# Incompatibility between local realism and quantum mechanics for pairs of neutral kaons

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A local realistic theory of single neutral kaons and of correlated neutral kaon pairs is formulated by deducing the most general consequences of three assumptions: (i) *the Einstein-Podolsky-Rosen (EPR) reality criterion*, (ii) *locality*, and (iii) *no retroactive causality*. Variables must be introduced for every kaon determining the stable $CP$ value and the variable value of strangeness $S$. Instantaneous $S$ jumps are shown to take place. If kaon pairs are produced in $\Phi$ meson decays, the local realistic probability of observing $\overline{K^0}\,\overline{K^0}$ pairs at certain different proper times necessarily differs by 30% from the quantum-mechanical predictions. The size of this difference justifies our systematic neglect of $CP$ violation. A $\Phi$ factory is thus shown to provide a unique tool for the study of the EPR problem [S1050-2947(97)05611-4]

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## I. INTRODUCTION

The strange nature of quantum correlations between separated systems, pointed out for the first time by Einstein, Podolsky, and Rosen (EPR) [1], has stimulated a lively debate over the past 60 years. The incompatibility between the predictions of quantum theory and some very general consequences of local realism became fully evident with the 1965 work of Bell [2], showing that a wide class of local hidden-variable models satisfies an inequality often violated by quantum mechanics.

In 1969, Clauser, Holt, Shimony and Horne [3] stressed that Bell's inequality could be checked experimentally with photon pairs emitted by single atoms, even with the available low efficiency photon counters, if suitable additional assumptions were made. Several experimental investigations of the EPR paradox have accordingly been performed, mostly with photon-polarization correlation measurements using radiative atomic cascade transitions [4]. In practically all these experiments the inequality was found to be violated, and the quantum-mechanical predictions turned out to agree with the data. It has been pointed out, however, that the introduction of additional assumptions had brought to the formulation of inequalities different from (and stronger than) Bell's original inequality [5]. The experimental results violated the stronger inequalities but were still compatible with Bell's original inequality, which was deduced from local realism alone without additional assumptions.

From a strictly logical point of view, the choice between local realism and the existing quantum theory has yet to be made. A more critical scrutiny of the incompatibility between quantum theory and local realism could come from the study of the EPR paradox in domains where highly efficient particle detectors can be used, and the additional assumptions are not needed. An appealing possibility is the decay of a $J^{PC}=1^{--}$ vector meson into a pair of neutral $K$ mesons [6–13]. The copious production of the $\Phi$ meson decays into two neutral kaons in a $\Phi$ factory provides a very useful tool for the study of the EPR problem. An experiment of this type is characterized by (a) almost perfect angular correlation between the two kaons, (b) nearly 100% efficient high-energy particle detectors, and (c) the absence of noise.

In quantum mechanics the state vector for a $J^{PC}=1^{--}$ system decaying into $K^0\overline{K^0}$, immediately after decay (i.e., at time zero) is given by

$$|\psi\rangle=\frac{1}{\sqrt{2}}\{|K^0\rangle_a|\overline{K^0}\rangle_b-|\overline{K^0}\rangle_a|K^0\rangle_b\}$$

$$=\frac{1}{\sqrt{2}}\{|K_S\rangle_a|K_L\rangle_b-|K_L\rangle_a|K_S\rangle_b\}, \tag{1}$$

where $a$ (left) and $b$ (right) denote the directions of motion of the kaons and $K_S$ and $K_L$ are the usual states for short- and long-lived kaons, respectively. The small effect of $CP$ nonconservation is neglected throughout this paper, and the $CP=\pm1$ eigenstates are identified with short long kaons, respectively. Their evolution is given by

$$|K_S(t)\rangle=|K_S\rangle\exp(-\alpha_S t),\quad |K_L(t)\rangle=|K_L\rangle\exp(-\alpha_L t), \tag{2}$$

where $t$ is the particle proper time and

$$\alpha_S=\tfrac{1}{2}\gamma_S+im_S,\quad \alpha_L=\tfrac{1}{2}\gamma_L+im_L. \tag{3}$$

In Eq. (3) $\gamma_S$ and $m_S$ ($\gamma_L$ and $m_L$) denote the decay rate and mass, respectively, of the $S(L)$ meson. Units $\hbar=c=1$ have been adopted. The time evolution operator of state (1) is the product of the time evolutions for the individual kaons, so that at proper times $t_a$ and $t_b$ one has

$$|\psi(t_a,t_b)\rangle=\frac{1}{\sqrt{2}}\{|K_S\rangle_a|K_L\rangle_b\exp(-\alpha_S t_a-\alpha_L t_b)$$
$$-|K_L\rangle_a|K_S\rangle_b\exp(-\alpha_L t_a-\alpha_S t_b)\}. \tag{4}$$

The difference between the two exponentials in Eq. (4) generates $K^0K^0$ and $\overline{K^0}\,\overline{K^0}$ components. The probability of $K^0K^0$ observation at times $t_a$ and $t_b$ is given by

 

$$P^{\mathrm{QM}}[\overline{K(t_a)};\overline{K(t_b)}]=\tfrac{1}{8}\{e^{-\gamma_S t_a-\gamma_L t_b}+e^{-\gamma_L t_a-\gamma_S t_b}-2e^{-(1/2)\gamma(t_a+t_b)}\cos\Delta m(t_a-t_b)\},\tag{5}$$

where $\gamma=\gamma_S+\gamma_L$, and $\Delta m=m_L-m_S$ is the $K_L-K_S$ mass difference. The right-hand side of Eq. (5) vanishes for $t_a=t_b$, as it must. The numerical parameters in units $\hbar=c=1$ are

$$\gamma_S=(1.121\pm0.002)\times10^{10}\ \mathrm{s}^{-1},$$

$$\gamma_L=(1.934\pm0.015)\times10^{7}\ \mathrm{s}^{-1},$$

$$\Delta m=m_L-m_S=(0.535\pm0.003)\times10^{10}\ \mathrm{s}^{-1}.$$

Taking $\gamma_S$ as inverse time unit, they can instead be written

$$\gamma_S=1,\quad \gamma_L=\frac{1}{579.6},\quad \Delta m=\frac{1}{2.10}.$$

In a real experiment the detection of $\overline{K^0}$'s can be achieved either via hyperon production in two suitably placed targets, or via $\Delta S=\Delta Q$ semileptonic decays at appropriate distances from the $\Phi$ decay region. The task of the present paper is to show that Eq. (5) is grossly incompatible with the predictions of any local realistic theory.

The paper is organized as follows: In Sec. II we review the results (obtained in the Appendix) concerning the strangeness and $CP$ ''elements of reality'' attributed to neutral kaons by the local realistic approach. The four neutral kaon states of local realism $K_i$ ($i=1$, 2, 3, and 4) are introduced. Sections III–V contain a possible (though not yet the most general) reinterpretation of quantum probabilities for single kaons in terms of the elementary probabilities of realism $p_{ji}(t|0)$: these are probabilities of observing a $K_j$ state at proper time $t$, conditional on the existence of a $K_i$ state at proper time zero. The $p_{ji}$ can be collected in a $4\times4$ matrix (''standard matrix''). The most general probability matrix is found in Secs. VII and VIII. A single, unknown quantity $\rho$ modifies all elements of the standard matrix by appearing as a $\pm\rho$ additional term. Upper and lower bounds are found for $\rho$ as consequences of the probabilistic interpretation of the $p_{ji}$ elements of the most general probability matrix. These bounds are essential in the final deduction of the range of possible local realistic predictions. Section IX deals with kaon pairs produced in the decay of $J^{\mathrm{PC}}=1^{--}\Phi$ mesons, and focuses on the probability $P[\overline{K};\overline{K}]$ for detecting two correlated neutral antikaons at different proper times. This is shown to have a very simple expression [Eq. (71)] in terms of the probabilities $p_{ji}$ for single kaons. As a curiosity we show that if a suitable assumption of non locality is made the quantum-mechanical prediction for $P[\overline{K};\overline{K}]$ is exactly reproduced (Sec. X). For a calculation of $P[\overline{K};\overline{K}]$ based on locality, $p_{ji}(t_b|t_a)$'s are needed for two proper times $t_b$ and $t_a$ possibly different from zero. Section XI develops a rate-equation method for finding these probabilities. The final result for $P[\overline{K};\overline{K}]$ (Sec. XII) contains two unknown quantities: one is the aforementioned $\rho$; the other one is a function $E$ of the time integral of the transition rate for strangeness jumps. A least value of $P[\overline{K};\overline{K}]$ is calculated by making $\rho$ and $E$ disappear, and it turns out (Sec. XIII) that this minimum is violated by quantum mechanics by about 30% in a rather broad proper time interval. The conclusion is that an experimental discrimination between local realism and quantum theory should be relatively easy at a $\Phi$ factory accelerator, and perhaps also with different techniques.

## II. ELEMENTS OF REALITY FOR KAONS

The EPR paradox arises from the incompatibility at the empirical level between the predictions of quantum theory and local realism. The latter consists of the following three assumptions.

(1) If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity (the EPR *reality criterion*).

(2) If two physical systems (e.g., two kaons) are separated by a large distance, an element of reality belonging to one of them cannot be created by a measurement performed on the other one (*locality*).

(3) If at a given time $t$ a physical system has an element of reality, the latter cannot be created by measurements on the same system performed at time $t'$, if $t'>t$ (*no retroactive causality*).

Local realism can be applied to kaon pairs described quantum mechanically by the state vector (4), by considering only those predictions of Eq. (4) to which the EPR reality criterion can be applied. These are the strict anticorrelations in strangeness $S$ and in $CP$. Our conclusions could not be correct if these anticorrelations were not found experimentally, but we will show that there is nothing paradoxical about them. If assumed to be exact the following conclusions hold (see the Appendix).

(1) Each kaon of every pair has an associated element of reality $\lambda_1$, which determines a well defined $CP$ value ($\lambda_1=\pm1$ corresponds to $CP=\pm1$, respectively).

(2) Each kaon of every pair has an associated element of reality $\lambda_2$, which determines a well-defined value of strangeness $S$ ($\lambda_2=\pm1$ corresponds to $S=\pm1$, respectively).

Furthermore $\lambda_1$ is a stable property, while $\lambda_2$ undergoes sudden jumps from $S=+1$ to $S=-1$, and vice versa, which are simultaneous for the two kaons of every pair, but happen more or less at random times in a statistical ensemble of many pairs: the theory of $S$ jumps is presented in Sec. XI. Notice that the application of local realism to the physical situation described by Eq. (4) has brought us at least formally outside quantum theory: no quantum-mechanical state vector exists, in fact, which can describe a kaon as having simultaneously well defined $CP$ and $S$ values. That this discrepancy is not only formal, but leads to empirically observable consequences, will be shown in the following.

## III. REALISM AND SINGLE KAON PHYSICS

We wish to reproduce the quantum-mechanical predictions for strangeness oscillations and decay of (single) $K^0$ mesons within the local realistic approach. That such a problem can be solved was shown in Ref. [14], but now we aim at the most general formulation of realism. The quantum-mechanical state vector of an initial $S=+1$ kaon evolves at proper time $t$ into

$$|K^0(t)\rangle=\frac{1}{\sqrt{2}}\,e^{-\alpha_S t}|K_S(0)\rangle+\frac{1}{\sqrt{2}}\,e^{-\alpha_L t}|K_L(0)\rangle$$
$$=\tfrac{1}{2}[e^{-\alpha_S t}+e^{\alpha_L t}]|K^0(0)\rangle$$
$$+\tfrac{1}{2}[e^{-\alpha_S t}-e^{-\alpha_L t}]|\overline{K^0}(0)\rangle, \tag{6}$$

so that, in general, both $K^0$ and $\overline{K^0}$ components turn out to be present at proper time $t$. Other quantum-mechanical time evolutions relevant to our problem are the following:

$$|\overline{K^0}(t)\rangle=\frac{1}{\sqrt{2}}\,e^{-\alpha_S t}|K_S(0)\rangle-\frac{1}{\sqrt{2}}\,e^{-\alpha_L t}|K_L(0)\rangle$$
$$=\tfrac{1}{2}[e^{-\alpha_S t}-e^{-\alpha_L t}]|K^0(0)\rangle$$
$$+\tfrac{1}{2}[e^{-\alpha_S t}+e^{-\alpha_L t}]|\overline{K^0}(0)\rangle, \tag{7}$$

$$|K_S(t)\rangle=e^{-\alpha_S t}|K_S(0)\rangle=e^{-\alpha_S t}\,\frac{1}{\sqrt{2}}\,\{|K^0(0)\rangle+|\overline{K^0}(0)\rangle\}, \tag{8}$$

$$|K_L(t)\rangle=e^{-\alpha_L t}|K_L(0)\rangle=e^{-\alpha_L t}\,\frac{1}{\sqrt{2}}\,\{|K^0(0)\rangle-|\overline{K^0}(0)\rangle\}. \tag{9}$$

For single kaons all these predictions are *a priori* nonparadoxical, as they do not refer to the case of correlated pairs where the Einstein-Podolsky-Rosen paradox could exist, but to individual quantum objects. We will see, in fact, that local realistic models exist reproducing the empirical consequences of Eqs. (6)–(9).

Following the ideas of the previous section we introduce four basic states:

$$K_1\equiv K_S:\ \text{state with } S=+1 \text{ and } CP=+1\ \text{(short-living kaon)},$$
$$K_2\equiv \overline{K_S}:\ \text{state with } S=-1 \text{ and } CP=+1\ \text{(short-living antikaon)},$$
$$K_3\equiv K_L:\ \text{state with } S=+1 \text{ and } CP=-1\ \text{(long-living kaon)},$$
$$K_4\equiv \overline{K_L}:\ \text{state with } S=-1 \text{ and } CP=-1\ \text{(long-living antikaon)}. \tag{10}$$

Next we introduce the probabilities of observing the previous states in a given physical situation:

$$p_i(t)=(\text{probability of } K_i \text{ at proper time } t)\ \ (i=1,2,3,4). \tag{11}$$

The initial conditions depend on the particular problem considered. We assume, as an example, that initially only the states with $S=+1$ are produced, and that the $CP=\pm1$ states are equiprobable. This corresponds to the situation described in quantum mechanics by the $S=+1$ ket (6). Therefore,

$$p_1(0)=p_3(0)=\tfrac{1}{2},\quad p_2(0)=p_4(0)=0. \tag{12}$$

In order to agree with the experimentally well-established validity of the quantum-mechanical probabilities (which refer to a well-defined initial $S$ without specifying $CP$) we must find a realistic model reproducing the following results for the chosen state:

$$p_1(t)+p_2(t)=|\langle K_S(0)|K(t)\rangle|^2=\tfrac{1}{2}E_S,$$
$$p_3(t)+p_4(t)=|\langle K_L(0)|K(t)\rangle|^2=\tfrac{1}{2}E_L \tag{13}$$

and

$$p_1(t)+p_3(t)=|\langle K(0)|K(t)\rangle|^2$$
$$=\tfrac{1}{4}[E_L+E_S+2\sqrt{E_LE_S}\cos\Delta mt], \tag{14}$$

$$p_2(t)+p_4(t)=|\langle\overline{K(0)}|K(t)\rangle|^2$$
$$=\tfrac{1}{4}[E_L+E_S-2\sqrt{E_LE_S}\cos\Delta mt],$$

where

$$E_S(t)=e^{-\gamma_S t},\quad E_L(t)=e^{-\gamma_L t}. \tag{15}$$

Notice that Eqs. (13) and (14) are compatible with Eq. (12). The sum of the two equations (13) gives the same as the sum of Eq. (14). Therefore we have really only three independent conditions for the four probabilities (11).

## IV. REINTERPRETATION OF QUANTUM PROBABILITIES

In order to fix all four probabilities, we start *provisionally* by assuming that they are linear in $\cos\Delta mt$, so that

$$p_1=a_1+b_1\cos\Delta mt,$$
$$p_2=a_2+b_2\cos\Delta mt,$$
$$p_3=a_3+b_3\cos\Delta mt,$$
$$p_4=a_4+b_4\cos\Delta mt, \tag{16}$$

where $a_i$, $b_i$, $(i=1,2,3,4)$ are independent of $\Delta m$, and are to be determined. By comparing Eqs. (16) with Eqs. (13) and (14), we obtain

$$a_1+a_2=\tfrac{1}{2}E_S, \quad b_1+b_2=0,$$
$$a_3+a_4=\tfrac{1}{2}E_L, \quad b_3+b_4=0,$$
$$a_1+a_3=\tfrac{1}{4}[E_S+E_L], \quad b_1+b_3=\tfrac{1}{2}\sqrt{E_SE_L},$$
$$a_2+a_4=\tfrac{1}{4}[E_S+E_L], \quad b_2+b_4=-\tfrac{1}{2}\sqrt{E_SE_L}. \tag{17}$$

These conditions are again unable to fix completely the $a_i$, $b_i$, $(i=1,2,3,4)$. It is, however, natural to adopt provisionally the symmetrical solutions

$$a_1=a_2=\tfrac{1}{4}E_S, \quad b_1=-b_2=\tfrac{1}{4}E_S\,\frac{2\sqrt{E_SE_L}}{E_L+E_S},$$
$$a_3=a_4=\tfrac{1}{4}E_L, \quad b_3=-b_4=\tfrac{1}{4}E_L\,\frac{2\sqrt{E_SE_L}}{E_L+E_S}, \tag{18}$$

because Eq. (18) is the simplest choice which satisfies all the conditions (17) while at the same time giving non-negative values to the probabilities. Therefore,

$$p_1=\tfrac{1}{4}E_S\left[1+\frac{2\sqrt{E_SE_L}}{E_L+E_S}\cos\Delta mt\right],$$
$$p_2=\tfrac{1}{4}E_S\left[1-\frac{2\sqrt{E_SE_L}}{E_L+E_S}\cos\Delta mt\right],$$
$$p_3=\tfrac{1}{4}E_L\left[1+\frac{2\sqrt{E_SE_L}}{E_L+E_S}\cos\Delta mt\right],$$
$$p_4=\tfrac{1}{4}E_L\left[1-\frac{2\sqrt{E_SE_L}}{E_L+E_S}\cos\Delta mt\right]. \tag{19}$$

These results can be rewritten in a physically more appealing way as

$$p_1(t)=\tfrac{1}{4}e^{-\gamma_S t}\,\frac{|\psi_L+\psi_S|^2}{|\psi_L|^2+|\psi_S|^2},$$
$$p_2(t)=\tfrac{1}{4}e^{-\gamma_S t}\,\frac{|\psi_L-\psi_S|^2}{|\psi_L|^2+|\psi_S|^2},$$
$$p_3(t)=\tfrac{1}{4}e^{-\gamma_L t}\,\frac{|\psi_L+\psi_S|^2}{|\psi_L|^2+|\psi_S|^2},$$
$$p_4(t)=\tfrac{1}{4}e^{-\gamma_L t}\,\frac{|\psi_L-\psi_S|^2}{|\psi_L|^2+|\psi_S|^2}, \tag{20}$$

where we introduced the ‘‘wave functions’’

$$\psi_L=\psi_0\exp(-\gamma_L t/2)\exp(-im_L t),$$
$$\psi_S=\psi_0\exp(-\gamma_S t/2)\exp(-im_S t). \tag{21}$$

Notice that the simple equations (20) do not exist within standard quantum theory. They suggest the following dualistic picture.

(1) All kaons are particles embedded in extended waves which are in all cases superpositions of $\psi_L$ and $\psi_S$. The time evolution of these wave functions is given by Eq. (21). Initially they have the same (unknown) value $\psi_0$.

(2) The physical quantity $CP$ is basically a particle property, and every neutral kaon is born with a fixed value $(\pm 1)$ of it. Neglecting $CP$ violation, every kaon maintains forever the same $CP$ with which it was born. Nevertheless both $\psi_L$ and $\psi_S$ waves are associated with every kaonic particle independently of its $CP$ value.

(3) Also, strangeness $S$ is basically a particle property and every kaon is born with a fixed value $(\pm 1)$ of it. Particles with $S=+1$ go always together with the wave $\psi_L+\psi_S$, particles with $S=-1$ go with $\psi_L-\psi_S$.

(4) Strangeness jumps from $S=+1$ to $S=-1$, or vice versa, are possible as sudden particle events and when a jump takes place also the wave is forced to change accordingly, e.g., with $\psi_S$ undergoing a sudden phase shift $\pi$. For example, the particle jump from $S=+1$ to $S=-1$ makes the wave go from $\psi_L+\psi_S$ to $\psi_L-\psi_S$.

(5) These jumps take place with a frequency such that the Born rule is always satisfied: the probabilities that any given kaon at a certain proper time $t$ has $S=\pm 1$ are proportional to $|\psi_L(t)\pm\psi_S(t)|^2$, respectively.

(6) Kaons are unstable particles and decay with constant rates $\gamma_S$ (if $CP=+1$) and $\gamma_L$ (if $CP=-1$). Thus a $CP=+1$ $(CP=-1)$ kaon has a probability of being undecayed at time $t$ which decreases exponentially, just like the squared modulus $E_S(E_L)$ of the corresponding wave function $\psi_S$ $(\psi_L)$. In this way Born's rule is easily satisfied.

Probabilities (20) find a complete physical interpretation within this model. For example,

$$p_1(t)=\tfrac{1}{2}e^{-\gamma_S t}\,\frac{|\psi_L+\psi_S|^2}{2\{|\psi_L|^2+|\psi_S|^2\}}$$

can be understood as follows: the factor 1/2 is the probability that the given kaon is born with $CP=+1$, in agreement with Eq. (12); the exponential factor is the probability that it has remained undecayed at time $t$; the final fraction is the probability that it has positive strangeness at time $t$. In this way the quantum-mechanical probabilities (13) and (14) are given a completely new physical interpretation within the local realistic approach. The corresponding ensembles are interpreted to be mixtures of other ensembles in which the basic states of local realism (10) apply. We stress again that Eqs. (19) and (20) do not give the most general probabilities within local realism: rather, they give only the most natural ones. A complete generalization will, however, be given in Sec. VII.

## V. PROBABILITIES FOR PURE STATES OF LOCAL REALISM

### A. Introduction

The local realistic probabilities (19) and (20) describe a particular mixture, that is, a statistical ensemble in which two of the basic states (10) are initially present with equal statistical weights, as shown by the initial conditions (12). However the physical reinterpretation given in Sec. IV allows us to extend very naturally our results to the case of ‘‘pure states’’ (only one kaonic state produced initially). Probabilities with two indices will be used, the second one specifying which one of the four states (10) was initially present. Naturally there are four possibilities, examined in Secs. V B–V E below. In the following we will use the shorter notation

$$Q_{\pm}(t) \equiv \frac{1}{2}\left[1 \pm \frac{2\sqrt{E_L E_S}}{E_L + E_S}\cos\Delta m t\right]. \tag{22}$$

### B. Initial state with $CP=+1$ and $S=+1$: $K_1(0)$

In a given physical situation the basic probabilities for the four states (10) at proper time $t$ can be considered *conditional* on the initial presence of $K_1(0)$. By using the symbol $p_{ji}(t|0)$ to denote the probability of a kaon in state $K_j$ at proper time $t$ conditional on the same kaon having been in state $K_i$ at proper time 0 ($j,i=1,2,3,4$) we can write

$$\begin{aligned} p_{11}(t|0) &= E_S(t)Q_+(t), \\ p_{21}(t|0) &= E_S(t)Q_-(t), \\ p_{31}(t|0) &= 0, \\ p_{41}(t|0) &= 0, \end{aligned} \tag{23}$$

which satisfy the initial conditions

$$p_{11}(0|0)=1, \quad p_{21}(0|0)=p_{31}(0|0)=p_{41}(0|0)=0. \tag{24}$$

Of course, in Eq. (23), $p_{11}(t|0)$ has a physical interpretation similar to that given for $p_1(t)$ at the end of Sec. IV, but relative to a different initial condition. In the case of $p_{11}(t|0)$ we can say that the given kaon is born with $CP=S=+1$, that $E_s(t)$ is the probability that it has not yet decayed at time $t$, and that $Q_+(t)$ is the probability of its still having $S=+1$ at time $t$. The interpretation of all $p_{ji}(t|0)$ ($j,i=1,2,3,4$) in Eq. (23) and in the coming equations is always similar.

### C. Initial state with $CP=+1$ and $S=-1$: $K_2(0)$

For this second initial condition one can write

$$\begin{aligned} p_{12}(t|0) &= E_S(t)Q_-(t), \\ p_{22}(t|0) &= E_S(t)Q_+(t), \\ p_{32}(t|0) &= 0, \\ p_{42}(t|0) &= 0, \end{aligned} \tag{25}$$

which satisfy

$$p_{12}(0|0)=0, \quad p_{22}(0|0)=1, \quad p_{32}(0|0)=p_{42}(0|0)=0. \tag{26}$$

### D. Initial state with $CP=-1$ and $S=+1$: $K_3(0)$

For this third initial condition one can write

$$\begin{aligned} p_{13}(t|0) &= 0, \\ p_{23}(t|0) &= 0, \\ p_{33}(t|0) &= E_L(t)Q_+(t), \\ p_{43}(t|0) &= E_L(t)Q_-(t), \end{aligned} \tag{27}$$

which satisfy

$$p_{13}(0|0)=p_{23}(0|0)=0, \quad p_{33}(0|0)=1, \quad p_{43}(0|0)=0. \tag{28}$$

### E. Initial state with $CP=-1$ and $S=-1$: $K_4(0)$

For this fourth initial condition one can write

$$\begin{aligned} p_{14}(t|0) &= 0, \\ p_{24}(t|0) &= 0, \\ p_{34}(t|0) &= E_L(t)Q_-(t), \\ p_{44}(t|0) &= E_L(t)Q_+(t), \end{aligned} \tag{29}$$

which satisfy

$$p_{14}(0|0)=p_{24}(0|0)=p_{34}(0|0)=0, \quad p_{44}(0|0)=1. \tag{30}$$

### F. Conclusions

The probabilities introduced in the present section through Eqs. (23), (25), (27), and (29) will be said to constitute ‘‘the standard set.’’ Of course these probabilities, albeit elegant, are to some extent arbitrary. A complete generalization will be given in Sec. VII.

## VI. SYSTEMATIC COMPARISON WITH QUANTUM MECHANICS

In this and in Sec. VII we will use, for the conditional probabilities, the shorter notation $p_{ji}$ instead of $p_{ji}(t|0)$. We can now check that all quantum-mechanical probabilities for single kaons are reproduced. First of all we can easily see that the quantum-mechanical conditions of $CP$ conservation are satisfied. Indeed,

$$|\langle K_L(0)|K_S(t)\rangle|^2 = 0 = \tfrac{1}{2}[p_{31}+p_{41}+p_{32}+p_{42}], \tag{31}$$

$$|\langle K_S(0)|K_L(t)\rangle|^2 = 0 = \tfrac{1}{2}[p_{13}+p_{23}+p_{14}+p_{24}], \tag{32}$$

because all probabilities appearing in the right-hand sides of Eqs. (31) and (32) vanish in Eqs. (23), (25), (27), and (29). From now on we will write only nonvanishing probabilities: this can be done by excluding all probabilities in which the first index is either 1 or 2 while at the same time the second index is either 3 or 4, and vice versa. It is then easy to check that the following 14 conditions [which can easily be deduced from the state vectors (6)–(9)] are satisfied by the probabilities of the standard set of equations (23), (25), (27), and (29).

$$|\langle K_S(0)|K(t)\rangle|^2 = \tfrac{1}{2}E_S = \tfrac{1}{2}[p_{11}+p_{21}], \tag{33}$$

$$|\langle K_L(0)|K(t)\rangle|^2 = \tfrac{1}{2}E_L = \tfrac{1}{2}[p_{33}+p_{43}], \tag{34}$$

$$|\langle K(0)|K(t)\rangle|^2 = \tfrac{1}{2}(E_L+E_S)Q_+ = \tfrac{1}{2}[p_{11}+p_{33}], \tag{35}$$

$$|\langle\overline{K(0)}|K(t)\rangle|^2=\tfrac{1}{2}(E_L+E_S)Q_-=\tfrac{1}{2}[p_{21}+p_{43}], \quad (36)$$

$$|\langle K_S(0)|\overline{K(t)}\rangle|^2=\tfrac{1}{2}E_S=\tfrac{1}{2}[p_{12}+p_{22}], \quad (37)$$

$$|\langle K_L(0)|\overline{K(t)}\rangle|^2=\tfrac{1}{2}E_L=\tfrac{1}{2}[p_{34}+p_{44}], \quad (38)$$

$$|\langle K(0)|\overline{K(t)}\rangle|^2=\tfrac{1}{2}(E_L+E_S)Q_-=\tfrac{1}{2}[p_{12}+p_{34}], \quad (39)$$

$$|\langle\overline{K(0)}|\overline{K(t)}\rangle|^2=\tfrac{1}{2}(E_L+E_S)Q_+=\tfrac{1}{2}[p_{22}+p_{44}], \quad (40)$$

$$|\langle K_S(0)|K_S(t)\rangle|^2=E_S=\tfrac{1}{2}[p_{11}+p_{21}+p_{12}+p_{22}], \quad (41)$$

$$|\langle K(0)|K_S(t)\rangle|^2=\tfrac{1}{2}E_S=\tfrac{1}{2}[p_{11}+p_{12}], \quad (42)$$

$$|\langle\overline{K(0)}|K_S(t)\rangle|^2=\tfrac{1}{2}E_S=\tfrac{1}{2}[p_{21}+p_{22}], \quad (43)$$

$$|\langle K_L(0)|K_L(t)\rangle|^2=E_L=\tfrac{1}{2}[p_{33}+p_{43}+p_{34}+p_{44}], \quad (44)$$

$$|\langle K(0)|K_L(t)\rangle|^2=\tfrac{1}{2}E_L=\tfrac{1}{2}[p_{33}+p_{34}], \quad (45)$$

$$|\langle\overline{K(0)}|K_L(t)\rangle|^2=\tfrac{1}{2}E_L=\tfrac{1}{2}[p_{43}+p_{44}]. \quad (46)$$

Therefore all the 16 physical conditions imposed by quantum mechanics are satisfied by the probabilities of local realism (23), (25), (27), and (29). These can be collected in a ‘‘standard probability matrix’’ $P_0$ as follows:

$$P_0=\begin{vmatrix} E_SQ_+ & E_SQ_- & 0 & 0 \\ E_SQ_- & E_SQ_+ & 0 & 0 \\ 0 & 0 & E_LQ_+ & E_LQ_- \\ 0 & 0 & E_LQ_- & E_LQ_+ \end{vmatrix}, \quad (47)$$

where $p_{11}(t|0)=E_S(t)Q_+(t)$, $p_{12}(t|0)=E_S(t)Q_-(t)$, etc., and $Q_\pm$ are of course given by Eq. (22), so that $Q_++Q_-=1$.

## VII. MOST GENERAL PROBABILITY MATRIX

With Eqs. (23), (25), (27), and (29), we obtained a remarkable solution for the probabilities $p_{ji}(t|0)(j,i=1,2,3,4)$. Mathematically speaking, however, this solution is not unique. In the present section we will study anew the problem of determining the most general set of local realistic probabilities compatible with the quantum-mechanical predictions. For simplicity we will not introduce new symbols, but will continue to denote our probabilities as $p_{ji}$ even when they do not belong to the standard set. Naturally conditions (31)–(46) must still be satisfied. Eqs. (31) and (32) imply the vanishing of eight probabilities:

$$p_{13}=p_{14}=p_{23}=p_{24}=p_{31}=p_{32}=p_{41}=p_{42}=0. \quad (48)$$

Taking Eq. (48) into account, the most general probability matrix $P$ is

$$P=\begin{vmatrix} p_{11} & p_{12} & 0 & 0 \\ p_{21} & p_{22} & 0 & 0 \\ 0 & 0 & p_{33} & p_{34} \\ 0 & 0 & p_{43} & p_{44} \end{vmatrix}. \quad (49)$$

There remain Eqs. (33)–(46), which can be considered as 14 (not independent) conditions for the eight nonvanishing probabilities. Writing

$$R_i=(\text{sum of the elements of the } i\text{th row}) \quad (i=1,2,3,4),$$

$$C_j=(\text{sum of the elements of the } j\text{th column}) \quad (j=1,2,3,4),$$

conditions (33)–(46) for the most general probabilities can respectively be written

$$C_1=E_S, \quad (33')$$

$$C_3=E_L, \quad (34')$$

$$p_{11}+p_{33}=(E_L+E_S)Q_+, \quad (35')$$

$$p_{21}+p_{43}=(E_L+E_S)Q_-, \quad (36')$$

$$C_2=E_S, \quad (37')$$

$$C_4=E_L, \quad (38')$$

$$p_{12}+p_{34}=(E_L+E_S)Q_-, \quad (39')$$

$$p_{22}+p_{44}=(E_L+E_S)Q_+, \quad (40')$$

$$C_1+C_2=2E_S, \quad (41')$$

$$R_1=E_S, \quad (42')$$

$$R_2=E_S, \quad (43')$$

$$C_3+C_4=2E_L, \quad (44')$$

$$R_3=E_L, \quad (45')$$

$$R_4=E_L, \quad (46')$$

Conditions (33′), (37′), (42′), and (43′) tell us that $R_1=R_2=C_1=C_2=E_S$. Conditions (34′), (38′), (45′), and (46′) tell us that $R_3=R_4=C_3=C_4=E_L$. Condition (41′) is a consequence of Eqs. (33′) and (37′), while condition (44′) is a consequence of Eqs. (34′) and (38′). A probability matrix satisfying the previous ten conditions is then

$$P=\begin{vmatrix} p_{11} & E_S-p_{11} & 0 & 0 \\ E_S-p_{11} & p_{11} & 0 & 0 \\ 0 & 0 & p_{33} & E_L-p_{33} \\ 0 & 0 & E_L-p_{33} & p_{33} \end{vmatrix}. \quad (50)$$

The conditions containing $Q_\pm$ [Eqs. (35′), (36′), (39′), and (40′)] remain to be analyzed. Written in terms of the probabilities appearing in Eq. (50), they are easily shown to reduce to the unique condition

$$p_{11}+p_{33}=(E_L+E_S)Q_+ . \tag{51}$$

The required agreement with quantum probabilities is thus seen to fix everything but the difference $p_{11}-p_{33}$. Writing

$$p_{11}-p_{33}=R, \tag{52}$$

where $R$ is unknown, we obtain

$$p_{11}=\tfrac{1}{2}[(E_L+E_S)Q_+ +R], \quad p_{33}=\tfrac{1}{2}[(E_L+E_S)Q_+ -R]. \tag{53}$$

The probability matrix (50) can then be written

$$P=\frac{1}{2}\begin{vmatrix} (E_L+E_S)Q_+ +R & E_SQ_- - E_LQ_+ -R & 0 & 0 \\ E_SQ_- - E_LQ_+ -R & (E_L+E_S)Q_+ +R & 0 & 0 \\ 0 & 0 & (E_L+E_S)Q_+ -R & E_LQ_- - E_SQ_+ +R \\ 0 & 0 & E_LQ_- - E_SQ_+ +R & (E_L+E_S)Q_+ -R \end{vmatrix}. \tag{54}$$

Restrictions on $R$ can be obtained by taking into account that every element of Eq. (54) is a probability and must lie between zero and one. This point will be discussed in Sec. VIII.

## VIII. DEVIATIONS FROM THE STANDARD SET

Equation (47) can be used to calculate the value of $R$ (call it $R_0$) predicted by the standard set of probabilities. By comparison with Eq. (54), one easily obtains

$$R_0=-(E_L-E_S)Q_+ .$$

Given that one can always introduce a $\rho$ such that

$$R=R_0+\rho, \tag{55}$$

the probability matrix (54) takes a form containing $\pm\rho$ corrections to the probabilities of the standard set (47):

$$P=\begin{vmatrix} E_SQ_+ +\rho & E_SQ_- -\rho & 0 & 0 \\ E_SQ_- -\rho & E_SQ_+ +\rho & 0 & 0 \\ 0 & 0 & E_LQ_+ -\rho & E_LQ_- +\rho \\ 0 & 0 & E_LQ_- +\rho & E_LQ_+ -\rho \end{vmatrix}. \tag{56}$$

Notice that every column refers to a well-defined initial state, one of the four cases (10). The sum of the elements of a column equals $E_S$ [for $K_1(0)$ and $K_2(0)$] and $E_L$ [for $K_3(0)$ and $K_4(0)$], corresponding to the appropriate population reductions due to spontaneous disintegration. Restrictions on $\rho$ can be obtained by assuming every probability to be positive. The first column gives

$$-E_SQ_+\leqslant\rho\leqslant E_SQ_- . \tag{57}$$

It is easy to see that from the second column the same result (57) follows. From the third and fourth columns, one similarly obtains

$$-E_LQ_-\leqslant\rho\leqslant E_LQ_+ . \tag{58}$$

The requirement that all elements of $P$ be less than one is automatically satisfied if Eqs. (57) and (58) hold, as one can easily verify. Conditions (57) and (58) must both be satisfied in any consistent local realistic theory. It is enough that one of them is violated by a given $\rho$ to conclude that the latter is incompatible with local realism. We checked numerically that for all times $E_SQ_- < E_LQ_+$. In other words of the two upper limits in Eqs. (57) and (58), it is enough to consider

$$\rho(t)\leqslant E_S(t)Q_-(t). \tag{59}$$

No simplification of this type exists for the lower limits of Eqs. (57) and (58).

The introduction of the unknown quantity $\rho$ is the most important difference between the present theory and that of Ref. [9]. Only the presence of $\rho$ allows us to state that we are working with the most general formulation of local realism.

## IX. CASE OF KAON PAIRS

### A. Introduction

Kaon pairs arising in the decay of the $\Phi$ meson, e.g., produced in $e^+e^-$ collisions, are described quantum mechanically by the $J^{PC}=1^{--}$ state vector (4). The probability (5) of a double $\overline{K^0}$ observation at proper times $t_a$ and $t_b$, written with notation (15), is

$$P^{\rm QM}[\overline{K(t_a)};\overline{K(t_b)}]=\tfrac{1}{8}[E_S(t_a)E_L(t_b)+E_L(t_a)E_S(t_b)-2\sqrt{E_S(t_a)E_L(t_b)E_L(t_a)E_S(t_b)}\cos\Delta m(t_a-t_b)]. \tag{60}$$

We will show that the local realistic approach leads necessarily to disagreement with prediction (60). The starting point is again the discussion in the Appendix where it is shown that local realism applied to the physical situation described by (4) implies — at equal proper times — a total anticorrelation both in strangeness and in $CP$ between the two kaons flying in opposite directions, the four possible physical configurations appearing at least initially with the same statistical weight ($\frac{1}{4}$). Given Eq. (10), we must then consider the following four cases for the calculation of $P^{\mathrm{LR}}[\overline{K}(t_a);\overline{K}(t_b)]$.

### B. Initial state with $K_1(0)$ on the left and $K_4(0)$ on the right

The probability that the initial $K_1(0)$ on the left evolves into a $S=-1$ state at proper time $t_a$ [then, given $CP$ conservation, into $K_2(t_a)$] is given by Eq. (56):

$$p_{21}(t_1|0)=E_S(t_1)Q_-(t_1)-\rho(t_a). \tag{61}$$

Correlated with the left-going antikaon $K_2(t_a)$, on the right-hand side of the physical process there will be at proper time $\tilde{t}_b=t_a$ either decay products, or a pure $K_3$ state. The probability of the latter is of course $E_L(\tilde{t}_b)$. The probability of its evolution into $K_4(t_b)$, conditional on the state $K_3(\tilde{t}_b)$ (with $t_b>\tilde{t}_b$) is

$$p_{43}(t_a|\tilde{t}_b)\equiv p[K_4(t_b)|K_3(\tilde{t}_b)]. \tag{62}$$

Therefore, in this first case the overall probability of double $S=-1$ observation at proper times $t_a$ (on the left) and $t_b$ (on the right) is clearly given by

$$\begin{aligned}P_1[K_2(t_a);K_4(t_b)]&=p_{21}(t_a|0)E_L(\tilde{t}_b)p_{43}(t_b|\tilde{t}_b)\\&=\{E_S(t_a)Q_-(t_a)-\rho(t_a)\}\\&\quad\times E_L(\tilde{t}_b)p_{43}(t_b|\tilde{t}_b).\end{aligned} \tag{63}$$

### C. Initial state with $K_2(0)$ on the left and $K_3(0)$ on the right

The probability that the initial $K_2(0)$ on the left remains a $S=-1$ state at proper time $t_a$ [then, given $CP$ conservation, that it becomes $K_2(t_a)$] is given by Eq. (56):

$$p_{22}(t_a|0)=E_S(t_a)Q_+(t_a)+\rho(t_a). \tag{64}$$

Correlated with the left-going antikaon $K_2(t_a)$, on the right-hand side of the physical process there will be, at the proper time $\tilde{t}_b=t_a$, either decay products or a pure $K_4$ state. The probability of the latter is of course $E_L(\tilde{t}_b)$. The probability of its evolution into $K_4(t_b)$ is again given by Eq. (62).

Therefore in this second case the overall probability of double $S=-1$ observation at proper times $t_a$ (on the left) and $t_b$ (on the right) is

$$\begin{aligned}P_2[K_2(t_a);K_4(t_b)]&=p_{22}(t_a|0)E_L(\tilde{t}_b)p_{43}(t_b|\tilde{t}_b)\\&=\{E_S(t_a)Q_+(t_a)+\rho(t_a)\}\\&\quad\times E_L(\tilde{t}_b)p_{43}(t_b|\tilde{t}_b).\end{aligned} \tag{65}$$

Notice that the term $\rho(t_a)$ disappears when Eqs. (63) and (65) are summed together, as will be done at the end of this section.

### D. Initial state with $K_3(0)$ on the left and $K_2(0)$ on the right

The probability that the initial $K_3(0)$ on the left evolves into a $S=-1$ state at proper time $t_a$ [then, given $CP$ conservation, into $K_4(t_a)$] is given by Eq. (56):

$$p_{43}(t_a|0)=E_L(t_a)Q_-(t_a)+\rho(t_a). \tag{66}$$

Correlated with the left-going antikaon $K_4(t_a)$, on the right-hand side of the physical process there will be, at a proper time $\tilde{t}_b=t_a$, either decay products or a pure $K_1$ state. The probability of the latter is of course $E_s(\tilde{t}_b)$. The probability of its evolution into $K_2(t_b)$, conditional on its having been a $K_1(\tilde{t}_b)$ (with $t_b>\tilde{t}_b$), is

$$p_{21}(t_b|\tilde{t}_b)\equiv p[K_2(t_b)|K_1(\tilde{t}_b)]. \tag{67}$$

Therefore in this third case the overall probability of double $S=-1$ observation at proper times $t_a$ (on the left) and $t_b$ (on the right) is

$$\begin{aligned}P_3[K_4(t_a);K_2(t_b)]&=p_{43}(t_a|0)E_S(\tilde{t}_b)p_{21}(t_b|\tilde{t}_b)\\&=\{E_L(t_a)Q_-(t_a)+\rho(t_a)\}\\&\quad\times E_S(\tilde{t}_b)p_{21}(t_b|\tilde{t}_b).\end{aligned} \tag{68}$$

### E. Initial state with $K_4(0)$ on the left and $K_1(0)$ on the right

The probability that the initial $K_4(0)$ on the left evolves into a $S=-1$ state at proper time $t_a$ [then, given $CP$ conservation, into $K_4(t_a)$] is given by Eq. (56):

$$p_{44}(t_a|0)=E_L(t_a)Q_+(t_a)-\rho(t_a). \tag{69}$$

Correlated with the left-going antikaon $K_4(t_a)$, on the right-hand side of the physical process there will be, at a proper time $\tilde{t}_b=t_a$, either decay products or a pure $K_1$ state. The probability of the latter is of course $E_s(\tilde{t}_b)$. The probability of its evolution into $K_2(t_b)$ is given by Eq. (67). Therefore in this fourth case the overall probability of double $S=-1$ observation at proper times $t_a$ (on the left) and $t_b$ (on the right) is

$$\begin{aligned}P_4[K_4(t_a);K_2(t_b)]&=p_{44}(t_a|0)E_S(\tilde{t}_b)p_{21}(t_b|\tilde{t}_b)\\&=\{E_L(t_a)Q_+(t_a)-\rho(t_a)\}\\&\quad\times E_S(\tilde{t}_b)p_{21}(t_b|\tilde{t}_b).\end{aligned} \tag{70}$$

Notice that the $\rho(t_a)$ term is once more going to disappear when Eqs. (68) and (70) will be summed together.

### F. Conclusion

The four elementary states of local realism must appear initially with the same weight ($\frac{1}{4}$) both on the left and on the right in the physical situation described quantum mechanically by the state vector (4), as shown in the Appendix. Therefore, given the results of Secs. IX B–IX E, we have

$$P^{\rm LR}[\overline{K(t_a)};\overline{K(t_b)}]=\tfrac{1}{4}\{P_1[K_2(t_a);K_4(t_b)]+P_2[K_2(t_a);K_4(t_b)]+P_3[K_4(t_a);K_2(t_b)]+P_4[K_4(t_a);K_2(t_b)]\}=\frac{E_S(t_a)E_L(\tilde{t}_b)}{4}\,p_{43}(t_b|\tilde{t}_b)+\frac{E_L(t_a)E_S(\tilde{t}_b)}{4}\,p_{21}(t_b|\tilde{t}_b).$$

Remembering that $\tilde{t}_b=t_a$, the last equation can be written

$$P^{\rm LR}[\overline{K(t_a)};\overline{K(t_b)}]=\frac{E_S(t_a)E_L(t_a)}{4}\,[p_{43}(t_b|t_a)+p_{21}(t_b|t_a)],\tag{71}$$

where $t_a$ is now used as time label also for the right-going kaon. Equation (71) will be the starting point of our further discussion. The probabilities $p_{43}$ and $p_{21}$ in Eq. (71) are not known in general: our previous considerations would fix them [up to the additive terms $\pm\rho(t_b)$] only if one had $t_a=0$. As seen in Sec. X, the assumption of nonlocality (action at a distance) leads to their determination.

## X. NONLOCAL MODEL FOR KAON PAIRS

We will show that a nonlocal model exists that reproduces prediction (60), as it does with all other consequences of the state vector (4). The (nonlocal) probability for the evolution of the right-going kaon either from $K_1(t_a)$ in $K_2(t_b)$, or from $K_3(t_a)$ in $K_4(t_b)$, can be obtained with a slight modification of Eq. (71):

$$P^{\rm NL}[\overline{K(t_a)};\overline{K(t_b)}]=\frac{E_S(t_a)E_L(t_a)}{4}\times[p_{21}(t_b-t_a)+p_{43}(t_b-t_a)].\tag{72}$$

Nonlocality has been introduced by assuming that the probabilities for the right-going kaon depend only on the time difference $t_b-t_a$, as if its history started again at the time $t_a$ when a measurement was performed on its coupled left-going kaon. By using this assumption together with Eqs. (23) and (27), we obtain

$$P^{\rm NL}[\overline{K(t_a)};\overline{K(t_b)}]=\frac{E_S(t_a)E_L(t_a)}{4}\,[E_L(t_b-t_a)+E_S(t_b-t_a)-2\sqrt{E_L(t_b-t_a)E_S(t_b-t_a)}\times\cos\Delta m(t_b-t_a)].\tag{73}$$

Given that from Eq. (15), one has

$$E_S(t_b-t_a)=\frac{E_S(t_b)}{E_S(t_a)},\quad E_L(t_b-t_a)=\frac{E_L(t_b)}{E_L(t_a)},\tag{74}$$

it is easy to obtain

$$P^{\rm NL}[\overline{K(t_a)};\overline{K(t_b)}]=\tfrac{1}{8}\,[E_L(t_b)E_S(t_a)+E_S(t_b)E_L(t_a)-2\sqrt{E_L(t_b)E_S(t_a)E_S(t_b)E_L(t_a)}\times\cos\Delta m(t_b-t_a)],\tag{75}$$

which is identical with the quantum-mechanical prediction (60). The above nonlocal model was found by Cobianco [13]. Its existence is of course a matter of curiosity: if the choice were only between quantum theory and a causal *nonlocal* approach, the former should probably be preferred.

## XI. RATE EQUATIONS

The remaining problem is calculating $p_{21}(t_b|t_a)$ and $p_{43}(t_b|t_a)$ in Eq. (71). The first one is the probability that a right-going kaon, that was with certainty a $K_1$ at time $\tilde{t}_b=t_a$, becomes a $K_2$ at time $t_b$. The meaning of the second probability is similar. The time evolution mixes opposite strangeness states without changing $CP$ (which we assume to be conserved). In the case of $CP=+1$ the interesting probabilities are

$$p_{11}(t|t_0)\equiv p[K_1(t)|K_1(t_0)],\quad p_{21}(t|t_0)\equiv p[K_2(t)|K_1(t_0)],\tag{76}$$

which must satisfy the conditions

$$p_{11}(t_0|t_0)=1,\quad p_{21}(t_0|t_0)=0.\tag{77}$$

These probabilities are not deducible from the results of Sec. VII, which refer only to initial time zero, but are calculable by means of the following equations:

$$\begin{aligned}p_{11}(t+dt|t_0)&=p_{11}(t|t_0)\{1-[T_1(t)+\gamma_S]dt\}+p_{21}(t|t_0)T_2(t)dt,\\ p_{21}(t+dt|t_0)&=p_{21}(t|t_0)\{1-[T_2(t)+\gamma_S]dt\}+p_{11}(t|t_0)T_1(t)dt,\end{aligned}\tag{78}$$

where $\gamma_S$ is the decay rate, $T_1(t)$ is the transition rate at proper time $t$ from $K_1$ to $K_2$, and $T_2(t)$ is the opposite transition rate from $K_2$ to $K_1$. The meaning of the two equations (78) is similar. The first one reads as follows: the probability of the state $K_1$ at proper time $t+dt$ equals the probability of having this same state at proper time $t$ times the probability that nothing happens (neither a transition nor a decay) in the time interval $(t,t+dt)$, added to the probability of having the state $K_2$ at proper time $t$ times the probability of a transition from $K_2$ to $K_1$ in the time interval $(t,t+dt)$.

From Eqs. (78) it is easy to obtain the differential equations

$$\begin{aligned}p'_{11}(t|t_0)&=-[T_1(t)+\gamma_S]p_{11}(t|t_0)+T_2(t)p_{21}(t|t_0),\\ p'_{21}(t|t_0)&=-[T_2(t)+\gamma_S]p_{21}(t|t_0)+T_1(t)p_{11}(t|t_0).\end{aligned}\tag{79}$$

These rate equations are more easily solved by writing

$$p_{11}(t|t_0)=w_{11}(t,t_0)E_S(t-t_0),$$

$$p_{21}(t|t_0)=w_{21}(t,t_0)E_S(t-t_0), \tag{80}$$

where, of course,

$$E_S(t-t_0)\equiv e^{-\gamma_S(t-t_0)}. \tag{81}$$

From Eqs. (79) and (80), we obtain equations free of the decay rates:

$$w'_{11}(t,t_0)=-T_1(t)w_{11}(t,t_0)+T_2(t)w_{21}(t,t_0),$$

$$w'_{21}(t,t_0)=-T_2(t)w_{21}(t,t_0)+T_1(t)w_{11}(t,t_0), \tag{82}$$

and these give

$$w'_{11}(t,t_0)+w'_{21}(t,t_0)=0,$$

$$w'_{11}(t,t_0)-w'_{21}(t,t_0)=-2T_1(t)w_{11}(t,t_0)+2T_2(t)w_{21}(t,t_0), \tag{83}$$

From the first of equations (83), one obtains

$$w_{11}(t,t_0)+w_{21}(t,t_0)=w_{11}(t_0,t_0)+w_{21}(t_0,t_0)$$

$$=p_{11}(t_0|t_0)+p_{21}(t_0|t_0)=1, \tag{84}$$

because of Eqs. (77), (80), and (81). It then becomes useful to express $w_{11}$ and $w_{21}$ in terms of their sum and their difference, the latter defined as

$$w(t,t_0)\equiv w_{11}(t,t_0)-w_{21}(t,t_0), \tag{85}$$

and one obtains

$$w_{11}(t,t_0)=\tfrac{1}{2}[1+w(t,t_0)], \quad w_{21}(t,t_0)=\tfrac{1}{2}[1-w(t,t_0)]. \tag{86}$$

The second of equations (83) then becomes

$$w'(t,t_0)=-w(t,t_0)A(t)+B(t), \tag{87}$$

where

$$A(t)\equiv T_1(t)+T_2(t), \quad B(t)\equiv T_2(t)-T_1(t). \tag{88}$$

In the present situation the following ''initial'' conditions must be satisfied:

$$p_{11}(t_0|t_0)=w_{11}(t_0,t_0)=1 \quad \Rightarrow \quad w(t_0,t_0)=1. \tag{89}$$

The solution of problem (87)–(89) is given next.

## XII. SOLUTION OF THE RATE EQUATIONS

The formal solution of the first-order differential equation (87) is well known to be

$$w(t,t_0)=\left\{w(t_0,t_0)+\int_{t_0}^{t}dt'E^{-1}(t',t_0)B(t')\right\}E(t,t_0), \tag{90}$$

where

$$E(t,t_0)\equiv\exp\left\{-\int_{t_0}^{t}dt'A(t')\right\}. \tag{91}$$

From Eqs. (86) and (89), we obtain

$$w_{11}(t,t_0)=\frac{1}{2}+\frac{1}{2}\left\{1+\int_{t_0}^{t}dt'E^{-1}(t',t_0)B(t')\right\}\times E(t,t_0),$$

$$w_{21}(t,t_0)=\frac{1}{2}-\frac{1}{2}\left\{1+\int_{t_0}^{t}dt'E^{-1}(t',t_0)B(t')\right\}E(t,t_0). \tag{92}$$

Remembering Eq. (80), the probabilities are

$$p_{11}(t|t_0)=\left[\frac{1}{2}+\frac{1}{2}\left\{1+\int_{t_0}^{t}dt'E^{-1}(t',t_0)B(t')\right\}E(t,t_0)\right]\times E_S(t-t_0),$$

$$p_{21}(t|t_0)=\left[\frac{1}{2}-\frac{1}{2}\left\{1+\int_{t_0}^{t}dt'E^{-1}(t',t_0)B(t')\right\}\times E(t,t_0)\right]E_S(t-t_0). \tag{93}$$

This solution is of no direct use because it contains the unknown rates $T_1(t)$ and $T_2(t)$. It will nevertheless be very useful in deducing the incompatibility between local realism and quantum mechanics in the case of correlated kaon pairs.

A symmetrical argument starting from rate equations closely similar to Eq. (79), but containing $\gamma_L$ instead of $\gamma_S$, can be developed for $p_{33}(t|t_0)$ and $p_{43}(t|t_0)$, and leads to the following results:

$$p_{33}(t|t_0)=\left[\frac{1}{2}+\frac{1}{2}\left\{1+\int_{t_0}^{t}dt'\widetilde{E}^{-1}(t',t_0)\widetilde{B}(t')\right\}\widetilde{E}(t,t_0)\right]\times E_L(t-t_0),$$

$$P_{43}(t|t_0)=\left[\frac{1}{2}-\frac{1}{2}\left\{1+\int_{t_0}^{t}dt'\widetilde{E}^{-1}(t',t_0)\widetilde{B}(t')\right\}\widetilde{E}(t,t_0)\right]\times E_L(t-t_0), \tag{94}$$

where

$$\widetilde{E}(t,t_0)\equiv\exp\left\{-\int_{t_0}^{t}dt'\widetilde{A}(t')\right\} \tag{95}$$

and

$$\widetilde{A}(t)\equiv T_3(t)+T_4(t), \quad \widetilde{B}(t)\equiv T_4(t)-T_3(t). \tag{96}$$

In Eq. (96), $T_3(t)$ is the transition rate at proper time $t$ from $K_3$ to $K_4$, and $T_4(t)$ is the opposite transition rate from $K_4$ to $K_3$.

If $E(t,t_0)$ is defined with $t=t_b$ and $t_0=t_a (t_b>t_a)$, from Eq. (90) it follows that

$$w(t_b,t_a)=\left\{1+\int_{t_a}^{t_b}dt'B(t')E^{-1}(t',t_a)\right\}E(t_b,t_a)$$
$$=E(t_b,t_a)+\left\{\int_{t_a}^{t_b}dt'B(t')E^{-1}(t',0)\right\}E(t_b,0). \tag{97}$$

The rate equation can also be solved by referring to the initial time zero. The solution can again be obtained from Eq. (90) by putting $t_0=0$. Considering two different ''final'' times $t=t_b$ and $t=t_a$, one obtains

$$w(t_b,0)=E(t_b,0)+\left\{\int_0^{t_b}dt'B(t')E^{-1}(t',0)\right\}E(t_b,0),$$
$$w(t_a,0)=E(t_a,0)+\left\{\int_0^{t_a}dt'B(t')E^{-1}(t',0)\right\}E(t_a,0), \tag{98}$$

from which

$$w(t_b,0)E^{-1}(t_b,0)=1+\int_0^{t_b}dt'B(t')E^{-1}(t',0),$$
$$w(t_a,0)E^{-1}(t_a,0)=1+\int_0^{t_a}dt'B(t')E^{-1}(t',0).$$

By taking the difference of these two equations, we have

$$\int_{t_a}^{t_b}dt'B(t')E^{-1}(t',0)=w(t_b,0)E^{-1}(t_b,0)-w(t_a,0)E^{-1}(t_a,0),$$

and thus we obtain the most important term of Eq. (97):

$$\left\{\int_{t_a}^{t_b}dt'B(t')E^{-1}(t',0)\right\}E(t_b,0)=w(t_b,0)-w(t_a,0)E(t_b,t_a). \tag{99}$$

It follows by comparison of Eqs. (99) and (97) that

$$w(t_b,t_a)=w(t_b,0)+[1-w(t_a,0)]E(t_b,t_a). \tag{100}$$

By introducing Eq. (100) in Eq. (86), rewritten with $t_0=t_a$ and $t=t_b$, one easily obtains

$$w_{21}(t_b,t_a)=w_{21}(t_b,0)-w_{21}(t_a,0)E(t_b,t_a). \tag{101}$$

From the second of equations (80), it follows that

$$w_{21}(t_b,t_a)=p_{21}(t_b|t_a)E_s^{-1}(t_b-t_a),$$
$$w_{21}(t_b,0)=p_{21}(t_b|0)E_S^{-1}(t_b), \tag{102}$$
$$w_{21}(t_a,0)=p_{21}(t_a|0)E_S^{-1}(t_a).$$

Therefore Eq. (101) can be used to deduce $p_{21}(t_b|t_a)$, which turns out to be

$$p_{21}(t_a|t_a)=E_S^{-1}(t_a)[p_{21}(t_b|0)-p_{21}(t_a|0)\times E_S(t_b-t_a)E(t_b,t_a)]. \tag{103}$$

Exactly the same approach can be used to deduce $p_{43}(t_b|t_a)$, which turns out to be

$$p_{43}(t_b|t_a)=E_L^{-1}(t_a)[p_{43}(t_b|0)-p_{43}(t_a|0)\times E_L(t_b-t_a)\tilde{E}(t_b,t_a)]. \tag{104}$$

It is now easy to reconstruct the interesting probability (71) and obtain

$$p^{\mathrm{LR}}[\overline{K(t_a)};\overline{K(t_b)}]=\frac{E_L(t_a)}{4}[p_{21}(t_b|0)-p_{21}(t_a|0)\times E_S(t_b-t_a)E(t_b,t_a)]+\frac{E_S(t_a)}{4}\times[p_{43}(t_b|0)-p_{43}(t_a|0)\times E_L(t_b-t_a)\tilde{E}(t_b,t_a)]. \tag{105}$$

This result is of fundamental importance because it allows us to use the single kaon theory of Sec. V–VIII, which described kaonic evolution starting from proper time zero. The probabilities $p_{21}(t_b|t_a)$ and $p_{43}(t_b|t_a)$ cannot in general be written as functions of $t_b-t_a$: this type of dependence is equivalent to an action at a distance from the first observed kaon to the second one, reducing the latter to the state of a newborn particle it had at proper time zero. This conclusion can be reached, for example, by considering the standard matrix (47); a numerical calculation shows then that $p_{21}(t_b|t_a)$ and $p_{43}(t_b|t_a)$ depend fully on their two time arguments, and not just on their difference.

## XIII. CONSTRAINTS FROM LOCAL REALISM

The unknown quantities in Eq. (105) are $E(t_b,t_a)$ and $\tilde{E}(t_b,t_a)$. They can be used to deduce upper and lower bounds for the left-hand side of Eq. (105). While a systematic investigation of this problem is left for a future paper, here we will consider a particularly interesting lower bound which turns out to be violated by the quantum-mechanical predictions. Given that both $A(t)$ and $\tilde{A}(t)$ are never negative, as follows from definitions (88) and (96), from Eqs. (91) and (95) one obtains

$$0\leqslant E(t_a,t_b),\quad \tilde{E}(t_a,t_b)\leqslant 1. \tag{106}$$

Observing that in Eq. (105), $E(t_a,t_b)$ and $\tilde{E}(t_a,t_b)$ multiply only negative terms, we can take for them the value 1, and obtain the inequality

$$P^{\mathrm{LR}}[\overline{K(t_a)};\overline{K(t_b)}]\geqslant\frac{E_L(t_a)}{4}[p_{21}(t_b|0)-p_{21}(t_a|0)\times E_S(t_b-t_a)]+\frac{E_S(t_a)}{4}\times[p_{43}(t_b|0)-p_{43}(t_a|0)E_L(t_b-t_a)]. \tag{107}$$

The elementary probabilities entering in Eq. (107) can be found in matrix (56), the result being, for time $t_b$,

$$p_{21}(t_b|0)=E_S(t_b)Q_-(t_b)-\rho(t_b),$$
$$p_{43}(t_b|0)=E_L(t_b)Q_-(t_b)+\rho(t_b). \tag{108}$$

Strictly similar results hold for time $t_a$. Therefore Eq. (107) becomes

$$P^{\mathrm{LR}}[\overline{K(t_a)};\overline{K(t_b)}]\geq\frac{E_L(t_a)}{4}\{[E_S(t_b)Q_-(t_b)-\rho(t_b)] -[E_S(t_a)Q_-(t_a)-\rho(t_a)]E_S(t_b-t_a)\} +\frac{E_S(t_a)}{4}\{[E_L(t_b)Q_-(t_b)+\rho(t_b)] -[E_L(t_a)Q_-(t_a)+\rho(t_a)]E_L(t_b-t_a)\}. \tag{109}$$

There are now two important remarks.

(1) $\rho(t_b)$ enters into Eq. (109) multiplied by $-[E_L(t_a)-E_S(t_a)]/4$, which is never positive because $\gamma_S>\gamma_L$. Therefore one can minimize Eq. (109) by taking the upper limit (59),

$$\rho_{\max}(t_b)=E_S(t_b)Q_-(t_b). \tag{110}$$

(2) $\rho(t_a)$ enters into Eq. (109) multiplied by a factor clearly vanishing for $t_b=2t_a$, the factor being $[E_L(t_a)E_S(t_b-t_a)-E_S(t_a)E_L(t_b-t_a)]/4$. Therefore it is convenient to make $\rho(t_a)$ disappear by considering Eq. (109) only for $t_b=2t_a$.

Thus we can write

$$P^{\mathrm{LR}}[\overline{K(t_a)};\overline{K(2t_a)}]\geq P^{\mathrm{LR}}_{\min}[\overline{K(t_a)};\overline{K(2t_a)}], \tag{111}$$

where

$$P^{\mathrm{LR}}_{\min}[\overline{K(t_a)};\overline{K(2t_a)}]\equiv\frac{E_S(t_a)}{4}\{[E_L(2t_a)+E_S(2t_a)]Q_-(2t_a) -E_L(t_a)[E_L(t_a)+E_S(t_a)]Q_-(t_a)\}. \tag{112}$$

The great advantage of Eq. (112) is that all terms on its right-hand side are known and calculable. A meaningful comparison can then be made with the quantum-mechanical expression (60), which for $t_b=2t_a$ becomes

$$P^{\mathrm{QM}}[\overline{K(t_a)};\overline{K(2t_a)}]=\frac{E_S(t_a)E_L(t_a)}{8}[E_L(t_a)+E_S(t_a) -2\sqrt{E_L(t_a)E_S(t_a)}\cos\Delta m t_a]. \tag{113}$$

A numerical comparison of Eqs. (112) and (113) for $t_a$ ranging between $0.2\gamma_S^{-1}$ and $1.6\gamma_S^{-1}$ is given in Table I.

In this region, quantum mechanics violates the local realistic limit by 28%, on the average. We take this to be a sufficient indication of the physical interest of our results and of the concrete feasibility of an experiment choosing between quantum mechanics and local realism.

TABLE I. Numerical comparison of the quantum-mechanical prediction with the smallest possible probability of all local realistic theories.

| $\gamma_s t_a$ | $P^{\mathrm{LR}}_{\min}[\overline{K(t_a)},\overline{K(2t_a)}]$ | $P^{\mathrm{QM}}[\overline{K(t_a)},\overline{K(2t_a)}]$ |
|---|---|---|
| 0.2 | 0.0044 | 0.0018 |
| 0.4 | 0.0118 | 0.0051 |
| 0.6 | 0.0171 | 0.0087 |
| 0.8 | 0.0195 | 0.0115 |
| 1.0 | 0.0192 | 0.0133 |
| 1.2 | 0.0174 | 0.0142 |
| 1.4 | 0.0148 | 0.0144 |
| 1.6 | 0.0119 | 0.0139 |

A critical discussion of the theoretical studies of the EPR paradox for $K^0\overline{K^0}$ pairs was made by Ghirardi, Grassi, and Weber [15]. Their general conclusion was that a $\Phi$-factory facility is not useful for opening new ways of testing quantum mechanics versus alternative general schemes of the local realistic type. This paper is a good example of how a wrong conclusion can be inferred from a set of generally correct premises. The main argument presented by the three authors is the fact that Bell's inequality, written in terms of four different times of flight of the kaons, is not violated by the quantum-mechanical two-time joint probability for correlated strangeness. This conclusion is correct, but this does not mean that a compatibility exists between local realism and quantum mechanics for kaon pairs: In fact, Bell's inequality is only one of the many consequences of local realism. Concerning Ref. [8], the three authors objected to the generality of the local realistic approach formulated with a left-right and particle-antiparticle symmetry. It is possible to agree with them once more, but not with their general conclusion that meaningful tests of local realism are impossible with a $\Phi$-factory facility. In fact, in the present paper we have shown that the opposite is true. A systematic numerical comparison between the quantum theoretical predictions and the upper and lower predictions of local realistic theories will be given in a forthcoming paper.

## APPENDIX: ELEMENTS OF REALITY FOR STRANGENESS AND *CP*

Local realism allows one to attribute elements of reality to each one of the two kaons belonging to $K^0\overline{K^0}$ pairs described by state vector (4), if this description is assumed correct at least in the predicted (nonparadoxical) correlations in $CP$ and in $S$. The reasoning starts from the following assumptions.

(1) If, without in any way disturbing a kaon, we can predict with certainty the value of one of its physical quantities, then there exists an element of physical reality corresponding to this physical quantity (the EPR *reality criterion*).

(2) If two kaons are separated by a large distance, an element of reality belonging to one of them cannot be created by a measurement performed on the other one (*locality*).

(3) If at a given time $t$ a kaon has an element of reality, the latter cannot be created by measurements on the same

kaon performed at time $t'$, if $t'>t$ (*no retroactive causality*).

If one measures $CP$ on the $a$ (left) kaon of a pair described by the state vector (4), one finds either $+1(K_S)$ or $-1(K_L)$ with the same frequency. This allows one to predict with certainty that a future $CP$ measurement on the $b$ (right) kaon will give the result $-1(K_L)$ or $+1(K_S)$, respectively. Using the EPR reality criterion one can attribute an element of reality $\lambda_1$ to the right-going kaon. Without loss of generality one can assume $\lambda_1$ to be dichotomic, with $\lambda_1=\pm 1$ corresponding to $CP=\pm 1$, respectively.

The element of reality $\lambda_1$, because of locality, is not created by the measurement made on the other kaon. Therefore it also exists (even if unknown in the individual case) if no such a measurement is made on the other kaon.

The element of reality $\lambda_1$, because of the assumed lack of retroactive causality, is not created by a future measurement on the same kaon to which it belongs. Therefore it exists also if no such a measurement is performed.

The situation is fully symmetrical between left and right. Therefore, the previous reasoning allows one to associate a $\lambda_1$ element of reality both with the $a$ (left) kaon and with the $b$ (right) kaon.

Conclusion: *each kaon of every pair has an associated element of reality* $\lambda_1$, *which determines a well-defined CP value* ($\lambda_1=\pm 1$ corresponds to $CP=\pm 1$, respectively). For both left- and right-going kaons the two cases $\lambda_1=+1$ and $\lambda_1=-1$ appear at random and with the same (50%) frequency.

A second element of reality, connected with strangeness $S$, can be introduced. The reasoning is now as follows: If one measures $S$ on the $a$ (left) kaon of a pair described by Eq. (4), one finds either $+1(K^0)$ or $-1(\bar{K}^0)$ with the same frequency. This allows one to predict with certainty that an $S$ measurement *at equal proper time* on the $b$ (right) kaon must give the result $-1(\bar{K}^0)$ or $+1(K^0)$, respectively. Using the EPR reality criterion one can attribute an element of reality $\lambda_2$ to the right kaon. Without loss of generality one can assume $\lambda_2$ to be dichotomic, with $\lambda_2=\pm 1$ corresponding to $S=\pm 1$, respectively.

The element of reality $\lambda_2$ cannot be created by the measurement made on the other kaon (locality). Therefore it also exists, at least instantaneously, if no measurement is made on the other kaon.

The element of reality $\lambda_2$ cannot be created by the measurement on the same kaon to which it belongs (the instrument works properly and could lead to a result different from the predicted one). Thus it exists also if no such a measurement is performed.

Conclusion: *each kaon of every pair has an associated element of reality* $\lambda_2$, *which determines a well-defined value of strangeness* $S$ ($\lambda_2=\pm 1$ corresponds to $S=\pm 1$, respectively). For both left- and right-going kaons the two cases $\lambda_2=+1$ and $\lambda_2=-1$ appear at random and with the same (50%) frequency.

The first element of reality $\lambda_1$ describes the objective property of a kaon to behave either as a $CP=+1$ $K_S$ or as a $CP=-1$ $K_L$. The second element of reality $\lambda_2$ describes the objective property of a kaon to behave either as a $S=+1$ $K^0$ or as a $S=-1$ $\bar{K}^0$. The important difference between $\lambda_1$ and $\lambda_2$ is that, while $\lambda_1$ describes a time-independent property, $\lambda_2$ describes an instantaneous property: if a kaon of a pair is known at time $t_0$ to have $S=+1$ one can expect that, at a later time, it could have acquired $S=-1$ because the well-known oscillations of $S$ modify the $S=\pm 1$ populations. Since at every instant the value of $\lambda_2$ is well defined, at every instant the kaon has either the $S=+1$ or the $S=-1$. Sudden jumps between the two situations are, therefore, possible, but they must be simultaneous for the two kaons in order to preserve the perfect anti correlation in $S$ that must hold at all equal proper times, as predicted by the state vector (4). This simultaneity can be compatible with locality, e.g., if the time of the jumps is predetermined when the kaons are produced in the decay of the $\Phi$ meson.

Quantum mechanics makes also the nonparadoxical prediction that a $|K_S\rangle$ can be observed with equal probability to be a $|K^0\rangle$ or a $|\bar{K}^0\rangle$, and that the same holds for a $|K_L\rangle$ [see Eqs. (8) and (9)]. Correspondingly, in our theory a kaon with $\lambda_1$ given is assumed to have with equal probability $\lambda_2=+1$ and $\lambda_2=-1$. We see thus that the four possible kaon types with $\lambda_1=\pm 1$ and $\lambda_2=\pm 1$ will appear initially with the same frequency 25% when Eq. (4) applies.

One of the virtues of the previous argument is that it shows the quantum-mechanical correlations in $CP$ and in $S$ predicted by the state vector (4) to be nonparadoxical, precisely because they are exactly reproduced by the local realistic approach. Other predictions of Eq. (4) are instead ''paradoxical,'' that is incompatible with local realism, as shown in the final part of this paper.

Notice that local realism implies that the two observables $S$ and $CP$, described quantum mechanically by noncommuting operators, must be simultaneously predetermined by elements of reality belonging to any given kaon. This is of course the standard treatment of ''incompatible'' observables in all ''hidden variable'' theories. The necessary codefinition of $S$ and $CP$ can be rigorously justified by applying local realism to a kaon belonging to a correlated kaon pair, as shown above. Nevertheless, it is very natural to believe that all kaons have the same basic properties and to extend the validity of Eq. (10) to single kaons even when they do not belong to an EPR pair.

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