Electromagnetically induced transparency with coherent and stochastic fields

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(Received 17 October 1996; revised manuscript received 14 May 1997)

This paper deals with electromagnetically induced transparency (EIT), i.e., a significant suppression of absorption at a material transition (in the form of a dip in an absorption spectrum), due to a strong laser field, coupling the excited level with an auxiliary excited (unpopulated) level. A comprehensive theory of EIT is developed for the cases of coherent and Markovian phase- and amplitude-phase-fluctuating coupling fields. For a coherent coupling field, a shift of the absorption minimum from the two-photon resonance is revealed in the off-resonance case. Two models of amplitude-phase fluctuating fields are considered: the chaotic field and the uncorrelated-jump field. Closed analytical expressions for EIT line shape are derived and exact limits of different regimes of EIT are obtained, the emphasis being on the near-resonance case. The main conclusion is that an amplitude-phase-fluctuating field can induce significant transparency, though reduced in comparison to a phase-fluctuating field of the same average intensity and bandwidth. EIT decreases with the increase of the bandwidth for all stochastic models considered. EIT with a chaotic field is generally less pronounced than EIT with an uncorrelated-jump field of the same intensity and bandwidth, the difference increasing with the field intensity. The possibility of experimental verification of the results obtained is discussed.
[S1050-2947(97)03709-8]

PACS number(s): 42.50.Gy, 42.50.Hz, 42.60.Mi, 42.62.Fi

I. INTRODUCTION

A great interest was shown recently in resonance optical phenomena in multilevel systems, which involve quantum interference [1]. The phenomena, which have been discussed, include lasing without inversion (LWI) [2,3], electromagnetically induced transparency (EIT) [4–8], enhancement of the index of refraction [9], and steep dispersion [10]. Of the above effects, EIT and, more generally, electromagnetically modified absorption (EMA) [11–14] are conceptually the simplest ones since they practically do not involve transitions between atomic levels: the strong field couples *empty* levels. As a result, the spectral manifestations of dressed states and their interference are unblurred by population effects in observations of EIT and EMA, in contrast, say, to LWI. Thus a good understanding of EIT and EMA, important by itself, can help to attain a deeper insight into more complicated phenomena. The study of EIT has also a practical significance. A number of applications of EIT have already been suggested, including second-harmonic generation [15], four-wave mixing [16], control of optical bistability [17], and isotope discrimination [18]. Other fields of current interest, which have much in common with EIT and EMA, are laser-induced continuum structures [19] and coherent control of chemical reactions [20].

Observations of the above processes would require strong laser fields, which are often stochastic. The effects of laser noise depend generally on the type of random modulation of the field. In particular, effect of phase diffusion of the coupling field on EIT can be readily taken into account [5]. In contrast, effects of amplitude-phase fluctuations are poorly known and a failure of attempts to produce EIT with a multimode laser was reported [4]. Vemuri *et al.* [3] demonstrated by numerical simulations that a chaotic coupling field can be used to produce lasing without inversion. This result cannot be readily extended to EIT due to the above differences between the two phenomena. The main purpose of this paper is to show the possibility of inducing transparency by amplitude-phase-fluctuating fields, the emphasis being made on the chaotic field model. A comprehensive theory of EIT with coherent, phase-fluctuating, and chaotic coupling fields, which allows for proper dephasing and arbitrary field bandwidth and detuning, is developed below. The analytic approach adopted here has allowed us to obtain explicit expressions for the EIT line shape as well as the exact limits for EIT and its different regimes.

The paper is organized as follows. In Sec. II the general expressions for electromagnetically modified susceptibility are derived for two atom-laser coupling schemes, under rather general assumptions about the atomic relaxation. In Sec. III a simple expression for EIT and the exact validity condition of EIT are obtained for the case of a coherent coupling field. These results are extended to the case of a Markovian phase-fluctuating field in Sec. IV. The remaining part of the paper is devoted to an amplitude-phase-fluctuating coupling field. Section V treats the case of a narrow-band coupling field with the Rayleigh intensity distribution. The effects of the field bandwidth are studied for two models: the chaotic field (Sec. VI) and the uncorrelated-jump model (Sec. VII). Section VIII presents numerical results and discussion. Section IX provides concluding remarks. The Appendixes contain details of the calculations.

II. GENERAL FORMALISM

In the previous treatments of EIT and EMA various specific three-level schemes were studied. In this section we consider EMA and EIT under rather general assumptions.

We assume that a weak probe field $\mathcal{E}_p(t) = E_p(t)e^{-i\omega t} + \text{c.c.}$ is nearly resonant to the transition from the ground state $|1\rangle$ to an excited state $|2\rangle$, which is coupled by a strong laser field $\mathcal{E}_c(t) = E_c(t)e^{-i\omega_c t} + \text{c.c.}$ to an auxil-

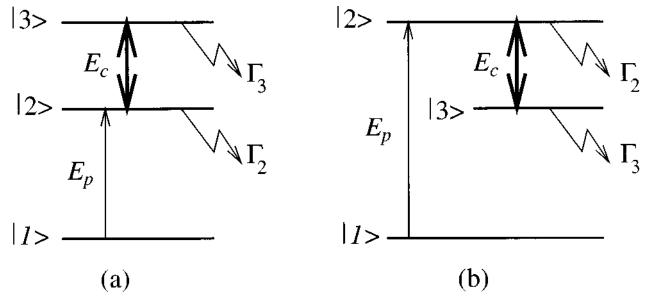

FIG. 1. Schemes of the atom-laser coupling: (a) ladder scheme and (b) Λ scheme.

iary excited state $|3\rangle$. The complex amplitudes $E_p(t)$ and $E_c(t)$ of the probe and coupling fields are assumed to vary slowly with time. Two cases of atom-laser coupling, the ladder and Λ schemes (Fig. 1), are considered. The three-level systems considered can be closed or open. Their relaxation, which can be induced by the field reservoir, matter, and/or configuration interaction in the atom [1], is described phenomenologically by arbitrary relaxation constants. The energies of the levels $|1\rangle$, $|2\rangle$, and $|3\rangle$ are ϵ_1, ϵ_2, and ϵ_3, respectively. Consider first the ladder scheme $\epsilon_2 < \epsilon_3$.

Equations for density matrix elements ρ_{ij} of the three-level atom in the rotating-wave approximation are written for the case of the ladder scheme as

$$\dot{\rho}_{11} = 2\,\mathrm{Im}[V_p^*(t)\sigma_{21}] + \Gamma_2'\rho_{22} + \Gamma_3'\rho_{33},$$

$$\dot{\rho}_{22} = 2\,\mathrm{Im}[V_c^*(t)\sigma_{32} - V_p^*(t)\sigma_{21}] - \Gamma_2\rho_{22} + \Gamma_3''\rho_{33},$$

$$\dot{\rho}_{33} = -2\,\mathrm{Im}[V_c^*(t)\sigma_{32}] - \Gamma_3\rho_{33}, \tag{2.1}$$

$$\dot{\sigma}_{21} = (i\Delta - \Gamma)\sigma_{21} + iV_p(t)(\rho_{22} - \rho_{11}) - iV_c^*(t)\sigma_{31},$$

$$\dot{\sigma}_{31} = (i\Delta' - \Gamma')\sigma_{31} + iV_p(t)\sigma_{32} - iV_c(t)\sigma_{21},$$

$$\dot{\sigma}_{32} = (i\Delta_c - \Gamma_{32})\sigma_{32} + iV_p^*(t)\sigma_{31} + iV_c(t)(\rho_{33} - \rho_{22}).$$

Here $\sigma_{21} = \rho_{21}e^{i\omega t}$, $\sigma_{31} = \rho_{31}e^{i(\omega+\omega_c)t}$, and $\sigma_{32} = \rho_{32}e^{i\omega_c t}$ are the density matrix elements in the rotating frame, $\sigma_{ij} = \sigma_{ji}^*$,

$$V_p(t) = -d_{21}E_p(t)/\hbar, \quad V_c(t) = -d_{32}E_c(t)/\hbar, \tag{2.2}$$

$\hbar$ is the Planck's constant, d_{ij} is a matrix element of the atom dipole moment,

$$\Delta = \omega - \omega_{21}, \quad \Delta' = \omega - \omega_{2p}, \quad \Delta_c = \omega_c - \omega_{32}, \tag{2.3}$$

Δ and Δ_c are, respectively, the probe and coupling field detunings, Δ' is the detuning from the two-photon resonance, $\omega_{2p} = \omega_{31} - \omega_c$ and $\omega_{ij} = (\epsilon_i - \epsilon_j)/\hbar$ $(i,j = 1,2,3)$; Γ_2 and Γ_3 are the decay rates of the states $|2\rangle$ and $|3\rangle$, respectively; Γ_2', Γ_3', and Γ_3'' are the rates of reservoir-induced transitions $|2\rangle \to |1\rangle$, $|3\rangle \to |1\rangle$, and $|3\rangle \to |2\rangle$, respectively (for a closed system $\Gamma_2 = \Gamma_2'$ and $\Gamma_3 = \Gamma_3' + \Gamma_3''$); and Γ, Γ', and Γ_{32} are the damping rates of ρ_{21}, ρ_{31}, and ρ_{32}, respectively. Assuming that the fields $\mathcal{E}_p(t)$ and $\mathcal{E}_c(t)$ are turned on at $t = 0$, the initial conditions for Eqs. (2.1) are

$$\rho_{ij}(0) = \delta_{1i}\delta_{1j}, \tag{2.4}$$

δ_{ij} being the Kronecker symbol.

The probe field induces the dipole moment per unit volume $P(t)e^{-i\omega t} + P^*(t)e^{i\omega t}$. Here

$$P(t) = N\omega d_{12}\sigma_{21}(t), \tag{2.5}$$

where N is the number of the atoms under consideration per unit volume. The solution of Eqs. (2.1) can be obtained perturbatively. In the zeroth order in E_p Eqs. (2.1) with the initial condition (2.4) yield $\rho_{ij}(t) = \rho_{ij}^{(0)}(t) = \delta_{1i}\delta_{1j}$. Using the latter values, in the first order in E_p one obtains the equations [21]

$$\dot{\sigma}_{21} = (i\Delta - \Gamma)\sigma_{21} - iV_c^*(t)\sigma_{31} - iV_p(t),$$

$$\dot{\sigma}_{31} = (i\Delta' - \Gamma')\sigma_{31} - iV_c(t)\sigma_{21}, \tag{2.6}$$

with the initial conditions $\sigma_{12}(0) = \sigma_{13}(0) = 0$. Formally solving Eqs. (2.6) for σ_{21} and inserting the result into Eq. (2.5) yields

$$P(t) = \frac{iN|d_{21}|^2}{\hbar}\int_0^t dt'\,\psi_a(t,t')E_p(t'). \tag{2.7}$$

Here $\psi_a(t,t')$ is a component of the vector

$$\psi = \begin{pmatrix} \psi_a \\ \psi_b \end{pmatrix}, \tag{2.8}$$

which as a function of t obeys the equation

$$\dot{\psi} = A(t)\psi, \tag{2.9}$$

with the initial conditions $\psi_a(t',t') = 1, \psi_b(t',t') = 0$. In Eq. (2.9)

$$A(t) = \begin{pmatrix} i\Delta - \Gamma & -iV_c^*(t) \\ -iV_c(t) & i\Delta' - \Gamma' \end{pmatrix}. \tag{2.10}$$

If the coupling field is a stationary random process, then $\overline{\psi(t,t')} = \overline{\psi(t-t')}$, where $\overline{\psi(t)}$ is the average over the field fluctuations of the solution $\psi(t)$ of Eq. (2.9) with the initial condition $\psi_a(0) = 1$ and $\psi_b(0) = 0$. In this case, as follows from Eq. (2.7), the average polarization amplitude is

$$\overline{P(t)} = \frac{iN|d_{21}|^2}{\hbar}\int_0^t dt'\,\overline{\psi_a}(t-t')E_p(t'). \tag{2.11}$$

Henceforth we assume that the probe field has a constant amplitude $E_p(t) = E_p$. Then for sufficiently long times Eq. (2.11) yields $\overline{P} = \overline{\chi(\omega)}E_p$, where the average susceptibility

$$\overline{\chi(\omega)} \equiv \overline{\chi'} + i\overline{\chi''} = \frac{iN|d_{21}|^2}{\hbar}\int_0^\infty dt\,\overline{\psi_a}(t). \tag{2.12}$$

In the case of a stochastic coupling field, assuming for simplicity that the sample is optically thin, the transmitted intensity of the probe field is attenuated by the factor $1 - \overline{\alpha}L$ ($\overline{\alpha}L \ll 1$), where L is the sample length along the direction of the probe propagation and $\overline{\alpha}$ is the average absorption coefficient. For a sufficiently dilute medium (i.e.,

$|\overline{\chi}|\ll 1)$, $\overline{\alpha}=(4\pi\omega/c)\overline{\chi''}$, where c is the vacuum speed of light. The latter equation and Eq. (2.12) yield

$$\overline{\alpha(\omega)}=K\mathrm{Re}\int_0^\infty \overline{\psi_a}(t)dt, \tag{2.13}$$

where $K=4\pi N\omega|d_{21}|^2/\hbar c=3\pi c^2 N\Gamma_2'/\omega^2$. Note that Eqs. (2.11)–(2.13) hold also for a coherent coupling field with a constant amplitude $E_c(t)=E_c$ if one drops the averaging in Eqs. (2.11)–(2.13).

Turning to the case of the Λ scheme $\epsilon_2>\epsilon_3$, one can show, on writing equations for the density matrix elements similar to Eq. (2.1), that Eqs. (2.7)–(2.13) remain valid upon the substitutions

$$\omega_c\rightarrow-\omega_c,\quad V_c(t)\leftrightarrow V_c^*(t). \tag{2.14}$$

Henceforth, for definiteness, the ladder system is discussed.

The above results (2.7) and (2.11)–(2.13) hold if the pumping of the excited levels $|2\rangle$ and $|3\rangle$ by the fields $\mathcal{E}_p(t)$ and $\mathcal{E}_c(t)$ is weak. This takes place if the rate W_{21} of transitions induced by the probe field is sufficiently small,

$$W_{21}=(2|V_p|^2/K)\overline{\alpha(\omega)}\ll\Gamma_2,\Gamma_3. \tag{2.15}$$

For Λ systems and open ladder systems the above treatment holds until the ground level is depopulated noticeably, i.e., for $t\ll W_{21}^{-1}$.

III. COHERENT COUPLING FIELD

Before we proceed to study effects of a stochastic coupling field, it is helpful to consider EIT with a coherent coupling field $E_c(t)=\mathrm{const}$. This case is important by itself. Moreover, we shall need it for a comparison with the results obtained below for a stochastic coupling field. Here we give an overview of EIT with the emphasis on those aspects of the problem that were not treated sufficiently or even overlooked in the previous studies.

For a coherent coupling field, Eqs. (2.9), (2.10), and (2.13) yield [11,12] the absorption coefficient

$$\alpha(\omega)=K\mathrm{Re}[\Gamma-i\Delta+I/(\Gamma'-i\Delta')]^{-1}, \tag{3.1}$$

where $I=|V_c|^2$ is proportional to the coupling field intensity. The spectrum is significantly modified under the condition

$$I\gg(\Gamma+|\Delta|)(\Gamma'+|\Delta'|), \tag{3.2}$$

which allows one to expand Eq. (3.1) to the second order in $(\Gamma-i\Delta)(\Gamma'-i\Delta')/I$ and obtain for $|\Delta'|\ll\sqrt{I},I/(\Gamma+|\Delta_c|)$ that

$$\alpha(\omega)=\frac{K}{I}\left[\Gamma'+\frac{\Gamma}{I}(\omega-\omega_m)^2\right]. \tag{3.3}$$

Here the position of the minimum

$$\omega_m=\omega_{2p}+\Gamma'\Delta_c/(\Gamma+2\Gamma'). \tag{3.4}$$

The absorption minimum is shifted with respect to ω_{2p} towards ω_{21} [22].

For the validity of Eq. (3.3) inequality (3.2) should hold at least at $\omega=\omega_m$, yielding the condition

$$I\gg\Gamma'[\Gamma+\Gamma'+\Delta_c^2/(\Gamma+\Gamma')]. \tag{3.5}$$

A very significant reduction of absorption in a vicinity of ω_m (i.e., EIT) occurs if the inequality $\alpha(\omega_m)\ll\alpha_0(\omega_m)$, where

$$\alpha_0(\omega)=K\Gamma/(\Gamma^2+\Delta^2) \tag{3.6}$$

is the absorption coefficient in the absence of the coupling field [cf. Eq. (3.1)], holds simultaneously with Eq. (3.5). As a result, one obtains that *EIT occurs in the region*

$$I\gg\Gamma'(\Gamma+\Gamma'+\Delta_c^2/\Gamma). \tag{3.7}$$

Equation (3.7) simplifies for the two alternative cases [23]

$$I\gg\Gamma'\Gamma+\Delta_c^2\Gamma'/\Gamma\quad(\text{if }\ \Gamma'\lesssim\Gamma), \tag{3.8a}$$

$$I\gg\Gamma'^2+\Delta_c^2\Gamma'/\Gamma\quad(\text{if }\ \Gamma'\gg\Gamma). \tag{3.8b}$$

EIT has been explained [5] as resulting from the ac Stark splitting combined with interference of the states dressed by the coupling field. The effect of interference is easy to estimate in the strong-field regime,

$$I\gg\Gamma^2+\Gamma'^2+\Delta_c^2, \tag{3.9}$$

when the spectrum is split into two Lorentzian peaks separated by $2\sqrt{I}$ with approximately equal widths $(\Gamma+\Gamma')/2$ and heights [11,12]. If the transitions to the two dressed states were independent, the spectrum would be a mere superposition of the two Lorentzians and the value of the absorption coefficient at the minimum would be

$$K(\Gamma+\Gamma')/2I. \tag{3.10}$$

Comparing the latter quantity with the correct minimal value $K\Gamma'/I$, which follows from Eq. (3.3), one infers that the interference [24] of the dressed states is destructive for $\Gamma'<\Gamma$, constructive for $\Gamma'>\Gamma$, and vanishes for $\Gamma'=\Gamma$. When $\Gamma'\sim\Gamma$ $(\Gamma'\gg\Gamma)$, the condition of EIT (3.8) is equivalent to (stricter than) the condition of the strong-field regime (3.9). By contrast, for $\Gamma'\ll\Gamma$ the destructive interference is significant enough to allow an observation of EIT even in the weak-field regime $\Gamma'(\Gamma^2+\Delta_c^2)/\Gamma\ll I\ll\Gamma^2+\Delta_c^2$.

To depict EIT graphically it is convenient to plot the length of absorption $l(\omega)\equiv1/\alpha(\omega)$, which is a peak in a vicinity of ω_m when EIT takes place (cf. Fig. 2). For $\Gamma'\ll\Gamma$ the peak is described approximately by the reciprocal of Eq. (3.3), which is a Lorentzian with the half-width at half maximum (HWHM) width $\Gamma_h=\sqrt{I\Gamma'/\Gamma}\ll\sqrt{I}$ (see curve 1 in Fig. 2), whereas for $\Gamma'\gtrsim\Gamma$ the width of the peak is of the order of the Stark splitting, $\Gamma_h\sim\sqrt{I}$, the reciprocal of Eq. (3.3) describing only the top of the peak.

Note that the shift $\Delta_m=\Gamma'\Delta_c/(\Gamma+2\Gamma')$ of the absorption minimum from the two-photon resonance frequency ω_{2p} [see Eq. (3.4)] was not revealed in the previous treatments of EIT. As follows from condition (3.7), $|\Delta_m|\gtrsim\Gamma_h$, i.e., the shift Δ_m is well discernible only in the off-resonance case with a strong destructive interference $|\Delta_c|\gg\Gamma\gg\Gamma'$,

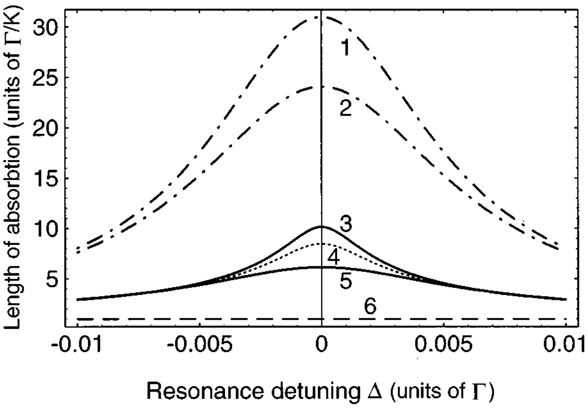

FIG. 2. Length of absorption $l(\omega)=1/\overline{\alpha(\omega)}$ (in units of Γ/K) as a function of the probe detuning Δ (in units of Γ) for the case of the exact resonance $\Delta_c=0$. Curve 1, coherent and phase-fluctuating ($\nu=10^{-7}\Gamma$) coupling fields; curve 2, phase-fluctuating field with $\nu=3\times10^{-4}\Gamma$; curve 3, the quasistatic limit as well as chaotic and uncorrelated-jump fields with $\nu=10^{-7}\Gamma$; curve 4, uncorrelated-jump field with $\nu=3\times10^{-4}\Gamma$; curve 5, chaotic field with $\nu=3\times10^{-4}\Gamma$; curve 6, no coupling field. The other parameters used are $\Gamma'=10^{-3}\Gamma$ and $I_0=0.03\Gamma^2$.

where $\Delta_m\approx\Gamma'\Delta_c/\Gamma$. For all other cases the shift is hardly noticeable and one can set $\omega_m\approx\omega_{2p}$. Note that the off-resonance case can be advantageous from the experimental point of view because for a sufficient detuning the effects of inhomogeneous (e.g., Doppler) broadening are not important.

IV. PHASE-FLUCTUATING COUPLING FIELD

For a coupling field with Markovian phase fluctuations (including the phase diffusion case) it was shown [5,12] that the average absorption coefficient $\overline{\alpha(\omega)}$ is described by the results for $\alpha(\omega)$ obtained for the case of a coherent coupling field (Sec. III), provided the substitution

$$\Gamma'\to\Gamma'+\nu \tag{4.1}$$

is made. In Eq. (4.1) ν is the HWHM width of the Lorentzian field band shape. Note that the EIT line shape in the present case, just as in the case of a coherent coupling field, is (approximately) symmetric, irrespective of the detuning Δ_c.

V. RAYLEIGH COUPLING FIELD

The remaining part of the paper is devoted to the case of an amplitude-phase-fluctuating coupling field. Henceforth we restrict ourselves to the case of a destructive or negligible interference of the dressed states, which requires, as mentioned in Sec. III, that $\Gamma'\ll\Gamma$. The general results will be obtained below for an arbitrary detuning of the coupling field. However, the analysis of the results will be made, for simplicity, for the (near-)resonance case $|\Delta_c|\ll\Gamma$.

In this section we consider the case when the field bandwidth ν is negligibly small (the quasistatic regime). In this case the probe absorption is independent of the details of the stochastic evolution of the coupling field, being determined only by the intensity distribution of the latter

$$\overline{\alpha(\omega)}=\int_0^\infty dI\,\phi(I)\,\alpha(\omega), \tag{5.1}$$

where $\phi(I)$ is the distribution of I. Here we consider the Rayleigh distribution

$$\phi(I)=I_0^{-1}\exp(-I/I_0), \tag{5.2}$$

where I_0 is the average of I. The Rayleigh distribution characterizes, in particular, the chaotic (i.e., complex Gaussian) field model, which is believed to describe well lasers operating on many uncoupled modes [25].

Inserting Eq. (3.1) into Eq. (5.1) yields

$$\overline{\alpha(\omega)}=KI_0^{-1}\mathrm{Re}\tilde{\Gamma}'e^{\tilde{\Gamma}\tilde{\Gamma}'/I_0}E_1(\tilde{\Gamma}\tilde{\Gamma}'/I_0), \tag{5.3}$$

where $\tilde{\Gamma}=\Gamma-i\Delta$, $\tilde{\Gamma}'=\Gamma'-i\Delta'$, and $E_1(\,)$ is the integral exponential function [26].

Consider the case of a significantly modified spectrum

$$I_0\gg(\Gamma+|\Delta|)(\Gamma'+|\Delta'|). \tag{5.4}$$

Then, using the expansion [26] of $E_1(z)$, one can get from Eq. (5.3) in the first approximation

$$\overline{\alpha(\omega)}=KI_0^{-1}\mathrm{Re}\tilde{\Gamma}'\ln[C_1I_0/(\tilde{\Gamma}\tilde{\Gamma}')] \tag{5.5}$$

or

$$\overline{\alpha(\omega)}=\frac{K}{I_0}\left[\Gamma'\ln\frac{C_1^2I_0^2}{(\Gamma^2+\Delta^2)(\Gamma'^2+\Delta'^2)}+\Delta'\left(\arctan\frac{\Delta}{\Gamma}+\arctan\frac{\Delta'}{\Gamma'}\right)\right]. \tag{5.6}$$

Here $C_1=e^{-\gamma}\approx0.56$, where $\gamma\approx0.58$ is the Euler constant [26].

An analysis of Eq. (5.6) shows that for the case of interest $\Gamma'\ll\Gamma$ and $|\Delta_c|\ll\Gamma$, the minimum of absorption is at ω_{2p}, the value at the minimum being

$$\overline{\alpha(\omega_{2p})}=K\frac{\Gamma'}{I_0}\ln\frac{C_1I_0}{\Gamma'\Gamma}. \tag{5.7}$$

For $|\Delta_c|\ll\Gamma$ one can set $\Delta\approx\Delta'$ in Eq. (5.6). Thereby one obtains that the shape of the EIT peak in the function $l(\omega)=1/\overline{\alpha(\omega)}$ is symmetric with the wings decaying as $1/|\Delta'|$ (Fig. 2, curve 3).

As follows from Eq. (5.7), *EIT can be achieved with the help of an amplitude-phase-fluctuating coupling field.* The magnitude of EIT is now reduced by a logarithmic factor, in comparison to the case of a coherent coupling field with $I=I_0$ [cf. Eq. (3.3)], due to contributions from small I in the integral (5.1). The comparison of Eq. (5.7) with Eq. (3.10) shows that destructive interference of the dressed states at $\Gamma'\ll\Gamma$ is preserved now (though its magnitude is less than for a coherent coupling field with the same mean intensity). The validity condition of EIT is obtained from the condition that Eq. (5.4) at $\omega=\omega_{2p}$ and the inequality

$\overline{\alpha(\omega_{2p})} \ll \alpha_0(\omega_{2p})$ hold simultaneously. In view of Eqs. (3.6) and (5.7), this means that *EIT takes place under the condition*

$$I_0 \gg \Gamma\Gamma', \tag{5.8}$$

which is a direct extension of Eq. (3.8a) in the present case $|\Delta_c| \ll \Gamma$.

The results of this section are rather general, being valid for all statistical models of an amplitude-phase-fluctuating coupling field, which share the Rayleigh distribution (5.2). The effects of the field bandwidth for two such models are considered in the Secs. VI and VII.

VI. CHAOTIC FIELD

A. Formalism

Consider now the effects of the field bandwidth for the case of chaotic field, which is the common model for multimode lasers [25]. Assume that the field line shape is a Lorentzian with the HWHM width ν, which means, by Doob's theorem [27], that the field is Gaussian and Markovian. Then one can use the theory of stochastic differential equations [27–29] and write the equation

$$\dot{\psi}(u,v,t) = [A(u,v) + L]\psi(u,v,t) \tag{6.1}$$

for $\psi(u,v,t)$, the partially averaged $\psi(t)$. In Eq. (6.1) $H(u,v)$ is given by Eq. (2.10) with $V_c(t) \to u + iv$ and the stochastic operator $L = L_u + L_v$ takes into account temporal fluctuations of the field. Here L_u and L_v are the Fokker-Planck operators

$$L_u = \nu\left(1 + u\frac{\partial}{\partial u} + \frac{I_0}{2}\frac{\partial^2}{\partial u^2}\right). \tag{6.2}$$

The initial condition for Eq. (6.1) is $\psi(u,v,0) = \psi(0)f(u,v)$, where $f(u,v) = f(u)f(v)$ is the distribution of the complex amplitude of the interaction, whereas $f(u)$ and $f(v)$ are the one-dimensional distributions

$$f(u) = \exp(-u^2/I_0)/\sqrt{\pi I_0}. \tag{6.3}$$

The integral of $\psi(u,v,t)$ over u and v yields $\overline{\psi(t)}$.

We define the vector

$$\Psi(u,v) \equiv \begin{pmatrix} \Psi_a(u,v) \\ \Psi_b(u,v) \end{pmatrix} = \int_0^\infty \psi(u,v,t)\,dt. \tag{6.4}$$

From Eqs. (6.1) and (6.4) one obtains the equations for the components of $\Psi(u,v)$,

$$(i\Delta - \Gamma)\Psi_a - iV_c^*\Psi_b + L\Psi_a = -f(u,v), \tag{6.5a}$$

$$(i\Delta' - \Gamma')\Psi_b - iV_c\Psi_a + L\Psi_b = 0, \tag{6.5b}$$

where $V_c = u + iv$. The average absorption coefficient can be written in terms of $\Psi_a(u,v)$ as

$$\overline{\alpha(\omega)} = K\,\mathrm{Re}\int d^2V_c\,\Psi_a(u,v), \tag{6.6}$$

where $d^2V_c = du\,dv$. The set of the second-order differential equations (6.5) cannot be solved in a closed form. Therefore, an approximate approach is developed below.

B. Approximate solution

If $\Gamma + |\Delta|$ is sufficiently large, one can neglect the term $L\Psi_a$ in Eq. (6.5a), yielding

$$\Psi_a(u,v) = \frac{f(u,v) - iV_c^*\Psi_b(u,v)}{\Gamma - i\Delta}. \tag{6.7}$$

Inserting Eq. (6.7) into Eq. (6.5b) yields the equation for $\Psi_b(u,v)$,

$$(i\Delta' - \Gamma')\Psi_b - \frac{u^2+v^2}{\Gamma - i\Delta}\Psi_b + L\Psi_b = \frac{iV_c f(u,v)}{\Gamma - i\Delta}. \tag{6.8}$$

The solution of Eq. (6.8) has the form

$$\Psi_b(u,v) = -i(\Gamma - i\Delta)^{-1}\int d^2V_c'\int_0^\infty dt\,f(u',v') \times g(u',v',u,v,t)e^{(i\Delta' - \Gamma')t}. \tag{6.9}$$

Here $d^2V_c' = du'\,dv'$ and $g(u',v',u,v,t)$ as a function of u, v, and t obeys the equation

$$\frac{\partial g}{\partial t} = -\frac{u^2+v^2}{\Gamma - i\Delta}g + Lg, \tag{6.10}$$

with the initial condition $g(u',v',u,v,0) = \delta(u-u')\delta(v-v')$. Inserting Eq. (6.9) into Eq. (6.7), one obtains from Eq. (6.6)

$$\overline{\alpha(\omega)} = K\,\mathrm{Re}(\Gamma - i\Delta)^{-1}\left[1 - \int_0^\infty dt\,G(t)e^{(i\Delta' - \Gamma')t}\right], \tag{6.11}$$

where

$$G(t) = \int\int d^2V_c'\,d^2V_c\,V_c'V_c^*f(u',v')g(u',v',u,v,t). \tag{6.12}$$

Equation (6.10) is a stochastic differential equation and hence $g(u',v',u,v,t)$ is the average of

$$\exp\left[-(\Gamma - i\Delta)^{-1}\int_0^t dt'|V_c(t')|^2\right] \tag{6.13}$$

over all realizations of the stochastic field, subject the conditions $V_c(0) = u' + iv'$ and $V_c(t) = u + iv$. The full average of quantity (6.13) was calculated in [29,30]. Here we are interested in a related average [cf. Eq. (6.12)]

$$G(t) = \left\langle V_c(0)V_c^*(t)\exp\left[-(\Gamma - i\Delta)^{-1}\int_0^t dt'|V_c(t')|^2\right]\right\rangle. \tag{6.14}$$

This average was calculated in [31] by the path integration method. Alternatively, $G(t)$ can be directly obtained from Eqs. (6.12) and (6.10) (see Appendix A). The result is

$$G(t)=4I_0\beta^2 e^{\nu t}[S(t)]^{-2}, \tag{6.15}$$

where $\beta=\sqrt{1+2I_0/\nu\tilde{\Gamma}}$ and

$$S(t)=2\beta\cosh\beta\nu t+(1+\beta^2)\sinh\beta\nu t. \tag{6.16}$$

Inserting Eq. (6.15) into Eq. (6.11) and reducing the integral to Eq. 3.194.1 of [32], one obtains

$$\overline{\alpha(\omega)}=\frac{K\Gamma}{\Gamma^2+\Delta^2}-\mathrm{Re}\frac{8KI_0\beta F(2,d;1+d;b)}{(\Gamma-i\Delta)^2(1+\beta)^4\nu d}, \tag{6.17}$$

where $F(\,)$ is the hypergeometric function [26], $b=(\beta-1)^2/(\beta+1)^2$, and

$$d=[(2\beta-1)\nu+\tilde{\Gamma}']/(2\beta\nu). \tag{6.18}$$

Equation (6.17) can be transformed to a somewhat simpler form with the help of Eq. 15.3.4 from [26]. As a result, one obtains

$$\overline{\alpha(\omega)}=\frac{K\Gamma}{\Gamma^2+\Delta^2}-KI_0\mathrm{Re}\frac{F(2,1;1+d;-z)}{\tilde{\Gamma}^2[\tilde{\Gamma}'+(2\beta-1)\nu]}, \tag{6.19}$$

where $z=(\beta-1)^2/4\beta$. As shown in Appendix B, Eqs. (6.17) and (6.19) hold for

$$\Gamma+|\Delta|\gg\nu. \tag{6.20}$$

If the coupling-field intensity is so high that both Eq. (5.4) and

$$I_0\gg\nu(\Gamma+|\Delta|) \tag{6.21}$$

hold, one should require also that

$$\sqrt{I_0\nu}\ll(\Gamma+|\Delta|)(\sqrt{\Gamma+|\Delta|}+\sqrt{\Gamma'+|\Delta'|}). \tag{6.22}$$

Equation (6.19) [or, equivalently, Eq. (6.17)] and its validity conditions represent one of the main results of this paper.

C. Special cases

It is shown in Appendix C that for $\nu\to 0$ the quasistatic limit (5.3) is recovered from Eq. (6.19), as one should expect. In the region (5.4) and (6.21) that is of interest here, Eq. (6.19) can be reduced in the first approximation to (see Appendix C)

$$\overline{\alpha(\omega)}=\frac{K}{I_0}\mathrm{Re}\tilde{\Gamma}'\left[\frac{1}{2}\ln\frac{C_1^2 I_0}{8\nu\tilde{\Gamma}}-\psi(a+1)+\frac{1}{2a}\right], \tag{6.23}$$

where $\psi(\,)$ is the logarithmic derivative of the Γ function [26] and $a=\tilde{\Gamma}'(\tilde{\Gamma}/8I_0\nu)^{1/2}$. For very small ν,

$$\sqrt{\nu}\ll\Gamma'\sqrt{(\Gamma+|\Delta_c|)/I_0}, \tag{6.24}$$

one can use the asymptotic expansion [26]

$$\psi(a+1)=\ln a+\frac{1}{2a}-\frac{1}{12a^2}+O\left(\frac{1}{|a|^4}\right) \tag{6.25}$$

to reduce Eq. (6.23) in the first approximation to Eq. (5.5). In this case the quasistatic results (5.3) and (5.5)–(5.7) are recovered approximately. In the opposite case

$$\sqrt{\nu}\gg\Gamma'\sqrt{(\Gamma+|\Delta_c|)/I_0}, \tag{6.26}$$

absorption significantly depends on ν.

The above results hold for arbitrary field detunings. Henceforth in this section we will focus on the near-resonance case $|\Delta_c|\ll\Gamma$. Now in the region (6.26) formula (6.23) yields that the absorption is minimal in a vicinity of the two-photon resonance frequency ω_{2p}, where one gets

$$\overline{\alpha(\omega)}\approx K\sqrt{2\nu/(I_0\Gamma)}\quad(|\Delta'|\ll\sqrt{I_0\nu/\Gamma}). \tag{6.27}$$

The line shape of the EIT window is symmetric (Fig. 2, curve 5), the wings of the line shape for $|\Delta'|\gg\sqrt{I_0\nu/\Gamma}$ being quasistatic (see Sec. V).

D. Limits of regimes of EIT

Consider now the validity conditions of different regimes of EIT for the near-resonance case. According to Eq. (6.26), the absorption minimum depends significantly on the field bandwidth if

$$\sqrt{I_0\nu}\gg\Gamma'\sqrt{\Gamma}. \tag{6.28}$$

In this case, as follows from Eqs. (6.22) and (6.27), the minimum absorption increases with ν as $\sqrt{\nu}$, while

$$\sqrt{I_0\nu}\ll\Gamma^{3/2}. \tag{6.29}$$

The EIT effect takes place if $\overline{\alpha(\omega_{2p})}\ll\alpha_0(\omega_{2p})$. In view of Eqs. (3.6) and (6.27), this corresponds in the region (6.28) and (6.29) to the condition

$$\sqrt{\nu/I_0}\ll 1/\sqrt{\Gamma}. \tag{6.30}$$

Figure 3(a) depicts graphically the region of existence of EIT and the boundaries between different regimes of EIT. In Fig. 3(a) boundaries 1–4 relate to the conditions (5.8), (6.28), (6.29), and (6.30), respectively. Region I in Fig. 3(a) depicts the quasistatic stage of EIT, whereas region II denotes the stage where Eq. (6.27) holds. Figure 3(a) shows that the necessary and sufficient condition for the existence of region II is $\Gamma'\ll\Gamma$, which suggests the idea of the presence of a strong destructive interference of the dressed states in region II. Indeed, region II allows for EIT with the weak and intermediate couplings $I_0\lesssim\Gamma^2$, which is a sign of a strong destructive interference, whereas for the strong-coupling case $I_0\gg\Gamma^2$ the minimum absorption (6.27) is much less than that given by Eq. (3.10). On boundary 3 of region II [Fig. 3(a)] the interference is negligible. Indeed, there Eq. (6.27) yields $\overline{\alpha(\omega_{2p})}\sim K\Gamma/I_0$, as in the case of a coherent coupling field with $\Gamma\sim\Gamma'$ [cf. Eq. (3.3)], which means negligible interference.

The approximation used in this section describes correctly EIT (at least, the maximum EIT value) to the left of boundary 3 and the negative ordinate half axis in Fig. 3(a), i.e., under the conditions (6.29) and $\nu\ll\Gamma$ [the latter follows from Eq. (6.20)]. One can show [33] that to the right of line 5 in Fig. 3(a), i.e., for

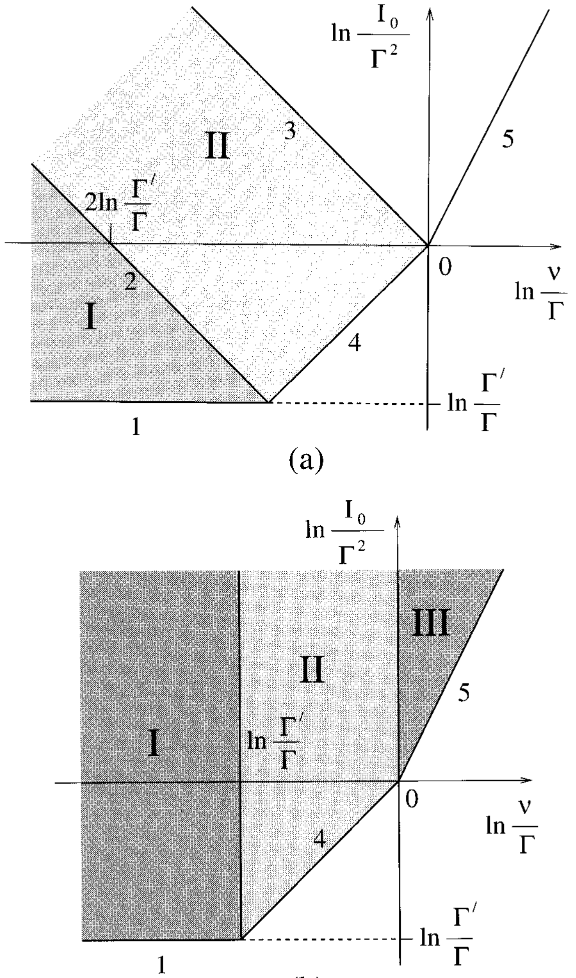

FIG. 3. Boundaries of different regimes of EIT in the parameter space for two models of the amplitude-phase fluctuating coupling field in the case $|\Delta_c|\lesssim\Gamma$: (a) chaotic field and (b) uncorrelated-jump field. Region I, the quasistatic regime; region II, the ν-dependent regime with strong destructive interference of the dressed states; region III, the ν-dependent regime without destructive interference of the dressed states.

$$I_0 \ll \nu^2 \quad (\nu \gtrsim \Gamma), \tag{6.31}$$

the stochastic perturbation theory holds. An analysis of the corresponding solution shows an absence of any EIT dip in the absorption spectrum [33]. Hence the only region for the possible existence of EIT, not covered by the available approximate methods, is the sector between lines 3 and 5 in Fig. 3(a), i.e., the regime of a strong-coupling field with the bandwidth in the interval $\Gamma^3/I_0 \lesssim \nu \lesssim \sqrt{I_0}$. To consider this case requires the full solution of the problem, with due regard for the stochastic operators in both Eqs. (6.5), which is beyond the scope of the present paper. Note that the above delimitation of various regimes of EIT holds actually for $|\Delta_c|\lesssim\Gamma$ and not only for $|\Delta_c|\ll\Gamma$.

VII. UNCORRELATED JUMP MODEL

In the model of Markovian uncorrelated jumps the complex amplitude of the field experiences at random times abrupt changes comparable by magnitude to the amplitude itself. This is in sharp contrast to a continuous evolution, expected for the field of a multimode laser. Therefore, the uncorrelated-jump model is considered to be much less suitable for real lasers than the model of chaotic field. However, the uncorrelated-jump model has the advantage that it is readily solvable in the most general form and therefore this model has been used [12,34] to obtain at least insight into the problem if not quantitative results. In view of this, it is important to compare the two models whenever it is possible. Another reason to consider the uncorrelated-jump model here is to check the sensitivity of EIT to statistical details of the field.

For the uncorrelated-jump model Eqs. (6.5) still hold with the only difference that the stochastic operator L is now [12]

$$L\Psi = -\nu\left[\Psi(u,v) - f(u,v)\int d^2V_c\Psi(u,v)\right], \tag{7.1}$$

where ν^{-1} is the average time between the jumps. As above, the field spectrum is Lorentzian with the HWHM width equal ν. For the model (7.1) Eqs. (6.5) can be solved in the general form, yielding [12]

$$\overline{\alpha(\omega)} = K\mathrm{Re}J(\omega)[1-\nu J(\omega)]^{-1}, \tag{7.2}$$

where

$$J(\omega) = \int_0^\infty dI\phi(I)[\Gamma+\nu-i\Delta+I/(\Gamma'+\nu-i\Delta')]^{-1}. \tag{7.3}$$

In particular, for the Rayleigh distribution (5.2) one obtains

$$J(\omega) = [(\tilde{\Gamma}'+\nu)/I_0]e^{(\tilde{\Gamma}+\nu)(\tilde{\Gamma}'+\nu)/I_0}E_1((\tilde{\Gamma}+\nu)(\tilde{\Gamma}'+\nu)/I_0), \tag{7.4}$$

Assume that

$$\nu|J(\omega)| \ll 1 \tag{7.5}$$

(this condition is verified in the last paragraph of the present section). Then Eq. (7.2) yields that

$$\overline{\alpha(\omega)} \approx K\mathrm{Re}J(\omega), \tag{7.6}$$

i.e., the absorption coefficient is given by the quasistatic Eq. (5.3), where the effect of the temporal field fluctuations is accounted for by the substitutions

$$\Gamma \to \Gamma+\nu, \quad \Gamma' \to \Gamma'+\nu. \tag{7.7}$$

Correspondingly, in the region (7.5) the results for EIT obtained in Sec. V are extended to the case of a nonvanishing ν by means of the substitution (7.7). In particular, under the condition of significantly modified spectrum [cf. Eq. (5.4)]

$$I_0 \gg (\Gamma+\nu+|\Delta|)(\Gamma'+\nu+|\Delta'|), \tag{7.8}$$

Eq. (7.6) becomes approximately

$$\overline{\alpha(\omega)}=K\mathrm{Re}\frac{\tilde{\Gamma}'+\nu}{I_0}\ln\frac{C_1 I_0}{(\tilde{\Gamma}+\nu)(\tilde{\Gamma}'+\nu)},\qquad(7.9)$$

which is an analog of Eq. (5.5).

Thus far in this section the results hold for arbitrary Δ_c. Henceforth we confine ourselves to the case $|\Delta_c|\ll\Gamma$. Now Eq. (7.9) implies that the absorption coefficient at the minimum is [cf. Eq. (5.7)]

$$\overline{\alpha(\omega_{2p})}=K\frac{\Gamma'+\nu}{I_0}\ln\frac{C_1 I_0}{(\Gamma'+\nu)(\Gamma+\nu)}.\qquad(7.10)$$

Note that condition (7.8) guarantees that the argument of the logarithm in Eq. (7.10) is much greater than one.

The different regimes of EIT for the uncorrelated-jump model are shown in Fig. 3(b), where the EIT boundaries 1, 4, and 5 correspond, respectively, to inequalities (5.8), (6.30), and

$$I_0\gg\nu^2.\qquad(7.11)$$

These boundaries are obtained from Eq. (7.8) and the condition $\overline{\alpha(\omega_{2p})}\ll\alpha_0(\omega_{2p})$, taking into account Eqs. (3.6) and (7.10).

The quasistatic regime [region I in Fig. 3(b)] holds for $\nu\ll\Gamma'$. The ν-dependent regime with the strong destructive interference of the dressed states (region II) takes place if $\Gamma'\ll\nu\ll\Gamma$. In this case one can use the results of Sec. V obtained for $\Gamma'\ll\Gamma$ with the only substitution $\Gamma'\to\nu$. In particular, as follows from Eq. (7.10), the minimum value of absorption,

$$\overline{\alpha(\omega_{2p})}=K\frac{\nu}{I_0}\ln\frac{C_1 I_0}{\nu\Gamma},\qquad(7.12)$$

increases with ν almost linearly. This can be compared with the $\sqrt{\nu}$ law for the field-bandwidth dependence of the minimum absorption coefficient in the case of the chaotic field model [Eq. (6.27)].

When the field bandwidth exceeds significantly the material relaxation constants $\nu\gg\Gamma$ one can use the results of Sec. V with the equal relaxation constants $\Gamma,\Gamma'\to\nu$, which implies negligible interference between the dressed states. In particular, Eq. (7.10) yields that the minimum absorption is

$$\overline{\alpha(\omega_{21})}=\frac{K\nu}{I_0}\ln\frac{C_1 I_0}{\nu^2}.\qquad(7.13)$$

One can estimate from Eqs. (7.3) and (7.9) that condition (7.5) is equivalent to the inequalities $I_0\gg\nu^2$ for $I_0\gtrsim\Gamma^2$ and $\nu\ll\Gamma$ for $I_0\lesssim\Gamma^2$, i.e., it holds to the left of boundary 5 and the negative part of the ordinate axis in Fig. 3(b). As in the case of the chaotic field model (see Sec. VI D), to the right of boundary 5 the stochastic perturbation theory can be used [33], implying an absence of an EIT dip. Thus the results of this section fully describe the EIT dip in the probe absorption in the frame of the uncorrelated-jump model.

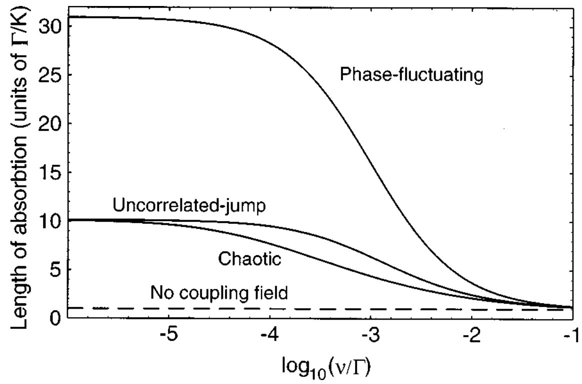

FIG. 4. Maximum length of absorption $1/\overline{\alpha(\omega_{21})}$ as a function of ν/Γ. The values of the parameters are $\Delta_c=0$, $\Gamma'=10^{-3}\Gamma$, and $I_0=0.03\Gamma^2$.

VIII. NUMERICAL RESULTS AND DISCUSSION

Figure 2 shows the dependence of the length of absorption on the probe-field frequency for a coherent coupling field, as well as for three stochastic models of the random coupling field: the Markovian phase-fluctuating, chaotic, and uncorrelated-jump fields. Here and henceforth the curves for the coherent, phase-fluctuating, and uncorrelated-jump models are calculated by the exact formulas, i.e., Eq. (3.1), Eq. (3.1) with the substitution Eq. (4.1), and Eq. (7.2) with the definition (7.4), respectively, whereas the curves for the chaotic field are obtained with the help of the most general available formula (6.17). Figure 2 illustrates the case of a weak coupling field $I_0\ll\Gamma^2$ (the lower half plane in Fig. 3), exactly resonant to the transition $|2\rangle\leftrightarrow|3\rangle$, for two values of the bandwidth. When the bandwidth is very small $\nu=10^{-7}\Gamma$, curve 1 obtained for the phase-fluctuating field coincides with the curve for the coherent field of the same intensity and frequency, whereas curves 3 obtained for chaotic and uncorrelated-jump fields coincide with each other and with the result (5.3) of the quasistatic limit $\nu\to 0$. Figure 4 shows the maximum value of the length of absorption as a function of the dimensionless bandwidth for the above stochastic models, the values of other parameters being the same as in Fig. 2. Figures 2 and 4 demonstrate the following facts. (i) Amplitude-phase fluctuations reduce EIT noticeably, even for $\nu\to 0$, but do not destroy the effect. (ii) EIT decreases with the increase of the field bandwidth, irrespective of the stochastic model. (iii) For a given bandwidth and intensity of the field, EIT is significantly lower for an amplitude-phase-fluctuating field than for a phase-fluctuating field [35] and is less pronounced for a chaotic field than for an uncorrelated-jump field.

Consider the case of a strong coupling field (the upper half plane in Fig. 3). Figure 5 shows (on a logarithmic scale) the field-bandwidth dependence of the maximum length of absorption in the exact-resonance case for three stochastic models of the strong-coupling field. The range of the ν dependence of the plot for the chaotic field is limited from above by the condition (6.29) (boundary 3 in Fig. 3). The comparison of Figs. 4 and 5 shows that the absolute and relative differences between the results for the chaotic and uncorrelated-jump models increase with the field intensity.

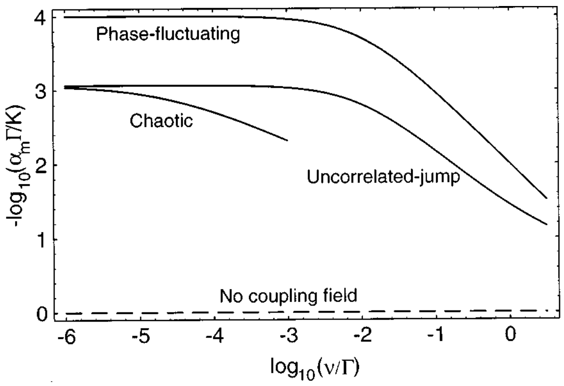

FIG. 5. Logarithm of the maximum length of absorption $-\log_{10}(\alpha_m\Gamma/K)$ [where $\alpha_m=\overline{\alpha(\omega_{21})}$] as a function of the dimensionless bandwidth for various models of the stochastic field. The values of the parameters are $\Gamma'=0.01\Gamma$, $\Delta_c=0$, and $I_0=100\Gamma^2$.

IX. CONCLUSION

In the previous sections the EIT dip in the probe-field absorption spectrum has been considered for the cases of coherent, phase-fluctuating, and amplitude-phase-fluctuating coupling fields with the Rayleigh distribution. In particular, two atom-field coupling schemes for the generation of EIT have been studied under rather general assumptions. The equations describing EMA and EIT have been derived and the conditions of their validity have been discussed. A comprehensive study of the EIT line shape for a coherent coupling field has been performed, which complements the previous treatments. A shift of the absorption minimum from the two-photon resonance frequency has been revealed in the off-resonance case. The role of quantum interference in EIT has been discussed and the dependence of the quantum interference on the relaxation constants has been obtained. Since the above treatment holds for an arbitrary relaxation scheme, the results obtained here allow one to study effects of dephasing collisions. Moreover, the results obtained above for a coherent coupling field can be readily used to describe effects of phase fluctuations of the coupling field, due to relation (4.1).

The above treatment puts the emphasis on the case of an amplitude-phase fluctuating coupling field since this practically important case was not considered previously. The case of narrow-band fields with the Rayleigh distribution (the quasistatic regime) has been considered quite generally. We have shown that *amplitude-phase-fluctuating fields are capable of inducing significant transparency*, though somewhat reduced as compared to the coherent field. The effects of the bandwidth have been considered for two stochastic models, which share the same intensity distribution and band shape, viz., the chaotic and uncorrelated-jump fields. Closed analytical solutions have been obtained and the validity conditions of different regimes of EIT have been derived. As follows from the above results, stochastic fluctuations are detrimental for EIT for all stochastic models considered.

The comparison of the two models of an amplitude-phase-fluctuating field considered above has shown a substantial difference between the results of the models, despite that the models have identical line shapes and phase-amplitude distributions. The differences concern all EIT features, including EIT line shapes, the dependence of the absorption spectrum on the relevant parameters, and the limits of various regimes of EIT. Thus EIT proves to be *highly sensitive to fine statistical details of the coupling-field fluctuations*. This can be compared to the population dynamics induced by a resonant Rayleigh field in the rate approximation [29]. This process proved to be not very sensitive to the difference between the chaotic and uncorrelated-jump fields. Thereby observations of EIT can be utilized for study of laser fluctuations.

Consider the experimental verification of the present predictions for the chaotic field. The results obtained above for this case hold if the field bandwidth ν is much less than the material width Γ [cf. Fig. 3(a)]. This condition may prove rather restrictive when transparency is induced with a multimode laser, since then the bandwidth is limited from below by the intermode frequency spacing $\pi c/n_0 d$. Here d is the laser resonator length and n_0 is the index of refraction. For $n_0\sim 1$ and $d=1$ m one gets $\nu>10^9$ s^{-1}, which requires $\Gamma>10^{10}$ s^{-1} to satisfy the above condition. This can be achieved by considering sufficiently broad material transitions, e.g., transitions to autoionizing states or homogeneously broadened absorption lines of impurities in condensed matter. Synthesized chaotic field [36,37] also can be employed in the experiment. In this case the condition $\nu\ll\Gamma$ is not so crucial since the bandwidth of synthesized fluctuations can be rather small.

ACKNOWLEDGMENTS

The author thanks Professor M. Shapiro, Professor R. J. Gordon, Professor C. S. Stroud, and Professor Y. Silberberg for useful discussions.

APPENDIX A: DERIVATION OF EQ. (6.15)

One can write

$$g(u',v',u,v,t)=h(u',u,t)h(v',v,t), \tag{A1}$$

where $h(u',u,t)$ obeys the equation [cf. Eq. (6.10)]

$$\frac{\partial h}{\partial t}=-\frac{u^2}{\Gamma-i\Delta}h+L_u h, \tag{A2}$$

with the initial condition $h(u',u,0)=\delta(u-u')$. Note that the change of the dependent variable that cancels the first derivatives with respect to u and v transforms Eqs. (6.10) and (A2) to the Schrödinger equations for, respectively, two- and one-dimensional harmonic oscillator with purely imaginary time.

To solve Eq. (A2) we extend the method of Ref. [38] by writing the ansatz

$$h(u',u,t)=\exp[A_0(t)+A_1(t)\alpha+A_2(t)\alpha^2], \tag{A3}$$

where $\alpha=u/\sqrt{I_0}$. Inserting Eq. (A3) into Eq. (A2) and taking into account Eq. (6.2) yields the set of equations

$$dA_0/d\tau=A_1^2/2+A_2+1, \quad dA_1/d\tau=A_1+2A_1A_2,$$
$$dA_2/d\tau=2A_2+2A_2^2-I_0/[(\Gamma-i\Delta)\nu], \tag{A4}$$

where $\tau = \nu t$. The singular initial condition for Eq. (A2) yields singular initial conditions for Eqs. (A4): for $t \to 0$ one gets $\sqrt{-\pi/A_2}\exp[A_0 - A_1^2/(4A_2)] \to 1$, $A_1/A_2 \to -2\alpha'$, and $A_2 \to -\infty$, where $\alpha' = u'/\sqrt{I_0}$. The solution of Eqs. (A4) yields

$$f(u')h(u',u,t) = \frac{\sqrt{\beta}}{\pi\sqrt{1-e^{-2\beta\tau}}}\exp\left[-\frac{(\beta-1)\tau}{2} + \left(\frac{\beta-1}{2} - \frac{\beta}{1-e^{-2\beta\tau}}\right)(\alpha^2+\alpha'^2) + \frac{\beta\alpha\alpha'}{\sinh\beta\tau}\right]. \tag{A5}$$

As follows from Eq. (A5), the function $\phi(u',u,t) = f(u')h(u',u,t)$ has the properties

$$\phi(u',u,t) = \phi(u,u',t) = \phi(-u',-u,t). \tag{A6}$$

Inserting Eq. (A1) into Eq. (6.12) and taking into account Eq. (A6) yields

$$G(t) = 2J_0(t)J_1(t), \tag{A7}$$

where

$$J_n(t) = \int_{-\infty}^{\infty} du' du u'^n u^n f(u')h(u',u,t). \tag{A8}$$

The function $J_0(t)$ was calculated in [29]. Inserting Eq. (A5) into Eq. (A8) and performing the integration yields

$$J_n(t) = \sqrt{2}I_0^n \beta^{n+1/2} e^{\nu t/2}[S(t)]^{-n-1/2} \quad (n=0,1). \tag{A9}$$

Equations (A7) and (A9) result in Eq. (6.15).

APPENDIX B: VALIDITY CONDITIONS OF EQS. (6.17) AND (6.19)

Consider corrections to the first-order approximation developed in Sec. VI B. In the second-order approximation Eq. (6.5a) becomes

$$-\tilde{\Gamma}\Psi_a^{(2)} + L\Psi_a^{(2)} = -Q(u,v), \tag{B1}$$

where

$$Q(u,v) = f(u,v) - iV_c^* \Psi_b^{(1)}(u,v). \tag{B2}$$

Here $\Psi_b^{(1)}(u,v)$ equals the first-order result (6.9). The solution of Eq. (B1) has the form

$$\Psi_a^{(2)}(u,v) = \int d^2V_c' Q(u',v') \int_0^\infty dt f(u',v',u,v,t) e^{-\tilde{\Gamma}t}, \tag{B3}$$

where $f(u',v',u,v,t)$ is the conditional probability of the Gaussian-Markovian process $V_c(t)$. $f(u',v',u,v,t)$ is the solution of the two-dimensional Fokker-Planck equation $\dot{f} = Lf$ with the initial condition $f(u',v',u,v,0) = \delta(u-u')\delta(v-v')$. It is easy to show that

$$f(u',v',u,v,t) = f_1(u',u,t)f_1(v',v,t), \tag{B4}$$

where $f_1(u',u,t)$ is the Green's function of the Fokker-Planck equation [27] $\dot{f}_1 = L_u f_1$,

$$f_1(u',u,t) = \frac{1}{\sqrt{\pi I_0(1-e^{-2\nu t})}}\exp\left[-\frac{(u-u'e^{-\nu t})^2}{I_0(1-e^{-2\nu t})}\right]. \tag{B5}$$

Inserting Eqs. (A1) and (A5) into Eq. (6.9) and integrating over u' and v' yields that

$$\Psi_b^{(1)}(u,v) = \int_0^\infty dt P(u,v,t) e^{-\tilde{\Gamma}' t}, \tag{B6}$$

where

$$P(u,v,t) = \frac{-iV_c\beta^2}{\pi I_0 \tilde{\Gamma} R^2(t)}\exp\left[\nu t - \frac{S(t)I}{2I_0R(t)}\right]. \tag{B7}$$

Here $I = u^2 + v^2$ and $R(t) = \beta\cosh\beta\nu t + \sinh\beta\nu t$. Equation (B7) yields the limiting cases

$$P(u,v,t) \approx -\frac{iV_c f(u,v)}{\tilde{\Gamma}}\exp(-It/\tilde{\Gamma}), \quad |\beta|\nu t \ll 1 \tag{B8a}$$

and

$$P(u,v,t) \approx -\frac{iV_c f(u,v)\beta^2}{\tilde{\Gamma}(1+\beta)^2}\exp\left[-\frac{(\beta-1)I}{2I_0} - 2\beta\nu t\right], \quad e^{|\beta|\nu t} \gg 1. \tag{B8b}$$

On inserting Eq. (B8a) into Eq. (B6), one obtains

$$\Psi_b^{(1)}(u,v) \approx -\frac{iV_c f(u,v)}{\tilde{\Gamma}\tilde{\Gamma}' + I}, \quad I \gg I_1, \tag{B9}$$

where $I_1 = \nu|\beta\tilde{\Gamma}|$. From Eqs. (B6) and (B8) one can estimate that

$$|\Psi_b^{(1)}(u,v)| \sim \frac{|V_c| f(u,v)}{|\tilde{\Gamma}(\tilde{\Gamma}' + \beta\nu)|}, \quad I \ll I_1. \tag{B10}$$

Combining Eqs. (B2), (B9), and (B10) yields

$$Q(u,v) \approx f(u,v)\begin{cases} \tilde{\Gamma}\tilde{\Gamma}'/(I + \tilde{\Gamma}\tilde{\Gamma}'), & I \gg I_1 \\ 1, & I \ll I_1. \end{cases} \tag{B11}$$

Hence $Q(u,v)$ as a function of I has the characteristic width $I_2 = \min\{I_1 + |\tilde{\Gamma}\tilde{\Gamma}'|, I_0\}$. On the other hand, as follows from Eqs. (B4) and (B5), under the condition (6.20) the characteristic width of the function $\int_0^\infty dt f(u',v',u,v,t)e^{-\tilde{\Gamma}t}$ in the (u',v') plane is $I_0\nu/|\tilde{\Gamma}|$. Therefore, if

$$I_0\nu/(\Gamma + |\Delta|) \ll I_2, \tag{B12}$$

one can write $f(u',v',u,v,t) \approx \delta(u-u')\delta(v-v')$ in Eq. (B3). This yields, taking into account Eq. (B2), that $\Psi_a^{(2)}(u,v) \approx Q(u,v)/f \approx \Psi_a^{(1)}(u,v)$, where the first-order re-

sult
$\Psi_a^{(1)}(u,v)$ is given by Eqs. (6.7) and (6.9). Thus the approximation described in Sec. VI B has been proved here to hold under conditions (6.20) and (B12). One can show that Eq. (B12) is equivalent to Eq. (6.22) if inequalities (5.4) and (6.21) hold simultaneously and to Eq. (6.20), otherwise.

APPENDIX C: ANALYSIS OF EQ. (6.19)

To show that Eq. (6.19) has the correct static limit $\nu\rightarrow 0$, we use Eq. 6.8(1) of [39]. Then one obtains that

$$\lim_{\nu\rightarrow 0} F(2,1;1+d;-z)=y^2\Psi(2,2;y), \tag{C1}$$

where $\Psi(\,)$ is the confluent hypergeometric function and $y=\tilde{\Gamma}\tilde{\Gamma}'/I_0$. On the other hand, Eqs. 13.6.28 [where $U(\,)=\Psi(\,)$], 5.1.45, and 5.1.14 of [26] yield that

$$\Psi(2,2;y)=y^{-1}-e^{y}E_1(y). \tag{C2}$$

With the help of Eqs. (C1) and (C2) one obtains that the $\nu\rightarrow 0$ limit of Eq. (6.19) equals Eq. (5.3).

In the case (6.20), which is of interest here, one gets $|\beta|\gg 1$, yielding

$$z\approx\beta/4-1/2\approx\sqrt{I_0/8\nu\tilde{\Gamma}}-1/2. \tag{C3}$$

Now one can use the expansion of the hypergeometric function for large $|z|$ (Eq. 15.3.14 in [26]) to obtain

$$F(2,1;1+d;-z)=d\left\{\frac{1}{z}-\sum_{n=0}^{\infty}\frac{(d-1)\cdots(d-1-n)}{n!z^{n+2}}\right.$$
$$\left.\times[\ln z+\psi(n+1)-\psi(d-1-n)]\right\}. \tag{C4}$$

Keeping in this expansion only the first two terms yields the expression

$$F(2,1;1+d;-z)\approx dz^{-1}\{1-(d-1)z^{-1}$$
$$\times[\ln z+\psi(1)-\psi(d-1)]\}. \tag{C5}$$

The smallness parameter in expansion (C4) is $|d/z|$. This means that Eq. (C5) is valid if conditions (5.4) and (6.21) hold simultaneously. Inserting Eq. (C5) into Eq. (6.19), using equalities [26] $\psi(1)=-\gamma$ and $\psi(d-1)=\psi(d)-(d-1)^{-1}$ and the formulas (6.18) and (C3) for d and z, and keeping only the lowest-order nonvanishing terms in the parameter $(\nu|\tilde{\Gamma}|/I_0)^{1/2}$ results in Eq. (6.23).

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