ARTICLES

Quantum-mechanical Maxwell's demon

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A Maxwell's demon is a device that gets information and trades it in for thermodynamic advantage, in apparent (but not actual) contradiction to the second law of thermodynamics. Quantum-mechanical versions of Maxwell's demon exhibit features that classical versions do not: in particular, a device that gets information about a quantum system disturbs it in the process. This paper proposes experimentally realizable models of quantum Maxwell's demons, explicates their thermodynamics, and shows how the information produced by quantum measurement and by decoherence acts as a source of thermodynamic inefficiency. [S1050-2947(97)02910-7]

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INTRODUCTION

In 1871, Maxwell noted that a being that could measure the velocities of individual molecules in a gas could shunt fast molecules into one container and slow molecules into another, thereby creating a difference in temperature between the two containers, in apparent violation of the second law of thermodynamics [1]. Kelvin called this being a ''demon:'' by getting information and being clever how it uses it, such a demon can in principle perform useful work. Maxwell's demon has been the subject of considerable discussion over the past century [2]. The contemporary view of the demon, spelled out in the past decade [3], is that a demon could indeed perform useful work k_BT ln2 for each bit obtained, but must increase entropy by at least $k_B$ln2 for each bit erased (a result known as ''Landauer's principle'' [4]). As a result, a demon that operates in cyclic fashion, erasing bits after it exploits them, cannot violate the second law of thermodynamics.

Up to now, Maxwell's demon has functioned primarily as a thought experiment that allows the exploration of theoretical issues. This paper, in contrast, proposes a model of a Maxwell's demon that could be realized experimentally using magnetic or optical resonance techniques. Any experimentally realizable model of a ''demon,'' like the molecules of Maxwell's original example, must be intrinsically quantum mechanical. The classic reference on quantum demons is Zurek's treatment of the quantum Szilard engine [5] (see also Refs. [6,7]). Zurek investigated a gedanken experiment consisting of a single quantum particle sitting in a classical cylinder, acted upon by a classical piston, and measured by a classical measuring device. The quantum nature of the particle allowed Zurek to give an accurate thermodynamic accounting of the heat taken in and work done by the particle as the piston expanded. Zurek's picture is semiclassical in that only the particle itself is taken to be quantum mechanical, while the remainder of the engine is classical; the semiclassical nature of this treatment does not allow Landauer's principle and the thermodynamics of the erasure process to be investigated in a quantum context.

This paper makes the following advances in treating Maxwell's demons.

(1) It uses well-established physics of the quantum electrodynamics of spin and optical resonance to present a fully quantum-mechanical model of a Maxwell's demon that unlike previous semiclassical models allows the thermodynamics of the demon's entire cycle of operation—heat absorption, measurement and decoherence, work generation, and erasure—to be treated within a unified quantum framework.

(2) Unlike other models of demons, the model presented here provides a detailed mechanism for erasing information and so allows Landauer's principle to be investigated in a realistic quantum context. In the proposed device, ''waste'' information is rejected by putting the spin that makes up the demon's quantum memory in contact with a low temperature mode of the electromagnetic field. Instead of violating the second law of thermodynamics, the demon operates as a quantum heat engine, doing work while pumping heat from hot modes to cold modes.

(3) The paper demonstrates the truth of Haus's conjecture [8] that processes such as quantum measurement and decoherence, by introducing information, effectively increase entropy and reduce the thermodynamic efficiency of the demon. As a result, quantum demons and quantum heat engines in general suffer from peculiarly quantum sources of inefficiency: each bit of information introduced by measurement or decoherence increases entropy by $k_B$ln2.

(4) Finally, the paper proposes experiments for realizing quantum demons using nuclear magnetic resonance on molecules.

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The outline of the paper is as follows. Section I explains how a nuclear or electronic spin can function as the ''working fluid'' for a quantum demon, and shows how Landauer's principle prevents the demon from violating the second law of thermodynamics. Section II shows how quantum measurement and decoherence make the demon less thermodynamically efficient and provides formulas that quantify the resulting reduction in efficiency: not only does the operation of the quantum demon fail to violate the second law, it may actually increase entropy beyond what is required classically. In Sec. III a nuclear magnetic resonance (NMR) model of a demon is presented. The model can be realized by a variety of NMR techniques on a variety of molecules. Section IV provides a detailed analysis of the thermodynamics and quantum statistical mechanics of the NMR model, and goes through the cyclic operation of the demon step by step, identifying the thermodynamic costs of information gathering and erasure. The mechanisms by which information is obtained, exploited, and erased are explored in detail. Landauer's principle is confirmed: such a device cannot violate the second law of thermodynamics, but if supplied with heat reservoirs at different temperatures it can undergo a cycle analogous to the Carnot cycle and function as a heat engine. Section V analyzes the effects of decoherence on the NMR demon and shows that each bit of information introduced by quantum measurement and decoherence functions as an extra $k_B \ln 2$ of entropy, decreasing the engine's efficiency. A quantum-mechanical engine that processes information is limited not by the Carnot efficiency, but by the potentially lower quantum efficiency ϵ_Q defined in Sec. II. Section VI discusses a variety of potential experimental realizations in addition to NMR. Finally, Sec. VII concludes and discusses future work.

I. A MAGNETIC RESONANCE MODEL OF A QUANTUM ''DEMON''

To see how a device that gets information about a quantum system can use that information to perform useful work, consider a spin in a magnetic field. If the spin points in the same direction as the field it has energy $-\mu B$, where μ is the spin's magnetic dipole moment, and B is the field strength. If it points in the opposite direction it has energy $+\mu B$. The state of the spin can be controlled by conventional magnetic resonance techniques: for example, the spin can be flipped from one energy state to the other by applying a π pulse at the spin's Larmor precession frequency $\omega = 2\mu B/\hbar$ [9,10].

When the spin flips, it exchanges energy with the oscillatory field. If the spin flips from the lower energy state to the higher energy state it coherently absorbs one photon with energy $\hbar\omega$ from the field; if it flips from the higher energy state to the lower, it coherently emits one photon with energy $\hbar\omega$ to the field. Either the field does work on the spin, or the spin does work on the field.

When the quantum nature of the electromagnetic field is taken into account, one might worry that the interaction that exchanges energy between spin and field also induces an exchange of information that effectively entangles the quantum state of the spin with the state of the field. A detailed quantum electrodynamic treatment of this interaction [11] shows that this worry is indeed justified when the field is in a nonclassical state such as a highly squeezed state. When the field is in a coherent state, however, as is the case for fields normally produced by lasers, masers, or rf coils, the energy exchange involves no information exchange, entropy increase, or loss of quantum coherence [11]. As a result, even though the oscillatory field that flips the spin is in fact quantum mechanical, it may be treated as if it were classical for the purpose of the experiments described below.

It is clear how a device that acquires information about such a spin could use the information to make the spin do work. Suppose that some device can measure whether the spin is in the low-energy quantum state $|\downarrow\rangle$ that points in the same direction as the field, or in the high-energy quantum state $|\uparrow\rangle$ that points in the opposite direction to the field, and if the spin is in the high-energy state, send in a π pulse to extract its energy. The device can then wait for the spin to come to equilibrium at temperature $T_1 \gg 2\mu B/k_B$ and repeat the operation. Each time it does so, it converts an average of μB of heat into work. The device gets information and uses that information to convert heat into work. The amount of work done by such a device operating on a single spin is negligible; but many such devices operating in parallel could function as a ''demon'' maser, coherently amplifying the pulse that flips the spins.

Landauer's principle prevents such a device from violating the second law of thermodynamics. To operate in a cyclic fashion, the device must erase the information that it has gained about the state of the spin. When this information is erased, entropy $S_{\mathrm{out}} \geq k_B \ln 2$ is pumped into the device's environment, compensating for the entropy $S_{\mathrm{in}} \approx k_B \ln 2$ of entropy in the spin originally. If the environment is a heat bath at temperature T_2, which may be different from the temperature T_1 of the spins' heat bath, heat $k_B T_2 \ln 2$ flows to the bath along with the entropy, decreasing the energy available to convert into work. The detailed mechanism by which the ''waste'' information is converted to heat is described in Sec. IV below.

The overall accounting of energy and entropy in the course of the cycle is as follows: heat in $Q_{\mathrm{in}} = T_1 S_{\mathrm{in}}$, heat out $Q_{\mathrm{out}} = T_2 S_{\mathrm{out}}$, work out $W_{\mathrm{out}} = Q_{\mathrm{in}} - Q_{\mathrm{out}}$, efficiency

$$\epsilon = W_{\mathrm{out}}/Q_{\mathrm{in}} = 1 - T_2 S_{\mathrm{out}}/T_1 S_{\mathrm{in}} \leq 1 - T_2/T_1 \equiv \epsilon_C, \tag{1.1}$$

where ϵ_C is the Carnot efficiency. Since $S_{\mathrm{out}} \geq S_{\mathrm{in}}$, W_{out} can be greater than zero only if $T_1 > T_2$. That is, Landauer's principle implies that instead of violating the second law of thermodynamics, the device operates as a heat engine, pumping heat from a high-temperature reservoir to a low-temperature reservoir and doing work in the process.

II. MEASUREMENT AND DECOHERENCE MAKE THE DEMON LESS EFFICIENT

Why quantum measurement introduces added inefficiency into the operation of such a device can be readily understood. Suppose that the spin is originally in the state $|\rightarrow\rangle = 1/\sqrt{2}(|\uparrow\rangle + |\downarrow\rangle)$. One way to extract energy from such a spin is to apply a $\pi/2$ pulse to rotate the spin to the state $|\downarrow\rangle$, extracting work μB in the process. A second way to extract energy is to repeat the process described above: measure the spin to see if it is in the state $|\uparrow\rangle$, and if it is, apply a π pulse to extract work $2\mu B$. Half the time the measurement will find the spin in the state $|\uparrow\rangle$ and half the time in the state $|\downarrow\rangle$; as a result, this process also generates work μB on average, but in addition generates a ''waste'' bit of information that costs energy $k_B T_2 \ln 2$ to erase. Quantum measurement introduces added inefficiency to the process of getting information about a quantum system and exploiting that information to perform work.

More generally, suppose the spin is initially described by a density matrix ρ. Let the spin interact with a measuring device or other system that decoheres the spin by destroying off-diagonal terms in the density matrix. The new density matrix is then

$$\rho' = \sum_i P_i \rho P_i , \tag{2.1}$$

where P_i are projection operators onto the eigenspaces of the operator corresponding to the measurement or onto the preferred subspaces of the decohering process [12]. The extra information generated by quantum measurement or decoherence is

$$\Delta I_Q = \Delta S_Q / k_B \ln 2 = -\mathrm{tr}\rho' \log_2 \rho' - (-\mathrm{tr}\rho \log_2 \rho) \geqslant 0. \tag{2.2}$$

The efficiency of the device in converting heat to work is limited not by the Carnot efficiency ϵ_C but by the quantum efficiency

$$\epsilon_Q = 1 - T_2(S_{\mathrm{in}} + \Delta S_Q)/T_1 S_{\mathrm{in}} = \epsilon_C - T_2 \Delta S_Q / T_1 S_{\mathrm{in}} . \tag{2.3}$$

Equation (2.3) quantifies the inefficiency due to any process, such as decoherence, that destroys off-diagonal terms of the density matrix ρ [12–14].

The degree to which the quantum efficiency differs from the Carnot efficiency depends on the amount of extra entropy ΔS_Q or information ΔI_Q introduced by measurement or decoherence. If ρ is already diagonal with respect to the projections P_i or diagonal in the preferred basis of the decohering process then no new information is introduced and $\epsilon_Q = \epsilon_C$. At the other extreme, the measurement or decohering process can completely randomize the system, giving

$$\rho' = I/2, \quad S_{\mathrm{in}} + \Delta S_Q = k_B \ln 2, \tag{2.4}$$

$$\epsilon_Q = 1 - T_2 k_B \ln 2 / T_1 S_{\mathrm{in}} . \tag{2.5}$$

Note that ϵ_Q can be negative, corresponding either to a process that generates heat but does no net work, or (as discussed in Secs. IV–VI below) to a refrigerator that pumps heat from a cold reservoir to a hot reservoir.

Equation (2.3) applies not only to ''demon''-like heat engines that operate by obtaining information about a quantum systems, thereby decohering them, it also applies to more conventional quantum heat engines such as lasers that undergo decoherence through interactions with their environment. For example, the reduction in quantum efficiency described by Eq. (2.3) could be observed by taking a laser in which a population inversion has been established between two levels, and ''tipping'' that inversion by an angle θ by applying a pulse at the resonant frequency between the levels with integrated intensity $\hbar \theta / 2D$, where D is the dipole moment between the levels. The amount by which the population of the higher level exceeds that of the lower level is proportional to $\cos\theta$, so that when the inversion has been tipped by $\pi/2$, there is no longer an effective inversion between the levels and further light will not be amplified. The maser analog of this effect is discussed in greater detail in Sec. V below.

III. A QUANTUM DEMON CAN BE REALIZED USING NUCLEAR MAGNETIC RESONANCE

To investigate more thoroughly how quantum measurement and decoherence introduce thermodynamic inefficiency in a quantum information-processing ''demon'' requires a more detailed model of how such a device gets and gets rid of information. One of the simplest quantum systems that can function as a measuring device is another spin. Magnetic resonance affords a variety of techniques, called spin-coherence double resonance, whereby one spin can coherently acquire information about another spin with which it interacts [9,10]. The basic idea is to apply a sequence of pulses that makes spin 2 flip if and only if spin 1 is in the excited state $|\uparrow\rangle_1$, while leaving spin 1 unchanged. If spin 2 is originally in the ground state $|\downarrow\rangle_2$, then after the conditional spin-flipping operation, the two spins will either be in the state $|\uparrow\rangle_1|\uparrow\rangle_2$ if spin 1 was originally in the state $|\uparrow\rangle_1$, or in the state $|\downarrow\rangle_1|\downarrow\rangle_2$ if spin 1 was originally in the state $|\downarrow\rangle_1$. Spin 2 has acquired information about spin 1. A variety of spin coherence double resonance techniques (going under acronyms such as INEPT and INADEQUATE) can be used to perform this conditional flipping operation [9,10], which can be thought of as an experimentally realizable version of Zurek's treatment of the measurement process in Ref. [5]. Readers familiar with quantum computation will recognize the conditional spin flip as the quantum logic operation ''controlled-NOT'' [15–17].

How can this information be used to extract energy from spin 1? Simply apply a second pulse sequence to flip spin 1 if and only if spin 2 is in the state $|\uparrow\rangle_2$, while leaving spin 2 unchanged. The energy transfer from spins to field is as follows. If spin 1 was originally in the state $|\downarrow\rangle_1$, then spin 1 and spin 2 remain in the state $|\downarrow\rangle$ through both pulse sequences and no energy is transferred to the field. If spin 1 was originally in the state $|\uparrow\rangle$, first spin 2 flips, then spin 1, yielding a transfer of energy from spins to field of $\hbar(\omega_1 - \omega_2) = 2(\mu_1 - \mu_2)B$, which is >0 as long as $\mu_1 > \mu_2$. The average energy extracted is half this value. As long as the conditional spin flips are performed coherently, the amount of energy extracted depends only on overall conservation of energy, and is independent of the particular double resonance technique used. Note that the entire process maintains quantum coherence and can be reversed simply by repeating the conditional spin flips in reverse order.

The steps just described have been realized experimentally in a variety of systems. One of the first realizations of such a double resonance experiment is the Pound-Overhauser effect [9] in which spin 1 is a proton and spin 2 is an electron. Here the amount of energy required to flip the electron is three orders of magnitude higher than the amount of energy gained by flipping the proton: instead of operating as an engine the double resonance process functions as a refrigerator, pumping heat from the proton to the electron.

IV. THERMODYNAMICS OF THE NMR DEMON

In the previous section it was shown how spins can get information about each other and put that information to use to do work. The spins are fulfilling their role as demon by extracting work without generating waste heat. They are not yet violating the second law of thermodynamics, however, which states that it is not possible for an engine to turn heat into work with no waste heat *while operating in a cyclic fashion*. To complete the cycle, the spins must be restored to their original state without generating waste heat. As will now be seen, this cannot happen.

To give a full treatment of the thermodynamics of this device and to understand the role of decoherence and quantum measurement in its functioning, we must investigate how the ''demon'' interacts with its thermal environment to take in heat and erase information. This section will show that a quantum device that interacts with a thermal environment can indeed get information and ''cash it in'' to do useful work, but not by violating the second law of thermodynamics: a detailed model of the erasure process confirms Landauer's principle (one bit of information ''costs'' entropy $k_B \ln 2$). As a result, instead of functioning as a perpetual motion machine, the device operates as a heat engine that undergoes a cycle analogous to a Carnot cycle.

The environment for our spins will be taken to consist of two sets of modes of the electromagnetic field, the first a set of modes at temperature T_1 with average frequency ω_1 and with frequency spread greater than the coupling constant $|\kappa|$ between the spins but less than $\omega_1 - \omega_2$, and the second a set of modes at temperature T_2 with average frequency ω_1 and the same frequency spread. Such an environment can be obtained, for example, by bathing the spins in incoherent radiation with the given frequencies and temperatures. The purpose of such an environment is to provide effectively separate heat reservoirs for spin 1 and spin 2: spin 1 interacts strongly with the on-resonance radiation at frequency ω_1, and weakly with the off-resonance radiation at frequency ω_2 and vice versa for spin 2. Over short times, to a good approximation spin 1 can be regarded as interacting only with mode 1, and spin 2 as interacting only with mode 2. A spin can be put in and out of ''contact'' with its reservoir by isentropically altering the frequency of the reservoir mode to put the spin in and out of resonance.

With this approximation, the initial probabilities for the state of the jth spin are (ignoring for the moment the coupling between the spins)

$$p_j(\uparrow) = e^{-\mu_j B/k_B T_j}/Z_j, \quad p_j(\downarrow) = e^{\mu_j B/k_B T_j}/Z_j, \qquad (4.1)$$

yielding energy

$$E_j = -\mu_j B \tanh(\mu_j B/k_B T_j), \qquad (4.2)$$

and entropy

$$S_j = -k_B \sum_{i=\uparrow,\downarrow} p_j(i) \ln p_j(i) = E_j/T_j + k_B \ln Z_j, \qquad (4.3)$$

where $Z_j = e^{-\mu_j B/k_B T_j} + e^{\mu_j B/k_B T_j} = 2\cosh(\mu_j B/k_B T_j)$. Even though it does not start out in a definite state, spin 2 can still acquire information about spin 1, and this information can be exploited to do electromagnetic work. The spins can function as a heat engine by going through the following cycle.

(1) Using spin coherence double resonance, flip spin 2 if spin 1 is in the state $|\uparrow\rangle_1$. This causes spin 2 to gain information $(\tilde{S}_2 - S_2)/k_B \ln 2$ about spin 1 at the expense of work $W_1 = p_1(\uparrow) 2\mu_2 B \tanh(\mu_2 B/k_B T_2)$ supplied by the oscillating field. Here $\tilde{S}_2 = -k_B \Sigma_{i=\uparrow,\downarrow} \tilde{p}_2(i) \ln \tilde{p}_2(i)$, where $\tilde{p}_2(\uparrow) = p_1(\uparrow) p_2(\downarrow) + p_1(\downarrow) p_2(\uparrow)$ and $\tilde{p}_2(\downarrow) = p_1(\downarrow) p_2(\downarrow) + p_1(\uparrow) p_2(\uparrow)$ are the probabilities for the states of spin 2 after the conditional spin flip.

(2) Flip spin 1 if spin 2 is in the state $|\uparrow\rangle_1$. This step allows spin 2 to ''cash in'' $(S_2 - S_1)/k_B \ln 2$ of the information it has acquired, thereby performing work $-\mu_1 B[\tanh(\mu_1 B/k_B T_1) - \tanh(\mu_2 B/k_B T_2)]$ on the field.

(3) Spin 2 still possesses information $(\tilde{S}_2 - S_1)/k_B \ln 2$ about spin 1, which can be converted into work by flipping spin 2 if spin 1 is in the state $|\uparrow\rangle_1$, thereby performing work $p_2(\uparrow) 2\mu_2 B \tanh(\mu_1 B/k_B T_2)$ on the field.

It is straightforward to verify that after these three conditional spin flips, spin 1 has probabilities $p_1'(i) = p_2(i)$ while spin 2 has probabilities $p_2'(i) = p_1(i)$. That is, the sequence of pulses has ''swapped'' the information in spin 1 with the information in spin 2. As a result, $S_1' = S_2$, $S_2' = S_1$, and the new energies of the spins are $E_1' = -\mu_1 B \tanh(\mu_2 B/k_B T_2)$ and $E_2' = -\mu_2 B \tanh(\mu_1 B/k_B T_1)$. The total amount of work done by the spins on the electromagnetic field is

$$\begin{aligned} W = -(E_1' + E_2' - E_1 - E_2) = -(\mu_1 - \mu_2) B [\tanh(\mu_1 B/k_B T_1) \\ - \tanh(\mu_2 B/k_B T_2)]. \end{aligned} \qquad (4.4)$$

At temperatures $T_i \gg \mu_i B/k_B$, Eq. (4.4) reduces to

$$W = -(\mu_1 - \mu_2)(\mu_1/T_1 - \mu_2/T_2) B^2/k_B. \qquad (4.5)$$

These formulas for the work done depend only on conservation of energy and do not depend on the specific set of coherent pulses that are used to ''swap'' the spins. Equations (4.4) and (4.5) shows that $W > 0$ if either $\mu_1 > \mu_2$, $\mu_1/T_1 < \mu_2/T_2$ or $\mu_1 < \mu_2$, $\mu_1/T_1 > \mu_2/T_2$. If $T_1 = T_2$, W is zero or negative: no work can be extracted from the spins at equilibrium. The device cannot function as a *perpetuum mobile* of the second kind. The cycle can be completed by letting the spins reequilibrate with their respective reservoirs. Each time steps (1)–(3) are repeated, heat $Q_{\text{in}} = E_1 - E_1'$ flows from reservoir 1 to spin 1 and heat $Q_{\text{out}} = E_2' - E_2$ flows from spin 2 into reservoir 2. The efficiency of this cycle is $W/Q_{\text{in}} = 1 - \mu_2/\mu_1 < 1 - T_2/T_1 = \epsilon_C$: the efficiency is less than the Carnot efficiency because when the spins equilibrate with their respective reservoirs, heat flows but no work is done.

The following steps can be added to the cycle to allow the spins to reequilibrate isentropically.

(5) Return spin 1 to its original state: (i) Take the spin out of ''contact'' with its reservoir by varying the frequency of the reservoir modes as above. (ii) Alter the quasistatic field from $B \rightarrow B_1 = BT_1/T_2$ adiabatically, with no heat flowing between spin and reservoir. (iii) Gradually change the

field from $B_1 \to B$ keeping the spin in ''contact'' with the reservoir at temperature T_1 so that heat flows isentropically between the spin and the reservoir. During this process, entropy $S_2 - S_1$ flows from the spin to the reservoir, while the spin does work $E_1 - E_1' - T_1(S_2 - S_1)$ on the field.

(6) Return spin 2 to its original state by the analogous set of steps.

The total work done by the spins on the electromagnetic field throughout the cycle is

$$W_C = (T_1 - T_2)(S_1 - S_2). \tag{4.6}$$

With the added steps (5) and (6) the demon undergoes a cycle analogous to the Carnot cycle and in principle operates at the Carnot efficiency $1 - T_2/T_1$. In practice, of course, the steps that go into operation of such an engine will be neither adiabatic nor isentropic, leading to an actual efficiency below the Carnot efficiency.

To attain the Carnot efficiency, it is important the demon ''know'' the temperatures T_1 and T_2. If the demon has incorrect values for these temperatures then after step [(5)(ii)] above spin 1 will be out of equilibrium with its reservoir and heat will flow with no work being done. An ordinary, macroscopic Carnot engine has a completely analogous source of inefficiency: altering the quasistatic field adiabatically for the spin is analogous to the adiabatic expansion-contraction stages for a Carnot engine. If the adiabatic expansion of the working fluid in a Carnot engine goes too far or not far enough, then the fluid will be at a temperature different from that of the low temperature reservoir, and when the fluid is put in contact with that reservoir the heat flow will not be isentropic but will increase entropy instead.

Equation (4.4) implies that when $\mu_1 > \mu_2$ and $\mu_1/T_1 > \mu_2/T_2$ the net work performed over the cycle is negative. In this case, the demon functions as a refrigerator, pumping net heat and entropy from the reservoir at temperature T_2 to the reservoir at temperature T_1 (note that T_1 need no longer be greater than T_2: indeed, T_1 and T_2 could now be equal). The efficiency of the refrigerator can be measured by the ratio between heat pumped and work done; for the idealized model given above this is equal to $T_2/(T_1 - T_2)$, which is just the usual Carnot coefficient of refrigerator performance. Naturally, any experimental realization of such a refrigerator will operate at an efficiency lower than the Carnot coefficient.

Such ''demon refrigerators'' have in fact been in operation since the 1950s. In the Pound-Overhauser effect [9], double resonance is used to ''swap'' information between an electron and a nucleon, thereby pumping entropy and energy from the nucleon to the electron. The Pound-Overhauser effect can be implemented by performing steps (1)–(3) above on an electron (spin 1) and a nucleon (spin 2). Swapping information between electron and nucleon interchanges their Boltzmann factors, reducing the effective temperature of the nucleon by a factor g_e/g_n where g_e, g_n are the gyromagnetic ratios of the electron and nucleon, respectively. In the Pound-Overhauser effect, entropy $S_2 - S_1$ is pumped from nucleon to electron at a cost in microscopic work given by Eq. (4.4). This entropy transfer is highly efficient: the work put into the transfer can be reextracted by swapping the information between the spins again. The only inefficiency arises from inaccuracies in the conditional spin flips. (At the macroscopic scale, of course, dissipation in the coils used to supply the pulses to flip the spins dwarfs the entropy transfers between spins.) Even at the microscopic level, when operated cyclically the Pound-Overhauser effect does not approach the Carnot coefficient of refrigerator performance, as the transfers of heat from lattice to nuclei and from electrons to electromagnetic field are not isentropic. Further examples of ''demon'' refrigerators will be discussed in Sec. VI.

V. THERMODYNAMIC COST OF QUANTUM MEASUREMENT AND DECOHERENCE

So far, although the demon has functioned within the laws of quantum mechanics, quantum measurement and quantum information have not entered in any fundamental way. Now that the thermodynamics of the demon have been elucidated, however, it is possible to quantify precisely the effects both of measurement and of the introduction of quantum information on the demon's thermodynamic efficiency. The simple quantum information-processing engine of the previous section can in principle be operated at the Carnot efficiency: as will now be shown, when the engine introduces information by a process of measurement or decoherence, it cannot be operated even in principle (let alone in practice) above the lower, quantum efficiency ϵ_Q of Eq. (2.3).

To isolate the effects of quantum information, let us first look more closely at the simple model of Sec. I above, in which spin 1 is initially in the state

$$|\to\rangle_1 = 1/\sqrt{2}(|\uparrow\rangle_1 + |\downarrow\rangle_1). \tag{5.1}$$

This state has nonminimum free energy which is available for immediate conversion into work: simply apply a $\pi/2$ pulse to rotate spin 1 into the state $|\downarrow\rangle_1$, adding energy $\hbar\omega_2/2 = \mu_1 B$ to the oscillating field in the process. Suppose, however, that instead of extracting this energy directly, the demon operates in information-gathering mode as above, using magnetic resonance techniques to correlate the state of spin 2 with the state of spin 1 (cf. Ref. [5]). Suppose spin 2 is initially in the state $|\downarrow\rangle_2$. In this case, coherently flipping spin 2 if spin 1 is in the state $|\uparrow\rangle_1$ results in the state

$$1/\sqrt{2}(|\uparrow\rangle_1|\uparrow\rangle_2 + |\downarrow\rangle_1|\downarrow\rangle_2), \tag{5.2}$$

a quantum ''entangled'' state in which the state of spin 2 is perfectly correlated with the state of spin 1. Continuing the energy extraction process by flipping spin 1 if spin 2 is in the state $|\uparrow\rangle_2$ as before allows on average an amount of energy $(\mu_1 - \mu_2)B$ to be extracted from the spin. The resulting state of the spins is $1/\sqrt{2}|\downarrow\rangle_1(|\uparrow\rangle_2 + |\downarrow\rangle_2) = 1/\sqrt{2}|\downarrow\rangle_1|\to\rangle_2$. Up until this point, no extra thermodynamic cost has been incurred. Indeed, since the conditional spin flipping occurs coherently, the process can be reversed by repeating the steps in reverse order to return to the original state $|\to\rangle_1$, with a net energy and entropy change of zero.

When is the cost of quantum measurement realized? When decoherence occurs. In the original cycle, decoherence takes place when spin 2 is put in contact with the reservoir to ''erase'' it [13–14]: the exchange of energy between the spin and the reservoir is an incoherent process during which the pure state

$$|\rightarrow\rangle_2 = 1/\sqrt{2}(|\uparrow\rangle_2 + |\downarrow\rangle_2) \quad (5.3)$$

goes to the mixed state described by the density matrix

$$(1/2)(|\uparrow\rangle_2\langle\uparrow| + |\downarrow\rangle_2\langle\downarrow|). \quad (5.4)$$

The time scale for this process of decoherence is equal to spin 2's dephasing time T_2^* and is typically much faster than the time scale for the transfer of energy [14]. In effect, interaction with the reservoir turns the process by which spin 2 coherently acquires quantum information about spin 1 into a decoherent process of measurement, during which a bit of information is created. This bit corresponds to an increase in entropy $k_B\ln 2$. During the process of erasure as in Sec. IV above, this entropy is transferred from spin 2 to the low-temperature reservoir, in accordance with Landauer's principle.

By decohering spin 2 and effectively measuring spin 1, the demon has increased entropy and introduced thermodynamic inefficiency. The amount of inefficiency can be quantified precisely by going to the Carnot cycle model of the demon above. In general, the state of spin 1 is described initially by a density matrix

$$\rho_1' = p_1(\uparrow')|\uparrow'\rangle_1\langle\uparrow'| + p_1(\downarrow')|\downarrow'\rangle_1\langle\downarrow'|, \quad (5.5)$$

where $|\uparrow'\rangle$ and $|\downarrow'\rangle$ are spin states along an axis at some angle θ from the z axis. Without loss of generality, T_1 and B can be taken to be such that

$$p_1(\uparrow') = e^{-\mu_1 B/k_B T_1}/Z_1, \quad p_1(\downarrow') = e^{\mu_1 B/k_B T_1}/Z_1. \quad (5.6)$$

This state is not at equilibrium, and possesses free energy that can be extracted by applying a tipping pulse that rotates the spin by θ and takes $|\uparrow'\rangle \rightarrow |\uparrow\rangle$ and $|\downarrow'\rangle \rightarrow |\downarrow\rangle$. The amount of work extracted is

$$W^* = E_1^* - E_1, \quad (5.7)$$

where

$$E_1 = \mu_1 B[p_1(\uparrow) - p_1(\downarrow)], \quad E_1^* = \mu_1 B[p_1^*(\uparrow) - p_1^*(\downarrow)] \quad (5.8)$$

and

$$p_1^*(\uparrow) = p_1(\uparrow)\cos^2\theta + p_1(\downarrow)\sin^2\theta,$$

$$p_1^*(\downarrow) = p_1(\downarrow)\cos^2\theta + p_1(\uparrow)\sin^2\theta. \quad (5.9)$$

Running the engine through a Carnot cycle by steps (1)–(5) above then extracts work $(T_1 - T_2)(S_1 - S_2)$. This process extracts the free energy of the spin isentropically, without increasing entropy.

By operating in this fashion—by only making measurements with respect to which the density matrix is diagonal—the demon reaches the upper limit of eq. (2.3) in which $\epsilon_Q = \epsilon_C$. Inefficiency due to measurement arises when instead of first applying the tipping pulse to extract spin 1's free energy, one simply operates the engine cyclically as before. In this case, the demon is making measurements with respect to which the density matrix has off-diagonal terms. As a result, the measurement introduces information and ϵ_Q is strictly less than ϵ_C.

The steps are as above: three conditional flips swap the states of spin 1 and spin 2, so that spin 1 is in the state ρ_2 and spin 2 is in the state $\rho_2' = \rho_1'$. The interaction with the heat reservoir then decoheres spin 2, destroying the off-diagonal terms in the density matrix so that

$$\rho_2' \rightarrow p_1^*(\uparrow)|\uparrow\rangle\langle\uparrow| + p_1^*(\downarrow)|\downarrow\rangle\langle\downarrow|, \quad (5.10)$$

with entropy

$$S_1^* = -k_B \sum_{i=\uparrow,\downarrow} p_1^*(i)\ln p_1^*(i), \quad (5.11)$$

$$\Delta S_Q = (S_1^* - S_1), \quad \Delta I_Q = (S_1^* - S_1)/k_B\ln 2 \quad (5.12)$$

are the ''extra'' entropy and information introduced by decoherence. The entropy $S_1^* - S_2 = \Delta S_Q + S_{\text{in}}$ that flows out to reservoir 2 is greater than the entropy $S_1 - S_2 = S_{\text{in}}$ that flowed in from reservoir 1. The total amount of work done is

$$T_1(S_1 - S_2) - T_2(S_1^* - S_2) + W^*, \quad (5.13)$$

$T_2(S_1^* - S_1)$ less than the work $(T_1 - T_2)(S_1 - S_2) + W^*$ done by simply undoing the tipping pulse and operating the engine as before. The overall efficiency with decoherence and measurement included is

$$\epsilon_Q = 1 - T_2(S_1^* - S_2)/T_1(S_1 - S_2) = 1 - T_2(S_{\text{in}} + \Delta S_Q)/T_1 S_{\text{in}} \leq \epsilon_C, \quad (5.14)$$

in accordance with Eq. (2.3) above. The extra information introduced by quantum measurement and decoherence has decreased the efficiency of the demon. The quantum efficiency ϵ_Q rather than the Carnot efficiency ϵ_C provides the upper limit to the maximum efficiency of such an engine.

Decoherence also decreases efficiency when the quantum demon is operated as a refrigerator. If the demon is used to pump heat from a reservoir at a low temperature T_2 to a reservoir at a high temperature T_1 in the presence of decoherence as above, then the upper limit to the coefficient of performance for the refrigerator is no longer given by the Carnot coefficient $\gamma_C = T_2/(T_1 - T_2)$ but by the quantum coefficient of performance

$$\gamma_Q = T_2/[(1 + \Delta S_Q/S_{\text{in}})T_1 - T_2], \quad (5.15)$$

where S_{in} is the net entropy pumped out of the low temperature reservoir, and ΔS_Q is the increase in entropy due to decoherence.

VI. EXPERIMENTAL REALIZATIONS

This paper has proposed several effects that could be realized experimentally.

First, Secs. I–IV showed how a quantum system such as a spin that gets information about another spin can function as a Maxwell's demon, ''cashing in'' that information to do useful work. Nucleon-nucleon double resonance methods of the sort described in Sec. III could be used to construct a ''demon maser'' that performs a net amplification of the pulses that flip the spins. Equations (4.4) and (4.5) imply that for the maser to provide a net amplification the two different species of nucleons must start the cycle at significantly different temperatures, which could be accomplished by preparing one of the species in a low temperature state using electron-nucleon double resonance as in the Pound-Overhauser effect, or by optical pumping as in sideband cooling [18].

Such a maser, though unlikely to set any power records, could be used to verify Landauer's principle directly. Boltzmann factors for the different spin species can be verified at each stage of the cycle by looking at the induction signal for the different species. In addition, it may be possible to measure directly the heating of the lattice caused by the transfer of ''waste'' information from spins to lattice.

Although the detailed model given above treated the quantum demon in terms of interacting spins, exactly the same thermodynamics applies to any interacting two-level quantum systems. At bottom, a quantum ''demon'' consists of nothing more than an interaction between two quantum systems that allows the controlled transfer of information from one to the other. In particular, any system that can provide the coherent quantum logic operation controlled-NOT that flips one quantum bit conditioned on the state of another could form the basis for an information-processing quantum heat engine [18,19]. In Wineland's ion-trap quantum logic gate, for example, one of the quantum bits consists of two hyperfine states of a trapped ion, while the other bit consists of the lowest two vibrational states of the ion in the trap. Optical pumping techniques can be used to flip one of the bits conditioned on the state of the other.

A quantum demon can also be used as a refrigerator to pump heat from a low temperature reservoir to a high temperature reservoir, using up work in the process. As noted above, a ''demon'' refrigerator was realized as long ago as the 1950s in the Pound-Overhauser effect [9]. Wineland's sideband cooling technique [18] gives a second example of such a ''Maxwell's refrigerator'' [20]: in this technique, the same types of laser pulses used to perform the conditional quantum logic operations described in the previous paragraph are used to pump entropy from the vibrational mode of the ion to an internal electronic state which radiates heat to the environment via spontaneous emission. Both of these demons operate using microscopic, quantum degrees of freedom as the effective measuring apparatus whereby the demon gets information about the state of a quantum system and acts on that information to obtain work or to pump heat. It is also possible for the measuring apparatus to be macroscopic, as in Ref. [21]. Without a detailed model of the microscopic dynamics of the measurement apparatus and its interaction with the environment, however, it is not possible to verify Landauer's principle or the decrease in efficiency due to decoherence.

The second significant theoretical claim of this paper is that measurement and decoherence necessarily introduce inefficiency in the operation of a quantum heat engine or quantum refrigerator as described by Eqs. (2.3), (5.14), and (5.15). This inefficiency due to decoherence could be measured in a variety of ways. In any of the systems described in the previous paragraphs, decoherence can be introduced as described in Secs. II and V by introducing an extra pulse to ''tip'' the state of one of the systems before it interacts incoherently with its environment. In the case of the Pound-Overhauser experiment, for example, tipping the electron then performing the conditional spin-flipping pulse sequence results in an effective temperature for the nucleon that is higher than the effective temperature $T_1 g_n / g_e$ that results from performing the experiment without decoherence. This increase in the effective temperature could be measured by performing the cooling, waiting for a time greater than the decoherence time of the nucleon but less than its decay time, then tipping the nucleon and monitoring the strength of the nuclear induction signal: the strength of the induction signal is determined by the relative size of the Boltzmann factors in Eq. (4.1) and can be used to gauge the effective temperature of the nucleon. In the case of sideband cooling, the rate at which the vibrational mode is cooled will be diminished if the electronic state of the ion is tipped before the sideband pulse is administered: this diminished cooling rate could be verified by using the ion to monitor the effective temperature of the vibrational mode as the cooling progresses. In all cases, decoherence implies an additional dissipation $T\Delta S$ when the extra entropy ΔS created by decoherence is pumped into a reservoir at temperature T. For systems that would otherwise operate at the Carnot efficiency, this decrease in efficiency is quantified by Eq. (5.14) for a quantum heat engine and by Eq. (5.15) for a quantum refrigerator.

The decrease in efficiency due to measurement and decoherence is not confined to ''demon'' heat engines that operate by processing information. A variety of quantum heat engines exist [22]: the best-known examples are lasers and masers. These heat engines should not be considered ''demons'' because they do not operate by acquiring information and then acting on it. (All heat engines operate by transferring entropy from one degree of freedom to another. The appellation ''demon'' should be reserved for systems that acquire information and then perform an action conditioned on the value of that information.) Nonetheless, even in the absence of measurement, decoherence can be a source of inefficiency in a conventional quantum heat engine such as a laser, or in an ''inverse laser'' that uses anti-Stokes emissions to perform refrigeration [23,24]. As noted in Sec. II, if a tipping pulse is applied to such a system before the stage in its cycle in which it interacts incoherently with its environment then the efficiency of the device is decreased in accordance with Eqs. (2.3), (5.14), and (5.15).

The use of a tipping pulse is an experimental convenience to put one of the quantum systems that make up the demon in a state in which its density matrix has off-diagonal terms and is susceptible to decoherence. In fact, the operation of demon heat engines and refrigerators necessarily puts the demon in an off-diagonal state at some point during its operation. Any decoherence that occurs during this time will also increase entropy and reduce the demon's efficiency.

CONCLUSION

This paper investigated Maxwell's demon in the context of realistic quantum models. As usual, such demons fail to violate the second law of thermodynamics. However, when operated between heat reservoirs at different temperatures, the demon can function either as a heat engine that pumps heat from a hot reservoir to a cold reservoir while doing work on the electromagnetic field, or as a refrigerator that pumps heat from a cold reservoir to a hot reservoir while absorbing work from the electromagnetic field. The paper derived general formulas that show how quantum measurement and decoherence decrease the efficiency of such heat engines.

The systems discussed here have several advantages over Zurek's quantum Szilard engine gedanken experiment [5]. First, the systems can be realized experimentally. Second, all aspects of the systems can be treated consistently within a unified quantum framework. The use of magnetic resonance techniques to describe a quantum Maxwell's demon was for the sake of convenience of exposition and potential experimental realizability: many other quantum systems could be suitable for performing the heat–information–energy conversion described above. Essentially any quantum system that can obtain information about another quantum system can form the basis for a quantum demon.

The work presented here suggests a variety of practical questions. First, we intend to investigate the circumstances under which a collection of many demons of the sort described could be used to pump macroscopic amounts of heat and do macroscopic amounts of work. In addition, we will examine the limitations to the thermodynamic efficiency of actual quantum devices that operate by getting and using information. Finally, having identified decoherence as a source of thermodynamic inefficiency in quantum devices, we can estimate the degree to which decoherence is responsible for low efficiency in existing quantum devices such as lasers, masers, and quantum cooling systems.

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[1] J. C. Maxwell, *Theory of Heat* (Appleton, London, 1871).

[2] H. S. Leff and A. F. Rex, *Maxwell's Demon: Entropy, Information, Computing* (Princeton University Press, Princeton, NJ, 1990).

[3] See, e.g., C. H. Bennett, Sci. Am. **257**, 108 (1987); W. H. Zurek, Nature (London) **341**, 119 (1989).

[4] R. Landauer, IBM J. Res. Dev. **5**, 183 (1961).

[5] W. H. Zurek, H. S. Leff, and A. F. Rex (Ref. [2]), pp. 249–259, reprinted from *Frontiers of Nonequilibrium Statistical Physics*, edited by G. T. Moore and M. O. Scully (Plenum, New York, 1986), pp. 151–161.

[6] S. Lloyd, Phys. Rev. A **39**, 5378 (1989).

[7] G. J. Milburn (unpublished); see also E. P. Gyftopoulos (unpublished).

[8] H. Haus (private communication).

[9] C. P. Slichter, *Principles of Magnetic Resonance*, 3rd ed. (Springer-Verlag, New York, 1990). A π pulse is a transversely polarized oscillatory pulse with integrated intensity $\hbar^{-1}\int \mu H(t) = \pi$, where $H(t)$ is the envelope function of the oscillatory field.

[10] O. R. Ernst, G. Bodenhausen, and A. Wokaun, *Principles of Nuclear Magnetic Resonance in One and Two Dimensions* (Oxford University Press, Oxford, 1987). There are a number of ways to flip one spin coherently conditioned on the state of another. For example, consider a second spin that interacts with the first in an Ising-like fashion, so that the Hamiltonian for the two spins is $2\mu_1 B\sigma_z^1 + 2\mu_2 B\sigma_z^2 + \hbar\kappa\sigma_z^1\sigma_z^2$, where $\mu_1 = \mu$ and μ_2 are the magnetic dipole moments for the two spins and κ is a coupling constant. To perform a controlled-NOT, (i) apply a $\pi/2$ pulse with frequency $\omega_2 = 2\mu_2 B/\hbar$ and width $\ll 2|\mu_1 - \mu_2|$ and $> 2\kappa$, (ii) wait for time $\pi/2\kappa$, and (iii) apply a second $\pi/2$ pulse with a phase delay of $3\pi/2$ from the first pulse. Step (i) rotates spin 2 by $\pi/2$, step (ii) allows spin 2 to acquire a phase of $\pm\pi/2$ conditioned on whether spin 1 is in the state $|\uparrow\rangle$ or $|\downarrow\rangle$, and step (iii) either rotates spin 2 back to its original state if spin $1 = |\downarrow\rangle$ or rotates spin 2 to an angle of π from its original state is spin $1 = |\uparrow\rangle$. Another way to flip spin 2 if spin $1 = |\uparrow\rangle$ is to apply a highly selective π pulse with frequency ω_2 and width $\ll \kappa$. Spin 1 is off-resonance and does nothing, while spin 2 is on-resonance and flips if spin 1 $= |\uparrow|\rangle$.

[11] S. Lloyd and W. H. Zurek, J. Stat. Phys. **62**, 819 (1991).

[12] See, e.g., J. A. Wheeler and W. H. Zurek, *Quantum Theory and Measurement* (Princeton University Press, Princeton, NJ, 1983).

[13] M. Gell-Mann and J. Hartle, in *Complexity, Entropy, and the Physics of Information*, edited by W. H. Zurek, Santa Fe Institute Studies in the Sciences of Complexity Vol. VIII (Addison-Wesley, Redwood City, CA, 1990).

[14] W. H. Zurek, Phys. Today **44** (10), 36 (1991); Phys. Rev. D **24**, 1516 (1981); **26**, 1516 (1981).

[15] D. Divincenzo, Science **270**, 255 (1995).

[16] D. Deutsch, A. Ekert, and R. Jozsa, Phys. Rev. Lett. **74**, 4083 (1995).

[17] T. Sleator and H. Weinfurter, Phys. Rev. Lett. **74**, 4087 (1995).

[18] C. Monroe, D. M. Meekhof, B. E. King, W. M. Itano, and D. J. Wineland, Phys. Rev. Lett. **75**, 4714 (1995).

[19] Q. A. Turchette, C. J. Hood, W. Lange, H. Mabuchi, and H. J. Kimble, Phys. Rev. Lett. **75**, 4710 (1995).

[20] J. Smith (unpublished).
[21] T. Sleator and M. Wilkens, Phys. Rev. A **48**, 3286 (1993).
[22] E. Geva and R. Kosloff, J. Chem. Phys. **96**, 3054 (1992); Phys. Rev. E **49**, 3903 (1994); J. Chem. Phys. **104**, 7681 (1996).
[23] For an example of a quantum-optical refrigerator, see R. I. Epstein, M. I. Buchwald, B. C. Edwards, T. R. Gosnell, and C. E. Mungan, Nature (London) **377**, 500 (1995). Inefficiency due to decoherence could be observed in their system and in the systems of Ref. [22] by applying intense laser pulses to ''tip'' the state of the ions during the operation of the refrigerator.
[24] S. Lloyd (unpublished).