Generation and properties of collective atomic Schrödinger-cat states

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We propose a method for generating collective atomic Schrödinger-cat states formed as superpositions of the Bloch (or atomic coherent) states in the context of cavity QED. We then study the properties of these states and show the effects of the superpositions on spontaneous and stimulated emission, the atomic dipole moment, spin squeezing, and coherence. [S1050-2947(97)07410-6]

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I. INTRODUCTION

In recent years there has been much interest in the generation and properties of superpositions of macroscopically distinguishable quantum states usually called Schrödinger-cat states [1]. Most of the theoretical and experimental activity has been associated with bosonic systems such as the quantized electromagnetic field [2] and with the quantized vibrational motion of the center of mass of a trapped ion [3]. In these cases, the cat states consist of superpositions of phase-shifted coherent states such as $|\alpha\rangle$ and $|-\alpha\rangle$ [2] or of $|\alpha \exp(i\phi/2)\rangle$ and $|\alpha \exp(-i\phi/2)\rangle$ [3].

In this paper, we study another kind of cat state, this time associated with the Dicke model [4] of the collective behavior of a large number of "two-level" atoms confined to a volume $V \ll \lambda^3$, where λ is the wavelength associated with the atomic transition. It is well known that for this system, the so-called Bloch states, also known as atomic coherent states or SU(2) coherent states [5,6], can possess a number of properties describing the collective behavior of the atoms [6]. Of particular interest are the properties of collective spontaneous emission (superradiance) and the existence of a macroscopic dipole moment that is able to radiate classically. These Bloch states can be generated by a classical field interacting with N atoms that are initially all in their ground states. We denote them here as $|\zeta,J\rangle$, where $\zeta=\tan(\vartheta/2)\exp(i\phi/2)$ as explained below, and the cooperation number $J=N/2$. Our collective atomic cat states then will consist of superpositions of the states $|\zeta,J\rangle$ and $|-\zeta,J\rangle$, which are phase shifted by π in the phase space given by the complex ζ plane. Previously, we have discussed Schrödinger-cat states associated with SU(2) [and with SU(1,1)] coherent states for a two-mode quantized electromagnetic field [7] making essential use of the Schwinger [8] realization of the angular momentum algebra in terms of two sets of boson operators. Such states have many nonclassical properties and we expect that striking nonclassical properties should appear for the atomic cat states as well.

This paper is organized as follows: In Sec. II, after briefly reviewing the Bloch states and their properties, we propose a method by which superpositions of $|\zeta,J\rangle$ and $|-\zeta,J\rangle$ can be generated in the context of cavity QED. In Sec. III we examine their properties. The paper closes in Sec. IV with a brief summary and some remarks.

II. GENERATION OF COLLECTIVE ATOMIC CAT STATES

We consider N two-level atoms in a cavity, of volume $V \ll \lambda^3$, where λ is the wavelength corresponding to the transition energy between the two levels. Under this condition we can introduce the three collective operators

$$J_{\pm} \equiv \sum_{i=1}^{N} \sigma_{\pm}^{(i)}, \quad J_3 \equiv \frac{1}{2}\sum_{i=1}^{N} \sigma_3^{(i)}, \tag{2.1}$$

where the $\sigma_{\pm}^{(i)}, \sigma_3^{(i)}$ are the usual Pauli operators for a single atom. The above operators close on the su(2) or angular momentum algebra

$$[J_+,J_-]=2J_3, \quad [J_3,J_{\pm}]=\pm J_{\pm}. \tag{2.2}$$

We denote the "angular momentum" states as $|J,M\rangle$, which in the present case are just the well-known Dicke states [4] and $J=N/2$ is the cooperation number. $|J,-J\rangle$ indicates a state for which all atoms are in the ground state while for $|J,J\rangle$ all atoms are in the excited state.

The SU(2) coherent states, or Bloch states, are defined as

$$|\zeta,J\rangle \equiv \exp(zJ_+ - z^*J_-)|J,-J\rangle = (1+|\zeta|^2)^{-J}\sum_{M=-J}^{J}\binom{2J}{J+M}^{1/2}\zeta^{J+M}|J,M\rangle, \tag{2.3}$$

where $z=\vartheta/2\exp(i\phi/2)$, $0\leq\vartheta\leq\pi$, $0\leq\phi\leq 2\pi$, and $\zeta=\tan(\vartheta/2)\exp(i\phi)$. Properties of the states have been discussed elsewhere [5,6,9] and we shall not review them at length here. However, we shall note that the states are in general nonorthogonal:

$$\langle\zeta',J|\zeta,J\rangle=\left[\frac{(1+\zeta'^*\zeta)^2}{(1+|\zeta'|^2)(1+|\zeta|^2)}\right]^J. \tag{2.4}$$

The probability distribution of the atoms in terms of the Dicke states $P_M^{(J)}$ is binomial:

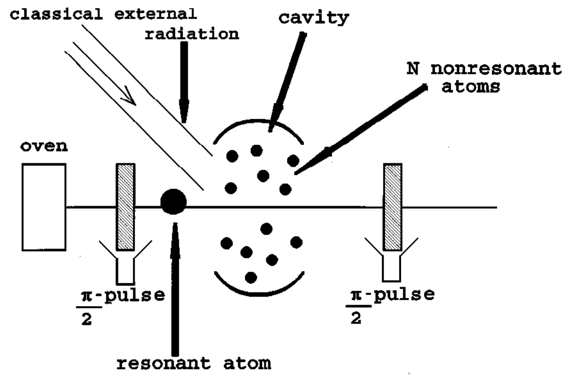

FIG. 1. Schematic of the experimental setup to generate the collective atomic Schrödinger-cat states $|\Psi_{\pm}\rangle$ [Eq. (2.18)]. The classical external field is resonant with the N two-level atoms and generates the Bloch state $|\zeta,J\rangle$.

$$P_M^{(J)} \equiv |\langle J,M|\zeta,J\rangle|^2 = (1+|\zeta|^2)^{-2J}\binom{2J}{J+M}|\zeta|^{2(M+J)}. \tag{2.5}$$

Arrechi *et al.* [6] have shown that these atomic coherent states contract to the usual harmonic oscillator states in the limit that $N\rightarrow\infty$. The above states can be generated by the application of a resonant classical driving field of a constant amplitude ε where the interaction Hamiltonian in the rotating frame and with the rotating-wave approximation is [10]

$$H_I = (i\kappa\varepsilon/2)(J_+ - J_-), \tag{2.6}$$

where κ is the dipole moment coupling constant of the atoms to the driving field. The SU(2) coherent state of Eq. (2.3) will be generated assuming all atoms are initially in their ground states.

In order to generate the desired superpositions of $|\zeta,J\rangle$ and $|-\zeta,J\rangle$ we consider a cavity supporting a single mode of frequency ω_c and containing N two-level atoms with a transition frequency ω_a as pictured in Fig. 1. We further assume that ω_c and ω_a are sufficiently different such that the cavity field and the atoms do not interact. We require another species of atoms whose states $|e\rangle$ and $|g\rangle$ are resonant with the cavity field. These atoms, after being properly prepared, are to be sent through the cavity, then after emerging are to be manipulated by a classical resonant field and then selectively ionized to generate the required states as explained in the following.

We start by assuming the cavity has been loaded with the N atoms, which are in the collective ground state $|J,-J\rangle$. An atom of the other type (we call it atom 1) resonant with the cavity field is prepared, by classical microwave fields, in the superposition $(|e_1\rangle+|g_1\rangle)/\sqrt{2}$ and the cavity field is assumed to be initially in the vacuum state $|0\rangle$. Thus our initial state for the combined system is

$$\frac{1}{\sqrt{2}}(|e_1\rangle+|g_1\rangle)|0\rangle|J,-J\rangle, \tag{2.7}$$

which upon passage of the single atom through the cavity becomes

$$\frac{1}{\sqrt{2}}[\cos(\Omega t)|e_1\rangle|0\rangle - i\,\sin(\Omega t)|g_1\rangle|1\rangle + |g_1\rangle|0\rangle]|J,-J\rangle, \tag{2.8}$$

where the standard Jaynes-Cummings interaction has been used and where Ω is the vacuum Rabi frequency. We next assume that atom 1 is velocity selected such that $\Omega t=\pi/2$, which results in

$$|\psi(t=\pi/2)\rangle = \frac{1}{\sqrt{2}}|g_1\rangle[-i|1\rangle+|0\rangle]|J,-J\rangle. \tag{2.9}$$

Then the detection (measurement) of the atom 1 projects the system into the state

$$\frac{1}{\sqrt{2}}[-i|1\rangle+|0\rangle]|J,-J\rangle. \tag{2.10}$$

Now simultaneously with the above procedure, or shortly after, an external classical field such as a laser, resonant with the frequency ω_a of the N atoms, by the interaction Hamiltonian of Eq. (2.6) drives the collective ground state $|J,-J\rangle$ into the atomic coherent state $|\zeta,J\rangle$ so that the system state is now

$$\frac{1}{\sqrt{2}}[-i|1\rangle+|0\rangle]|\zeta,J\rangle. \tag{2.11}$$

At this point a small electric field is applied inside the cavity to Stark shift the atomic levels of the N atoms to a transition frequency detuned enough from that of the cavity yielding a dispersive interaction described by the Hamiltonian [11]

$$H_I = \hbar\chi a^+ a J_3, \tag{2.12}$$

where χ depends on the square of the dipole moment and the inverse of the detuning. Through this interaction, the state of Eq. (2.11) evolves as

$$\exp(-iH_I t/\hbar)\frac{1}{\sqrt{2}}[-i|1\rangle+|0\rangle]|\zeta,J\rangle$$
$$= \frac{1}{\sqrt{2}}[-i|1\rangle\exp(-i\chi tJ)|\zeta\,\exp(i\chi t),J\rangle+|0\rangle|\zeta,J\rangle]. \tag{2.13}$$

We assume that the electric field is turned off at the time when $\chi t=\pi$ such that our state is then

$$\frac{1}{\sqrt{2}}[-i|1\rangle\exp(-i\pi J)|-\zeta,J\rangle+|0\rangle|\zeta,J\rangle]. \tag{2.14}$$

Now, a second resonant atom, in the ground state $|g_2\rangle$, is sent through the cavity resulting in

$$\frac{1}{\sqrt{2}}\{-i[\cos(\Omega t)|1\rangle|g_2\rangle - i\,\sin(\Omega t)|0\rangle|e_2\rangle]$$
$$\times\exp(-i\pi J)|-\zeta,J\rangle+|g_2\rangle|0\rangle|\zeta,J\rangle\}. \tag{2.15}$$

We again assume that $\Omega t=\pi/2$ so that we have

TABLE I. Sequence of interactions to generate the collective atomic Schrödinger-cat states $|\Psi_{\pm}\rangle$.

(1) $|\psi(t=0)\rangle=\frac{1}{\sqrt{2}}(|e_1\rangle+|g_1\rangle)|0\rangle|J,-J\rangle$	
$\downarrow$	After passing atom 1 through cavity in time $\Omega t=\pi/2$ and detection of $|g_1\rangle$
(2) $\frac{1}{\sqrt{2}}[-i|1\rangle+|0\rangle]|J,-J\rangle$	
$\downarrow$	external driving field creates a Bloch state
(3) $\frac{1}{\sqrt{2}}[-i|1\rangle+|0\rangle]|\zeta,J\rangle$	
$\downarrow$	Stark shift for the N atoms so that the interaction Hamiltonian is $H_I=\hbar\chi a^+ aJ_3$ up to the time $\chi t=\pi$
(4) $\frac{1}{\sqrt{2}}[-i|1\rangle\exp(-i\pi J)|-\zeta,J\rangle+|0\rangle|\zeta,J\rangle]$	
$\downarrow$	send in a second atom in state $|g_2\rangle$ for time $\Omega t=\pi/2$
(5) $\frac{1}{\sqrt{2}}[-|e_2\rangle\exp(-i\pi J)|-\zeta,J\rangle+|g_2\rangle|\zeta,J\rangle]|0\rangle$	
$\downarrow$	$\pi/2$ pulse on the atom
(6) $\frac{1}{2}[|g_2\rangle\{|\zeta,J\rangle-\exp(-i\pi J)|-\zeta,J\rangle\}-|e_2\rangle\{|\zeta,J\rangle+\exp(-i\pi J)|-\zeta,J\rangle\}]$	
$\downarrow$	detection of $|e_2\rangle$ yields
(7) $|\Psi_+\rangle\equiv\mathcal{N}_+[|\zeta,J\rangle+\exp(-i\pi J)|-\zeta,J\rangle]$	
	or of $|g_2\rangle$ yields
$|\Psi_-\rangle\equiv\mathcal{N}_-[|\zeta,J\rangle-\exp(-i\pi J)|-\zeta,J\rangle]$	

$$\frac{1}{\sqrt{2}}[-|e_2\rangle\exp(-i\pi J)|-\zeta,J\rangle+|g_2\rangle|\zeta,J\rangle]|0\rangle, \tag{2.16}$$

where the cavity field is again in the vacuum. Finally, as this second atom emerges from the cavity it is subjected to $\pi/2$ pulse by a resonant classical field producing the transformations $|e_2\rangle\to(|e_2\rangle+|g_2\rangle)/\sqrt{2}$ and $|g_2\rangle\to(|g_2\rangle-|e_2\rangle)/\sqrt{2}$. This results in

$$\tfrac{1}{2}[|g_2\rangle\{|\zeta,J\rangle-\exp(-i\pi J)|-\zeta,J\rangle\}-|e_2\rangle \times\{|\zeta,J\rangle+\exp(-i\pi J)|-\zeta,J\rangle\}]|0\rangle. \tag{2.17}$$

Detection of the atom in state $|e_2\rangle$ projects the N atoms into the collective atomic cat state

$$|\Psi_+\rangle\equiv\mathcal{N}_+[|\zeta,J\rangle+\exp(-i\pi J)|-\zeta,J\rangle] \tag{2.18a}$$

and detection in state $|g_2\rangle$ produces the atomic state

$$|\Psi_-\rangle\equiv\mathcal{N}_-[|\zeta,J\rangle-\exp(-i\pi J)|-\zeta,J\rangle]. \tag{2.18b}$$

The real normalization factors are given as

$$\mathcal{N}_{\pm}\equiv\frac{1}{\sqrt{2}}\left[1+\cos(\delta_{\pm}-\pi J)\left(\frac{1-|\zeta|^2}{1+|\zeta|^2}\right)^{2J}\right]^{-1/2}, \tag{2.19}$$

and where $\delta_+\equiv 0$ and $\delta_-\equiv\pi$. In the case where the cooperation number J is an even integer the states $|\Psi_{\pm}\rangle$ are the analog of the even and odd coherent states, respectively, of the field [12]. This is also true in the case when J is an odd integer except that $|\Psi_{\pm}\rangle$ are odd and even, respectively. On the other hand, if J is half odd integer the states are analogous to the Yurke-Stoler states of the field [13] and the quantum distribution of the Dicke states remains the same as for the atomic coherent state. The sequence for generating these states is summarized in Table I.

The manner in which the superposition states above are formed has some similarity to the method for forming even and odd field coherent states in the context of cavity QED [14]. As in that case, we must be concerned about the relevant decay and decoherence of the cavity field and the atomic states. The cavity lifetime $T_{\rm cav}=Q/\omega_c$ is of the order of 10^{-2} sec for a cavity with Q factor 10^8 and of frequency $\omega_c/2\pi\sim 50$ GHz for Rydberg atoms of high angular momentum quantum number with principal quantum number $n\cong 50$, the radiative life times $T_{\rm rad}$ are typically also on the order of 10^{-2} sec. For a collection of N atoms their energy would be radiatively damped in a time $T_{\rm rad}/N$, thus limiting the number of atoms that could sustain an atomic coherent state during the manipulations required from the superpositions although the lack of resonant cavity mode into which to decay could possibly increase the lifetime of the states. In the

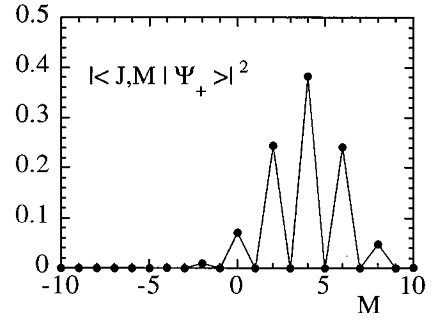

FIG. 2. The distribution of collective atomic cat states $|\Psi_+\rangle$ [Eq. (2.18)] over the Dicke states $|\langle J,M|\Psi_+\rangle|^2$ for the case with $N=20$ and $\zeta=1.5$.

balance of this paper we shall ignore the issue of decay and decoherence of our states but rather we shall study some of their properties.

III. PROPERTIES OF THE ATOMIC CAT STATES

We have already mentioned that for J an even or odd integer, the states of Eq. (2.18) are analogous to the even and odd coherent field state. The latter states are characterized by oscillations in the photon number distributions that can be explained as resulting from interference in phase space. For the atomic cat states, a similar interpretation can be given for the oscillations in the atomic population, but the phase space is in this case that of the Bloch sphere. For example, in the case J is an even integer, the distribution over the Dicke states for $|\Psi_+\rangle$ is given as

$$P_M^{(J)} \equiv |\langle J,M|\Psi_+\rangle|^2 = |\mathcal{N}_+|^2(1+|\zeta|^2)^{-2J}\binom{2J}{J+M}|\zeta|^{2(M+J)}[1+(-1)^M]^2, \tag{3.1}$$

which vanishes for M odd. Thus the Dicke states for M odd are removed from the superposition. This is like a binomial distribution but every other term is missing. For $\zeta=1$, $P_M^{(J)}$ is centered around $M=0$ and this center shifts towards $M=J$ with increasing ζ and towards $M=-J$ for decreasing ζ. In Fig. 2 we plot $P_M^{(J)}$ for the case with $N=20$, $J=10$ where we note the oscillations in the distribution. Similar oscillating distributions are also known to occur for the atomic squeezed states [15] and this has recently been interpreted in terms of interfering ''pathways'' [16].

These oscillations are indicative of correlations between the atoms. The Dicke states $|J,M\rangle$ are highly ''nonclassical'' in the same sense as for the number states of a quantized field. In the latter case the expectation value of the electric field vanishes whereas in the former, the expectation value of the total dipole moment vanishes. This in turn is related to the correlations between the atoms. For the Dicke states the atoms are highly correlated. In contrast, for the Bloch states, the atoms are uncorrelated [6] but it seems that for superposition states of the type in Eq. (2.18) with J an even or odd integer, correlations between the atoms can exist.

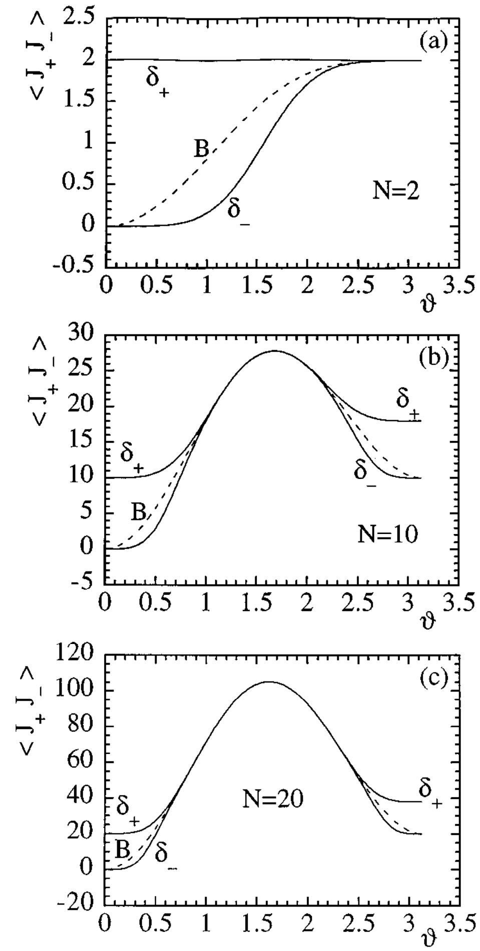

FIG. 3. The intensity for spontaneous emission $\langle J_+J_-\rangle$ for the Bloch states $|\zeta,J\rangle(B)$ [Eq. (2.3)] and the collective atomic cat states $|\Psi_\pm\rangle$ as a function of ϑ. The parameter ζ is chosen to be real ($\phi=0$). (a) $N=2$ atoms. (b) $N=10$ atoms. (c) $N=20$ atoms.

As a simple illustrative example, consider the case of two atoms, $N=2$. The usual Dicke states in this case are ($|J=1,\ M=1,0,-1\rangle$)

$$|1,1\rangle = |e_1e_2\rangle,$$
$$|1,0\rangle = \frac{1}{\sqrt{2}}[|e_1g_2\rangle + |g_1e_2\rangle], \tag{3.2}$$
$$|1,-1\rangle = |g_1g_2\rangle,$$

where $e_{1,2}$ ($g_{1,2}$) denote the excited (ground) states of atoms 1 and 2, respectively (not to be confused with the atoms used to form the states in Sec. II). The associated Bloch state has the form

$$|\zeta,1\rangle = (1+|\zeta|^2)^{-1}[|g_1g_2\rangle + \zeta(|e_1g_2\rangle + |g_1e_2\rangle) + \zeta^2|e_1e_2\rangle], \tag{3.3}$$

which is factorizable into the form $|\Psi_1\rangle|\Psi_2\rangle$, where

$$|\Psi_1\rangle = (1+|\zeta|^2)^{-1/2}[|g_1\rangle + \zeta|e_1\rangle],$$
$$|\Psi_2\rangle = (1+|\zeta|^2)^{-1/2}[|g_2\rangle + \zeta|e_2\rangle], \tag{3.4}$$

and thus the atoms are uncorrelated [17]. Then, up to normalization factors, the superposition states are

$$|\zeta,1\rangle+|-\zeta,1\rangle\sim|g_1g_2\rangle+\zeta^2|e_1e_2\rangle, \tag{3.5a}$$

$$|\zeta,1\rangle-|-\zeta,1\rangle\sim|e_1g_2\rangle+|g_1e_2\rangle. \tag{3.5b}$$

The latter state is completely correlated for all ζ while the former is maximally correlated (or entangled) for $|\zeta|=1$. Both states are of the type that violate Bell's inequalities in Bohm's version of the Einstein, Podolsky, and Rosen experiment [18]. Obviously, in the limits $|\zeta|\rightarrow 0$ or $|\zeta|\rightarrow\infty$, the state of Eq. (3.5a) becomes disentangled.

For the cases when J is a half-odd integer, the type of interference discussed above does not occur and it appears, as we show below, that these states have properties almost, but not quite, identical to the Bloch states.

We first consider spontaneous emission. The intensity of spontaneous emission for an arbitrary N-atom state $|\Psi\rangle$ is given by [6]

$$I_\Psi=\langle\Psi|J_+J_-|\Psi\rangle I_0, \tag{3.6}$$

where I_0 is the intensity for one atom. For the Dicke state $|\Psi\rangle=|J,M\rangle$ the emission intensity is $I_D=(J+M)(J-M+1)I_0$ and, of course for $M\cong 0$ one has $I_D\cong J^2I_0$, the well-known Dicke superradiance. For a Bloch state $|\zeta,J\rangle$ we obtain [6]

$$I_B=\frac{4J^2|\zeta|^2+2J|\zeta|^4}{(1+|\zeta|^2)^2}\,I_0=[J^2\sin^2\vartheta+2J\,\sin^2(\vartheta/2)]I_0, \tag{3.7}$$

which is also superradiant for $\vartheta\cong\pi/2$ ($|\zeta|\cong 1$). But for our cat states $|\Psi_\pm\rangle$ we have

$$\begin{aligned}I_C&=\langle\Psi_\pm|J_+J_-|\Psi_\pm\rangle I_0\\&=|\mathcal{N}_\pm|^2\{[2J^2\sin^2\vartheta+4J\,\sin^2(\vartheta/2)]+\cos(\delta_\pm-\pi J)[-2J^2\sin^2\vartheta+4J\,\sin^2(\vartheta/2)]\cos^{(2J-2)}\vartheta\,\cos^4(\vartheta/2)\}I_0.\end{aligned} \tag{3.8}$$

For the case where $\vartheta\cong\pi/2$ we again obtain a superradiant state owing to the fact that the states $|\zeta,J\rangle$ and $|-\zeta,J\rangle$ are nearly orthogonal. But for other values of ϑ it is possible to have spontaneous emission intensities that are quite different than those of the Bloch states as illustrated in Fig. 3, where we plot $\langle J_+J_-\rangle$ as a function of ϑ for various N for both Bloch and cat states. For J a half-odd integer, the spontaneous emission is the same as for Bloch states [note that $\cos(\delta_\pm-\pi J)=0$ for J half-odd integer]. Consider the case for two atoms ($N=2,J=1$). With $\delta_-\equiv\pi$, the emission is weaker than for the corresponding Bloch state. For $\delta_+\equiv 0$ the emission is constant with ϑ. This has a simple explanation. As discussed previously, due to the interference, the state $|\Psi_+\rangle$ contains only the Dicke state $|1,0\rangle$ and so $|\Psi_+\rangle$ is independent of ϑ and this state is superradiant. For the case with $N=10$ ($J=5$) we see that for $|\Psi_-\rangle$ the intensity is also lower than the Bloch state but is higher for $|\Psi_+\rangle$ except near $\vartheta\cong\pi/2$. But for $N=20$ ($J=10$) we find the situation reversed, that is, for $|\Psi_-\rangle$ the intensity is higher than for the corresponding Bloch state and lower for $|\Psi_-\rangle$. This is simply due to the change in the sign of the factor $\exp(-i\pi J)$ in Eqs. (2.18). It is of interest that the greatest rate of spontaneous emission occurs when $|\zeta|=1$ where the states $|\zeta,J\rangle$ and $|-\zeta,J\rangle$ are orthogonal.

We next examine the stimulated emission rate, which is proportional to the atomic inversion: $2\langle J_3\rangle$. For the Dicke states the inversion is simply $2M$ and for the Bloch states $-2J\,\cos\vartheta$ [6]. But for the cat states we obtain

$$2\langle J_3\rangle=-4|\mathcal{N}_\pm|^2[\cos\vartheta+\cos(\delta_\pm-\pi J)\cos^{(2J-1)}\vartheta]. \tag{3.9}$$

Thus, for example, with $\delta_+\equiv 0$ and J an integer, the stimulated emission rate will be enhanced or diminished if J is even or odd, respectively. If J is half-odd integer, the rate will be unaffected by the superposition. In Fig. 4 we plot $\langle J_3\rangle$ as a function of ϑ for $N=2$, 10, and 20. We find that for $N=2$ and 10 ($J=1$ and 5, respectively) the stimulated emission is lower than for the Bloch states for $\vartheta<\pi/2$ but higher for $\vartheta>\pi/2$ for the state $|\Psi_-\rangle$. But for $|\Psi_+\rangle$ the opposite is true (for $N=2$, there is no stimulated emission in this case). For $N=20$ ($J=10$), all of this is reversed with respect to the states $|\Psi_\pm\rangle$.

The total dipole moment of the collective atomic system is given by [6]

$$\mathbf{D}=\mathbf{p}\,\exp(i\omega t)J_++\mathbf{p}^*\exp(-i\omega t)J_-. \tag{3.10}$$

For the Dicke states one obtains $\langle J,M|\mathbf{D}|J,M\rangle=0$ whereas for the Bloch states [6] the dipole moment is

$$\langle\zeta,J|\mathbf{D}|\zeta,J\rangle=J\,\sin\vartheta(\mathbf{p}\,\exp(i\omega t+i\phi)+\mathbf{p}^*\times\exp(-i\omega t-i\phi)). \tag{3.11}$$

The Bloch state is characterized by a macroscopic dipole moment that is able to radiate classically. However, for the atomic cat states we obtain

$$\begin{aligned}\langle\Psi_\pm|\mathbf{D}|\Psi_\pm\rangle=&4J\,\sin(\delta_\pm-\pi J)|\mathcal{N}_\pm|^2\sin\vartheta\,\cos^{(2J-2)}\vartheta\\&\times(\mathbf{p}\,\exp(i\omega t+i\phi+i\pi/2)\\&+\mathbf{p}^*\exp(-i\omega t-i\phi-i\pi/2)).\end{aligned} \tag{3.12}$$

Thus, for example, with $\delta_+\equiv 0$ and J an integer, the dipole moment will vanish as a result of interference.

Note that even though the dipole moment vanishes in this case, the state is still superradiant. One might be tempted to imagine the situation as being one in which there is a superposition of two macroscopic dipole moments radiating out of phase by 180° and thus canceling each other's field contrary to the predictions of Eq. (3.8). However, the superposition of Bloch states involves correlations of the atomic states and it

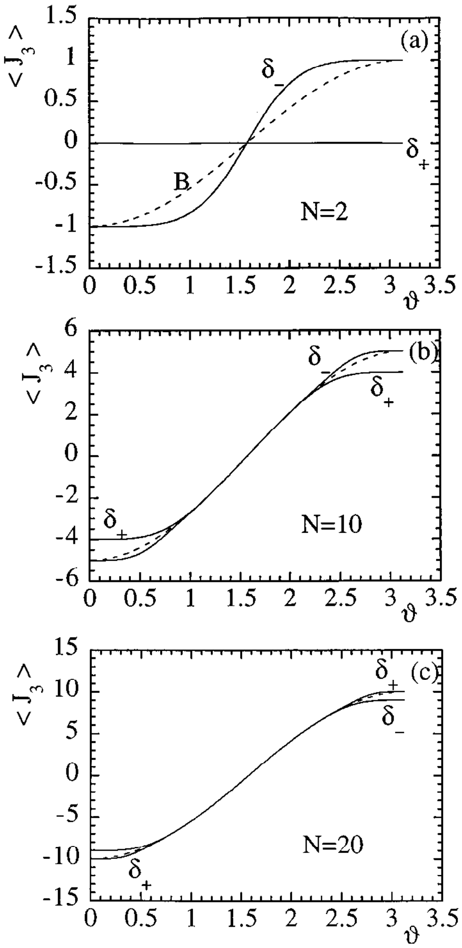

FIG. 4. The intensity for stimulated emission $\langle J_3\rangle$ for the Bloch states $|\zeta,J\rangle(B)$ [Eq. (2.3)] and the collective atomic cat states $|\Psi_\pm\rangle$ as a function of ϑ. The parameter ζ is chosen to be real $(\phi=0)$. (a) $N=2$ atoms. (b) $N=10$ atoms. (c) $N=20$ atoms.

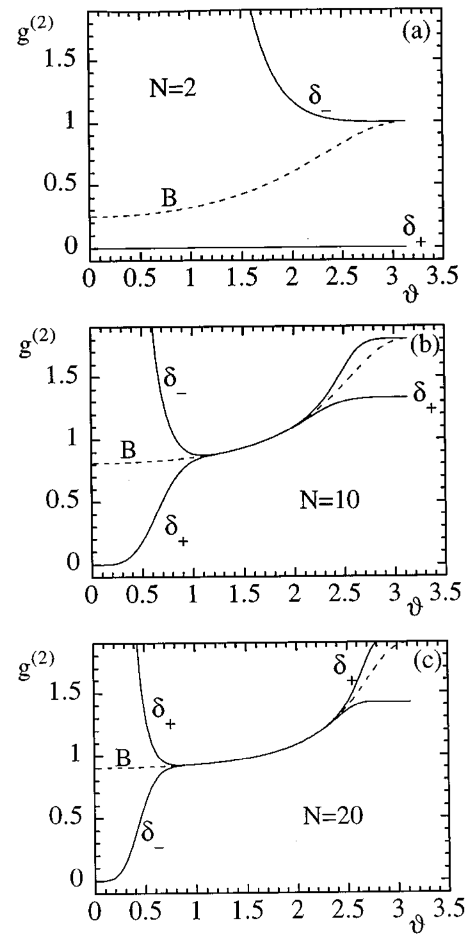

FIG. 5. The normalized second-order correlation function $g^{(2)}$ [Eq. (3.14)] for the Bloch states $|\zeta,J\rangle(B)$ [Eq. (2.3)] and the collective atomic cat states $|\Psi_\pm\rangle$ as a function of ϑ. The parameter ζ is chosen to be real $(\phi=0)$. (a) $N=2$ atoms. (b) $N=10$ atoms. (c) $N=20$ atoms.

is this that is responsible for the vanishing of the dipole. Recall that the Dicke states are also correlated atomic states for which the dipole moment vanishes but they are still superradiant. However, the emitted radiation is incoherent [19] whereas the superradiant Bloch states emit coherent radiation. In the present case, because of the vanishing of the dipole moment we expect the emitted radiation to again be incoherent. For J a half-odd integer, the dipole moment will not generally vanish but does oscillate 90° out of phase with that of the Bloch state. This seems to be the only effect for the superpositions with an odd number of atoms that differentiates them from the Bloch states.

The coherence properties of the states $|\Psi_\pm\rangle$ can be characterized by the normalized second-order correlation function

$$g^{(2)}\equiv\frac{\langle J_+J_+J_-J_-\rangle}{\langle J_+J_-\rangle^2}. \tag{3.13}$$

For the Bloch states one has [20]

$$g^{(2)}=\frac{2J-1}{J}\,\frac{\tan^4(\vartheta/2)+(2J-1)\tan^2(\vartheta/2)+J(2J-1)}{[\tan^2(\vartheta/2)+2J]^2}, \tag{3.14}$$

which has the limits $g^{(2)}=0$ for one atom $(J=1/2)$ and $g^{(2)}\cong 1$ for many atoms $(J\gg 1)$. In the former case we see the dipole operators exhibit the antibunching properties of the fluorescent light while in the latter the dipole operators are coherent. The results for the cat states are displayed in Fig. 5. We find for $N=2$ and 10 that $g^{(2)}$, for ϑ away from $\pi/2$, is lower than for the Bloch states for the states $|\Psi_+\rangle$ thus exhibiting the antibunching effect for $\vartheta<\pi/2$. For $|\Psi_-\rangle$ we generally obtain $g^{(2)}>0$, which is the bunching effect. Again for $N=20$, these properties are reversed for the $|\Psi_\pm\rangle$ states.

Finally we examine the possibility that the atomic Schrödinger-cat states exhibit squeezed fluctuations. One way to define squeezing for the atomic system is in terms of angular momentum uncertainty relations $\Delta J_1\Delta J_2\geqslant|\langle J_3\rangle|/2$ where $J_1=(J_++J_-)/2$ and $J_2=(J_+-J_-)/2i$. Squeezing then exists whenever $\Delta J_i\leqslant|\langle J_3\rangle/2|^{1/2}$ for $i=1$ or 2 [10]. The Bloch states possess squeezing of this sort [19]. Elsewhere [7], we have shown that such squeezing can also be present for the two-mode SU(2) cat states and is thus present in the atomic cat states as well, at least for the cases where J is integer. However, it has been pointed out by Kitagawa and Ueda [15] that the above definition of squeezing does not identify a standard quantum limit to be overcome and, furthermore, the Bloch states may be squeezed simply as a result of being placed in a particular coordinate system. It has been shown by Wineland *et al.* [15] that the parameter

$$\xi=(2J)^{1/2}(\Delta J_i^{\perp})^2/\langle J_i\rangle^2, \tag{3.15}$$

where $J_i^{\perp}$ is a spin component perpendicular to J_i, has a minimum value of 1 for all Bloch states. A squeezed spin state is then defined as a state for which $\xi<1$ for some component of the spin. This definition of squeezing is very useful in the context of Ramsey spectroscopy [15]. In our case, we have found squeezing in the ''2'' direction; that is, if

$$\xi=(2J)^{1/2}(\Delta J_2)^2/\langle J_3\rangle^2, \tag{3.16}$$

then $\xi<1$ for some values of ϑ, as indicated in Fig. 6 for the case with $N=20$. Only for $\delta=0$ do we obtain spin squeezing according to this definition.

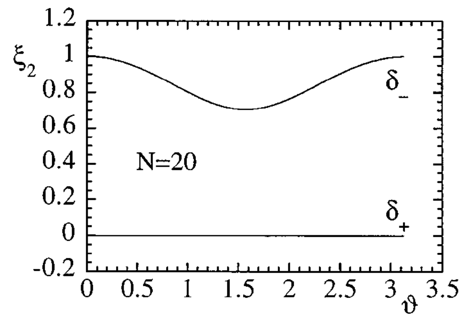

FIG. 6. The degree of squeezing ξ in the ''2'' direction [Eq. (3.16)] for the collective atomic cat states $|\Psi_{\pm}\rangle$ as a function of $\vartheta[N=20]$.

IV. CONCLUSION

In this paper we have proposed a cavity QED based method of generating collective atomic Schrödinger-cat states consisting of superpositions of Bloch states. We have then examined a number of their properties and have shown that they may exhibit enhanced or reduced rates of spontaneous and stimulated emission rates and a vanishing dipole moment resulting from the interference of the two Bloch state components. We examined the coherence properties of the states and found that they can display the properties of antibunching as well as bunching. Finally, we showed that, in some cases, it is possible to obtain ''spin'' squeezing for the superposition states. In this paper we have ignored dissipative effects assuming that the time required for their formation is short compared to the relevant decay times. After being formed, the states would rapidly decohere, a process described by a master equation involving the angular momentum operators in much the same way that decoherence is brought about for the usual cat state of a single mode field. We plan to discuss this decoherence elsewhere.

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