

## Dynamic electron correlations in single capture from helium by fast protons

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We employ the four-body continuum distorted-wave (CDW-4B) method to compute the *total* cross sections for the single electron-capture process  $H^+ + He(1s^2) \rightarrow H(1s) + He^+(1s)$  at incident energies ranging from 20 to 20 000 keV. The purpose of the present work is twofold: (i) to verify whether the CDW-4B approximation for one-electron transfer is as successful as for double charge exchange in fast proton-helium collisions and (ii) to search for evidence of dynamic electron correlation effects as a function of increased projectile energy. The validity of our findings is critically assessed in comparisons with the available experimental data in the quoted impact energy range. [S1050-2947(97)01311-5]

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### I. INTRODUCTION

The simple four-body problem of the basic single charge exchange  $p + He(1s^2) \rightarrow H(1s) + He^+(1s)$  offers an intriguing task of establishing the relative importance of the inter-electron and electron-projectile potentials. In principle, both interactions can lead to one-electron capture, but the invoked mechanisms differ considerably from each other. The interaction  $V_{p1}$  of the “active” electron  $e_1$  to be captured with the incident proton  $p$  leads to single charge exchange and forms the basis of the so-called “velocity-matching” mechanism [1]. This mechanism can be best conceived by resorting to the transition probability in terms of the initial  $\tilde{\varphi}_{n_i, l_i, m_i}$  and final  $\tilde{\varphi}_{n_f, l_f, m_f}$  bound-state wave functions in momentum space. When a fast impinging proton  $p$  of mass  $m_p$  passes by the helium target with a large momentum  $m_p \vec{v}_p$  and velocity  $\vec{v}_p$ , the electron  $e_1$  of mass  $m_e \ll m_p$ , which is orbiting about the  $\alpha$  particle with the classical velocity  $v_{e_1} \ll v_p$ , could be captured via a single binary collision  $p-e_1$  only if considerable momentum, of the order of  $m_e \vec{v}_p$ , is imparted onto  $e_1$ . Since this displacement  $m_e \vec{v}_p$  in the momentum components of  $\tilde{\varphi}_{n_f, l_f, m_f}$  will increase with augmentation of  $v_p$ , it is clear that only the largest components of the momentum-space wave functions would be able to provide the “velocity matching,” which should yield a nonvanishing overlap of the initial and final orbitals. Nevertheless, the resulting probability is exceedingly small, giving a cross section with the typical behavior  $v_p^{-12-2l_i-2l_f}$  for large  $v_p$ , since both orbitals  $\tilde{\varphi}_{n_i, l_i, m_i}$  and  $\tilde{\varphi}_{n_f, l_f, m_f}$  fall off too rapidly with increasing values of their momentum variables. A large momentum  $m_e \vec{v}_p$  could be transferred to  $e_1$  only if the impinging proton  $p$  comes close to electron  $e_1$ , which possesses a very small initial momentum in the target ( $m_e v_{e_1} \ll m_e v_p$ ). Hence, the velocity-matching mechanism is expected to be operative mainly at small impact parameters  $\rho$ . This is a common feature of all first Born-type models [2].

One can alternatively choose the potential  $V_{p1}$  as a perturber of the asymptotic state  $\Phi_i$  in the entrance channel,

with  $\Phi_i$  given by the product of the two-electron target wave function  $\varphi_i$  and the plane wave for the relative motion of the projectile. This is accomplished through the usual multiplication of the channel state  $\Phi_i$  by the appropriate Coulomb wave and an automatic cancellation of  $V_{p1}$  from the entrance channel perturbation  $V_i$  in the transition  $T$  matrix. This procedure is reminiscent of the four-body continuum distorted-wave (CDW-4B) theory [3–6], which represents a rigorous first-order approximation of the Dodd and Greider [7,8] distorted-wave perturbation expansion without any divergencies, such as disconnected Feynman diagrams and the like. In the CDW-4B model, the projectile does not directly impart a large momentum of the order  $\sim m_e \vec{v}_p$  onto the “active” electron  $e_1$ . Instead, this momentum is transferred indirectly via the long-range distortion effects and, hence, close encounters between  $p$  and  $e_1$  are not mandatory any longer. This illustrates that the small size of the impact parameter  $\rho$  cannot be used as an unambiguous signature or a characterization of the velocity-matching mechanism. A much clearer situation emerges from the analysis of recoiled particle  $He^+$  of the target remainder. Namely, in order to conserve the total momentum of the whole four-body collision system, the residual  $He^+$  ion in the exit channel must recoil in the backward direction. This is readily observed by means of the well-established experimental technique known as the recoiled ion momentum spectroscopy (RIMS) [9,10].

Additionally, the interaction  $V_{p2}$  of the presumably “passive” electron  $e_2$  with the projectile can also lead to capture of electron  $e_1$ . Here, electron  $e_2$  receives a large momentum  $m_e \vec{v}_p$ , which is afterwards transferred onto  $e_1$ . This is possible only if the static correlation of the two electrons in the helium target plays a non-negligible role. The relative importance of  $V_{p2}$  as a perturbation in a  $T$  matrix has not been previously estimated and this will be thoroughly investigated in the present work by means of the CDW-4B method.

We shall mainly focus on the role of the interelectronic ( $e_1-e_2$ ) potential  $V_{12}$  from the *dynamic* point of view. To achieve this goal within the CDW-4B method, one ought to employ the “post” formalism with the appropriate full per-

turbation  $V_f$  in the exit channel. A contribution from  $V_{12}$  to single capture in the  $p$ -He collisions has not been previously assessed and it is our main goal to bridge this gap with the help of the CDW-4B model. The mechanism of the dynamic electron correlation does not necessitate close encounters with a projectile. Moreover, when the dynamic correlations are operating, the target remainder  $\text{He}^+$  is at rest as a mere spectator, in sharp contrast to the backward recoil in the velocity-matching mechanism [11].

The above-listed mechanisms should be completed by yet another pathway, commonly known as the Thomas [12] double scattering ( $p$ - $e_1$ - $\alpha$ ). In this two-step mechanism of the  $p$ -He single capture, the electron  $e_1$  first collides with the incident proton  $p$  and then scatters elastically on the  $\alpha$  particle of the target, acquiring finally the ejection momentum  $m_e \vec{v}_p$ , which is sufficient to form atomic hydrogen. Such a mechanism operates only at very high energies, yielding the  $v^{-11}$  Thomas-type behavior of the cross section. This is correctly predicted by the CDW-4B theory. The purely classical mechanism of the Thomas double encounter is comprised of two consecutive Rutherford scatterings each yielding the  $v_p^{-4}$  behavior of the cross section. When the ensuing result  $v_p^{-8}$  is further multiplied by the volume element  $v_p^{-3}$  of momentum space, the overall Thomas cross section  $v_p^{-11}$  is obtained. The second-Born approximation describes this classical effect by means of the transition operator  $V_{\alpha 1} G_0^+ V_{p1}$ . Here,  $V_{\alpha 1}$  is the Coulomb interaction between the  $\alpha$  particle and  $e_1$ , whereas  $G_0^+$  is the free-particle Green's operator. A slower decrease of the Thomas cross section with rising  $v_p$  in comparison with the velocity-matching mechanism can be mathematically interpreted as an enlarged overlap between the initial and final bound states. This is also expected from the physical viewpoint, since the stringent condition for velocity matching in the  $p$ - $e_1$  encounter is partially relaxed via the subsequent  $e_1$ - $\alpha$  collision, where the target nucleus could carry some excess of energy. Clearly, this surplus of energy and momentum taken by the  $\alpha$  particle is severely limited by the heavy mass of the target nucleus. A much more pronounced reduction of the cross section, however, is expected if the energy excess is taken by, e.g., a photon, which indeed yields the  $v_p^{-5}$  behavior [13] of the cross sections for the ensuing radiative electron capture (REC). A similar argument could be evoked to anticipate a considerably weaker dependence than the  $v_p^{-11}$  asymptote of the second Born cross section [14] for transfer ionization (TI) in the  $p$ -He collision, where  $e_1$  is captured by the proton and  $e_2$  is simultaneously ionized. Here, the second electron  $e_2$  could readily carry away the excess of energy and momentum and, therefore, maximally relax the strict velocity-matching requirement. Indeed, a very recent experiment by Mergel *et al.* [15] provides evidence that the total cross section for TI in the  $p$ -He collision behaves like  $v_p^{-7.4 \pm 1.0}$  with increasing  $v_p$  at impact energies ranging from 300 to 1400 keV.

Atomic units will be used throughout unless stated otherwise.

## II. THEORY

It is important to investigate the influence of the dynamic electron correlation effects on *total* cross sections for single

capture in the  $p$ -He collision, particularly with respect to the increasing values of the incident proton velocity  $v_p$ , but still below the region of the dominance of the Thomas double scattering  $p$ - $e_1$ - $\alpha$ . In this way, one could monitor an interplay between the *single*-step velocity-matching mechanism and the dynamic electron correlations and, hence, determine their relative importance. Here, the electron dynamics enter the composite process under investigation via a *two-step*  $p$ - $e_1$ - $e_2$  Thomas-type mechanism, which corresponds to the propagator  $V_{12} G_0^+ V_{p1}$  of the second Born approximation. The first  $p$ - $e_1$  step in the CDW-4B method is not accomplished directly via an explicit appearance of the electrostatic interaction  $V_{p1}$  in the transition  $T$  operator, but rather indirectly through dressing of the unperturbed states  $\Phi_i$  and  $\Phi_f$  with the Coulomb waves centered on  $p$  and  $\alpha$ , respectively. The transfer of large momentum  $m_e \vec{v}_p$  from  $p$  to  $e_1$ , mediated by these long-range distortions of the asymptotic channel states, enables the two electrons to interact strongly in the second  $e_1$ - $e_2$  step of the above  $p$ - $e_1$ - $e_2$  mechanism via the explicit  $V_{12}$  potential in the  $V_f$  perturbation of the exit channel. Although  $e_2$  remains on the target rest  $\text{He}^+$ , as opposed to TI, the  $e_1$ - $e_2$  scattering should be able to considerably relax the velocity-matching condition of the preceding  $p$ - $e_1$  step. Our goal is to determine whether such an effect could be detected already on the level of the total cross sections, since these observables are easier to measure experimentally. There exist several measurements of differential cross sections for the Thomas double scattering  $p$ - $e_1$ - $Z_T$  in the  $p$ -He or  $p$ -H<sub>2</sub> single capture at high energies [16,17]. These data indicate that the width of the Thomas  $p$ - $e_1$ - $Z_T$  peak is larger for  $p$ -He or  $p$ -H<sub>2</sub> than in the case of the  $p$ -H collision. Such a phenomenon could be due a broader momentum distribution of the initial state of the “active” electron in  $p$ -He than in  $p$ -H case. In addition, the dynamic correlation effect might change the shape of the conventional Thomas peak in the  $p$ - $e_1$ - $Z_T$  encounter. This is supported by our preliminary computation for  $p$ -He single charge exchange within the CDW-4B method, which shows that the Thomas peaks for the double  $p$ - $e_1$ - $e_2$  and  $p$ - $e_1$ - $\alpha$  collisions occur at the same critical angle.

In a search for the signature of the  $p$ - $e_1$ - $e_2$  mechanism in the  $p$ -He single-electron transfer, we will also be in a position to verify whether a four-body distorted-wave formalism, such as the presently used CDW-4B model for this process, would be of comparable adequacy in regard to the results of a previously reported study on double charge exchange for the same colliding system [3]. This is important in view of some evidence within another four-body formalism known as the forced impulse approximation (FIA), which is successful for double ionization, but has very recently been reported to break down for single ionization in  $p$ -He collisions [18].

In all the previous studies of a general one-electron capture from a heliumlike atomic systems of the nuclear charge  $Z_T$  by a completely stripped projectile of the charge  $Z_P$ :

$$Z_P + (Z_T; e_1, e_2)_i \rightarrow (Z_P, e_1)_{f1} + (Z_T, e_2)_{f2}, \quad (1)$$

the electron  $e_2$  has been considered as being “passive” in the sense of occupying the same orbital before and after collision. Moreover, all the residual interactions of  $e_2$  with  $Z_P$  and  $e_1$  in the form  $V_{p2}$  and  $V_{12}$  mentioned in the Introduction

have systematically been neglected from distorted-wave models in order to bypass several complicated bound-free multicenter atomic form factors with Coulomb functions. The net result of such a ‘‘frozen-core’’ approximation is an explicit reduction of the four-body formalism to essentially a three-body problem [19,20]. Here, the electron  $e_2$  is *de facto* ‘‘phased out’’ from the otherwise onset four-body starting point. This is customarily accomplished by using, e.g., the Slater-screened hydrogenic wave function [21] for a helium-like target:  $\varphi_i(\vec{x}_1, \vec{x}_2) = (\lambda^3/\pi) \exp[-\lambda(x_1+x_2)]$ , where  $\lambda = Z_T - \lambda_S$  and  $\lambda_S = 0.3125$ . In such a way, all the interesting features of the pure dielectronic dynamics are washed out from the four-particle formalism, which, therefore, does not introduce anything new with respect to a pure three-body problem. If instead of the hydrogenic wave functions [21], some more elaborate orbitals are used via configuration interactions (CI) to describe  $\varphi_i(\vec{x}_1, \vec{x}_2)$ , one will end up with a reduction of a four-body  $T$  matrix to a linear combination of three-body  $T$  matrices and again no new insight would be gained on the electronic dynamics [22]. Moreover, such a four-body formalism that neglects  $V_{p2}$  and  $V_{12}$  would be, in a sense, equivalent to the independent particle model (IPM), which discards the dynamic electron correlations from the

onset [23,24]. Nowadays, however, with the advent of fast computers, there is no need any longer to overlook the contributions from  $V_{p2}$  and  $V_{12}$ . Taking the Fourier transforms of these residual potentials, one finally arrives at convenient distorted-wave four-body  $T$  matrices for Eq. (1) in terms of some readily manageable triple numerical integrations. A similar technique has already been implemented in double charge exchange [3], resonant or nonresonant transfer excitation [4,5] and in transfer ionization [6] treated by means of the CDW-4B approximation. For this reason, there should be no need to give here any details of the derivation, but merely to quote the final expressions of the total cross sections for the process (1) in the prior ( $Q_{if}^-$ ) and post ( $Q_{if}^+$ ) forms:

$$Q_{if}^\pm(a_0^2) = \int d\vec{\eta} \left| \frac{R_{if}^\pm(\vec{\eta})}{2\pi v} \right|^2, \quad (2)$$

where  $\vec{\eta}$  is the usual transverse momentum transfer and  $\vec{v} = \vec{v}_p$  is the incident velocity ( $\vec{\eta} \cdot \vec{v} = 0$ ). The integrals  $R_{if}^-(\vec{\eta})$  and  $R_{if}^+(\vec{\eta})$  in Eq. (2) are given by the following matrix elements with the *complete* prior  $V_i$  and post  $V_f$  perturbations, respectively:

$$R_{if}^-(\vec{\eta}) = N_{PT} \int \int d\vec{R} d\vec{x}_1 d\vec{x}_2 e^{i\vec{q} \cdot \vec{s}_1 + i\vec{p} \cdot \vec{x}_1} \varphi_{f_1}^*(\vec{s}_1) \varphi_{f_2}^*(\vec{x}_2) {}_1F_1(i\nu_T; 1; i\nu x_1 + i\vec{v} \cdot \vec{x}) [V_i \varphi_i(\vec{x}_1, \vec{x}_2) \times {}_1F_1(i\nu_P; 1; i\nu s_1 + i\vec{v} \cdot \vec{s}_1)], \quad (3)$$

$$V_i = \Delta V_{P2} - \vec{\nabla}_{x_1} \ln \varphi_i(\vec{x}_1, \vec{x}_2) \cdot \vec{\nabla}_{s_1}, \quad (4)$$

$$\Delta V_{P2} = V_{P2} - V_{P2}^\infty, \quad V_{P2} = -\frac{Z_P}{s_2}, \quad V_{P2}^\infty = -\frac{Z_P}{R}, \quad (5)$$

$$R_{if}^+(\vec{\eta}) = N_{PT} \int \int d\vec{R} d\vec{s}_1 d\vec{s}_2 e^{i\vec{q} \cdot \vec{s}_1 + i\vec{p} \cdot \vec{x}_1} \varphi_i(\vec{x}_1, \vec{x}_2) \varphi_{f_2}^*(\vec{x}_2) {}_1F_1(i\nu_P; 1; i\nu s_1 + i\vec{v} \cdot \vec{s}_1) [V_f \varphi_{f_1}^*(\vec{s}) {}_1F_1(i\nu_T; 1; i\nu x_1 + i\vec{v} \cdot \vec{x}_1)], \quad (6)$$

$$V_f = \Delta V_{P2} + \Delta V_{12} - \vec{\nabla}_{s_1} \ln \varphi_{f_1}^*(\vec{s}_1) \cdot \vec{\nabla}_{x_1}. \quad (7)$$

$$\Delta V_{12} = V_{12} - V_{12}^\infty, \quad V_{12} = \frac{1}{x_{12}} = \frac{1}{|\vec{x}_1 - \vec{x}_2|}, \quad V_{12}^\infty = \frac{1}{x_1}, \quad (8)$$

The symbol  ${}_1F_1$  in Eqs. (3) and (6) denotes the confluent hypergeometric Kummer function, whereas  $\varphi_i$  and  $\varphi_{f_j}$  ( $j = 1, 2$ ) are the initial and final bound-state orbitals. The quantity  $N_{PT}$  in  $R_{if}^\pm(\vec{\eta})$  is given by  $N_{PT} = N^+(\nu_P) N^{-*}(\nu_T)$  and  $N^\pm(\nu_K) = \Gamma(1 \mp i\nu_K) \exp(\pi\nu_K/2)$  ( $K = P, T$ ), where  $N^+(\nu_P)$  and  $N^-(\nu_T)$  are the standard normalization Coulomb constants with the Sommerfeld parameters  $\nu_P = Z_P/v$  and  $\nu_T = (Z_T - 1)/v$ . The two momentum transfers  $\vec{p}$  and  $\vec{q}$  in Eqs. (3) and (6) are defined as follows:

$$\vec{q} = \vec{\eta} - \left( \frac{v}{2} - \frac{\Delta E}{v} \right) \vec{v}, \quad \vec{p} = -\vec{\eta} - \left( \frac{v}{2} + \frac{\Delta E}{v} \right) \vec{v}, \quad (9)$$

$$\Delta E = E_i - (E_{f_1} + E_{f_2}),$$

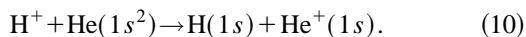
where  $E_i$  and  $E_{f_{1,2}}$  are the initial (helium) and final (hydrogenic) binding energies, respectively. In the course of deriving Eq. (2) from the corresponding  $T$  matrices  $T_{if}^\pm(\vec{\eta})$ , the term  $\rho^{2iZ_P(Z_T-1)/v}$  disappears from the total cross sections  $Q_{if}^\pm$ . This phase, however, is present in the corresponding differential cross sections  $dQ_{if}^\pm/d\Omega$ , describing the  $Z_P - Z_T$  Rutherford internuclear scattering [1]. Notice that the  $Z_P - e_2$  Coulomb interaction of the ‘‘passive’’ electron  $e_2$  at finite and infinite distances is present in the same fashion in both  $V_i$  and  $V_f$  from Eqs. (4) and (7) through  $\Delta V_{P2}$ . This is because the potential  $V_{P2}$  is chosen as inactive in the course of dynamic distortion of total scattering states in the entrance and exit channels. However, the  $e_1 - e_2$  potential appears only in  $V_f$ , since  $V_{12}$  emerges in the definition of the exit channel

perturbation through the difference between the total interaction  $V = Z_P Z_T / R - Z_P / s_1 - Z_P / s_2 - Z_T / x_1 - Z_T / x_2 + 1/x_{12}$  and the binding potentials in the noninteracting hydrogenic atomic systems  $(Z_P, e_1)_{f_1}$  and  $(Z_T, e_2)_{f_2}$ . Additionally, a residual potential  $V_{12}^\infty = 1/x_1$ , as the limiting value of  $V_{12}$  at infinitely large  $x_1$  and finite  $x_2$ , consistently appears in  $V_f$ . This is because at infinitely large  $x_1$ , the “active” electron  $e_1$  from  $(Z_P, e_1)_{f_1}$  cannot discern the individual constituents in  $(Z_T, e_2)_{f_2}$ , which is, therefore, conceived as the net charge  $Z_T - 1$ . In order to account for this correct screened nuclear charge, the genuine potential  $V_{T1} = -Z_T/x_1$  is written as  $-(Z_T - 1)/x_1 + 1/x_1 \equiv -(Z_T - 1)/x_1 + V_{12}^\infty$ . Here, the term  $-(Z_T - 1)/x_1$  is used to produce the distortion  $\Gamma(1 + i\nu_T)e^{\pi\nu_T/2}{}_1F_1(i\nu_T; 1; i\nu_T x_1 + i\vec{\nu} \cdot \vec{x}_1)$  with  $\nu_T = (Z_T - 1)/v$ , whereas the potential  $V_{12}^\infty$  is joined together with  $V_{12}$  to yield  $\Delta V_{12}$ . On the other hand, the term  $V_{12}$  does not appear in  $V_i$ , since the  $e_1$ - $e_2$  interaction is, in principle, *exactly* included in the initial eigenvalue problem:  $(-\nabla_{x_1}^2/2 - \nabla_{x_2}^2/2 - Z_T/x_1 - Z_T/x_2 + 1/x_{12} - E_i^{\text{ex}})\varphi_i^{\text{ex}}(\vec{x}_1, \vec{x}_2) = 0$ . In practice, however, the exact wave function  $\varphi_i^{\text{ex}}(\vec{x}_1, \vec{x}_2)$  is unavailable and the nonzero term  $O_i \equiv (-\nabla_{x_1}^2/2 - \nabla_{x_2}^2/2 - Z_T/x_1 - Z_T/x_2 + 1/x_{12} - E_i^{\text{ex}})\varphi_i(\vec{x}_1, \vec{x}_2) \neq 0$  should be incorporated in  $V_i$  as an additional perturbation for any particular approximation  $\varphi_i$  of  $\varphi_i^{\text{ex}}$  [25]. For example, using the Slater-screened hydrogenic wave function [21], we shall have  $O_i = (1/x_{12} - \lambda_S/x_1 - \lambda_S/x_2 + E_i - E_i^{\text{ex}})\exp[-\lambda(x_1 + x_2)]$ , where  $E_i = -\lambda^2$  and  $E_i^{\text{ex}}$  is the “exact” initial binding energy, the best estimate of which is given by the CI ansatz of Drake [26]. In this way,  $V_{12}$  would also appear in  $V_i$ , but only in the role of static electron correlations. The initial eigenvalue correction  $O_i$  was recently found to be negligible at high energies for  $\alpha$ -He transfer ionization treated by means of the CDW-4B method [6]. Assuming that this will also be the case with single-electron capture (1), we shall not consider the correction  $O_i$  in the present work.

It is clear now from Eqs. (3) and (6) that the use of the uncorrelated closed-shell  $(1s)^2$  Slater-screened hydrogenic wave function [21], accompanied with the neglect of  $\Delta V_{p2}$  and  $\Delta V_{12}$ , will effectively reduce the matrix elements of the CDW-4B method to those of its three-body counterpart, i.e., the CDW-3B model. In the present computations, however, we shall employ the radially correlated CI wave function  $(1s1s')$  of Silverman *et al.* [27] for  $\varphi_i$  defined as  $\varphi_i(\vec{x}_1, \vec{x}_2) = (\mathcal{N}/\pi)[\exp(-ax_1 - bx_2) + \exp(-bx_1 - ax_2)]$ , where  $\mathcal{N}^{-2} = 2[(ab)^{-3} + (a/2 + b/2)^{-6}]$ . Despite this very simple form of the open shell of the helium ground-state wave function [27], the radial correlations are taken into account to a very high degree of the order of nearly 95%.

### III. THE RESULTS

With the purpose of illustrating the CDW-4B method, we shall consider single-electron capture from helium by fast protons:



The results of the computations of prior and post total cross sections for the process (10) are summarized on Figs. 1–5. In

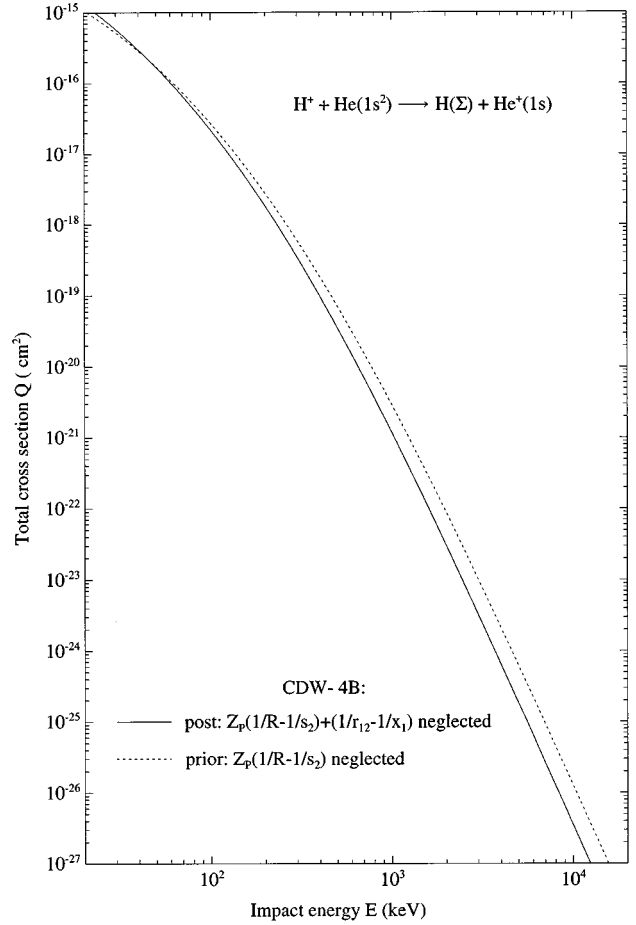


FIG. 1. Total cross sections for single capture from helium by protons as a function of the laboratory impact energy  $E$  computed in the four-body continuum distorted wave (CDW-4B) approximation. The symbol  $\Sigma$  represents the summation over all the bound states of atomic hydrogen (H). The explicit computations of  $Q_{1s}^\pm \equiv Q_{1s}^{\pm}$  are carried out only for the ground state of  $\text{H}(1s)$  and the  $n^{-3}$  Oppenheimer scaling law is used for  $\text{H}(n)$  to estimate the contribution from the whole spectrum of the excited states  $\text{H}(\Sigma)$  via  $Q^\pm = 1.202Q_{1s}^\pm$ . The initial wave functions ( $\varphi_i$ ) of Silverman *et al.* [27] is used. The full and the dashed lines correspond to the case where the potentials  $\Delta V_{p2} + \Delta V_{12}$  and  $\Delta V_{p2}$  are neglected in the *abbreviated*  $V_i$  and  $V_f$  according to Eq. (11), in a test of the post-prior discrepancy.

our discussions, emphasis will be placed upon the relative role of various terms in the full prior  $V_i$  and post  $V_f$  perturbations from Eqs. (4) and (7), along the lines traced in the Introduction. The simplest forms of the perturbations  $V_i$  and  $V_f$ , used in all the previous computations of the CDW method for Eq. (10), as well as for other processes of the general type (1), are given by

$$V_i \approx -\vec{\nabla}_{x_1} \varphi_i(\vec{x}_1, \vec{x}_2) \cdot \vec{\nabla}_{s_1}, \quad V_f \approx -\vec{\nabla}_{s_1} \varphi_f^*(\vec{s}_1) \cdot \vec{\nabla}_{x_1}. \quad (11)$$

In comparison with the full prior perturbation  $V_i$  from Eq. (4), the choice (11) corresponds to neglect of  $\Delta V_{p2} = V_{p2} - V_{p2}^\infty = -1/s_2 + 1/R$ , where  $P=p$  for  $Z_P=1$ . However, keeping only the scalar product of the gradient operators in the interaction  $V_f$  via the selection (11), will result in a more

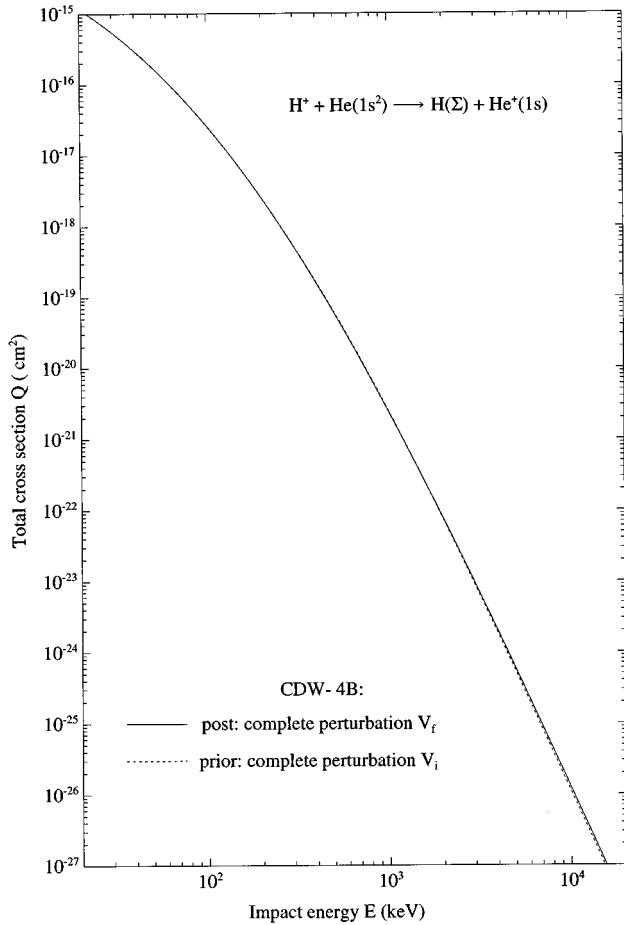


FIG. 2. The same as in Fig. 1, except for the *complete* perturbations  $V_i$  and  $V_f$ , according to Eqs. (4) and (7), respectively.

severe post approximation through omission of  $\Delta V_{p2} + \Delta V_{12}$ . The extent to which the dynamic electron correlation term  $\Delta V_{12} = V_{12} - V_{12}^\infty = 1/x_{12} - 1/x_1$  is ignored can be at once seen on Fig. 1. It follows that above 100 keV, where the CDW-4B model is expected to be valid, the so-called “post-prior” discrepancy between  $V_i$  and  $V_f$  from Eq. (11) is seen as huge. In this region, the post cross sections are always smaller than those of the prior form. The discrepancy increases with augmentation of the incident energy. This indicates that the role of the dynamic electron correlation effect  $\Delta V_{12}$  becomes more prominent at higher incident energies. Such an indication will be checked subsequently when we limit ourselves exclusively to the post approximation, considering the cases *with* and *without*  $\Delta V_{12}$ . Once the complete perturbations  $V_i$  and  $V_f$  are retained in the computations, according to Eqs. (4) and (7), the full CDW-4B model emerges with the feature of a dramatically reduced post-prior discrepancy (see Fig. 2). This is an excellent property of the method, since the same physical assumptions are involved in both prior and post forms. Although the difference between the post and prior results with the complete  $V_i$  and  $V_f$  is small, it should be noticed that the two curves in Fig. 2 are *not parallel* to each other. This difference becomes more significant at higher energies, where the post results are larger than the prior ones, as opposed to the preceding situation in Fig. 1. Despite the fact that the post-prior discrepancy with the complete  $V_i$  and  $V_f$  from Eqs. (4) and (7) does

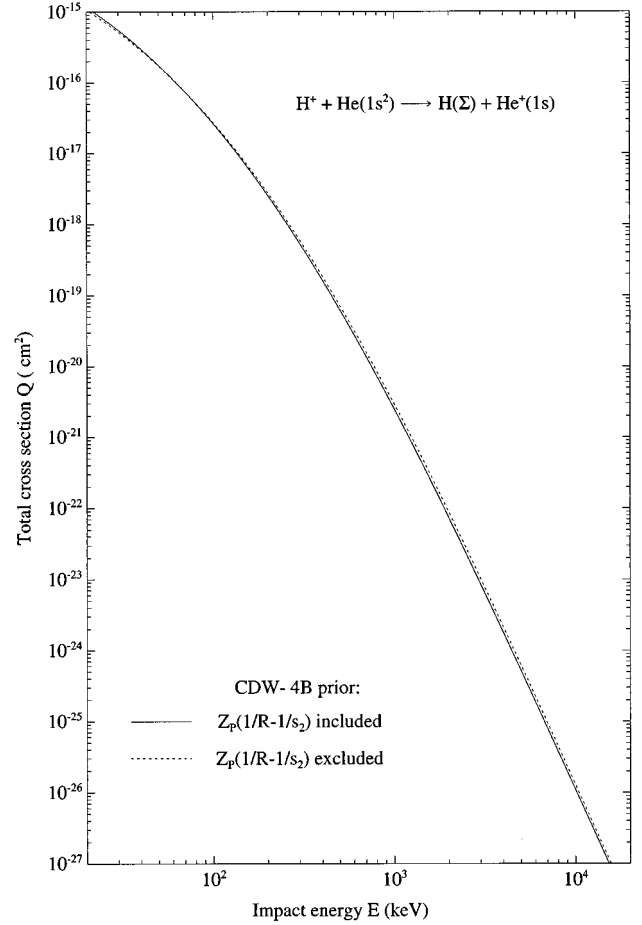


FIG. 3. The same as in Fig. 1, except for the limitation to the prior results only. The full and the dashed lines correspond to the case where the potential  $\Delta V_{p2}$  is included or excluded from  $V_i$ , respectively.

not exceed 15%, the trend of this discrepancy is significant, indicating again an increasing role of the dynamic electron correlations at higher energies. Next we examine the role of the “passive” electron by displaying the results with and without  $\Delta V_{p2}$  in Figs. 3 and 4. Above 100 keV, the cross sections accounting for  $\Delta V_{p2}$  in  $V_i$  and  $V_f$  are smaller by some 10% than those neglecting this term. However, their difference remains constant with increasing impact energy. In order to critically assess the usefulness of the present model, a comparison with measurements is required, and this is carried out on Fig. 5. One can see from Fig. 5 that the CDW-4B approximation is in excellent agreement with the available experimental data, provided that the full perturbation  $V_f$  is included in the post formulation. This comparison, which extends over three orders of magnitude of the impact energy  $E$  (20–20000 keV), where the cross sections vary within 12 orders of magnitude ( $10^{-15} - 10^{-27}$  cm<sup>2</sup>), establishes validity of the CDW-4B method at  $E \geq 70$  keV. Moreover, it can be observed from Fig. 5 that the dynamic electron correlations are essential, since exclusion of the relevant term  $\Delta V_{12}$  from  $V_f$  yields results that grossly underestimate the experiments at all energies above 100 keV.

#### IV. CONCLUSION

We have studied single charge exchange in proton-helium collisions at intermediate and high impact energies. This is

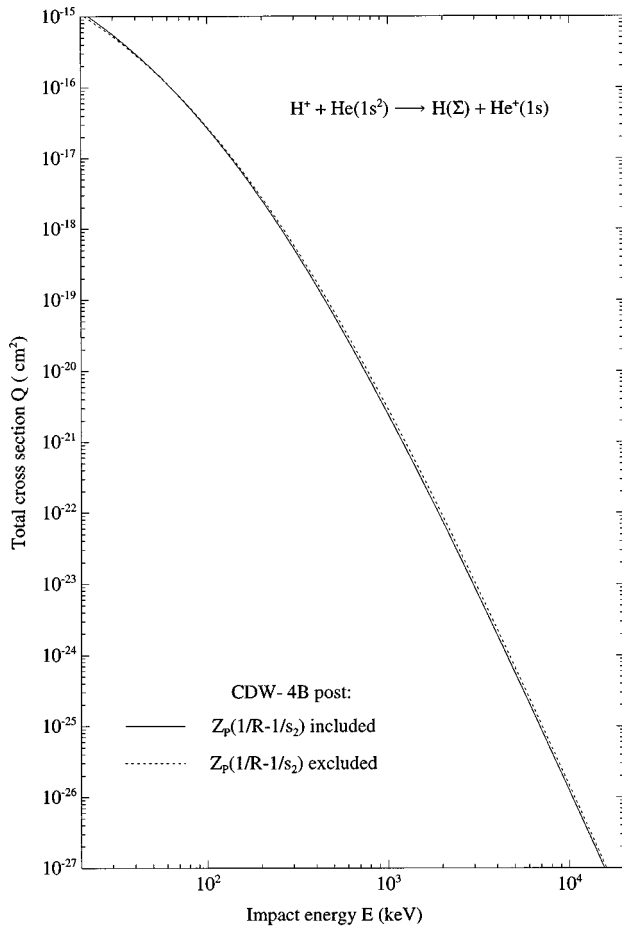


FIG. 4. The same as in Fig. 1, except for the limitation to the post results only. The full and the dashed lines correspond to the case where the potential  $\Delta V_{p2}$  is included or excluded from  $V_f$ , respectively.

accomplished by using the four-body continuum distorted wave (CDW-4B) theory, which was originally formulated for two-electron transitions, such as double charge exchange, transfer excitation, and transfer ionization. The obtained total cross sections for investigated  $p$ -He one-electron capture are presently found to be in excellent agreement with the available experimental data. Such a finding could also be used as an *a posteriori* justification of the physical assumptions of the original formulation of the CDW-4B model for more complex problems involving two active electrons. This is not surprising, since the four-body version of the CDW approximation is obtained as a direct extension of its well established three-body counterpart. Both three- and four-body formalisms of the same CDW theory are consistently derived as a first-order approximation of their respective Dodd-Greider perturbation developments. Such power series expansions are free from singular terms and this guarantees a meaningful interpretation of their first-order estimates. However, the employed CDW-4B method for the  $p$ -He single electron transfer is not unique, due to a multitude of choices for distorting potentials, and hence the need for a judicious choice of the prior and post perturbations  $V_i$  and  $V_f$ . The present selection of these perturbations, which are responsible for the transition under study, is guided by the correct boundary conditions as well as by the relative role of the competitive mecha-

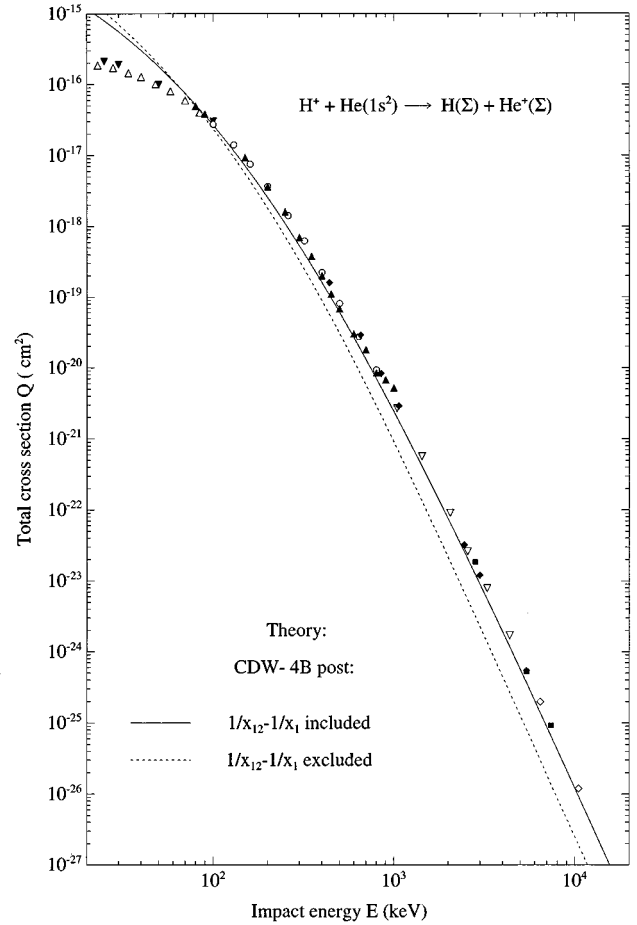


FIG. 5. Total cross sections for single capture from helium by protons as a function of the laboratory impact energy  $E$ . The symbol  $\Sigma$  represents the summation over all the bound states of atomic hydrogen (H) and helium ion ( $\text{He}^+$ ) as measured experimentally. In the computations of the displayed post cross sections  $Q^+$  only the ground states of  $\text{H}(1s)$  and  $\text{He}^+(1s)$  are accounted for. However, all the excited states  $\text{H}(\Sigma)$  of atomic hydrogen are approximately included via  $1.202Q_{1s}$  in terms of the  $n^{-3}$  Oppenheimer scaling law for  $\text{H}(n)$ . The initial wave function  $\varphi_i$  of Silverman *et al.* [27] is used. The full and the dashed lines obtained by means of the four-body continuum distorted-wave (CDW-4B) method correspond to the case where the potential  $\Delta V_{12}$  is included or excluded from the complete perturbation  $V_f$ , respectively. Experimental data:  $\nabla$ : Schryber [28];  $\triangle$ : Shah *et al.* [29];  $\circ$ : Shah and Gilbody [30];  $\diamond$ : Berkner *et al.* [31];  $\blacktriangle$ : Williams [32];  $\bullet$ : Horsdal-Pedersen *et al.* [33];  $\blacktriangledown$ : Martin *et al.* [34];  $\blacklozenge$ : Welsh *et al.* [35].

nisms of “velocity matching” (electron-projectile) and dynamic correlation effect (electron-electron interaction). The reported theoretical data provide evidence of the prevailing importance of the interelectron potential over the electron-nucleus interaction at high impact energies. This invalidates the widely accepted concept of considering the noncaptured electron as being “passive” in proton-helium single charge exchange. We emphasize that the present conclusion is reached on the level of the *total* cross sections. Since these observables are easier to measure, it is hoped that our findings would motivate further experimental studies on similar collisional systems at high energies, especially by means of storage rings equipped with the recoiled ion momentum spectroscopy.

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