Coherence properties of Bose-Einstein condensates and atom lasers

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(Received 21 May 1997)

We discuss various aspects of coherence of a Bose-Einstein condensate, and summarize the experimental evidence obtained thus far. It is shown that the mean-field energy of a condensate is a measure of second-order coherence. ''Release-energy'' measurements therefore provide direct evidence of the suppression of density fluctuations in a condensate compared to a thermal cloud. [S1050-2947(97)00810-X]

PACS number(s): 03.75.Fi, 05.30.Jp, 05.70.Fh

One fascinating aspect of Bose-Einstein condensation (BEC) is the nature of coherence in a macroscopic quantum system. Theoretically, the condensate should be described by a macroscopic wave function, behaving like a ''giant matter wave'' which is characterized by long-range order. The first experiments on BEC focused on the energetics of Bose-Einstein condensation, i.e., they showed that Bose-Einstein condensates spread out with a very narrow momentum distribution when released from the atom trap [1,2]. The coherence properties of the Bose-Einstein condensate have been directly addressed only in very recent experiments [3,4]. It is the purpose of this Brief Report to discuss various aspects of coherence and their experimental evidence. We show in a rigorous derivation that the interaction energy of a condensate is proportional to the second-order spatial correlation function (at zero relative position), which is a measure of the local-density fluctuations. We will show that previous experiments on the mean-field energy of a condensate can be reinterpreted as a determination of its second-order coherence.

Observation of a low-energy cloud: The first evidence of Bose-Einstein condensation was obtained by observing the sudden appearance of a low-energy component of the cloud in addition to the thermal component [1,2]. Subsequently, similar experiments were done by direct imaging where the sudden appearance of a dense core was observed [5,6]. These results provided clear evidence of quantum occupancies (phase-space densities) much larger than one which is one aspect of coherence. However, it was not possible to interpret these results as the macroscopic occupation of a single quantum state or even a few quantum states, since the observed energy release was considerable larger than the zero-point energy of the trap. The largest part of the release energy came from the repulsive interactions between the atoms. In the case of large condensates of sodium atoms in the cloverleaf trap [7], the number of harmonic-oscillator levels with an energy less than the system's chemical potential was about 10^5 (the chemical potential is 3.5 times the observed release energy of typically 100 nK). For condensates of 4000 Rb atoms, the number of harmonic-oscillator levels below the chemical potential was about 100 [8]. The actual level density is even higher when the repulsive interactions are taken into account, since they effectively weaken the trap.

A system which has ''condensed'' to energy levels with energies smaller than the mean-field energy can be locally coherent, but globally incoherent, and is then called a quasi-condensate [9]. Once the system establishes long-range order [10], it is a ''true'' condensate which is described by a macroscopic wave function, where (almost) all the atoms occupy the Hartree-Fock ground state of the system [11]. The observations of a bimodal density distribution with a dense core did not distinguish between condensates and quasicondensates.

The phase fluctuations of a quasicondensate cost additional kinetic energy. Careful energy release measurements can therefore distinguish between ''true'' and quasicondensates, and may give a lower bound on the coherence length [12]. This method should work for small condensates [8], but not for large ones: In the cloverleaf trap, an additional axial kinetic energy of about 1 pK is a negligible contribution compared to the 100-nK mean-field energy [7].

Mean-field energy: Quantitative agreement has been found between the energy release of a Bose-Einstein condensate obtained from time-of-flight measurements and the predictions of mean-field theory [7,8] As we show below, these measurements can be reinterpreted as a direct measurement of the second-order correlation function. We first express the potential-energy operator $\hat{U}$ in second quantization using field operators $\hat{\Psi}(r)$ [13,14]:

$$\hat{U} = \tfrac{1}{2} \int dr_1 dr_2 \hat{\Psi}^\dagger(r_1) \hat{\Psi}^\dagger(r_2) U(r_1 - r_2) \hat{\Psi}(r_2) \hat{\Psi}(r_1).$$

The expectation value for the interaction energy is

$$\langle U \rangle = \tfrac{1}{2} \int dr_1 dr_2 U(r_1 - r_2) n(r_1) n(r_2) g^{(2)}(r_1, r_2), \quad (1)$$

where the second-order correlation function $g^{(2)}(r_1, r_2)$ is defined as

$$g^{(2)}(r_1, r_2) = \langle \hat{\Psi}^\dagger(r_1) \hat{\Psi}^\dagger(r_2) \hat{\Psi}(r_2) \hat{\Psi}(r_1) \rangle / n(r_1) n(r_2),$$

and $n(r) = \langle \hat{\Psi}^\dagger(r) \hat{\Psi}(r) \rangle$ is the atomic density.

For a short-range potential, we can use the pseudopotential $U(r) = (4\pi\hbar^2 a/m)\delta(r)$, to obtain the leading-order term from Eq. (1),

$$\langle U \rangle = (2\pi\hbar^2/m) a g^{(2)}(0) \int dr[n(r)]^2, \quad (2)$$

where a is the s-wave scattering length and m the atomic mass, and we have used $g^{(2)}(r) = g^{(2)}(R, R+r)$, assuming that $g^{(2)}(r_1, r_2)$ depends only on $r_1 - r_2$.

For a pure condensate, $g^{(2)}(0) = 1$, and Eq. (2) reduces to the normal expression for the mean field energy used in the nonlinear Schrödinger equation [15]. Thus the validity of this equation and of the ''standard'' mean-field expression for the condensate is based on the assumption that $g^{(2)}(0) = 1$, i.e., that density fluctuations are absent. Thermal density fluctuations are characterized by $g^{(2)}(0) = 2$, as recently observed for neon atoms by measuring the intensity correlation function [16]. The value of 2 for $g^{(2)}(0)$ for a thermal cloud can be traced back to an exchange term in the interaction matrix element. This exchange term arises whenever the system occupies different quantum states which have spatial overlap [14]. $g^{(2)}(0) = 1$ implies that the system can be described *locally* by a single wave function, but it does not rule out the population of numerous nonoverlapping quantum states. The factor of 2 in the interaction energy already appears for a pure state which has many singly occupied quantum levels; the thermal average is performed by averaging over many such configurations. Therefore, the factor of 2 is not due to thermal fluctuations, but rather due to quantum fluctuations.

An important conclusion from this derivation is that the interaction energy of a condensate is a direct measure of $g^{(2)}(0)$. This is intuitively obvious, because for a short-range potential, the interaction energy is proportional to the probability that two atoms are at the same position, which is proportional to $g^{(2)}(0)$.

Previous measurements of the release energy of sodium condensates were compared to Eq. (2) assuming $g^{(2)}(0) = 1$, and used to determine the scattering length a. Since $g^{(2)}(0) = 1$ had not been experimentally verified before, these experiments actually determined the product $g^{(2)}(0)a$. Combining our earlier result of $a = 65 \pm 30 a_0$ [7] with the recent spectroscopic determination of the scattering length, $a = (52 \pm 5) a_0$ [17], leads to $g^{(2)}(0) = 1.25 \pm 0.58$. Castin and Dum analyzed similar time-of-flight data and extracted $a = (42 \pm 15) a_0$ [18], implying $g^{(2)}(0) = 0.81 \pm 0.29$. Even more accurate time-of-flight measurements were done with Rb condensates [8]. The authors quoted that assuming a scattering length which deviates by more than 20% from the spectroscopic value would be inconsistent with the results. This implies $g^{(2)}(0) = 1.0 \pm 0.2$. Thus all measurements of the interaction energy are consistent with the prediction of $g^{(2)}(0) = 1$ for a pure condensate. The experimental $g^{(2)}(0)$ values are significantly smaller than 2, and are therefore strong evidence for the suppression of local density fluctuations in a Bose-Einstein condensate.

Inelastic collisions: Analysis of the mean-field energy yields an *absolute* value for $g^{(2)}(0)$. This determination of $g^{(2)}(0)$ is based on elastic collisions (which give rise to the mean-field energy). The parameter describing the elastic collisions, the scattering length a, is precisely known from photoassociation spectroscopy. Similarly, n-body inelastic collisions can be used to determine higher-order atom correlations, since the rate of n-body collisions (per unit volume) is proportional to $g^{(n)}(0)$ times the density to the power n [19]. However, since the rate coefficients for inelastic collisions at very low temperature are not accurately

known, one can only compare the rate of inelastic collisions in a thermal cloud and a condensate and obtain a *relative* value of $g^{(n)}(0)$ for a thermal cloud compared to a condensate. For a noninteracting Bose gas, this ratio was predicted to be $n!$, and simply reflects the counting statistics of bosons. The $n!$ change in the (density-normalized) rate of inelastic collisions was suggested as a method to monitor the formation of a condensate [19].

In an important experiment, Burt *et al.* recently compared the trap loss due to three-body recombination of a Rb condensate to that of a thermal cloud, and obtained 7.4 ± 2 for the ratio of the $g^{(3)}(0)$ values in good agreement with the predicted value of 6 [4]. This experiment is a clear demonstration of the third-order coherence of a Bose-Einstein condensate. The condensate density was inferred from the number of atoms in the condensate assuming that the density profile is an inverted parabola. The factor $a \, g^{(2)}(0)$ in Eq. (2) which determines the width of the parabola, was taken from Ref. [8], assuming that $g^{(2)}(0)$ has the same value as for the small condensates of Ref. [8]. The peak density of the condensate scales with the assumed value x for $g^{(2)}(0)$ as $x^{-3/5}$ and the $g^{(3)}(0)$ ratio as $x^{6/5}$. A determination of $g^{(3)}(0)$ which does not rely on release-energy measurements is possible when the density of a condensate is directly measured. Direct density measurements have been recently done using phase-contrast imaging [20]. Any model or experiment which determines the density profile of a trapped condensate contains information about the density fluctuations, and provides an absolute value of $g^{(2)}(0)$ when combined with the spectroscopic determination of the scattering length a.

Interference of two condensates: Measurements of mean-field energy and collisions probe only *short-range* correlations in a Bose-Einstein condensate, and cannot distinguish between quasicondensates, which lack long-range correlations, and ''true'' condensates [9]. The recently reported interference of two condensates [3] provides direct evidence for *long-range* coherence in a Bose-Einstein condensate. It was a direct measurement of first-order coherence, and provided evidence that the correlations extended over the whole sample. The contrast of the matter wave interference was estimated from the observed fringe contrast to be between 50% and 100%. A more precise value could not be given because finer details of the probing and imaging of the interference pattern were not quantitatively accounted for. Theoretical simulations of the interference showed that the results were consistent with numerical solutions of the nonlinear Schrödinger equation, which assumed 100% coherence [21].

If one assumes that the condensate is 100% spatially coherent, then the pulse-to-pulse fluctuations in the number of condensed atoms contain further information on the atom statistics. This is similar to the comparison of a pulsed thermal source which is passed through a single-mode filter and a pulsed single-mode laser beam. In both cases, the pulses are 100% coherent (single mode), but they differ in the shot-to-shot fluctuations. The thermal-like statistics is characterized by an exponential distribution for which the most probable value is zero, whereas the laserlike distribution is a much narrower Poissonian distribution [22,23]. The number fluctuations in creating Bose-Einstein condensates or coupling out pulses of a Bose-Einstein condensate were small (typically 10%, probably determined by the reproducibility

of loading the atom trap), and therefore ''laserlike'' rather than ''thermallike.'' Of course, this just reflects the fact that the Bose-Einstein condensate is not filtered out of a large reservoir, which would result in large fluctuations in the atom number.

This Brief Report has summarized the current evidence for coherence of Bose-Einstein condensates, and emphasized that there are many different aspects of coherence. The experiments discussed here have provided several pieces of evidence for first-, second-, and third-order coherence, and for long-range first-order correlations. All these measurements are consistent with the standard interpretation that the Bose-Einstein condensate is characterized by a single macroscopic wave function, but more experiments are needed to reduce the ''error bar'' on this statement and to fully characterize the coherence of Bose-Einstein condensates.

All the measurements done so far show the strong analogy between Bose-Einstein condensed atoms and laser photons, and suggest that ''atom laser'' is a descriptive name for atom sources based on Bose-Einstein condensates. We have used this name to describe several advances towards an ''ideal'' atom laser [24] which we have made recently: the

direct evidence of long-range correlations [3], the realization of an output coupler for a Bose-Einstein condensate [25], the generation of multiple pulses of coherent matter waves [25], and the explicit demonstration of the coherence of beams extracted from a Bose-Einstein condensate [3]. However, whether one wants to adopt this use of ''atom laser,'' or use this name already for trapped Bose-Einstein condensates (''intracavity'' atoms), or reserve it for the demonstration of a cw source of coherent matter waves is rather a question of personal taste, since a rigorous definition of a laser does not exist, even for the optical laser.

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