Electron-atom ionizing collisions in the presence of a bichromatic laser field

D. B. Milošević
Faculty of Science and Mathematics, Department of Physics, University of Sarajevo, 71000 Sarajevo, Bosnia and Herzegovina and Faculty of Mechanical Engineering, Department of Precise Mechanics and Optics, University of Sarajevo, 71000 Sarajevo, Bosnia and Herzegovina

F. Ehlotzky
Institute for Theoretical Physics, University of Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria
(Received 25 February 1997)

A theory of electron-atom ionizing collisions in the presence of a bichromatic laser field is developed. The incident and scattered electron of high energy is described by Volkov states, the atomic ground state is dressed by the laser field to first order in the electric-field strength, and the atomic continuum state that corresponds to the ejected slow electron is dressed by the laser field to all orders in the field strength, but in an approximate way. It is shown that the dressing of these continuum states is mainly responsible for the different effects that a bichromatic laser field introduces into this process. These effects are more pronounced compared to similar effects recently observed in an analysis of the target dressing in free-free transitions in a bichromatic laser field. The possibility of the coherent phase control is analyzed and it is shown that the triple differential cross section (TDCS) can be changed by almost one order of magnitude by changing the relative phase between the two laser field components. The TDCS as a function of the relative phase, the number of absorbed (emitted) photons, and the laser electric-field strength shows a specific behavior that is mainly determined by the properties of the generalized Bessel functions appearing in our results. It is also shown that these effects are particularly pronounced for rather small scattering angles of the fast electron. [S1050-2947(97)04310-2]

PACS number(s): 34.50.Rk, 34.80.Qb, 32.80.−t

I. INTRODUCTION

The investigation of atomic processes in multicolor laser fields has recently become of particular interest. It has been shown that the rates of laser-assisted and laser-induced atomic and molecular processes can be considerably enhanced or modified in such fields. The important parameter in such investigations is the phase difference ϕ between the laser field components (usually the laser fields considered are bichromatic and consist of two components of frequencies ω and 2ω or ω and 3ω). It was found that for particular values of ϕ the molecular reaction rates can be suppressed or enhanced and the angular distributions modified (see [1–5] and references therein). This effect was coined coherent phase control (CPC). The investigations of the CPC of molecular reactions were followed by similar considerations of the CPC of multiphoton ionization (both experimentally [6–8] and theoretically [9–20]), autoionization [21,22], high-order harmonic generation [23–30], and free-free transitions in a laser field [31–40].

The electron-impact ionization of atoms in the absence of a laser field is well described in textbooks [41,42]. It is interesting that this reaction has its prehistory in nuclear physics, where the first $(p,2p)$ experiments were proposed forty years ago (see, for example, [43]). The electron-impact ionization of atoms or $(e,2e)$ reactions are important for the electron momentum spectroscopy of atoms, molecules, and solids. An initial experiment of this kind was performed in 1969 (see [43] and references therein).

An initial treatment of electron-impact ionization in the presence of a laser field was given in [44]. This paper was followed by the work of many authors [45–53], who gave a more detailed treatment of this process (see also Mittleman's book [54]). Essentially, the results for the triple differential cross sections (TDCSs), presented in these papers, have a structure similar to the cross sections of other laser-assisted atomic collision processes derived in the first Born approximation (FBA) [55] or low-frequency approximation [56], i.e., the TDCS is expressed as a product of the square of an ordinary Bessel function and the TDCS for the electron-impact ionization without the laser field but with laser-field-dependent momenta and energy shifts. Later on it was recognized [57–61] that the influence of the dressing of the atomic states by a laser field cannot be neglected. In our paper we shall follow the approach of these papers and this will be explained in more detail in Sec. II.

Because we are interested in the CPC effect, we shall simplify our considerations by analyzing the case of $(e,2e)$ reactions for atomic hydrogen only and, moreover, in Ehrhardt's asymmetric coplanar geometry [62,63]. Ehrhardt's group has studied the situation in which a fast incident electron of momentum $\vec{k}_i$ and energy $E_{k_i}\approx 250$ eV scatters at the target and the outgoing fast electron of momentum $\vec{k}_A$ is detected in coincidence with a slow ejected electron of momentum $\vec{k}_B$. All three momenta $\vec{k}_i$, $\vec{k}_A$, and $\vec{k}_B$ are taken in the same plane and the scattering angle θ_A of the fast electron is fixed and small ($\approx 3°$), so that the momentum transfer $\vec{K}=\vec{k}_i-\vec{k}_A$ is relatively small. The angle θ_B of the slow electron ($E_{k_B}\approx 5$ eV) is varied. We shall also choose a linearly polarized laser field in which both field components have the same field strength and polarization, i.e.,

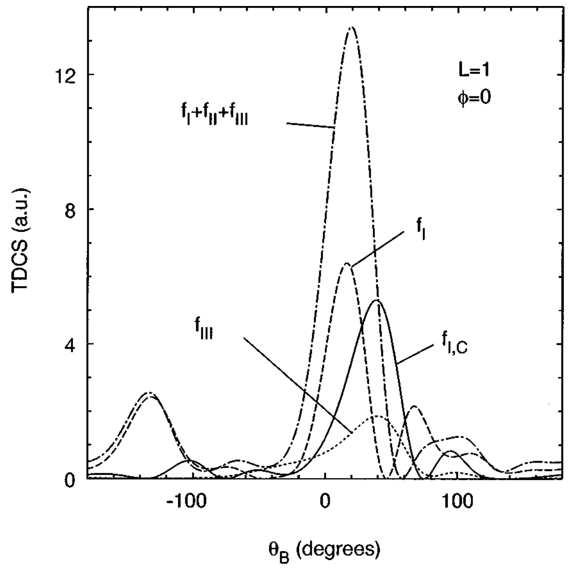

FIG. 1. TDCS (in a.u.) for the ionization of atomic hydrogen from the ground state by electron impact in the presence of a bichromatic laser field as a function of the angle θ_B of the ejected electron. The incident electron kinetic energy is $E_{k_i}=250$ eV, the ejected electron energy is $E_{k_B}=5$ eV, and the scattering angle is $\theta_A=3°$. The laser electric field strength is $E_0=5\times10^9$ V/m and the laser photon energy is $\omega=1.17$ eV. The electric-field vector $\vec{E}_0$ is taken parallel to the incident momentum $\vec{k}_i$. The relative phase between the laser field components is $\phi=0$ and the number of photons absorbed in the reaction is $L=1$. The complete TDCS is presented by the dot-dashed curve ($f_{\rm I}+f_{\rm II}+f_{\rm III}$). The continuous curve ($f_{\rm I,C}$) represents the contribution of the first part of the scattering amplitude $f_{\rm I}$ to the complete TDCS. The contributions of the amplitudes $f_{\rm I}$ and $f_{\rm III}$ are presented by dashed ($f_{\rm I}$) and dotted ($f_{\rm II}$) curves, respectively. The amplitude $f_{\rm II}$ is negligibly small in comparison to $f_{\rm I}$ and $f_{\rm III}$ and therefore is not presented in the figure.

$$\vec{E}(t)=\vec{E}_0[\sin\,\omega t+\sin(2\omega t+\phi)],\tag{1}$$

with the electric field vector $\vec{E}_0$ parallel to the incident momentum $\vec{k}_i$ and the fundamental laser field frequency equal to the Nd:YAG laser frequency $\omega=1.17$ eV (where YAG denotes yttrium aluminum garnet).

In Sec. II we present our theory, which is essentially a generalization to the case of a bichromatic laser field of the results of previous work [57,58]. We shall also discuss the closure approximation that we shall apply in our numerical calculations. In Sec. III we present and discuss our numerical results. Finally, Sec. IV is devoted to the conclusions. All our results were derived in SI units and the final expressions presented in this paper are in the atomic system of units ($\hbar=e=m=1$).

II. THEORY

We shall consider the $(e,2e)$ reaction $e^-+{\rm H}(1s)\rightarrow{\rm H}^+ +2e^-$, during which L photons are transferred between the electron-atom system and the field so that the energy conserving condition is

$$E_{k_i}+E_0+L\omega=E_{k_A}+E_{k_B},\tag{2}$$

where $E_0=-1/2$ a.u. is the ground-state energy of the hydrogen atom and $E_{k_j}=\vec{k}_j^2/2$, $j=i,A,B$. Positive values of L correspond to the photon absorption process. With reference to the discussion in [58] and taking into account that we are mainly interested in the effects introduced by a bichromatic laser field, we assume that the FBA is adequate for the description of our process. The FBA S-matrix element for the $(e,2e)$ reaction in the presence of the laser field is

$$S_{\rm ion}^{\rm FBA}=-i\int_{-\infty}^{\infty}dt\langle\chi_{\vec{k}_A}(t)|\langle\Phi_{\vec{k}_B}(t)|\times(V_0+V_{01})|\chi_{\vec{k}_i}(t)\rangle|\Phi_0(t)\rangle.\tag{3}$$

If we denote the coordinates of the projectile and target electrons by $\vec{r}_0$ and $\vec{r}_1$, respectively, then $V_0=-1/r_0$ and $V_{01}=1/|\vec{r}_0-\vec{r}_1|$. The incident and scattered electrons are described by the Volkov wave functions $\chi_{\vec{k}_i}(\vec{r}_0,t)$ and $\chi_{\vec{k}_A}(\vec{r}_0,t)$, which, for the bichromatic laser field (1), have the form

$$\chi_{\vec{k}}(\vec{r}_0,t)=(2\pi)^{-3/2}\exp\{i[\vec{k}\cdot\vec{r}_0-\vec{k}\cdot\vec{\alpha}(t)-E_kt]\},\tag{4}$$

where $E_k=\vec{k}^2/2$ and

$$\vec{\alpha}(t)=\int^t dt'\vec{A}(t')=\vec{\alpha}_0[\sin\,\omega t+\tfrac{1}{4}\,\sin(2\omega t+\phi)],$$

$$\alpha_0=A_0/\omega=E_0/\omega^2.\tag{5}$$

$\vec{A}(t)$ is the vector potential of the laser field $[\vec{E}(t)=-\partial\vec{A}(t)/\partial t]$,

$$\vec{A}(t)=\vec{A}_0[\cos\,\omega t+\tfrac{1}{2}\,\sin(2\omega t+\phi)].\tag{6}$$

The Volkov states (4) are in the momentum gauge and the A^2 term has been removed by performing a standard unitary transformation. The wave functions $\Phi_0(\vec{r}_1,t)$ and $\Phi_{\vec{k}_B}(\vec{r}_1,t)$ appearing in Eq. (3) are the laser modified atomic hydrogen-ground and continuum states, respectively. Similarly, as in [58], we suppose that the laser field is not too strong so that we can use the first-order time-dependent perturbation theory to obtain explicit expressions for these states. Omitting the details of the derivation, we present here the final results for these states in the case of the bichromatic laser field (1). The dressed ground-state wave function of the hydrogen atom is given by

$$\Phi_0(\vec{r}_1,t)=\exp\{-i[\vec{A}(t)\cdot\vec{r}_1+E_0t]\}\times\left\{\psi_0(\vec{r}_1)+\frac{i}{2}\sum_n\psi_n(\vec{r}_1)\langle\psi_n|\vec{r}_1\cdot\vec{E}_0|\psi_0\rangle\times\left[\frac{\exp(i\omega t)}{E_n-E_0+\omega}-\frac{\exp(-i\omega t)}{E_n-E_0-\omega}+\frac{\exp(2i\omega t+i\phi)}{E_n-E_0+2\omega}-\frac{\exp(-2i\omega t-i\phi)}{E_n-E_0-2\omega}\right]\right\},\tag{7}$$

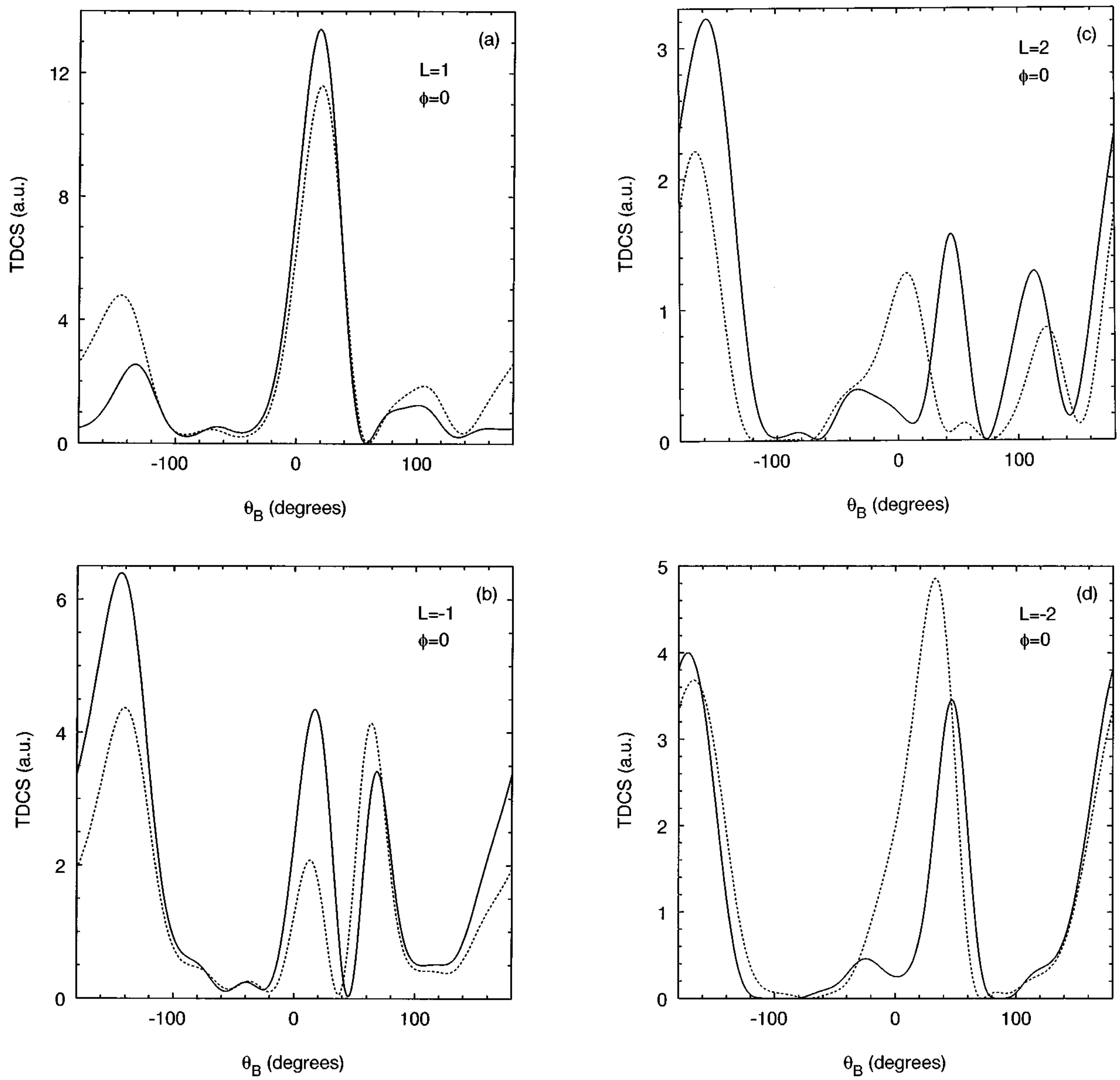

FIG. 2. Comparison of the results for the TDCS as a function of the angle θ_B of the ejected electron in the presence of a monochromatic (dotted curve) and a bichromatic (continuous curve) laser field, respectively, for (a) $L=1$, (b) $L=-1$, (c) $L=2$, and (d) $L=-2$ absorbed or emitted photons. The other parameters are as in Fig. 1.

where $\psi_n(\vec{r}_1)$ are the target states of energy E_n in the absence of the laser field and the summation is over the discrete and continuum states of the hydrogen atom. The factor $\exp[-i\vec{A}(t)\cdot\vec{r}_1]$ is introduced to ensure the gauge consistency with the Volkov wave functions, (4), which are given in the momentum gauge, while Eq. (7) was derived in the $\vec{r}\cdot\vec{E}$ gauge. Similarly, by the use of time-dependent perturbation theory in a way analogous to the low-frequency analysis of [47], for the continuum wave function we obtain

$$\Phi_{\vec{k}_B}(\vec{r}_1,t)=\exp\{-i[\vec{A}(t)\cdot\vec{r}_1+\vec{k}_B\cdot\vec{\alpha}(t)+E_{k_B}t]\}$$
$$\times\Bigg\{\psi_{\vec{k}_B}(\vec{r}_1)[1+i\vec{k}_B\cdot\vec{\alpha}(t)]+\frac{i}{2}\sum_n \psi_n(\vec{r}_1)$$
$$\times\langle\psi_n|\vec{r}_1\cdot\vec{E}_0|\psi_{\vec{k}_B}\rangle$$
$$\times\Bigg[\frac{\exp(i\omega t)}{E_n-E_{k_B}+\omega}-\frac{\exp(-i\omega t)}{E_n-E_{k_B}-\omega}$$
$$+\frac{\exp(2i\omega t+i\phi)}{E_n-E_{k_B}+2\omega}-\frac{\exp(-2i\omega t-i\phi)}{E_n-E_{k_B}-2\omega}\Bigg]\Bigg\}. \tag{8}$$

In the solutions (7) and (8) $\psi_0(\vec{r})$ is the wave function of the ground state of the hydrogen atom $\psi_0(\vec{r})=\pi^{-1/2}\exp(-r)$ and $\psi_{\vec{k}_B}(\vec{r})$ is the Coulomb wave function

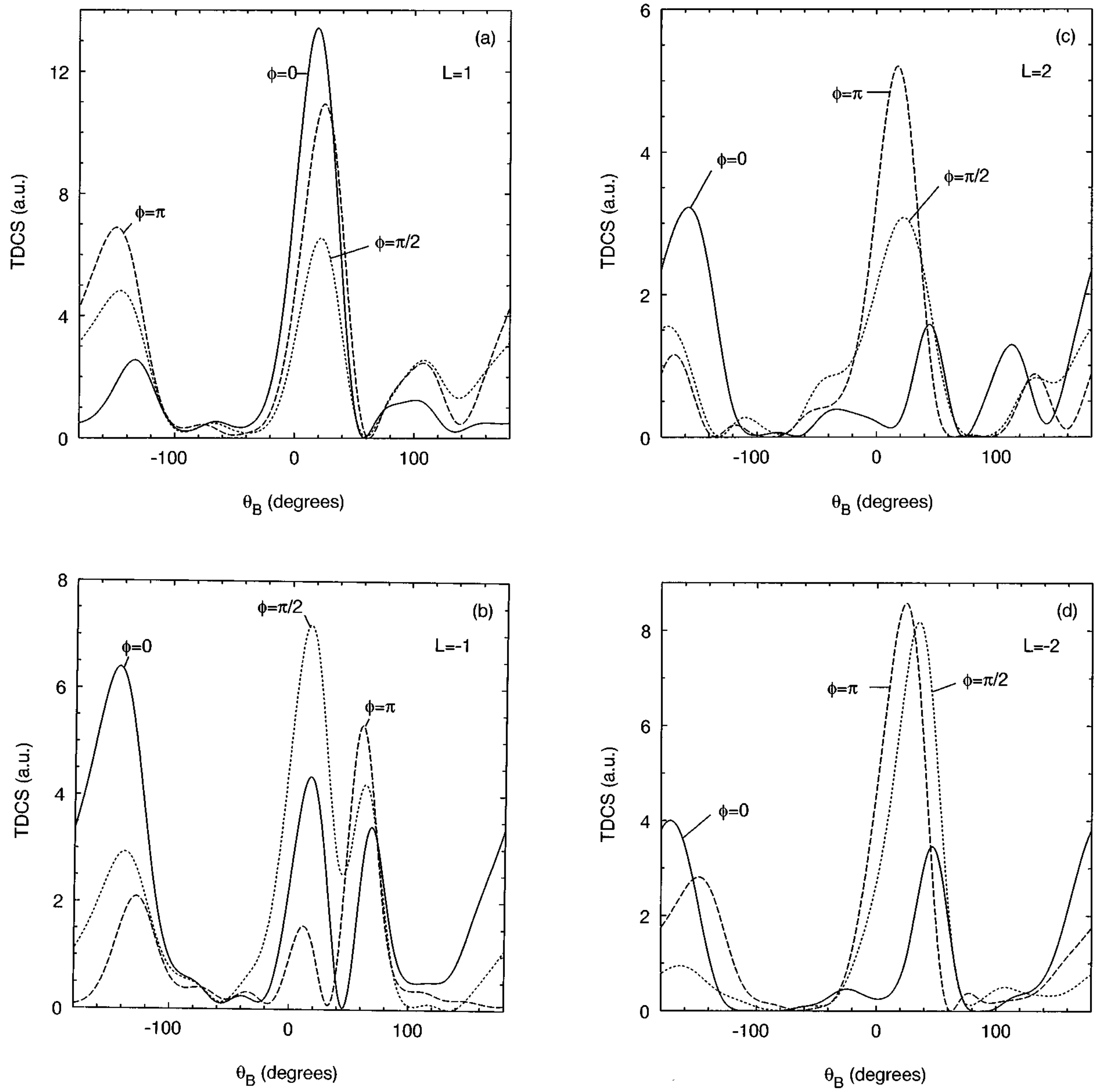

FIG. 3. TDCS as a function of the angle θ_B of the ejected electron for different values of the relative phase ϕ between the bichromatic laser field components and for a different number of photons L, absorbed or emitted in the reaction. The other parameters are as in Fig. 1. Continuous curve, $\phi=0$; dotted curve, $\phi=\pi/2$; and dashed curve, $\phi=\pi$. (a) $L=1$, (b) $L=-1$, (c) $L=2$, and (d) $L=-2$.

$$\psi^{-}_{\vec{k}_B}(\vec{r})=(2\pi)^{-3/2}\exp[\pi/(2k_B)]\Gamma(1+i/k_B)\times\exp(i\vec{k}_B\cdot\vec{r})\,{}_1F_1(-i/k_B,1,-i(k_Br+\vec{k}_B\cdot\vec{r})). \tag{9}$$

It should be mentioned that the A^2 term cannot be removed from these dressed states as it was done with the Volkov states. This can only be achieved within a perturbative approach, which puts an upper limit on the laser field intensity for which this approximation is valid. For our Nd:YAG laser and the energy chosen for the slow electron $E_{k_B}=5$ eV, the parameter U_p/E_{k_B} is less than 10% if the laser field intensity is less than 4.8×10^{12} W/cm^2 ($U_p=E_0^2/4\omega^2$ is the ponderomotive energy corresponding to the A^2 term). Hence our limit on the laser electric field strength is $E_0<6\times10^9$ V/m. The Volkov states, as well as the state (8), contain the laser field to all orders via the factor $\exp[-i\vec{k}\cdot\vec{\alpha}(t)]$. The dressing of the atomic states by the laser field is included in the states (7) and (8) only to first order in E_0.

Introducing the wave functions (4), (7), and (8) into the S-matrix element (3), we obtain the following result after integration over the time and the coordinates $\vec{r}_0$:

$$S^{\rm FBA}_{\rm ion}=\frac{i}{2\pi}\sum_{L=-\infty}^{\infty}\delta(E_{k_A}+E_{k_B}-E_{k_i}-E_0-L\omega)f^{{\rm FBA},L}_{\rm ion},$$

$$f^{{\rm FBA},L}_{\rm ion}=f_{\rm I}+f_{\rm II}+f_{\rm III}, \tag{10}$$

where

$$f_{\mathrm{I}}=-\frac{1}{K^2}\langle\psi_{\vec{k}_B}|e^{i\vec{K}\cdot\vec{r}_1}|\psi_0\rangle\{2b_L+\vec{k}_B\cdot\vec{\alpha}_0[b_{L-1}-b_{L+1}+\tfrac{1}{4}(e^{-i\phi}b_{L-2}-e^{i\phi}b_{L+2})]\},\tag{11}$$

$$f_{\mathrm{II}}=\frac{i}{K^2}\sum_n\langle\psi_{\vec{k}_B}|e^{i\vec{K}\cdot\vec{r}_1}|\psi_n\rangle\langle\psi_n|\vec{E}_0\cdot\vec{r}_1|\psi_0\rangle\left(\frac{b_{L-1}}{E_n-E_0-\omega}-\frac{b_{L+1}}{E_n-E_0+\omega}+\frac{e^{-i\phi}b_{L-2}}{E_n-E_0-2\omega}-\frac{e^{i\phi}b_{L+2}}{E_n-E_0+2\omega}\right),\tag{12}$$

$$f_{\mathrm{III}}=\frac{i}{K^2}\sum_n\langle\psi_{\vec{k}_B}|\vec{E}_0\cdot\vec{r}_1|\psi_n\rangle\langle\psi_n|e^{i\vec{K}\cdot\vec{r}_1}|\psi_0\rangle\left(\frac{b_{L-1}}{E_n-E_{k_B}+\omega}-\frac{b_{L+1}}{E_n-E_{k_B}-\omega}+\frac{e^{-i\phi}b_{L-2}}{E_n-E_{k_B}+2\omega}-\frac{e^{i\phi}b_{L+2}}{E_n-E_{k_B}-2\omega}\right).\tag{13}$$

The functions b_L are defined by

$$b_L\equiv B_L^*(\lambda,\lambda/4;\phi)=\sum_{\mu=-\infty}^{\infty}J_{L-2\mu}(\lambda)J_\mu(\lambda/4)\exp(i\mu\phi),$$

$$\lambda=(\vec{K}-\vec{k}_B)\cdot\vec{\alpha}_0,\tag{14}$$

where $B_L(a,b;\phi)$ are generalized Bessel functions defined in [31–37,40]. We are using the functions b_L instead of B_L because, using the formula $B_L^*(\lambda,0;\phi)=J_L(\lambda)$, we can easily establish the connection between our results and those of [57,58]. In the following we shall use the closure approximation in order to simplify the expressions for the scattering amplitudes f_{II} and f_{III}. In the amplitude f_{II} we replace E_n-E_0 by the mean value $\bar{E}=4/9$ a.u. According to [64,65,40], this is a reasonable choice. It is more difficult to determine the value of $\bar{E}$ for the amplitude f_{III}. We expect that the dominant contribution to the scattering amplitude comes from the term with $E_n\approx E_{k_B}$ [58] and therefore we shall choose in the amplitude f_{III} $\bar{E}=E_n-E_{k_B}=0$. In this case, using the closure relation $\Sigma_n|\psi_n\rangle\langle\psi_n|=1$, in both amplitudes f_{II} and f_{III} the following matrix elements appear: $\langle\psi_{\vec{k}_B}|\vec{r}_1\cdot\vec{E}_0\exp(i\vec{K}\cdot\vec{r}_1)|\psi_0\rangle=-i\vec{E}_0\cdot(\partial/\partial\vec{K})\langle\psi_{\vec{k}_B}|\exp(i\vec{K}\cdot\vec{r}_1)|\psi_0\rangle$, and we therefore only need the analytical result for the matrix element $\langle\psi_{\vec{k}_B}|\exp(i\vec{K}\cdot\vec{r}_1)|\psi_0\rangle$ for the computation of the whole scattering amplitude. We justify our use of the closure approximation by stressing that in this paper we are mainly interested in those effects that a bichromatic laser field introduces into the electron-atom ionizing collision process as well as the possibility of the CPC of this process. We can go beyond this approximation using the Sturmian expansion of the Coulomb Green's functions that appear in Eqs. (12) and (13). We do not expect any significant change for the amplitude f_{II} because our laser field frequency is much smaller than any excitation energy from the ground state of the hydrogen atom and our laser electric field is not too strong. As concerns the amplitude f_{III}, the Sturmian expansion may not be adequate because for our values of the laser field and the scattering parameters the energy of the Green's function is positive and the Sturmian expansion may diverge. In this case, one can apply the Dalgarno method (see, for example, [58]), but there are still some difficulties that should be overcome.

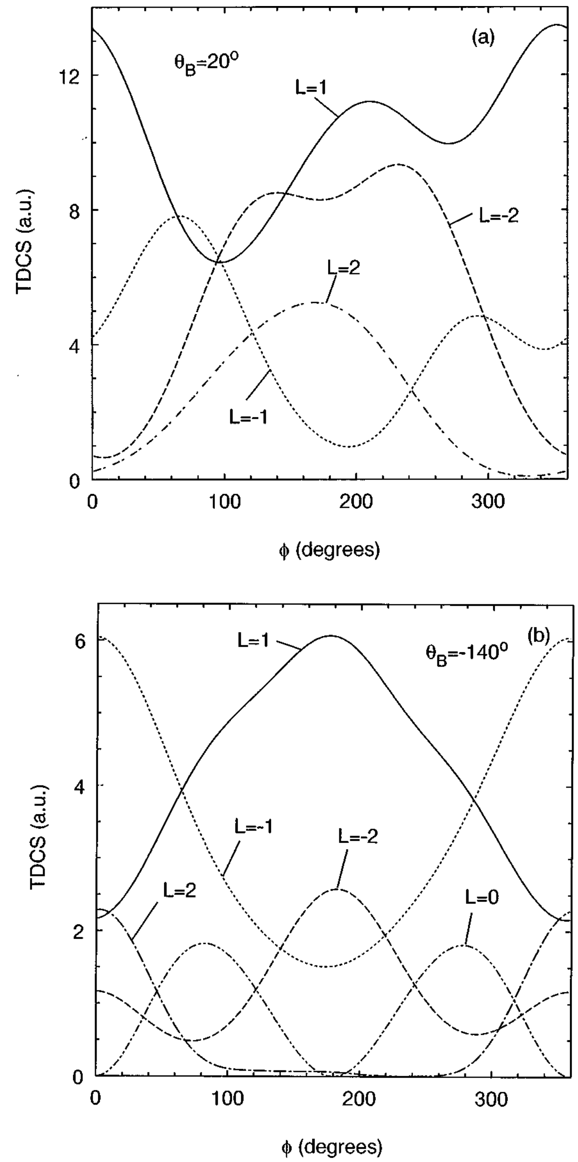

FIG. 4. TDCS for the ionization of atomic hydrogen from the ground state by electron impact in the presence of a bichromatic laser field as a function of the relative phase ϕ between the laser field components. The ejected electron angle is (a) $\theta_B=20°$ and (b) $\theta_B=-140°$. L is the number of photons absorbed or emitted in the reaction (continuous curve, $L=1$; dotted curve, $L=-1$; dot-dashed curve, $L=2$; dashed curve, $L=-2$; and double-dot–dashed curve, $L=0$ [in (b) only]. The other parameters are as in Fig. 1.

The TDCS for the electron-impact ionization with the absorption of L photons, which corresponds to the S-matrix element (10), is given by

$$\frac{d^3\sigma_{\mathrm{ion}}^{\mathrm{FBA},L}}{d\Omega_A d\Omega_B dE}=\frac{k_Ak_B}{k_i}|f_{\mathrm{ion}}^{\mathrm{FBA},L}|^2.\tag{15}$$

If we only retain the first part of the amplitude f_{I} and neglect the amplitudes f_{II} and f_{III}, then we obtain in the monochro-

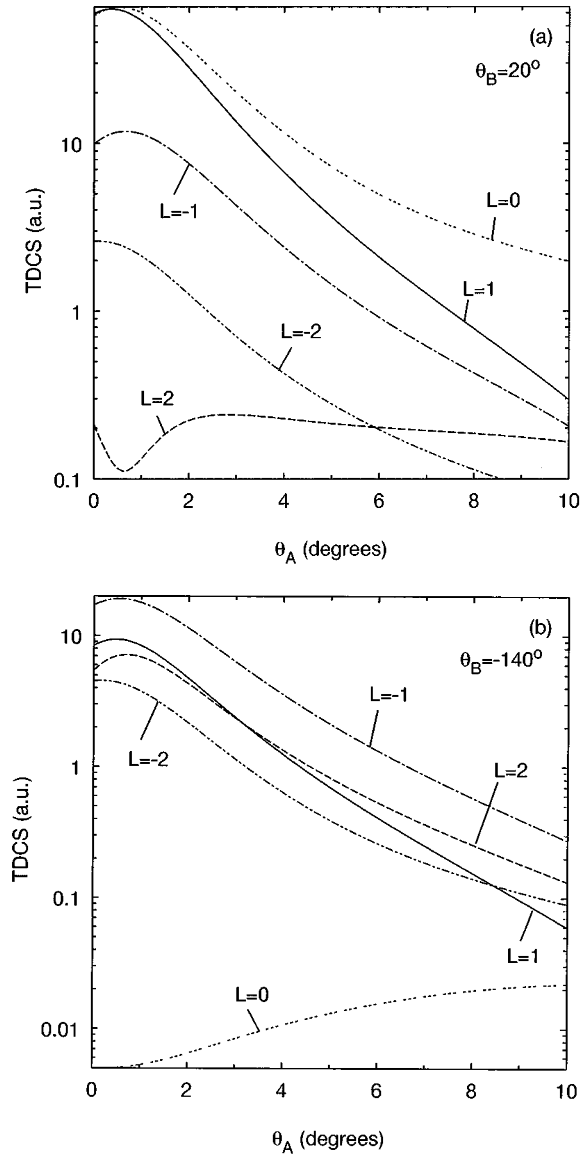

FIG. 5. TDCS as a function of the scattering angle θ_A for a given angle of the ejected electron (a) $\theta_B = 20°$ and (b) $\theta_B = -140°$ and for a different number of absorbed or emitted photons: $L=1$ (continuous curve), $L=-1$ (dot-dashed curve), $L=2$ (dashed curve), $L=-2$ (double-dot–dashed curve), and $L=0$ (triple-dotted curve). The other parameters are as in Fig. 1.

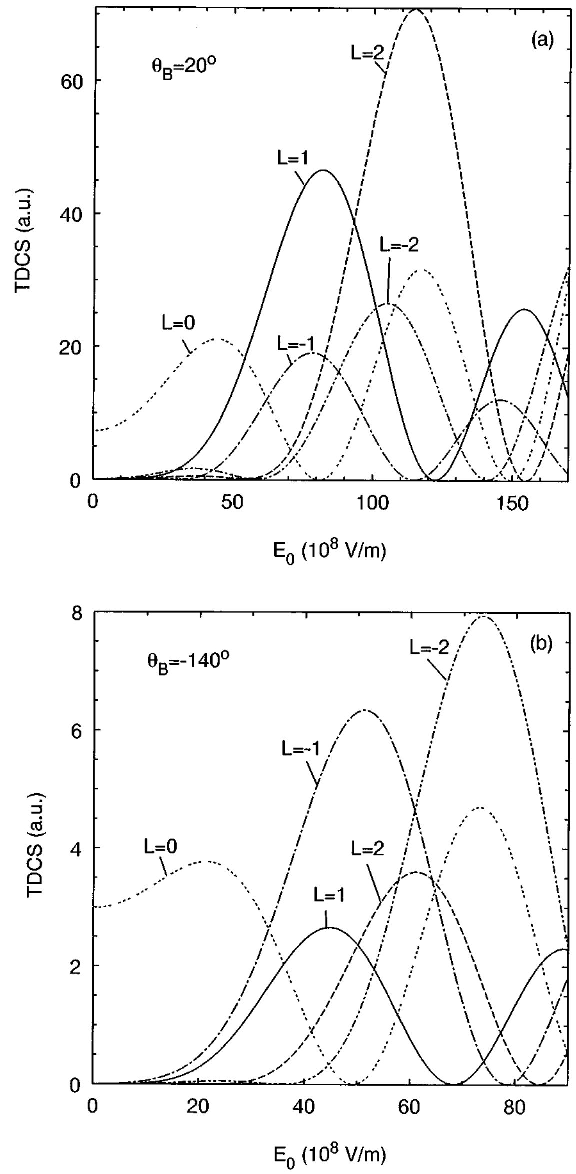

FIG. 6. TDCS as a function of the laser electric field strength E_0 for a given angle of the ejected electron (a) $\theta_B = 20°$ and (b) $\theta_B = -140°$ and for a different number of absorbed or emitted photons: $L=1$ (continuous curve), $L=-1$ (dot-dashed curve), $L=2$ (dashed curve), $L=-2$ (double-dot–dashed curve), and $L=0$ (triple-dotted curve). The other parameters are as in Fig. 1.

matic case $f_{\rm ion}^{{\rm FBA},L} = -2J_L(\lambda)\langle \psi_{\vec{k}_B}^- | \exp(i\vec{K}\cdot\vec{r}_1)|\psi_0\rangle / K^2 \equiv f_{I,{\rm C}}$, which is equivalent to the result of Cavaliere *et al.* [45] mentioned in the Introduction. Keeping all three terms $f_{\rm I}$, $f_{\rm II}$, and $f_{\rm III}$, we reproduce in the monochromatic case the results of [57,58]. Our choice of $\overline{E}$ in $f_{\rm II}$ and $f_{\rm III}$ was made such as to reproduce fairly well the numerical data in [57,58].

III. NUMERICAL RESULTS AND DISCUSSION

In our numerical calculations the following parameters were fixed: the incident electron kinetic energy $E_{k_i} = 250$ eV, the ejected electron energy $E_{k_B} = 5$ eV, the laser field frequency $\omega = 1.17$ eV, and the electric field vector $\vec{E}_0$ is taken parallel to the incident momentum $\vec{k}_i$. The laser electric field strength is $E_0 = 5\times 10^9$ V/m for all figures, except Fig. 6, and the scattering angle is $\theta_A = 3°$ for all figures, except Fig. 5.

We shall first analyze separately the contributions of the amplitudes $f_{I,{\rm C}}$ [defined below Eq. (15)], $f_{\rm I}$, $f_{\rm II}$, and $f_{\rm III}$, to the complete TDCS (15). In Fig. 1 we present the results for the TDCS as a function of the angle θ_B of the ejected electron for $L=1$, i.e., one absorbed photon, and the relative phase $\phi=0$ between the laser field components. From this figure it follows that the main contribution to the complete TDCS comes from the term $f_{\rm I}$. The term $f_{\rm II}$ is not presented because it is negligibly small. The term $f_{\rm III}$ is important because it can enhance the complete TDCS through a constructive interference with $f_{\rm I}$ as is the case for the results presented in Fig. 1. The term $f_{I,{\rm C}}$ of Cavaliere *et al.* [45] is important, but it is not sufficient to explain the whole process because the other term in $f_{\rm I}$, as well as the term $f_{\rm III}$, are of the same order of magnitude. We conclude that all terms, except maybe $f_{\rm II}$, should be taken into account in computing the complete TDCS. The term $f_{\rm II}$ is proportional to $1/(E_n - E_0 \pm \mu\omega)$, $\mu = 1,2$. This term has its maximum for $n = 1$, but it is still small in comparison to the term $f_{\rm III} \sim 1/(E_n - E_{k_B} \pm \mu\omega) \sim 1/\omega$. Furthermore, in [40] it was shown that the CPC effect for the target dressing in free-free transitions becomes important only for high laser field intensities ($E_0 > 10^{10}$ V/m). Taking into account that the term $f_{\rm II}$ corresponds to dressing of the atomic ground state, it follows that its contribution to the TDCS should be small for the laser field intensities we are considering.

In Fig. 2 we present the comparison of the results for the TDCS as a function of the angle θ_B of the ejected electron in the presence of a monochromatic (dotted curve) and a bichromatic (continuous curve) laser field for (a) $L = 1$, (b) $L = -1$, (c) $L = 2$, and (d) $L = -2$ absorbed or emitted photons. It is evident that the influence of the second field component is considerable. Some of the peaks in the TDCS are enhanced, while others are suppressed. Even the positions of the peaks are shifted [see, for example, the case $L = 2$ in Fig. 2(c)]. In the case $L = 0$ (no photons absorbed) we only get one strong peak at $\theta_B \approx 40°$ and the influence of the second field component is small (we therefore do not present this case in Fig. 2).

In Fig. 3 we present the TDCS as a function of the angle θ_B of the ejected electron for different values of the relative phase ϕ and for a different number of absorbed (emitted) photons L. One can see that by changing the phase the TDCS can be increased or decreased by a factor of 5. Hence the CPC effect is significant. As in Fig. 2, we do not present the results for $L = 0$ because the phase effects are small in this case. We also have noticed that the peak positions are not affected much by the change of the phase. Therefore, for our further analysis we fix the angle of the ejected electron to those values that approximately correspond to the maxima in the TDCS.

In Fig. 4 we show the TDCS as a function of the relative phase ϕ for fixed $\theta_B = 20°$ [Fig. 4(a)] and $\theta_B = -140°$ [Fig. 4(b)] and for different values of L. In addition to a significant CPC effect on the order of magnitude of the TDCS, one can also notice some symmetry behavior of the TDCS as a function of ϕ. This symmetry is the consequence of the symmetry properties of the generalized Bessel functions, as shown in [31,32,36,37,39]. In the case of $\theta_B = -140°$ [Fig. 4(b)], we recognize the symmetry $d\sigma_L(\phi) = d\sigma_L(2\pi - \phi)$ [we denote the TDCS for fixed L and ϕ by $d\sigma_L(\phi)$ and express ϕ in radians]. There is also a mirror symmetry for $d\sigma_{|L|}$ and $d\sigma_{-|L|}$ with respect to $L = 0$. This means in particular that $d\sigma_{-1}(\phi)$ has its maxima for $\phi = 0$ and 2π and its minimum for $\phi = \pi$, whereas $d\sigma_1(\phi)$ has its minima at $\phi = 0$ and 2π and its maximum for $\phi = \pi$. The curves $d\sigma_{|L|}(\phi)$ and $d\sigma_{-|L|}(\phi)$ are shifted in phase by π relative to each other. The situation is more complicated for $\theta_B = 20°$ [Fig. 4(a)]. There is no longer a $\phi \leftrightarrow 2\pi - \phi$ symmetry, but the mirror symmetry for the case $L = \pm 1$ is still present. The fact that in this case we have only approximate symmetry properties is probably due to the shift of the maxima of the TDCS as a function of θ_B. According to Fig. 6, another explanation is that we are in a different ''regime'' of behavior of the generalized Bessel function, i.e., for $E_0 = 5 \times 10^9$ V/m the dominant maximum in case (a) is for $L = 0$, while in case (b) the dominant maximum is for $L = \pm 1$. This is so because the argument λ of the generalized Bessel function depends on θ_B.

The results presented in Fig. 5 show the behavior of the TDCS as a function of the scattering angle θ_A for a given angle θ_B of the ejected electron in (a) $\theta_B = 20°$ and (b) $\theta_B = -140°$ and for a different number of the absorbed photons: $L = 1$ (continuous curve), $L = -1$ (dot-dashed curve), $L = 2$ (dashed curve), $L = -2$ (double-dot–dashed curve), and $L = 0$ (triple-dotted curve). The results presented are consistent with the choice of the Ehrhardt geometry [62,63], i.e., they show that the TDCSs are in general dominant for small values of θ_A. Furthermore, we can increase the TDCS by choosing values of θ_A lower than 3° (as chosen for all other results presented). We expect that the effects of a bichromatic laser field are more pronounced for $\theta_A = 1°$, as shown in [40].

Finally, in Fig. 6 we present the TDCS as a function of the laser electric field strength E_0 for a given angle of the ejected electron (a) $\theta_B = 20°$ and (b) $\theta_B = -140°$ and for different numbers L of the absorbed (emitted) photons. The results of Fig. 6(a) show that for $E_0 < 5 \times 10^9$ V/m $d\sigma_0$ is larger than $d\sigma_L$, $L = \pm 1, \pm 2$. With increasing laser field strength the $d\sigma_{\pm 1}$ become dominant, while $d\sigma_0$ decreases. With a further increase of the laser field intensity the $d\sigma_{\pm 2}$ become dominant, $d\sigma_0$ is increased, and $d\sigma_{\pm 1}$ become suppressed. The results of Fig. 6(b) show similar behavior, except that now the effect of the increase and decrease of $d\sigma_L$ is noticeable for lower laser field intensities. The contribution of $d\sigma_0$ now has a maximum for $E_0 \approx 2 \times 10^9$ V/m and gets completely suppressed for $E_0 \approx 5 \times 10^9$ V/m, where $d\sigma_{\pm 1}$ have their maxima. $d\sigma_2$ has its maximum around $E_0 \approx 6 \times 10^9$ V/m, while $d\sigma_{-2}$ becomes dominant above $E_0 = 7 \times 10^9$ V/m, where we also find a noticeable contribution of $d\sigma_0$. The explanation for this oscillatory behavior of $d\sigma_L(E_0)$ is similar to the one given in [39]. The arguments of the generalized Bessel functions are proportional to E_0 and also depend on the reaction parameters (such as θ_B), which explains the differences between the results presented in Figs. 6(a) and 6(b). Therefore, with the increase of E_0 the parameter λ also increases and becomes of order L. But for $\lambda \sim L$ the generalized Bessel functions have their maxima, which explains the appearance of maxima for $L = \pm 1$ and later on for $L = \pm 2$.

IV. CONCLUSION

We have presented analytical and numerical results for the electron-atom ionizing collisions in the presence of a bichromatic, linearly polarized laser field. The incident and scattered electrons were described by taking into account the laser field to all orders through the Volkov states for a bichromatic laser field. We described the second electron by the laser modified ground and continuum states of the hydrogen atom including the laser field to the first order for the ground state and to all orders (but in an approximate way) for the continuum state. Our final result for the TDCS for the electron-impact ionization with the absorption or emission of L photons in the FBA is given by Eq. (15). The amplitude $f_{\rm ion}^{{\rm FBA},L}$, which appears in this result, consists of three terms. The behavior of the TDCS is mainly determined by the behavior of the generalized Bessel functions (14), that appear in all these amplitudes. The first term $f_{\rm I}$ in the amplitude $f_{\rm ion}^{{\rm FBA},L}$ is given by Eq. (11) and consists of two parts. The first one is proportional to a generalized Bessel function with index L and, in the monochromatic case, reproduces the results by Cavaliere *et al.* [45]. The second part of $f_{\rm I}$ has its origin in the laser dressing of the state of the ionized electron and has terms with $b_{L\pm1}$ and $b_{L\pm2}$. For a sufficiently strong laser field this second part can become dominant. The second term $f_{\rm II}$ in Eq. (12) has its origin in the dressing of the ground state. For the laser field intensity we are considering and within the closure approximation, the contribution of this term is small [40]. Finally, the third term $f_{\rm III}$ in Eq. (13) can give a significant contribution to the TDCS because it has terms proportional to $1/(E_n-E_{k_B}\pm\mu\omega)$, $\mu=1,2$, which are large for $E_n\approx E_{k_B}$. In our computation of the terms $f_{\rm II}$ and $f_{\rm III}$ we have used the closure approximation, adjusting the two average energies $\bar{E}$ such that we get sufficiently good agreement with the results of [57,58] for a monochromatic laser field.

Our numerical results have shown that the introduction of the second field component can significantly change the results for the TDCS. We have presented a detailed analysis of the influence of the relative phase between the two field components on the TDCS and we have noticed that the CPC effect is important and that it is possible to increase or decrease the TDCS by almost one order of magnitude by changing the relative phase. We also have noticed the symmetry properties of the TDCS due to the presence of the generalized Bessel functions, which are in agreement with previous findings [31,32,36,37,39]. Moreover, we have analyzed the influence of the scattering angle θ_A of the fast electron on the TDCS and we have found that the TDCS has its maximum for small values of θ_A, which is consistent with the reaction geometry described in [62,63] and the results of [40].

Finally, we showed that the TDCS as a function of the laser electric field strength E_0 has an oscillatory behavior. There are regions of E_0 in which the reactions for a fixed L are suppressed or enhanced, depending on the behavior of the generalized Bessel functions.

The general conclusion of this paper is that the influence of the laser field on the atomic processes is more pronounced if the atomic continuum states take part in the process. The influence of the laser field on the electrons in such states becomes comparable to the influence of the electric field of the atomic nucleus on these states and therefore the laser field induced effects, as is the CPC in our case, become more pronounced.

ACKNOWLEDGMENTS

D.M. thanks the Institute for Theoretical Physics, University of Innsbruck, for its hospitality and financial support. This work was partially supported financially by the Büro für Austauschprogramme mit Mittel- und Osteuropa des Österreichischen Akademischen Austauschdientstes.

[1] A. Giusti-Suzor, F. H. Mies, L. F. DiMauro, E. Charron, and B. Yang, J. Phys. B **28**, 309 (1995).

[2] J. J. L. Ting, Phys. Rev. A **51**, 2641 (1995).

[3] B. Sheahy, B. Walker, and L. F. DiMauro, Phys. Rev. Lett. **74**, 4799 (1995).

[4] Z. Chen, M. Shapiro, and P. Brumer, Phys. Rev. A **52**, 2225 (1995).

[5] E. Charron, A. Giusti-Suzor, and F. H. Mies, Phys. Rev. Lett. **75**, 2815 (1995).

[6] H. G. Muller, P. H. Bucksbaum, D. W. Schumacher, and A. Zavriev, J. Phys. B **23**, 2761 (1990).

[7] C. Chen and D. S. Elliott, Phys. Rev. Lett. **65**, 1737 (1990).

[8] Y. Y. Yin, C. Chen, and D. S. Elliott, Phys. Rev. Lett. **69**, 2353 (1992).

[9] A. Szöke, K. C. Kulander, and J. N. Bardsley, J. Phys. B **24**, 3165 (1991).

[10] R. M. Potvliege and P. H. G. Smith, J. Phys. B **24**, L641 (1991).

[11] K. J. Schaffer and K. C. Kulander, Phys. Rev. A **45**, 8026 (1992).

[12] R. M. Potvliege and P. H. G. Smith, J. Phys. B **25**, 2501 (1992).

[13] M. Pont, R. M. Potvliege, R. Shakeshaft, and P. H. G. Smith, Phys. Rev. A **46**, 555 (1992).

[14] R. M. Potvliege and P. H. G. Smith, Phys. Rev. A **49**, 3110 (1994).

[15] V. A. Pazdzersky and V. A. Yurovsky, Phys. Rev. A **51**, 632 (1995).

[16] V. Véniard, R. Taïeb, and A. Maquet, Phys. Rev. Lett. **74**, 4161 (1995).

[17] D. A. Telnov, J. Wang, and S. I. Chu, Phys. Rev. A **51**, 4797 (1995).

[18] G. G. Paulus, W. Becker, and H. Walther, Phys. Rev. A **52**, 4043 (1995).

[19] R. A. Blank and M. Shapiro, Phys. Rev. A **52**, 4278 (1995).

[20] M. Protopapas and P. L. Knight, J. Phys. B **28**, 4459 (1995).

[21] T. Nakajima and P. Lambropoulos, Phys. Rev. Lett. **70**, 1081 (1993).

[22] T. Nakajima and P. Lambropoulos, Phys. Rev. A **50**, 595 (1994).

[23] M. D. Perry and J. K. Crane, Phys. Rev. A **48**, R4051 (1993).

[24] M. Protopapas, P. L. Knight, and K. Burnett, Phys. Rev. A **49**, 1945 (1994).

[25] S. Watanabe, K. Kondo, Y. Nabekawa, A. Sagisaka, and Y. Kobayashi, Phys. Rev. Lett. **73**, 2692 (1994).

[26] T. Zuo, A. D. Bandrauk, M. Ivanov, and P. B. Corcum, Phys. Rev. A **51**, 3991 (1995).

[27] S. Long, W. Becker, and J. K. McIver, Phys. Rev. A **52**, 2262 (1995).

[28] M. Protopapas, A. Sanpera, P. L. Knight, and K. Burnett, Phys. Rev. A **52**, R2527 (1995).

[29] D. A. Telnov, J. Wang, and S. I. Chu, Phys. Rev. A **52**, 3988 (1995).
[30] D. B. Milošević and B. Piraux, Phys. Rev. A **54**, 1522 (1996).
[31] S. Varró and F. Ehlotzky, Phys. Rev. A **47**, 715 (1993).
[32] S. Varró and F. Ehlotzky, Opt. Commun. **99**, 177 (1993).
[33] J. Z. Kamiński and F. Ehlotzky, Phys. Rev. A **50**, 4404 (1994).
[34] F. Ehlotzky, Nuovo Cimento D **16**, 453 (1994).
[35] J. Z. Kamiński, A. Jaroń, and F. Ehlotzky, J. Phys. B **28**, 4895 (1995).
[36] S. Varró and F. Ehlotzky, J. Phys. B **28**, 1613 (1995).
[37] S. Varró and F. Ehlotzky, J. Phys. B **28**, 2729 (1995).
[38] I. Rabadán, L. Mendez, and A. S. Dickinson, J. Phys. B **29**, 163 (1996).
[39] D. B. Milošević, J. Phys. B **29**, 875 (1996).
[40] S. Varró and F. Ehlotzky, J. Phys. B **30**, 1061 (1997).
[41] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Clarendon, Oxford, 1971).
[42] M. R. C. McDowell, and J. P. Coleman, *Introduction to the Theory of Ion-Atom Collisions* (North-Holland, Amsterdam, 1970).
[43] I. E. McCarthy, Can. J. Phys. **74**, 703 (1996).
[44] M. Mohan and P. Chand, Phys. Lett. **65A**, 399 (1978).
[45] P. Cavaliere, G. Ferrante, and C. Leone, J. Phys. B **13**, 4495 (1980).
[46] P. Cavaliere, C. Leone, R. Zangara, and G. Ferrante, Phys. Rev. A **24**, 910 (1981).
[47] J. Banerji and M. H. Mittleman, J. Phys. B **14**, 3717 (1981).
[48] P. Cavaliere, G. Ferrante, and C. Leone, J. Phys. B **15**, 475 (1982).
[49] R. Zangara, P. Cavaliere, C. Leone, and G. Ferrante, J. Phys. B **15**, 3881 (1982).
[50] M. Zarcone, D. L. Moores, and M. R. C. McDowell, J. Phys. B **16**, L11 (1983).
[51] M. H. Mittleman, J. Phys. B **16**, 1089 (1983).
[52] S. K. Mandal and A. S. Ghosh, Phys. Rev. A **30**, 2759 (1984).
[53] R. Burlon, P. Cavaliere, and G. Ferrante, Nuovo Cimento D **4**, 19 (1984).
[54] M. H. Mittleman, *Introduction to the Theory of Laser-Atom Interactions*, 2nd ed. (Plenum, New York, 1993).
[55] F. V. Bunkin and M. V. Fedorov, Zh. Eksp. Teor. Fiz. **49**, 1215 (1965) [Sov. Phys. JETP **22**, 844 (1966)].
[56] N. M. Kroll and K. M. Watson, Phys. Rev. A **8**, 804 (1973).
[57] C. J. Joachain, P. Francken, A. Maquet, P. Martin, and V. Véniard, Phys. Rev. Lett. **61**, 165 (1988).
[58] P. Martin, V. Véniard, A. Maquet, P. Francken, and C. J. Joachain, Phys. Rev. A **39**, 6178 (1989).
[59] X. J. Chen, Phys. Rev. A **40**, 1795 (1989).
[60] P. Francken, and C. J. Joachain, J. Opt. Soc. Am. B **7**, 554 (1990).
[61] R. Taïeb, V. Véniard, A. Maquet, S. Vučić, and R. M. Potvliege, J. Phys. B **24**, 3229 (1991).
[62] H. Ehrhardt, K. H. Hesselbacher, K. Jung, M. Schulz, and K. Willmann, J. Phys. B **5**, 2107 (1972).
[63] H. Ehrhardt, K. Jung, G. Knoth, and P. Schlemmer, Z. Phys. D **1**, 3 (1986).
[64] F. W. Byron, Jr., P. Francken, and C. J. Joachain, J. Phys. B **20**, 5487 (1987).
[65] P. Francken, Y. Attaourti, and C. J. Joachain, Phys. Rev. A **38**, 1785 (1988).