Atomic Schrödinger cat states

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We consider a physical process that leads to the generation of atomic Schrödinger cat states for a system of two level atoms. The effective interaction between atoms in a dispersive cavity leads to the superposition of atomic coherent states. We study in detail the quasidistributions for these states and demonstrate how the Schrödinger cat states can lead to interesting interferences over the surface of a sphere.
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I. INTRODUCTION

The concept of superposition of macroscopically distinct quantum states, i.e., the Schrödinger cat states, plays an important role in understanding the conceptual foundations of quantum mechanics. The methods for generation of such superposition states are, therefore, of fundamental interest. A lot of attention has been paid to the problem of generation and properties of Schrödinger cat states of a harmonic oscillator or the electromagnetic field [1–4]. The macroscopic states in that case are the harmonic oscillator or electromagnetic field coherent states with large number of photons. A field coherent state evolving under the influence of a Hamiltonian linear in the field quadrature a and $a^\dagger$ and number operator $a^\dagger a$ evolves to another field coherent state. A nonlinear interaction can generate a Schrödinger cat state from a coherent state [3–5]. Brune *et al.* [6] have realized Schrödinger cat states of the radiation field in cavity QED. A nonlinearity of particular interest for generating a field cat state is of Kerr type [2–4]. The Kerr nonlinearity corresponds to a Hamiltonian that is quadratic in the field number operator. A number of state reduction techniques [7,8] can also yield superposition of such states. Recently the mesoscopic Schrödinger cat states for vibrational motion of ions in traps have been discussed and realized [9,10].

In this paper we discuss the issue of generating Schrödinger cat states of a system of spins or equivalently a system of N two-level atoms. The macroscopic states in this case are, in analogy with the case of the electromagnetic field, the spin or atomic coherent states (ACS) $|\theta,\varphi\rangle$. As in the case of the electromagnetic field, we search for a nonlinear interaction that can generate a superposition of the ACS as from an initial atomic coherent state. Here we show that the interaction *nonlinear in the operator for atomic population* inversion can generate a superposition of ACS, i.e., an atomic Schrödinger cat state. This nonlinear interaction is shown to be *realizable* in the experiments on two-level atoms in a low-Q cavity highly detuned from the atomic transition frequency. Here the light shift due to the vacuum field induces an effective interaction which is nonlinear.

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The organization of the paper is as follows. In Sec. II we introduce the model of interacting atom-cavity system and derive the expression for the effective atomic Hamiltonian by adiabatically eliminating the field variables. In Sec. III, we study the evolution of an initial atomic coherent state under the action of that effective Hamiltonian and we show the emergence of cat states at particular times. In Sec. IV we show polar and contour plots for the quasidistributions for Schrödinger cat states. These plots elucidate the properties of the Schrödinger cat states.

II. THE DYNAMICS OF ATOMS IN A DISPERSIVE CAVITY

Consider a system of N identical two-level atoms each of frequency ω_0 interacting collectively with a single mode electromagnetic field in a cavity whose characteristic frequency nearest to ω_0 is ω_c. We show that under certain conditions the dynamics of atoms is described by an effective Hamiltonian [Eq. (12) below] in atomic variables. The effective Hamiltonian is reminescent of the effective Hamiltonian for radiation field in a Kerr medium. If the system of atoms is described by the spin $S_\pm, S_z$ and the cavity field mode by the bosonic operators $(a,a^\dagger)$ then the atom-field interaction is governed by the Hamiltonian

$$H_{\rm af}=\hbar\omega_0 S_z+\hbar\omega_c a^\dagger a+\hbar g(S_+a+a^\dagger S_-), \tag{1}$$

where g is the atom-field coupling constant. It may be noted that microcavities could be ideal candidates for studying such collective interactions. In addition to the reversible interaction with the atoms, the field suffers irreversible losses on account of leakage out of the cavity described by the Liouvillian

$$\Lambda_f\rho=\kappa(\overline{n}+1)(2a\rho a^\dagger-a^\dagger a\rho-\rho a^\dagger a)$$
$$+\kappa\overline{n}(2a^\dagger\rho a-aa^\dagger\rho-\rho aa^\dagger), \tag{2}$$

where 2κ is the rate of the loss of photons and $\overline{n}$ is the average number of thermal photons in the cavity. We assume that the dissipation due to atomic spontaneous emission is negligibly small on the time scale of interaction. That is consistent with the conditions in the experiments on atoms undergoing transitions between Rydberg levels in a cavity. The evolution of the system in a *frame rotating with* ω_0 is then described by the master equation

$$\frac{d\rho}{dt}=L_{\rm af}\rho+L_f\rho \tag{3}$$

for the density matrix ρ where

$$L_{\rm af}\rho=-ig[S_+a+a^\dagger S_-,\rho], \tag{4}$$

$$L_f\rho=-i\delta_c[a^\dagger a,\rho]+\Lambda_f\rho,\quad \delta_c=\omega_c-\omega_0.$$

We now derive an equation for the density matrix ρ_a of the atoms alone. A systematic approach for obtaining the master equation for a subsystem from that for the complete system is offered by the projection operator technique. However, for the sake of simplicity and in order to highlight the physics aspects, we follow a method that is essentially the same in its content but not in rigor as the method of projection operators. The results so obtained are the same as those arrived at by more rigorous approaches.

To that end, we write the formal solution of Eq. (3) in the form

$$\rho(t)=\exp(L_ft)\rho(0)+\int_0^t d\tau\,\exp[L_f(t-\tau)]L_{\rm af}\rho(0)+\int_0^t d\tau\int_0^\tau d\tau_1\exp[L_f(t-\tau)]L_{af}\times\exp[L_f(\tau-\tau_1)]L_{af}\rho(\tau_1). \tag{5}$$

The Liouvillian $L_{\rm af}$ determines the rate of exchange of energy between the field and the atoms. For large N, that rate is known to be of the order of $g\sqrt{N}$. On the other hand, the rate of the process described by the Liouvillian L_f is clearly of the order of $|i\delta_c+\kappa|$. Hence, if $|i\delta_c+\kappa|\gg g\sqrt{N}$ then the evolution of the field is dominated by L_f and it is reasonable to assume that the field remains for all times adiabatically in the state determined by L_f. Furthermore, if the time scale of observation is much longer than $|i\delta_c+\kappa|$ then that state is the steady state $\rho_f^{\rm ss}$ of L_f, which is the state of thermal equilibrium given by

$$\rho_f^{\rm ss}=\exp(-\beta a^\dagger a)/{\rm Tr}[\exp(-\beta a^\dagger a)],\quad \exp(-\beta)=\frac{\overline{n}}{\overline{n}+1}. \tag{6}$$

We, therefore, write the density matrix $\rho(t)$ in Eq. (5) as the outer product of the time-dependent density matrix $\rho_a(t)$ of the atoms and the steady-state density matrix $\rho_f^{\rm ss}$ for the field; take the trace of the two sides over the field to arrive at the following equation for $\rho_a(t)$:

$$\frac{d\rho_a}{dt}=\int_0^t d\tau{\rm Tr}_f\{L_{\rm af}\exp[L_f(t-\tau)]L_{\rm af}(\tau)\rho_a(\tau)\rho_f^{\rm ss}\}, \tag{7}$$

where $L_{\rm af}(\tau)$ is the Liouville operator $L_{\rm af}$ in the interaction picture. Next, we evaluate the integral in Eq. (7) by using the results

$$\exp(L_ft)a\rho=\exp(i\delta_ct)[\{(\overline{n}+1)\exp(\kappa t)-\overline{n}\exp(-\kappa t)\}a\rho-\overline{n}\{\exp(\kappa t)-\exp(-\kappa t)\}\rho a;$$

$$\exp(L_ft)\rho a=\exp(i\delta_ct)[(\overline{n}+1)\{\exp(\kappa t)-\exp(-\kappa t)\}a\rho-\{\overline{n}\exp(\kappa t)-(\overline{n}+1)\exp(-\kappa t)\}\rho a], \tag{8}$$

along with their Hermitian conjugates and obtain

$$\frac{d\rho_a}{dt}=\frac{g^2}{\kappa^2+\delta_c^2}[-i\delta_c\{[S_+S_-,\rho_a]+2\overline{n}[S_z,\rho_a]\}+\kappa\{(\overline{n}+1)(2S_-\rho_aS_+-S_+S_-\rho_a-\rho_aS_+S_-)+\overline{n}(2S_+\rho_aS_--S_-S_+\rho_a-\rho_aS_-S_+)\}]. \tag{9}$$

We of course have also assumed that the atomic density matrix $\rho_a(t)$ evolves slowly on the scale $1/\omega_0$. Now, if $\delta_c\gg\kappa$ then the contribution due to damping in Eq. (9) is negligibly small and it reduces to

$$\frac{d\rho_a}{dt}=-i\eta[S_+S_-+2\overline{n}S_z,\rho_a], \tag{10}$$

where

$$\eta=\frac{g^2\delta_c}{\kappa^2+\delta_c^2}, \tag{11}$$

and $Ng^2\ll\kappa^2+\delta_c^2$. For a given value of $\sqrt{N}g$ and κ, the adiabatic condition can be satisfied by increasing the detuning $|\omega_0-\omega_c|$. Equation (10) shows that in a cavity, highly detuned from the atomic transition frequency, the evolution of the atomic system is approximately unitary and is governed by the effective Hamiltonian, which essentially arises from the effect of vacuum light shifts,

$$H_{\rm eff}=\hbar\eta[S_+S_-+2\overline{n}S_z]\equiv\hbar\eta\left[\frac{N}{2}\left(\frac{N}{2}+1\right)-S_z^2+(2\overline{n}+1)S_z\right]. \tag{12}$$

The Hamiltonian (12) is quadratic [11] in the population inversion operator S_z. It is, therefore, analogous to the Hamiltonian quadratic in the number operator of the single mode field propagating through a Kerr medium like an optical fibre. In the next section we show that the Hamiltonian (12) leads to the generation of atomic Schrödinger cat states from an initial atomic coherent state. Note that the temperature-dependent term corresponds to a simple rotation and therefore we drop it in our further considerations, i.e., we set $\overline{n}=0$

III. ATOMIC SCHRÖDINGER CAT STATES PRODUCED BY DISPERSIVE INTERACTION IN A CAVITY

In this section we show that the unitary evolution generated by the Hamiltonian (12) transforms an atomic coherent

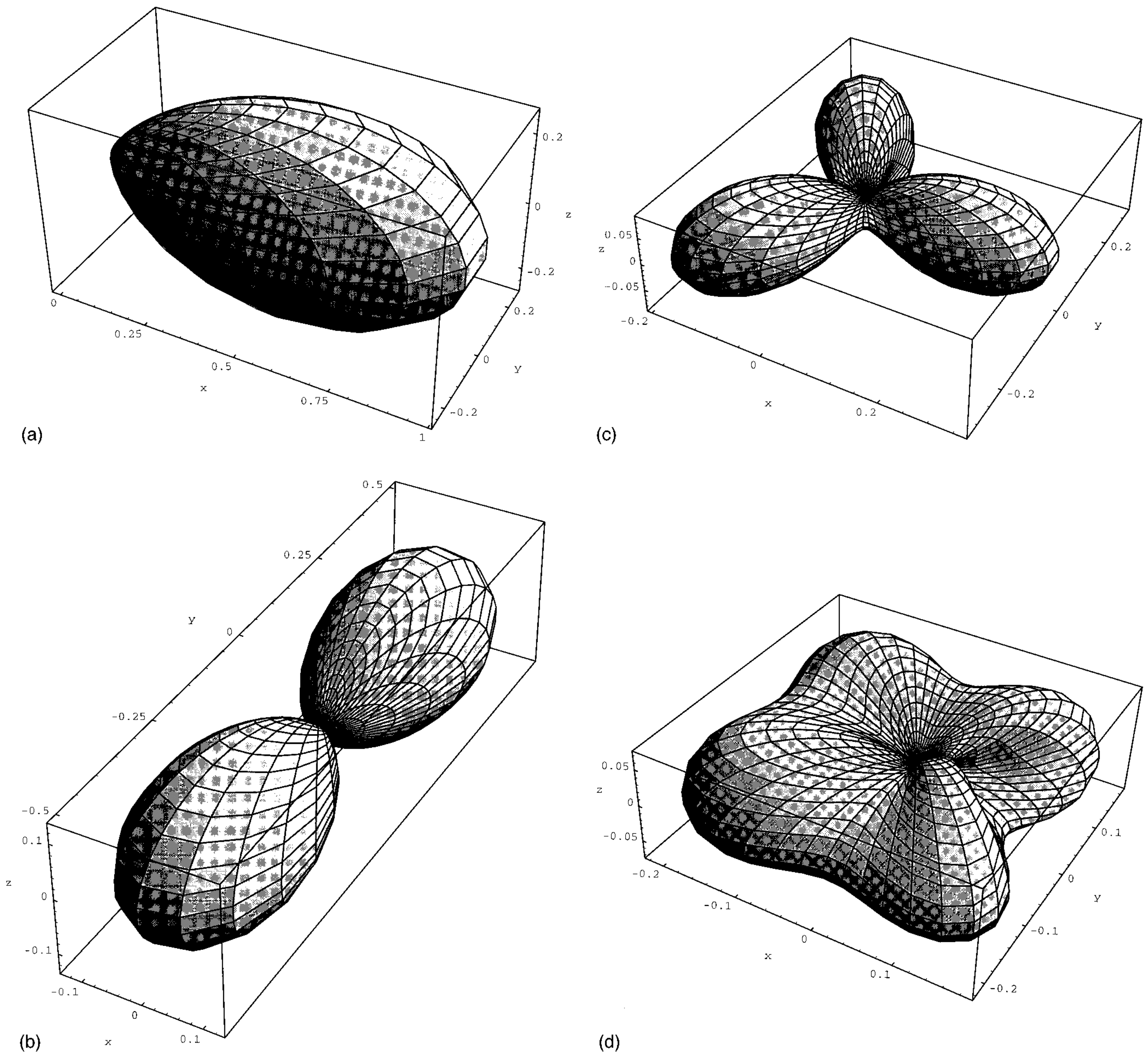

FIG. 1. Spherical polar plots of the quasidistribution $Q(\alpha,\beta)$ for (a) $\tau=0$, (b) $\tau=\pi/2$, (c) $\tau=\pi/3$, and (d) $\tau=\pi/4$; Number of atoms: $N=10$. $x\rightarrow Q(\alpha,\beta)\sin\alpha\cos\beta$, $y\rightarrow Q(\alpha,\beta)\sin\alpha\sin\beta$, $z\rightarrow Q(\alpha,\beta)\cos\alpha$.

state into a superposition of distinct atomic coherent states. The superposition of atomic coherent states is an atomic Schrödinger cat state in the same sense as the superposition of electromagnetic field coherent states is a Schrödinger cat state.

Consider an atomic system prepared in the atomic coherent state [12]

$$|\psi(0)\rangle \equiv |\theta,\phi\rangle = \sum_{k=0}^{N} \sqrt{\frac{N!}{(N-k)!k!}} \exp(ik\phi) \times \sin^{N-k}\left(\frac{\theta}{2}\right)\cos^{k}\left(\frac{\theta}{2}\right)\left|\frac{N}{2}-k\right\rangle . \tag{13}$$

The state (13) is an eigenstate of the component of the angular momentum in the (θ,ϕ) direction with eigenvalue $N/2$. The atoms in the state (13) are uncorrelated as (13) can be written as a product state for individual atoms. In particular, one will have

$$\langle S_i^+ S_j^- \rangle \equiv \langle S_i^+\rangle\langle S_j^-\rangle, \quad i\neq j. \tag{14}$$

An atomic coherent state, like a harmonic oscillator coherent state, is most classical in the sense that it is a minimum uncertainty state of a pair of operators for two mutually orthogonal spin components orthogonal to the average direction of spin in the atomic coherent state. An evolution generated by a linear combination of the atomic operators transforms an atomic coherent state to another one—this is equivalent to motion on a Bloch sphere. The atomic coherent state (13), under the action of the nonlinear unitary evolution generated by Eq. (12), transforms to

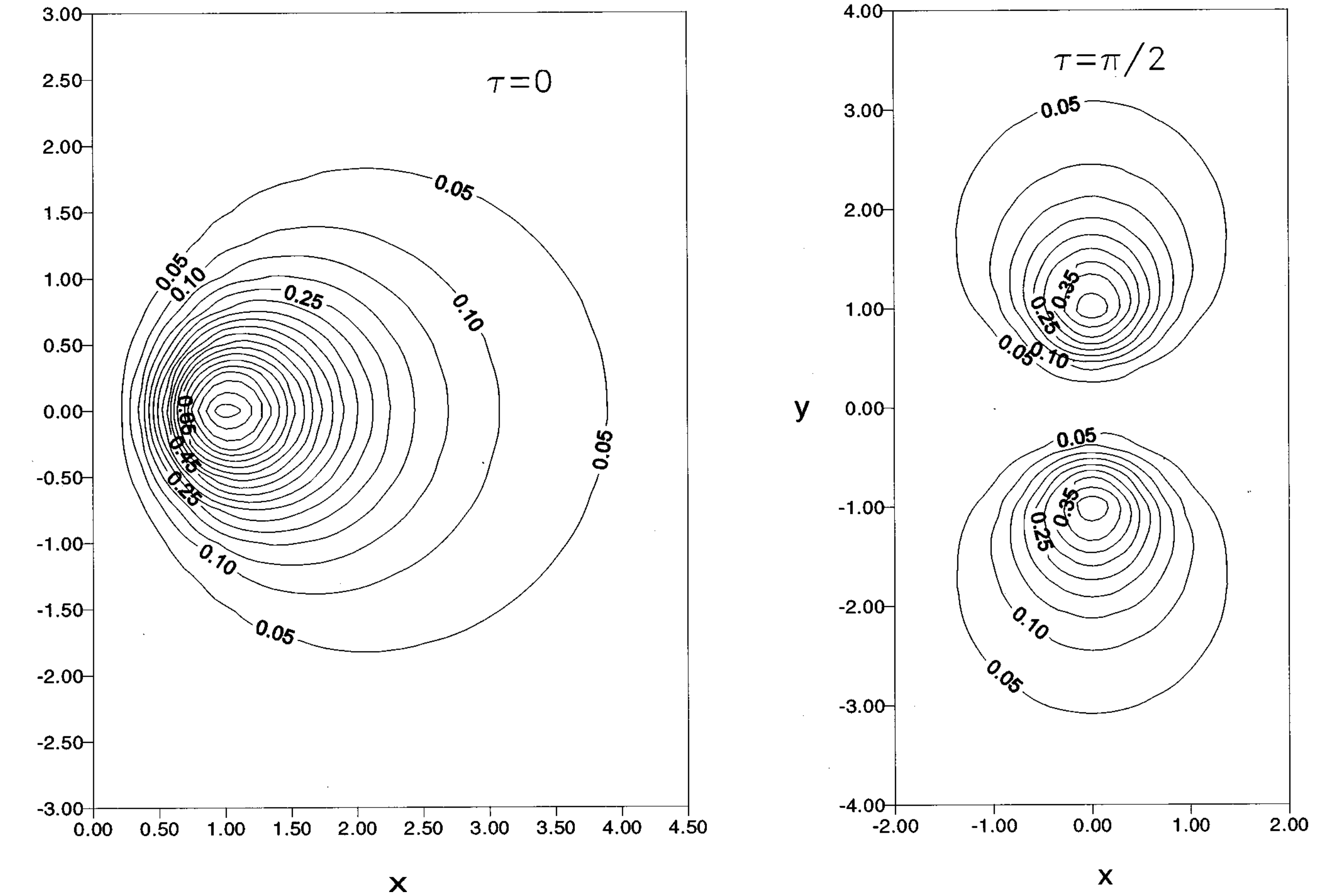

FIG. 2. Contour plots of $Q(x,y)$ for $\tau=0$ and $\tau=\pi/2$.

$$|\psi(t)\rangle \equiv \exp[-iHt]|\theta,\phi\rangle = \sum_{k=0}^{N} \sqrt{\frac{N!}{(N-k)!k!}} \times \exp(ik\phi)\sin^{N-k}\left(\frac{\theta}{2}\right)\cos^{k}\left(\frac{\theta}{2}\right) \times \exp[-i\tau\{N+(N-1)k-k^2\}]\left|\frac{N}{2}-k\right\rangle,$$

$$\tau \equiv \eta t. \qquad (15)$$

From now on we consider Eq. (15) at special times $\tau=\pi/m$, where m is an integer,

$$|\psi(t)\rangle = \exp\left[-\frac{i\pi N}{m}\right]\sum_{k=0}^{N}\left\{\sqrt{\frac{N!}{(N-k)!k!}} \times \exp(ik\phi')\sin^{N-k}\left(\frac{\theta}{2}\right)\cos^{k}\left(\frac{\theta}{2}\right) \times \exp\left[i\frac{\pi}{m}k(k+1)\right]\left|\frac{N}{2}-k\right\rangle\right\}, \qquad (16)$$

where

$$\phi' = \phi - \frac{\pi}{m}N. \qquad (17)$$

Now use the expansion [2]

$$\exp\left[\frac{i\pi}{m}k(k+1)\right] = \sum_{q=0}^{m-1} f_q^{(o)}\exp\left[\frac{2\pi iq}{m}k\right],$$

$$\exp\left[\frac{i\pi}{m}k^2\right] = \sum_{q=0}^{m-1} f_q^{(e)}\exp\left[\frac{2\pi iq}{m}k\right] \qquad (18)$$

for m odd and even, respectively along with the inversion relations

$$f_q^{(o)} = \frac{1}{m}\sum_{k=0}^{m-1} \exp\left[-\frac{2\pi iq}{m}k\right]\exp\left[\frac{i\pi}{m}k(k+1)\right],$$

$$f_q^{(e)} = \frac{1}{m}\sum_{k=0}^{m-1} \exp\left[-\frac{2\pi iq}{m}k\right]\exp\left[\frac{i\pi}{m}k^2\right]. \qquad (19)$$

These expansions are based on the periodicity of the left-hand side of Eq. (18). The importance of Eq. (18) lies in the fact that an exponentially quadratic form has been converted into sums of exponentials linear in k. On combining Eqs. (15) and (18) we obtain

$$\exp[-iHt]|\theta,\phi\rangle=\exp\left[-\frac{i\pi N}{m}\right]\sum_{q=0}^{m-1} f_q^{(o)} \times\left|\theta,\phi+\pi\frac{2q-N}{m}\right\rangle,\quad t=\frac{\pi}{m\eta} \tag{20}$$

for m odd and

$$\exp[-iHt]|\theta,\phi\rangle=\exp\left[-\frac{i\pi N}{m}\right]\sum_{q=0}^{m-1} f_q^{(e)} \times\left|\theta,\phi+\pi\frac{2q-N+1}{m}\right\rangle,$$

$$t=\frac{\pi}{m\eta} \tag{21}$$

for m even. The expressions (19) and (20) show that at the particular time $t=\pi/m\eta$, an atomic coherent state evolves to a superposition of the atomic coherent states; i.e., it becomes an atomic cat state. The states in the superposition *differ by phase*. In particular, for $m=2$ it follows that

$$\exp[-iHt]|\theta,\phi\rangle=\frac{\exp[-iN\pi/2]}{\sqrt{2}} \times\left[\exp\left(\frac{i\pi}{4}\right)\left|\theta,\phi-\pi\frac{N-1}{2}\right\rangle +\exp\left(\frac{-i\pi}{4}\right)\left|\theta,\phi-\pi\frac{N-3}{2}\right\rangle\right]. \tag{22}$$

One can similarly obtain expressions for the Schrödinger cat states for other values of m.

IV. QUASIDISTRIBUTIONS FOR ATOMIC SCHRÖDRINGER CAT STATES

Next we present the form for the quasidistribution $Q(\alpha,\beta)$ for Schrödinger cat states defined by

$$Q(\alpha,\beta)\equiv\langle\alpha,\beta|\rho(t)|\alpha,\beta\rangle. \tag{23}$$

Consider the state (15) with an initial state $|\theta,\phi\rangle\equiv|\pi/2,0\rangle$. Then the $Q(\alpha,\beta)$ function is

$$Q(\alpha,\beta,\tau)\equiv\left(\frac{1}{2}\right)^N\sum_{k=0}^{N}\sum_{k'=0}^{N}\left(\frac{N!}{(N-k)!k!}\right) \times\left(\frac{N!}{(N-k')!k'!}\right)\sin^{2N-k-k'}\left(\frac{\alpha}{2}\right) \times\cos^{k+k'}\left(\frac{\alpha}{2}\right)\exp[-i(k-k')\beta] \times\exp[-i(k-k')(N-k-k'-1)\tau]. \tag{24}$$

We show in Figs. 1(a)–1(d) the spherical polar plots [13] of the quasidistribution for a system of 10 atoms and for $\tau=0$, $\pi/2$, $\pi/3$, and $\pi/4$. The Schrödinger cat character of the state is clearly seen for $\tau=\pi/2$ and $\pi/3$. For $\tau=\pi/4$ there is too much overlap between different components of the state and that is why splittings are not clearly seen.

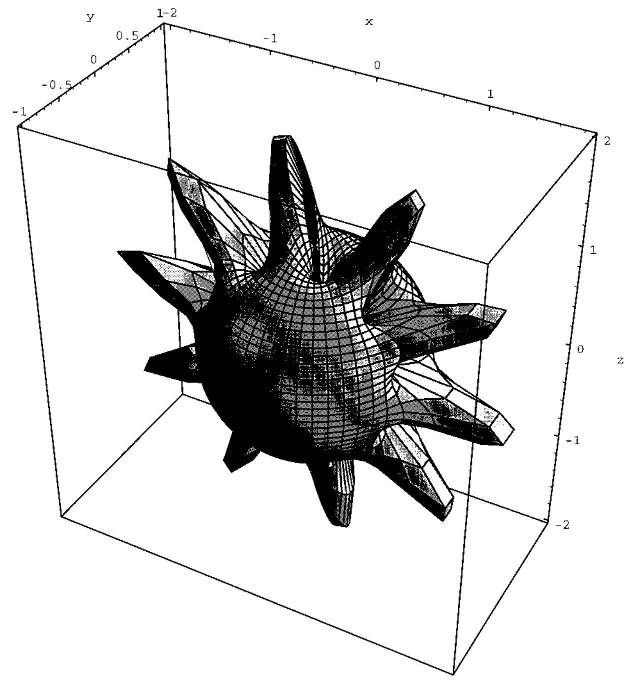

FIG. 3. Plot of Q/Q_0 [Eq. (31)] showing interference structures for atoms in a Schrödinger cat state.

The contour plots of these states are also interesting. For countour plots *we map from polar coordinates* (α,β) *to planar coordinates* (x,y) via

$$x+iy=\left[\tan\left(\frac{\alpha}{2}\right)\right]e^{i\beta},\quad \alpha\rightarrow 2\tan^{-1}\sqrt{x^2+y^2},$$

$$\beta\rightarrow\tan^{-1}\frac{y}{x}. \tag{25}$$

Thus $Q(x,y)$ becomes a function of x and y. The contour plots correspond to $Q(x,y)$ = const. These are shown in Fig. 2. For the state $|\pi/2,0\rangle$ the contour plots are circles—this is similar to harmonic oscillator coherent states. However, the center of the circle moves. This can be seen from the explicit formula for the Q function for the state $|\pi/2,0\rangle$:

$$Q(x,y)\equiv\left(\frac{1}{2}\right)^N\left[\frac{1+x^2+y^2+2x}{1+x^2+y^2}\right]^N, \tag{26}$$

which can be written as

$$x^2+y^2+1+\frac{2x}{1-a}=0,\quad a=2Q^{1/N}. \tag{27}$$

Note that $Q>0$ and a is real. The expression (27) represents a system of circles, where the *center of the circle moves with* a. Clearly the center will move according to the value of Q. The contour plots of Schrödinger cat state (22) for $\pi/2$

are quite suggestive. These clearly correspond to two coherent states shifted in phase by π. In fact $Q(x,y)$ for the state at $\tau=\pi/2$ is found to be

$$Q(x,y)=\left(\frac{1}{2}\right)^{N+1}\left[\left(\frac{1+x^2+y^2+2y}{1+x^2+y^2}\right)^N + \left(\frac{1+x^2+y^2-2y}{1+x^2+y^2}\right)^N + \left(i\left(\frac{x^2+y^2-1-2ix}{1+x^2+y^2}\right)^N + \text{c.c.}\right)\right], \qquad (28)$$

which shows the existence of a pair of circles in contour plot. The Schrödinger cat states are correlated and are quantum in nature.

The Schrödinger cat states for harmonic oscillator systems are known to exhibit interesting quantum interferences [1,14]. We now investigate the question of quantum interferences for atomic Schrödinger cat states. For this purpose we write the Q function as

$$Q=\langle \alpha,\beta|\psi\rangle\langle\psi|\alpha,\beta\rangle, \qquad (29)$$

where $|\psi\rangle$ represents Eq. (22), which we rewrite as

$$|\psi\rangle=\frac{(|\psi_1\rangle+|\psi_2\rangle)}{\sqrt{2}}. \qquad (30)$$

Note that for (22), $\langle\psi_1|\psi_2\rangle=0$. Thus Q function will become

$$Q=Q_0+Q_1, \qquad (31)$$

$$Q_0\equiv\tfrac{1}{2}(|\langle\alpha,\beta|\psi_1\rangle|^2+|\langle\alpha,\beta|\psi_2\rangle|^2), \qquad (32)$$

$$Q_1\equiv\tfrac{1}{2}(\langle\alpha,\beta|\psi_1\rangle\langle\psi_2|\alpha,\beta\rangle+\text{c.c.}). \qquad (33)$$

Note that Q_0 will correspond to a situation when there are no interferences and Q_1 gives the interference contribution. Thus a plot of (Q/Q_0) will exhibit interference structures, i.e., maxima and minima. We show this behavior in Fig. 3. If there were no interferences then the polar plot will be a sphere. However, we find well defined maxima and minima on the surface of the sphere. We have thus interferences on a sphere. It is also clear from Eq. (22) that it could not be expressed as a product of states of individual spins. Thus atom-atom correlations are important in the Schrödinger cat state.

In conclusion, we have shown how the Schrödinger cat states for a system of atoms can be generated using the dispersive interactions in a cavity. We show the existence of interesting interferences by studying the quasidistributions. The present work can be generalized to construct more general superposition states as well to construct t Schrödinger cat states for multilevel systems. Finally we note that one can also use state reduction methods [15,16] for generating atomic Schrödinger cat states.

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