Atom localization via Ramsey interferometry: A coherent cavity field provides a better resolution

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We investigate the position localization of a polarized atom interacting with an off-resonant quantized standing-wave field. We show that a coherent cavity field achieves a higher resolution than a classical field. An almost perfect localization is possible when the atom passes through several identically prepared cavities. [S1050-2947(97)04010-9]

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I. INTRODUCTION

The problem of localizing an atom by scattering light, the Heisenberg microscope [1], dates back to the early days of quantum mechanics. The modern tools of quantum optics have made such gedanken experiments reality [2,3]. Moreover, most recently, the localization of atoms in domains smaller than the wavelength of light has been discussed theoretically [4] and achieved experimentally [5] using a classical standing-wave light field. In the present paper we show that an even more dramatic localization results from the use of a quantum field.

A variation of the Heisenberg microscope has been demonstrated by Gardner *et al.* [6]. In this method an inhomogeneous light intensity causes a spatially varying atomic-level shift that correlates the atomic resonance frequency with the atomic position. Several proposed methods [7–11] for high-precision measurements of atomic position rely on the interaction of an atom with a standing wave inside the cavity. This interaction leads to a quantum entanglement between the atomic degrees of freedom including the center-of-mass motion and the state of the field. This entanglement arises as the position of the atom is highly correlated with the phase of the field inside the cavity. A measurement of the internal state of the atom or the state of the field leads to a quantum-state reduction of the center-of-mass motion and hence the atom localization.

Reference [12] utilizes these ideas to localize the position of a polarized atom by passing it through a classical standing wave and measuring the phase of the atomic dipole moment in a Ramsey-type experiment [13,14]. In this scheme a relatively large narrowing of the atomic position distribution is achieved by increasing the intensity of the field. This increased narrowing is, however, accompanied by an increase in the number of peaks in the position distribution of the atom.

In this paper we analyze the interaction of a single-mode quantized standing-wave cavity field with a three-level atom moving in the Raman-Nath regime and far off resonance. We show that for a coherent-state field inside the cavity, the quantum interference due to different photon number excitations would substantially alter the results for the atom localization, resulting in a *collapse* of the atom position around the nodes of the field. This collapse is reminiscent of the collapse of the population inversion in the Jaynes-Cummings model of the atom-field interaction. A substantial localization takes place when the atom is passed through a number of identically prepared setups. It is then possible, under certain conditions, that almost complete localization is obtained around the field nodes.

In Sec. II we present the model and give results for a single atom passing through the Ramsey-type experimental setup with a quantized field inside the cavity. In Sec. III we extend these results to include the passage through an array of Ramsey setups. We conclude in Sec. IV by summarizing our main results and briefly outlining a possible experiment. We emphasize that the currently available technology allows to perform this experiment.

II. LOCALIZATION OF AN ATOM: A COHERENT-STATE FIELD IS BETTER THAN A CLASSICAL FIELD

In the present section we summarize the essential ingredients of atom localization using Ramsey interferometry. The measurement of the atomic dipole provides a tool to localize an atom in a standing wave beyond a wavelength of the light. In contrast to earlier work [12], however, we focus on a quantum field and the resulting position distribution. We show that a coherent cavity field significally enhances the resolution compared to a classical field.

We start our discussion by briefly discussing the Ramsey setup shown in Fig. 1(a). We consider a three-level atom with two near-degenerate ground states $|a\rangle$ and $|b\rangle$ and an optically excited state $|c\rangle$ crossing a Ramsey-type experimental setup shown in Fig. 1(a) and used recently in Refs. [12, 13]. The experimental setup includes a standing-wave cavity light field that is sandwiched between two microwave Ramsey fields. The off-resonant cavity field couples both lower levels $|a\rangle$ and $|b\rangle$ to the upper level $|c\rangle$ via electric-dipole transitions. The resonant Ramsey fields induce transitions between the lower levels $|a\rangle$ and $|b\rangle$ via magnetic-dipole coupling. We neglect damping of the cavity and the

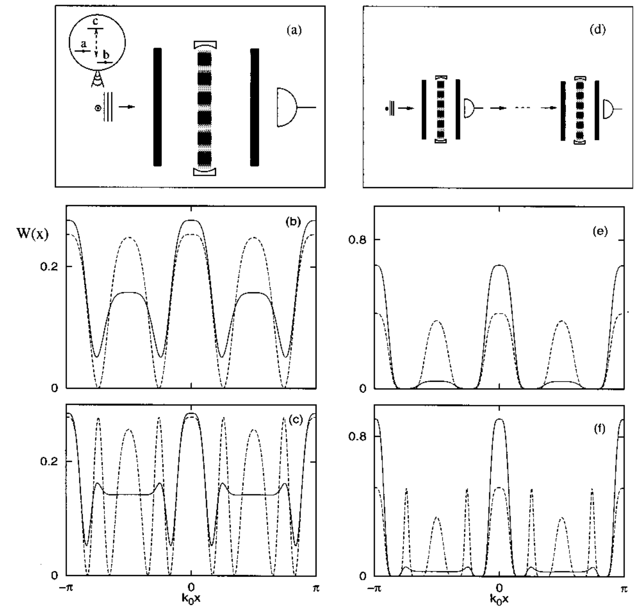

FIG. 1. Localization of an atom by a single Ramsey setup (left column) and an array of Ramsey setups (right column). A classical field prepares a dipole between levels $|a\rangle$ and $|b\rangle$ of a three-level atom before it crosses a cavity field. This field is in either a Fock state or a coherent state of an identical average number of photons. Another classical field with a fixed phase relative to the first classical field and a detector measuring the population of either level $|a\rangle$ or $|b\rangle$ read out the change of this dipole. This setup (a) leads to a conditional position distribution $W(x)$ shown in (b) and (c) for two different interaction parameters $\kappa=0.3$ and $\kappa=0.6$, respectively. In the position distribution corresponding to a coherent state of $|\alpha|^2=10$ shown in (b) and (c) by the solid curves, the maxima at $k_0x=0,\pm\pi,\ldots,$ that is, at the nodes of the field, are enhanced compared to the corresponding distributions of a Fock state indicated by the dashed curves. In the coherent-state case the other maxima are suppressed and the minima are filled, creating a background. In the case of a Fock state an increase of the interaction strength leads to more maxima, as apparent in (c). In contrast, the corresponding position distribution for a coherent state displays larger maxima at the nodes while the background is essentially the same as in (b). The increase of the interaction strength from $\kappa=0.3$ (e) to $\kappa=0.6$ (f) also eliminates the many side maxima of the Fock state case. Here we have chosen $k=5$ Ramsey setups with identically prepared cavity fields and identically prepared atomic states. In all figures we have taken the probability amplitudes for the levels in the prepared and detected internal states of the atom $C_a=C_b=\widetilde{C}_a=\widetilde{C}_b=1/\sqrt{2}$, corresponding to $V=1/2$, $G=1/4$, and $\phi=0$.

spontaneous emission of the atom and take the dipole and rotating-wave approximations. The initial momentum of the atomic motion along the longitudinal z direction is assumed to be large enough so that it can be treated classically. The motion of the atom in the transverse x direction along the standing-wave mode, the internal degrees of freedom of the atom, and the cavity light field are quantized. We consider the Raman-Nath regime where the kinetic energy of the transverse motion of the atom can be ignored.

Suppose that the initial state of the atomic transverse motion is a pure state described by the state vector

$$|\psi_{\text{motion}}\rangle=\int_{-\infty}^{\infty}dx f(x)|x\rangle. \tag{1}$$

The Ramsey zones employ microwave fields in resonance with the transitions between the lower levels $|a\rangle$ and $|b\rangle$. The first Ramsey field prepares the atom in a coherent-superposition state

$$|\psi_{\text{atom}}\rangle=C_a|a\rangle+C_b|b\rangle. \tag{2}$$

The initial state of the standing-wave light field in the cavity is given by

$$|\psi_{\text{field}}\rangle=\sum_{n=0}^{\infty} w_n|n\rangle. \tag{3}$$

The frequency of the standing-wave light field is detuned from the atomic frequency for the transition between $|c\rangle$ and $|j\rangle$ $(j=a,b)$. For very large detuning, the atom practically remains in the ground states during the interaction with the standing-wave light field and the effective interaction Hamiltonian is [13,15,16]

$$\hat{H}=-2\hbar\,\sin^2(k_0\hat{x})(g_a|a\rangle\langle a|+g_b|b\rangle\langle b|)\hat{a}^\dagger\hat{a}. \tag{4}$$

Here the operator $\hat{x}$ describes the transverse position of the atom, $\hat{a}$ and $\hat{a}^\dagger$ are the annihilation and creation operators of the cavity mode, and k_0 is the wave number of the cavity mode. The effective coupling parameters for the transition channels $|a\rangle\leftrightarrow|c\rangle$ and $|b\rangle\leftrightarrow|c\rangle$ are given by $g_a=\Omega_a^2/2\Delta_a$ and $g_b=\Omega_b^2/2\Delta_b$, respectively, where Ω_a and Ω_b are the associated vacuum Rabi frequencies and Δ_a and Δ_b are the corresponding detunings. In the case when the vacuum Rabi frequencies of the two channels are the same, that is, $\Omega_a=\Omega_b$, and the field is detuned to the middle point between the two lower levels, that is, $\Delta_a=-\Delta_b$, we have $g_a=-g_b$.

The time-dependent state vector

$$|\psi_{\text{total}}(t)\rangle=\exp(-i\hat{H}t/\hbar)|\psi_{\text{motion}}\rangle\otimes|\psi_{\text{atom}}\rangle\otimes|\psi_{\text{field}}\rangle \tag{5}$$

of the combined system in the interaction picture follows from Eqs. (1)–(5) and reads

$$|\psi_{\text{total}}(t)\rangle=\int_{-\infty}^{\infty}dx\sum_{j=a,b}\sum_{n=0}^{\infty} f(x)C_j w_n \times\exp[2ing_jt\,\sin^2(k_0x)]|x\rangle\otimes|j\rangle\otimes|n\rangle. \tag{6}$$

Behind the cavity, we make a measurement of the internal state of the atom by using the second Ramsey field together with a state-selective detector. If the atom is detected in the internal state

$$|\widetilde{\psi}_{\text{atom}}\rangle=\widetilde{C}_a|a\rangle+\widetilde{C}_b|b\rangle, \tag{7}$$

the distribution of the transverse position of the atom conditioned by such a measurement is

$$W(x)=\mathcal{P}^{-1}\sum_{n=0}^{\infty}|\langle x|\otimes\langle\widetilde{\psi}_{\text{atom}}|\otimes\langle n|\psi_{\text{total}}(\tau)\rangle|^2. \tag{8}$$

Here τ is the transit time through the standing-wave light field, while the state-reduction normalization constant $\mathcal{P}$ is the probability of finding the atom exiting the cavity in the internal state $|\widetilde{\psi}_{\text{atom}}\rangle$. On substituting Eqs. (6) and (7) into Eq. (8), we find the conditional position distribution

$$W(x)=\mathcal{P}^{-1}\mathcal{F}(x;|\psi_{\text{field}}\rangle)W_0(x), \tag{9}$$

where $W_0(x)=|f(x)|^2$ is the initial distribution of the transverse position of the atom and

$$\mathcal{F}(x;|\psi_{\text{field}}\rangle)=V+2G\sum_{n=0}^{\infty}P_n(|\psi_{\text{field}}\rangle)\times\cos[2n\kappa\sin^2(k_0x)+\phi]. \tag{10}$$

Here we have introduced the abbreviation $V=|C_a|^2|\widetilde{C}_a|^2+|C_b|^2|\widetilde{C}_b|^2$, the modulus G and the phase ϕ of $C_aC_b^*\widetilde{C}_a^*\widetilde{C}_b$, that is, $C_aC_b^*\widetilde{C}_a^*\widetilde{C}_b=G\exp(\mathrm{i}\phi)$, and the interaction parameter $\kappa=(g_a-g_b)\tau$. Note that $\mathcal{F}$ depends on the quantum state $|\psi_{\text{field}}\rangle$ of the cavity field through the photon distribution $P_n(|\psi_{\text{field}}\rangle)=|\langle n|\psi_{\text{field}}\rangle|^2=|w_n|^2$.

Since $V\geqslant 2G\geqslant 0$, the function $\mathcal{F}$ is non-negative. Thus, after the interaction with the field and the measurement of the internal state, the position distribution of the atom is multiplied by a non-negative oscillating function, which leads to the decimation of the initial position distribution, and hence the localization, of the atom. Therefore, $\mathcal{F}$ plays the role of a filter function.

We conclude this section by discussing the atomic position distribution resulting from a Fock state or a coherent state of the cavity field. In the case when the field is initially in a Fock state $|n_0\rangle$, the filter function $\mathcal{F}$ reduces to

$$\mathcal{F}(x;|n_0\rangle)=V+2G\cos[2n_0\kappa\sin^2(k_0x)+\phi]. \tag{11}$$

The function $\mathcal{F}$ displays oscillations in x and has one or more peaks within half a period of the standing-wave field, that is, within the length of $\lambda/2=\pi/k_0$. The larger the size of $n_0\kappa$, the smaller the widths of the peaks. According to Eq. (9), the position distribution W is the product of the filter function $\mathcal{F}$ and the initial position distribution W_0. Hence the filter function $\mathcal{F}$ selects from the initial position distribution of the atom those domains where $\mathcal{F}$ has dominant peaks. This results in a localization of the atom. We note that the filter function $\mathcal{F}$ [Eq. (11)] of a Fock state is identical to that [12] of a classical cavity field. We emphasize that Ref. [5] reports the observation of this localization phenomenon by a classical field.

An undesirable feature of the localization of the atom using a Fock-state field or a classical field is that the decrease of the widths of the peaks is associated with the increase of the number of the peaks. To overcome this problem, we next consider the case when the field is initially in a coherent state $|\alpha\rangle$. The photon distribution of this state is

$$P_n(|\alpha\rangle)=\exp(-|\alpha|^2)|\alpha|^{2n}/n!. \tag{12}$$

On inserting Eq. (12) into Eq. (10) and performing the summation, we find the filter function

$$\mathcal{F}(x;|\alpha\rangle)=V+2G\exp\{-2|\alpha|^2\sin^2[\kappa\sin^2(k_0x)]\}\times\cos\{|\alpha|^2\sin[2\kappa\sin^2(k_0x)]+\phi\} \tag{13}$$

for the cavity field in a coherent state. In comparison to the case of the Fock state or the classical field, the oscillatory cosine term is now multiplied by an exponential factor. In the neighborhood of the nodes this exponential term is close to unity. However, away from the nodes it damps out the oscillations. Hence the peaks positioned near the nodes are dominant compared to the other peaks. Note that, for $\kappa\cong\pi/2$, the most substantial reduction of the peaks positioned away from the nodes takes place. When $\kappa\ll 1$ and $|\alpha|\kappa\ll 1$, the result for the coherent state of the field reduces to that for the Fock state.

In Figs. 1(b) and 1(c) we show the conditional distribution $W(x)$ of the transverse position of the atom after passage through the setup. For the sake of simplicity, we have chosen the probability amplitudes for the levels in the prepared and detected atomic internal states to be $C_a=C_b=\widetilde{C}_a=\widetilde{C}_b=1/\sqrt{2}$, corresponding to $V=1/2$, $G=1/4$, and $\phi=0$. The interaction parameter for Fig. 1(b) is $\kappa=0.3$, whereas in Fig. 1(c) it is $\kappa=0.6$. The initial transverse-position distribution of the atom is uniform over a wavelength of the light field. The solid curves correspond to the case when field is initially in a coherent state of mean photon number $|\alpha|^2=10$, while the dashed curves correspond to the case when the field is initially in a Fock state with the same number $n_0=10$ of photons. As seen from the figures, in the case when the field is initially in a coherent state, some localization peaks of the atom are damped; the central peak becomes higher, although its width remains almost the same compared to the case when the field is initially in a Fock state.

It should be emphasized here that when the mean photon number $|\alpha|^2$ of the initial coherent state $|\alpha\rangle$ of the cavity field is large, that is, when the initial quantum field is closest to a classical field, the expression (13) for the filter function does not reduce to that for a classical field. In contrast, the filter function (11) for the case of an intrinsically quantum Fock state is identical to that for a classical field. These features are due to the fact that the filter function of the quantized field depends on the initial photon distribution spread, which disappears for a Fock state but is finite for a coherent state. Very similar features have been observed and discussed in the framework of the Jaynes-Cummings model. It has been shown in this model that the atomic response is purely classical if the initial field state is a Fock state and the complicated quantum collapse and revival effects occur if the initial field is coherent [17]. Note that when we consider the field modes of the laser beams irradiating the atoms with no cavity, the dynamics of the field modes is usually assumed to be determined externally rather than by the irradiated atoms. In such a case, the field quantization of the laser modes adds nothing except for calculation convenience. However, when we consider the excited field modes in a cavity, the behavior of the modes is completely governed by the interaction with the atoms inside the cavity. In such a case, the dynamics of the atom-field system can depend on the discreteness of the photon numbers, the spread of the photon distribution, and the strength of the reaction of the atoms back to the field.

III. LOCALIZATION OF AN ATOM: AN ARRAY OF RAMSEY SETUPS IS BETTER THAN A SINGLE ONE

In the preceding section we have shown that the measurement of the atomic dipole using a Ramsey setup allows us to localize the atom. In order to enhance this localization, we now use the array of Ramsey setups shown in Fig. 1(d) and show that such an arrangement does indeed sharpen the position distribution of the atom [12]. We emphasize that we cannot apply the result of Sec. II in a straightforward way: This treatment relies on the center-of-mass motion being initially in a pure state. However, the state of the transverse motion of the atom after leaving the first setup is no longer a pure state. The reason is that we measured the atomic internal state but did not probe the field and therefore had to project the state of the total system onto the detected atomic internal state and trace over the field. We hence use in this section the density-matrix approach to study the array of Ramsey setups.

The effective Hamiltonian describing the interaction between the atom and the off-resonant standing-wave light field in the kth cavity reads

$$\hat{H}^{(k)} = -2\hbar\, \sin^2(k_0\hat{x})(g_a|a\rangle\langle a| + g_b|b\rangle\langle b|)\hat{a}_k^\dagger \hat{a}_k . \tag{14}$$

Let the atom enter the kth cavity at $t_{k-1}=(k-1)\tau$, stay there for time interval τ, and exit at $t_k=k\tau$. The standing-wave field in the kth cavity is in the state

$$|\psi_{\text{field}}^{(k)}\rangle = \sum_{n=0}^{\infty} w_n^{(k)}|n\rangle. \tag{15}$$

Before entering the kth cavity, the atom is prepared in an internal coherent-superposition state

$$|\psi_{\text{atom}}^{(k)}\rangle = C_a^{(k)}|a\rangle + C_b^{(k)}|b\rangle \tag{16}$$

and after exiting the cavity the atom is detected in an internal state

$$|\tilde{\psi}_{\text{atom}}^{(k)}\rangle = \tilde{C}_a^{(k)}|a\rangle + \tilde{C}_b^{(k)}|b\rangle. \tag{17}$$

The density operator

$$\hat{\varrho}_{\text{motion}}^{(k)} = \int dx \int dx' \,\varrho_{\text{motion}}^{(k)}(x,x')|x\rangle\langle x'| \tag{18}$$

for the atomic transverse motion after the internal-state measurement behind the kth cavity follows from the density operator

$$\hat{\varrho}_0^{(k)} = |\psi_{\text{atom}}^{(k)}\rangle\langle\psi_{\text{atom}}^{(k)}| \otimes \hat{\varrho}_{\text{motion}}^{(k-1)} \otimes |\psi_{\text{field}}^{(k)}\rangle\langle\psi_{\text{field}}^{(k)}| \tag{19}$$

of the atom before the kth setup and the field in the kth cavity as

$$\hat{\varrho}_{\text{motion}}^{(k)} = \frac{1}{\mathcal{P}^{(k)}} \,\text{Tr}_{\text{field}}\langle\tilde{\psi}_{\text{atom}}^{(k)}|\hat{U}^{(k)}\hat{\varrho}_0^{(k)}\hat{U}^{(k)\dagger}|\tilde{\psi}_{\text{atom}}^{(k)}\rangle. \tag{20}$$

Here the normalization constant $\mathcal{P}^{(k)}$ is the probability of finding the atom exiting the kth cavity in the internal state $|\tilde{\psi}_{\text{atom}}^{(k)}\rangle$ and the evolution operator $\hat{U}^{(k)}$ is

$$\hat{U}^{(k)} = \exp(-i\hat{H}^{(k)}\tau/\hbar). \tag{21}$$

Since we do not perform measurements on the field we trace over it.

When we substitute the Hamiltonian $\hat{H}^{(k)}$ for the kth atom-field interaction into Eq. (21) and insert the resulting expression into Eq. (20), we arrive at

$$\varrho_{\text{motion}}^{(k)}(x,x') = \frac{1}{\mathcal{P}^{(k)}} F^{(k)}(x,x';|\psi_{\text{field}}^{(k)}\rangle)\,\varrho_{\text{motion}}^{(k-1)}(x,x'). \tag{22}$$

Here the function

$$\begin{aligned} F^{(k)}&(x,x';|\psi_{\text{field}}^{(k)}\rangle) \\ &= \sum_{n=0}^{\infty} \sum_{j,j'=a,b} P_n(|\psi_{\text{field}}^{(k)}\rangle) C_j^{(k)} C_{j'}^{(k)*} \tilde{C}_j^{(k)*} \tilde{C}_{j'}^{(k)} \\ &\quad \times \exp\{2in\tau[g_j \sin^2(k_0x) - g_{j'} \sin^2(k_0x')]\} \end{aligned} \tag{23}$$

relates the density matrix $\varrho_{\text{motion}}^{(k)}(x,x')$ for the transverse motion of the atom after passage through the kth cavity to the corresponding matrix $\varrho_{\text{motion}}^{(k-1)}(x,x')$ after passage through the $(k-1)$th cavity.

The transverse-position distribution $W^{(k)}(x) = \varrho_{\text{motion}}^{(k)}(x,x)$ of the atom after the internal-state measurement behind the kth cavity follows from Eq. (22) when we set $x = x'$ in this equation. We find

$$W^{(k)}(x) = \frac{1}{\mathcal{P}_{\text{joint}}^{(k)}} \mathcal{F}^{(k)}(x;\{|\psi_{\text{field}}^{(k)}\rangle\})W_0(x), \tag{24}$$

where the filter function

$$\mathcal{F}^{(k)}(x;\{|\psi_{\text{field}}^{(k)}\rangle\}) = \prod_{l=1}^{k} F^{(l)}(x,x;|\psi_{\text{field}}^{(l)}\rangle) \tag{25}$$

is the product of the generalized filter functions $F^{(l)}$ [Eq. (23)] evaluated at $x'=x$. Note that $\mathcal{F}^{(k)}$ depends on the field states $|\psi_{\text{field}}^{(l)}\rangle$ $(l=1,2,\ldots,k)$ of all k cavities as indicated by the symbol $\{|\psi_{\text{field}}^{(k)}\rangle\}$ in the argument of $\mathcal{F}^{(k)}$. The joint probability

$$\mathcal{P}_{\text{joint}}^{(k)} = \prod_{l=1}^{k} \mathcal{P}^{(l)} \tag{26}$$

of finding the atom in a sequence of selected internal states $|\tilde{\psi}_{\text{atom}}^{(l)}\rangle$ behind the setups is the product of all individual probabilities $\mathcal{P}^{(l)}$ of finding the atom in those states.

We now simplify this result for the case where the probability amplitudes $C_j^{(k)}$ and $\tilde{C}_j^{(k)}$ in the prepared and detected atomic internal states are the same for all k. Moreover, we assume that all the cavity fields are prepared in the same state $|\psi_{\rm field}\rangle$. In this case, all the individual filter functions $F^{(l)}(x,x;|\psi^{(l)}_{\rm field}\rangle)$ of the Ramsey setups are identical and hence Eq. (25) reduces to

$$\mathcal{F}^{(k)}(x;\{|\psi^{(k)}_{\rm field}\rangle\})=\mathcal{F}^{k}(x;|\psi_{\rm field}\rangle), \tag{27}$$

where $\mathcal{F}$ is defined by Eq. (10). The position distribution

$$W^{(k)}(x)=\frac{1}{\mathcal{P}^{(k)}_{\rm joint}}\,\mathcal{F}^{k}(x;|\psi_{\rm field}\rangle)W_0(x) \tag{28}$$

involves the kth power of the filter function $\mathcal{F}$. Therefore, in an array of Ramsey setups, that is, for $k>1$, the position distribution of the atom gets sharpened and leads to a more effective localization of the atom.

We illustrate this phenomenon in Figs. 1(e) and 1(f), where we show the conditional distribution $W^{(k)}(x)$ of the transverse position of the atom after passage through the experimental setup with $k=5$ cavities. All the other parameters are the same as for Figs. 1(b) and 1(c). The figures show a dramatic difference between the position distributions resulting from a coherent state or a Fock state of the field: The usage of the Ramsey array in the case when these fields are initially prepared in a coherent state has essentially increased the localization of the atom into the narrow regions around the nodes of the light fields.

IV. SUMMARY AND DISCUSSION

In summary, we have investigated the position localization of a polarized atom that interacts with an off-resonant quantized standing-wave field inside a cavity. We have shown that for a coherent state inside the cavity, the atom is localized around the nodes of the field. Moreover, an almost perfect localization around the field nodes is possible when the atom passes through several identically prepared cavities. This scheme relies on conditional measurements of the atomic dipole. Therefore, we can achieve such a narrow distribution only with a probability $\mathcal{P}^{(k)}_{\rm joint}$ given by Eq. (26). For the parameters of Figs. 1(e) and 1(f) and for the initial coherent state of the field inside the cavity, this probability is $\mathcal{P}^{(k)}_{\rm joint}=0.24$ and 0.18, respectively. Moreover, we emphasize that this scheme is not sensitive to the efficiency of the Ramsey detector.

We conclude by briefly outlining a method to create such an array of Ramsey setups. The experiment of Ref. [5] employs atoms from a magneto-optical trap, which are optically pumped into state $|a\rangle$. The localization is then performed in two steps. First, a sequence of three pulsed fields is used to encode the position information in the internal atomic states. As was described above, this is achieved with a properly detuned light field sandwiched between two microwave Ramsey fields resonant with the transition from state $|a\rangle$ to state $|b\rangle$. Second, laser light resonant with the transition from state $|a\rangle$ to state $|c\rangle$ or from state $|b\rangle$ to state $|c\rangle$ is used for the internal state measurement. Note that short pulses and slow atoms are employed. Therefore, atomic motion can be neglected during the pulse sequence.

It is straightforward to extend this scheme and apply a series of sequences of pulsed fields for both the encoding and the measurement. Each sequence consists of three pulses for the position encoding and one pulse for the internal state measurement. Note that this scheme employs an optical cavity field pulsed on for a short time only. The whole series of sequences of pulses is applied while the atom is in the cavity. In addition, the following trick can be used in order to avoid the (unwanted) random recoil kicks associated with the atomic fluorescence during the internal state measurement: For example, the measurement of atoms in state $|a\rangle$ can be achieved by tuning the laser frequency to the transition from state $|b\rangle$ to state $|c\rangle$. Atoms in state $|a\rangle$ manifest themselves by the absence of fluorescence. This scheme makes it possible to measure the internal atomic state without a momentum change, thus allowing the creation of a dramatic localization of the atoms.

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