being discussed as the mechanism responsible for the emergence of classicality in quantum measurement, and also for the absence, in the real physical world, of Schrödinger-cat-like states [3]. Decoherence results from the irreversible coupling of the system (or the apparatus in a measurementlike scenario) to its environment. The emergence of classicality via decoherence is marked by the dynamical transition of the reduced density matrix of the system of interest (after tracing over the environment degrees of freedom) from a pure state to a statistical mixture, for which all information on the system becomes classically interpretable. This line of approach was initiated by Zeh [4] and Zurek [3]. Most studies of decoherence in the literature deal with an environment modeled by a collection of oscillators, and the dynamics of the reduced density matrix of the system of interest is then studied via the corresponding master equation [5–7]. For our pupose we shall concern ourselves with the master equation derived by Caldeira and Leggett [7] using the Feynman-Vernon [8] influence functional technique. For the reduced density matrix of a free particle, this equation in the high-temperature (Markovian) limit can be written in the position representation as:

$$\frac{\partial\rho(x,x',t)}{\partial t}=\left[-\frac{\hbar}{2im}\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial x'^2}\right)-\gamma(x-x')\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)-D/4\hbar^2(x-x')^2\right]\rho(x,x',t), \tag{1}$$

where m is the mass of the particle, $\hbar$ is Planck's constant, γ is the Langevin friction coefficient, and D has the usual interpretation of the diffusion constant. γ and D are related to the parameters of the Hamiltonian of the total system. For a high-temperature thermal bath, $D=4m\gamma k_BT$. Zurek has argued that, out of all the terms in the above equation, it is the last term that is the dominant term for decoherence. Since one seeks to explain the emergence of classicality, it can be argued that in that limit, Planck's constant will be small relative to the actions involved and if the object of interest is massive, the last term naturally dominates. By considering only the last term in the equation, Zurek [3] showed that an initial coherent superpostion of two Gaussians separated by a distance Δx decoheres over a time scale given by

$$\tau_d=\gamma^{-1}[\lambda_d/\Delta x]^2, \tag{2}$$

where $\lambda_d=\hbar/\sqrt{2mk_BT}$ is the thermal de Broglie wavelength of the particle. Thus quantum coherence, which is signified by the presence of the off-diagonal elements of the density matrix of the initial superposition state, decays on a time scale that goes inversly as the separation between the two

*Present address: Laser Physics and Quantum Optics Group, Physical Research Laboratory, Navrangpura, Ahmedabad-380 009, India. Electronic address: anu@prl.ernet.in

parts of the superpostion. The further apart the two wave packets are, the faster the decoherence to a statistical mixture.

It has been shown [9] that, for an initial Gaussian wave packet, the exact solution to Eq. (1) when all the terms are included is such that, at large times ($\gamma t \gg 1$), there is a complete diagonalization of the density matrix in the momentum basis while the position space density matrix only diagonalizes to the extent of the de Broglie wavelength of the particle. It is believed [10] that the decoherence phenomenon is dominant during time scales which are short compared to the thermal relaxation time (γ^{-1}), which implies that decoherence operates in regimes where friction is neglegible. The exact solutions of Eq. (1) for an initial Gaussian wave packet when examined on short-time scales show that there is diagonalization in the position basis, and the time scale for this is similar to Eq. (2), supporting the fact that the decoherence mechanism illustrated by Zurek's approximate solution to Eq. (1) is indeed significant at time scales much shorter than the thermal relaxation time of the system. Note that this seems to be the case for the particular example considered here of the free particle described by the high-temperature Markovian master equation, where the system-heat bath coupling is a linear coordinate-coordinate coupling. In general, however, if we are to understand the decoherence time as the time scale over which the reduced density matrix of the system of interest diagonalizes, then it could depend on several factors like the form of the coupling to the reservoir etc. In general, decoherence is a consequence of both fluctuations and dissipation in the system.

Recently, Brune *et al.* [11] experimentally created a mesoscopic superposition of quantum states involving radiation fields with classically distinct phases, and observed its progressive decoherence to a statistical mixture. Such a superposition is an equivalent of an ''atom plus meter'' system in which the meter is simultaneously pointing in two different directions, i.e., in a Schrödinger-cat-like superposition. Decoherence here is a consequence of dissipation brought about by the linear coupling of the field mode with a bath of thermal oscillators at zero temperature [12]. Decoherence is then monitored using two-atom correlation experiments [11]. The dynamics of the reduced density matrix here is described by the Markovian master equation at zero temperature for a harmonic oscillator coupled to a bath of oscillators [6,7,13]: The master equation for the reduced density operator under the Born and Markov approximations is

$$\frac{\partial\hat{\rho}}{\partial t} = -i\omega[a^{\dagger}a,\hat{\rho}] - \frac{i\gamma}{\hbar}[\hat{X},\hat{P}\hat{\rho}+\hat{\rho}\hat{P}] - \frac{i\gamma}{2\hbar}[\hat{P}\hat{X}-\hat{X}\hat{P},\hat{\rho}] - \frac{2\gamma}{\hbar}(\overline{n}+1/2)m\omega[\hat{X},[\hat{X},\hat{\rho}]], \tag{3}$$

where these operators correspond to the system alone, as all the bath degrees of freedom have been averaged out. γ is the damping constant, $\hat{P}$ is the system momentum observable, and $\overline{n}$ is the expected number of quanta in a harmonic oscillator of frequency ω at equilibrium at temperature T:

$$\overline{n} = [\exp(\hbar\omega/k_B T)-1]^{-1}. \tag{4}$$

If one assumes that the heat bath is Markovian even at low temperatures, Eq. (3) is valid at arbitrary temperatures. At $T=0$, $\overline{n}=0$. For an initial superposition of two coherent states (Schrödinger-cat state) of the form

$$|\psi\rangle = 1/N\,(|\alpha\rangle + |-\alpha\rangle), \tag{5}$$

which evolves according to Eq. (3) at zero temperature, the transition to a statistical mixture is goverened by the exponential factor $\exp[-2|\alpha|^2(1-e^{-\gamma t})]$ [14]. For $\gamma t \ll 1$ (i.e., the regime in which decoherence is considered relevant), this factor becomes $\exp(-2|\alpha|^2\gamma t)$. Thus, as in the case for the free particle considered by Zurek, here the decoherence time goes inversly as the separation between the two parts of the superposition, which in this case is quantified by the scalar product $|\alpha|^2$. The further apart these meter states are, the faster is the decoherence. In the experiment of Brune *et al.* [11], the atom-meter state is

$$|\psi\rangle_{A+M} = 1/\sqrt{2}\,(|e,\alpha e^{i\phi}\rangle + |g,\alpha e^{-i\phi}\rangle), \tag{6}$$

where the separation $\Delta = 2\sqrt{n}\,\sin\phi$, with $\phi = \Omega^2 t/\delta$, with Ω corresponding to the Rabi frequency and δ the detuning [11]. $|\alpha e^{i\phi}\rangle$ and $|\alpha e^{-i\phi}\rangle$ are like macroscopic pointers (when $|\alpha|^2 \gg 1$), which are related to the microscopic atomic states, the field being left in the state $|\alpha e^{i\phi}\rangle$ when the atom crosses the cavity in state $|g\rangle$ or in state $|\alpha e^{-i\phi}\rangle$ if the atom is in state $|e\rangle$. In their experiment, Brune *et al.* saw that decoherence, which is marked by a decrease in the fringe contrast in their measurement scheme [11], occurs over a time scale which goes inversly as Δ^2. This behavior is seen in time scales which are much shorter compared to the thermal relaxation times (γ^{-1}) of the system, i.e., in the regime ($\gamma t \ll 1$). If one waits for longer times, it can be seen that the fields relax toward vacuum, and no longer cease to be orthogonal, and their overlap becomes important [14]. This is a consequence of field dissipation, and soon enough one loses the one-to-one correlation between the atom and the meter states. Thus in these experiments the study of decoherence is to be confined to extremely short-time scales, and, since the cavity relaxation times are very fast, there is a need to have low dissipation cavities with very large damping times [11].

We [15] have analyzed the Stern-Gerlach model for the measurement of spin-$\frac{1}{2}$ using the decoherence approach. In this model a spin-$\frac{1}{2}$ particle is in an inhomogeneous magnetic field, and the whole setup is in contact with an external environment. Here the spin constitutes the system of interest, while the position or momentum degrees of freedom of the particle is like the apparatus. The Hamiltonian of the combined system and apparatus with the environment is

$$H^{SAE} = (p^2/2m) + \lambda\sigma_Z + \epsilon x\sigma_Z + H^{AE} + H^E. \tag{7}$$

Here x and p denote the position and momentum of the particle of mass m, $\lambda\sigma_Z$ the Hamiltonian of the system, ϵ the product of the field gradient and the magnetic moment of the particle, H^{AE} the interaction of the environmental degrees of freedom with the coordinate x, and H^E the Hamiltonian for the environmental degrees of freedom. The model of the environment is the usual oscillator heat bath model as discussed above, and they deal directly with the master equation, now corresponding to four elements of the spin space, $(\uparrow\uparrow,\uparrow\downarrow,\downarrow\uparrow,\downarrow\downarrow)$ [15]:

$$\frac{\partial \rho_{ss'}(x,x',t)}{\partial t}=\left[-\frac{\hbar}{2im}\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial x'^2}\right)-\gamma(x-x')\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial x'}\right)-\frac{D}{4\hbar^2}(x-x')^2+\frac{i\epsilon(xs-x's')}{\hbar}+\frac{i\lambda(s-s')}{\hbar}\right]\rho_{ss'}(x,x',t), \tag{8}$$

where $s,s'=+1$ (for $\uparrow$) or -1 (for $\downarrow$). The details of the calculations can be seen in Ref. [15]. If the initial condition for the system and apparatus combination is a product of a Gaussian wave packet and a superposition of spin states of the form

$$\psi(x,o)=\frac{1}{\sqrt{\sigma\sqrt{\pi}}}\exp(ipx-x^2/2\sigma^2)\otimes(a|\uparrow\rangle+b\downarrow\rangle), \tag{9}$$

then one can easily check that in the absence of environmental interaction, the state of the system is indeed a Schrödinger-cat-like state whose density matrix can be written as

$$\rho=|a|^2|\uparrow\rangle\langle\uparrow|\psi_+^*(x,t)\psi_+(x',t)+|b|^2|\downarrow\rangle\times\langle\downarrow|\psi_-^*(x,t)\psi_-(x',t)+ab^*|\uparrow\rangle\langle\downarrow|\psi_+^*(x,t)\psi_-(x',t)+a^*b|\downarrow\rangle\langle\uparrow|\psi_-^*(x,t)\psi_+(x',t). \tag{10}$$

The spin-up and -down states are thus correlated with $\psi_+^*(x,t)\psi_+(x',t)$ and $\psi_-^*(x,t)\psi_-(x',t)$, which correspond to the position distributions:

$$|\psi|_\pm^2(x,t)=\left(\frac{\pi}{\sigma^2+\frac{\hbar^2t^2}{\sigma^2m^2}}\right)^{1/2}\exp\left[-\frac{1}{\sigma^2+\frac{\hbar^2t^2}{\sigma^2m^2}}\times\left(x\pm\frac{\epsilon t^2}{2m}\right)^2\right], \tag{11}$$

which corresponds to the distance

$$\Delta_x=\epsilon t^2/m \tag{12}$$

between the ''pointers'' which are, at this point, in a coherent superposition. If Δ_x is macroscopic, we have a Schrödinger-cat-state. It may be noted that the wave packet would spread in time in the position space, and so Eq. (10) would represent a macroscopic superposition in the true sense only if the widths of the wave packets are smaller than the separation between their peaks. Since both increase with time and the separation additionally depends on ϵ, one can ensure such a condition by adjusting ϵ. In the momentum space also, it can be seen that Eq. (10) is a Schrödinger-cat-like state where, unlike the position case, there will be no spread in the width of the wave packet with time. When environmental interactions are included, one can see from the results of Ref. [15] that Eq. (10) diagonalizes at large times ($\gamma t\gg 1$) to a statistical mixture:

$$\rho\rightarrow|a|^2|\uparrow\rangle\langle\uparrow|\rho_{\uparrow\uparrow}+|b|^2|\downarrow\rangle\langle\downarrow|\rho_{\downarrow\downarrow}, \tag{13}$$

where the pointers $\rho_{\uparrow\uparrow}$ and $\rho_{\downarrow\downarrow}$ are no longer in a coherent superposition, but correspond to diagonal distributions in momentum:

$$\rho_{\uparrow\uparrow,\downarrow\downarrow}(u)=\left(\frac{\pi}{N(t)}\right)^{1/2}\exp\left[-\frac{1}{N(t)}\left(u\mp\frac{\epsilon}{\hbar\gamma}\right)^2\right]. \tag{14}$$

where

$$N(t)=\frac{D}{2\hbar^2\gamma}(1-e^{-2\gamma t})+\frac{1}{\sigma^2}e^{-2\gamma t}, \tag{15}$$

and *approximate* [15] diagonal distributions in position,

$$\rho_{\uparrow\uparrow,\downarrow\downarrow}(x)=\left(\frac{\pi}{M(t)}\right)^{1/2}\exp\left[-\frac{1}{M(t)}\left(x-\frac{\overline{p\hbar}}{m\gamma}\mp\frac{\epsilon t}{m\gamma}\pm\frac{\epsilon}{m\gamma^2}\right)^2\right], \tag{16}$$

where

$$M(t)=\sigma^2+\frac{\hbar^2}{\sigma^2m^2\gamma^2}(1-e^{-2\gamma t})+\frac{D}{2m^2\gamma^3}(2\gamma t-3+4e^{-\gamma t}-e^{-2\gamma t}). \tag{17}$$

As has been pointed out before [9], for the distribution in position space the off-diagonal elements are not strictly zero in the large time limit, though initially (at shorter times) they decay faster than the momentum off-diagonal elements. The separation between the mean pointer positions is $\Delta_x=2\epsilon t/m\gamma$, and that between the mean pointer momenta is $\Delta_p=2\epsilon/\hbar\gamma$. The interesting point, now, is that $\rho_{\uparrow\downarrow}$ and $\rho_{\downarrow\uparrow}$, which are correlated with the off-diagonal elements in spin space decay to zero at large times ($\gamma t\gg 1$), as seen in the exact solutions in the partial fourier transform representation where the leading decay term A goes as [15]

$$A\sim\exp(-\epsilon^2Dt^3/3m^2\gamma^2\hbar^2). \tag{18}$$

An examination of this term clearly shows that

$$A\sim\exp\left(-\frac{\epsilon^2Dt^3}{3m^2\gamma^2\hbar^2}\right)=\exp\left(-\frac{\Delta_x^2Dt}{12\hbar^2}\right)=\exp\left(-\frac{\Delta_p^2Dt^3}{12m^2}\right). \tag{19}$$

Thus the decoherence corresponding to the decay of the elements correlated with the off-diagonals in spin space increases as the square of the *macroscopic* separation in mean position Δ_x and momentum Δ_p. Note that this result is a consequence of the *exact* solutions to the high-temperature master equation at large times, and includes the effect of *both* dissipation and fluctuations. Though it is quite straightforward to see that decoherence increases as the square of the macroscopic separations, since the separation in mean position Δ_x itself is time dependent, it is not easy to directly extract a decoherence time. The separation between the two components of the superposition in the experiment of Brune *et al.* is also time dependent [11], as pointed out above. However, in their experiment, the transit times across the cavity of the first atom which creates the superposition and

of the second which probes it are lower than the delay. The phase ϕ evolves only during the interaction times, and is left constant during the delay between the two atoms. It is thus assumed that the two coherent components are instantaneously separated by the first atom, left free to relax for the a fixed time interval T, and then probed, again instantaneously, by the second atom [16]. In this way it is possible to extract a decoherence time as the separation is no longer time dependent. In the Stern-Gerlach example considered here, the time dependence of the macroscopic separation in position space makes it slightly more complicated. However, it is quite clear that the same behavior in terms of the dependence of decoherence on the macroscopic separation emerges even in this case. In terms of the macroscopic separation in the momentum space, however, one can always extract a time scale

$$\tau_{d_p} \sim [12m^2/D\Delta_p^2]^{1/3}, \tag{20}$$

since the separation Δ_p is not time dependent. Note that in this case we see this feature of decoherence increasing with the square of the macroscopic separation even at large times for the exact solutions, and have not taken any short-time limit. This thus proves Zurek's earlier result in a much more general framework. Note also that unlike the system studied experimentally by Brune *et al.* [11], here the system-meter correlations are permanent, and will not be lost due to energy dissipation as was the case there. The parameters involved, i.e., ϵ and γ, can also be externally controlled. A possible experimental scenario could be a spin-recombination set up in which the first Stern-Gerlach spits the spin-$\frac{1}{2}$ beam and the second one recombines these split beams in a reversed magnetic field. If, in such a setup, we introduce the environment, say, in form of a certain amount of gas which decoheres the total density matrix, then the following consequences emerge. When an x-polarized beam is passed through the first setup, the beam splits into two z-polarized beams. In the absence of any decoherence, if these two beams are recombined in the second setup, one would again obtain the x polarization. On the other hand, in the presence of decoherence, one will obtain a statistical mixture of two z-polarized beams at the end of the second Stern-Gerlach setup. A quantitative study of the effects induced by varying the pressure and temperature of the gas (and hence affecting γ), and ϵ will then provide an understanding of the decoherence phenomenon in general, and a means to estimate the decoherence time. The analysis in this paper has been in the context of the experiment done by Brune *et al.* It may be noted that other experiments like the atom optical Stern-Gerlach effect using cold atoms and laser dipole forces observed by Sleator *et al.* [17], are closer to the spirit of this paper, and are well worth analyzing in the context of this work.

To conclude, a Stern-Gerlach setup for the measurement of spin-$\frac{1}{2}$ [15] produces Shrödinger-cat-like states where the meter positions are the particle trajectories correlated with up- and down-spins and macroscopically separated in terms of position and momentum. These states decohere when the environment is included in the form of the Caldeira-Leggett heat bath model. An analysis of the exact solutions to the master equation show that the decoherence increases directly as the square of the *macroscopic separation* between mean position as well as the mean momenta of the meter. This behavior is seen at times larger than the thermal relaxation time, unlike previous estimates of the decoherence times where one had to look for decoherence in the limit of negligible friction (i.e., $\gamma t \ll 1$). These exact results are also important for the short time regime since they do not involve neglecting any term in the master equation. These solutions show the same kind of dependence of the decoherence rate on the separation between the two parts of the superpositions as in Zurek's earlier approximate result [3]. Also in this example, the fact that the system-meter correlations are permanent, and do not disappear due to dissipation, makes this an interesting system to look at experimentally from the point of view of quantum measurement.

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